Emergent Spacetime

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May 14, 2018

Abstract

There have been recent theoretical hints that spacetime should not be thought of as a fundamental concept but rather as an emergent property of an underlying microscopic theory. In this paper we give an overview of proposed microscopic models that suggest that of spacetime should emerge as an effective description.

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1 Introduction

1.1 Emergence

The reductionist philosophy seeks to find the "true" laws of nature by breaking down its constituents into its most basic forms. However, this is not the complete story because even if one knows the fundamental description of nature the challenge still remains to construct exact descriptions of large systems using these fundamental laws [1].

One can attempt to understanding fish behavior first by dissection to understand the individual fish parts. The model that explains how the fish moves would be a complicated mechanical mess of fins and muscles. Even after this respectable effort the model wouldn't adequately describe all phases of fish behavior. For example, if one wanted to explain the schooling of fish, this model would be incomprehensibly complicated to use. Instead one should view the school as a new being because on average it behaves differently than any individual fish. This fish phase then has to be described by an emergent theory that is not obviously determined in terms to the microscopic theory of fish dynamics. This is the beautiful principle of emergence.

It happens in physics as well. For example, one can attempt to solve the many-body problem of 10^{23} interacting electrons in a metal or one can understand that electrons are fermions which leads to the idea of a Fermi surface and finally Fermi liquid theory. When viewing a metal in this way, one easily finds other microscopic systems (not electrons in a metal) whose emergent description is the same! This perspective is completely opposite that of the reductionist approach where to understand a metal we would have to construct each electron as an excitation in a quantum field. This would be an incomprehensibly intractable problem both analytically and qualitatively. One understands a metal to have some effective phenomena such as conduction with a finite resistivity which would not be evident by constructing the theory of a metal in terms of QED. Rather, the emergent property of a Fermi liquid gives a much clearer picture of a metal while only having to know a few experimentally measured parameters rather the individual behavior of all electrons and atoms in the lattice. The idea of emergence in physics is pervasive especially in condensed matter and cold atom physics. For example, superconductivity and superfluidity are both described by a condensate, which is an emergent state of matter with finite fraction of particles in the ground state and long range order.

1.2 Emergent Spacetime

There is an idea amongst theoretical physicists that spacetime is not fundamental but itself an effective description. More importantly, they believe that the notion of particles and fields living in a spacetime is an emergent property from the dynamics of some other underlying microscopic theory. Most believe that these degrees of freedom of the underlying theory must be also be quantum mechanical in nature. We know that a fundamental feature of quantum mechanics is the notion of entanglement between systems. In the picture of emergent spacetime, a coarse-grained view of the entanglement structure between these quantum degrees of freedom might be most economically described in terms of smooth spacetime. This means that at some scale the notion of a smooth spacetime has to break down.

However, we know that fundamental quantum field theories such as those in the Standard Model rely on a background spacetime to define a notion of locality and causality. These notions tell us that quantum states in these theories can be well localized in the infinite past and future. We do exactly this in particle accelerators; locality and causality are well-tested features of quantum field theories describing particles in the Standard Model, which has itself been tested rigorously to the TeV scale. Thus spacetime is a key concept in defining consistent quantum field theories that describe the interactions of these particles.

This implies that the scale at which spacetime breaks down has to be much smaller than the scales probed by particle accelerators. This scale might be anywhere between the TeV scale and the Planck scale. Thus a theory of emergent spacetie could be related to a theory of quantum gravity, which is the purported UV completion (which means the underlying microscopic theory) of general relativity. Therefore it is fruitful to try to understand what quantum gravity is before we can analyze whether a description of the universe in terms of a smooth spacetime is emergent or fundamental.

2 Quantum Gravity

Quantum gravity is hard. Many attempts have been made to quantize ordinary gravitational theories. Most of these ideas are plagued with the lack of a consistent conceptual framework or a lack of analytic control. For example, if we simply take a well tested known theory of gravity, Einstein gravity, and interpret it as an ordinary quantum field theory by attempting to canonically quantize classical fields we get divergences that cannot be removed. Ordinarily we choose to trust a quantum field theory even if divergences appear as long as the theory has a property known as renormalizability. This means that once we know a finite number of experimental parameters of a theory (particle masses, interaction couplings) at a certain scale, everything else can be predicted.

This is not the case for Einstein gravity and hence we call it a non-renormalizable theory. If one attempts to quantize this theory using the standard Feynman path integral prescription, the quantum corrections to the classical theory diverge and cannot be removed by renormalization can be done in the Standard Model. Renormalizable theories are sometimes viewed as more fundamental than non-renormalizable ones because these divergences can be systematically removed to sensical predictions (given that we ask the right questions). More specifically this means there are a finite number of independent parameters that have to be measured to fully specify the theory. In the case of a non-renormalizable theory one would need to experimentally measure an infinite number of parameters which demolishes all hope of making sensible predictions from such a theory. Therefore it seems to be the case that attempting to interpret Einstein gravity as an ordinary quantum field theory isn't consistent with the typical framework of a quantum field theory. This means that merging together quantum mechanics and gravity is more complicated than we once hoped.

One thing we know for certain about quantum gravity is that its semiclassical approximation should include virtual effects on top of the classical theory of gravity. This leads to the conceptually clean idea that if we linearize the equations of motion of a theory of gravity, we should get a gauge theory with a massless particle that mediates the force of gravity. This particle is is a massless spin-2 particle known as a graviton. Essentially we employ the background field method of quantum field theory. Choose a particular background metric and linearize fluctuations around it:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu} \tag{1}$$

where $g_{\mu\nu}$ is the full metric that splits up into the background $\tilde{g}_{\mu\nu}$ and the fluctuations $h_{\mu\nu}$. We can then linearize the action around this and find an equation of motion for the graviton. Schematically, this looks like

$$S = \frac{1}{16\pi G_N} \int \sqrt{g} R \tag{2}$$

$$S \int \partial h \partial h + h \partial h \partial h + \mathcal{O}(\partial^2 h) \tag{3}$$

The first term is just the standard kinetic term of a quantum fields. The following terms are self-interactions, which follows from the fact that gravitons themselves gravitate. Thus for any theory that claims to be a theory of emergent spacetime must have a graviton in its spectrum.

2.1 Hints from String Theory: Ambiguity of Spacetime

String theory is a potential candidate for a UV completion of General Relativity, or a theory of quantum gravity. We know from popular science that string theory says that all particles and forces can be reproduced from vibrating strings. If one works out the modes of a quantum string, it is "easy" to see that a massless spin-2 particle always exists in the excitation spectrum of any string theory! If we want the emergence of spacetime to happen at the Planck scale, we would expect that string theory has something to say about emergent spacetime. In string theory, geometry is emergent from the collective behavior of the low-energy modes of strings. In some sense, spacetime itself is a coherent state of gravitons, which we know always exist as modes of the vibrating strings.

One of the reasons that string theory receives so much attention is because a description of spacetime (or Einstein's equations) emerges quite naturally from string theory. If strings are the fundamental building blocks of nature, they should themselves be featureless and it doesn't make much sense to ask the question, "What are strings made of?". This is done mathematically by imposing that the coordinates that parameterize strings should not be "physical", meaning that Lagrangian describing the dynamics of strings are parameterization invariant. An analogous statement can be made for the parameterization of time along a particle's worldline. For a string this amounts to imposing conformal invariance (invariance under local "streching" of spacetime that preserve angles between vectors). If we want conformal invariance to persist at the quantum level we need to look at the fixed points of the theory, or when the beta functions go to zero. In a quantum field theory the beta functions tell you how the couplings, or the interaction strengths, change with energy scale. In fact, Einstein's equations (along with additional matter fields) fall right out of imposing conformal invariance at the quantum level [3]!

$$\sum_i \beta^i_{\mu\nu} \approx (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - T^{matter}_{\mu\nu} = 0 \tag{4}$$

Furthermore, even after this spacetime is set by the dynamics of the string it is not in any sense unique. For example, in string theory there is an exact symmetry known as a T-duality. What it tells us is that the spectrum of a string that is wrapped around a compactified circle of a certain radius R is exactly the same as that of a string wrapped around a circle of radius proportional to 1/R. This leads to a dual description of the string theory in these distinct spacetimes. In this case, both have the same topology, but in one of them, if R happens to be very small, meaning this compactified dimension is immeasurably small and is not accessible in the low energy theory. In the other spacetime, the radius of this compact dimension is 1/R, which is a large extra dimension. And yet they are somehow described by exactly the same solution of string theory. This suggests that there is an ambiguity in the description of the world in terms of a unique spacetime. This suggest further evidence that spacetime emerges from a more fundamental theory [2].

2.2 Gravitons and Weinberg-Witten theorem

Given that we know a theory of gravity has to contain a graviton, it is appealing to think that if gravity should be emergent from a different microscopic theory then the graviton might be a composite particle of an underlying quantum field theory living in flat space. This is analogous to how cooper pairs are composite quasi-particles. This line of reasoning was analyzed by Weinberg and Witten in 1980. They found that massless particles in a quantum field theory with spin greater than 1 cannot carry a conserved stress-energy tensor [4]. If the theory doesn't have gravity to begin with (no diffeomorphism invariance), the stress energy tensor is gauge-invariant and this theorem applies.

This is to say that if we have a theory with quantum fields on flat spacetime, an emergent/composite graviton cannot exist. To get emergent spacetime there needs to be some mechanism that allows the proposed theory to evade the Weinberg-Witten theorem. This of course happens for General Relativity because we do have diffeomorphism invariance and the stress energy tensor isn't gauge invariant.

2.3 Gravity from Thermodynamics

Another interesting direction is the idea that Einsteins equations are actually a thermodynamic equation of state. It is generally true that in thermodynamics, if one has information about the dependence of the entropy on other thermodynamic variables it is possible to easily find the equation of state. In gravitational theories we know about how to relate the entropy

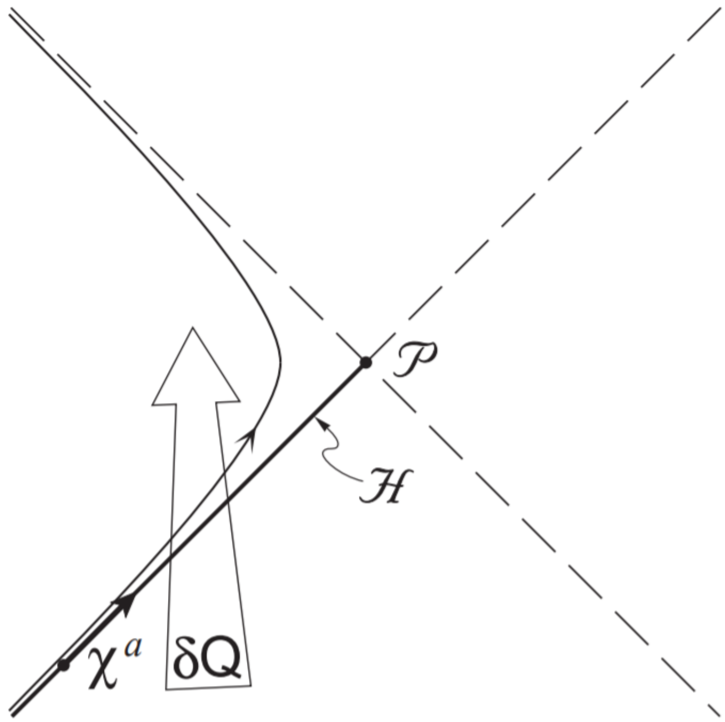

Figure 1: δQ is matter flux that crosses the horizon of a codimension 2 space-like surface $\mathcal{P}$.

to geometric data in certain situations from Bekensteins original argument that black holes should have an entropy associated with them that is proportional to the area of the horizon.

$$S = \frac{A}{4G_N}/, . \tag{5}$$

We make the assumption that this true of all horizons. We also need the first law of thermodynamics in the gravitational context

$$\begin{aligned} \delta Q &= T\delta S & (6) \\ &= \int_{\mathcal{H}} T_{ab}\chi^a d\Sigma^b\,, & (7) \end{aligned}$$

where T_{ab} is the stress energy tensor, χ^a is vector field that generates the horizon and the integration region is over the horizon.

Given that we have information about the matter flux through the horizon, the author uses Raychudhuri's equations (which are a statement about how the curvature of a spacetime affects how light bends) to derive Einstein's equations as a thermodynamic equation of state. One starts with Raychudhuri's equations:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b \tag{8}$$

where k^a are the null vectors describing the light-like geodesic. θ is function of the affine parameter λ (which parameterizes the light-like curve) at which the light-like geodesic "bends" locally due to the curvature of spacetime. We can ignore the first two terms, and R_{ab} is the Ricci tensor describing the curvature of spacetime. Then relate this to the variation of the entropy (which is related to the variation in the area) and one can extract Einstein's equations.

If Einstein's equations are thermodynamic in nature, this suggests that trying to quantize gravity by canonical quantization of the metric makes about as much sense as trying to canonically quantize the equation of state of hydrodynamics of fluids (Navier-Stokes equations) by quantizing the velocity field. Since the assumptions of this model involve pure thermodynamical statements, this theory doesn't necessarily rule out a connection to a deeper underlying microscopic theory. If we make the further identification of the thermodynamic entropy with boltzmann entropy (in equilibrium situations), as long as the microscopic theory can reproduce the area law the thermodynamic argument of Jacobson is not ruled out.

3 Emergence of Gravity in AdS/CFT

Another example of emergent spacetime comes from the program of AdS/CFT. The statement of the correspondence suggests that the degrees of freedom of gravity should have a description in the quantum field theory that lives on the boundary of the spacetime where the gravitational theory lives.

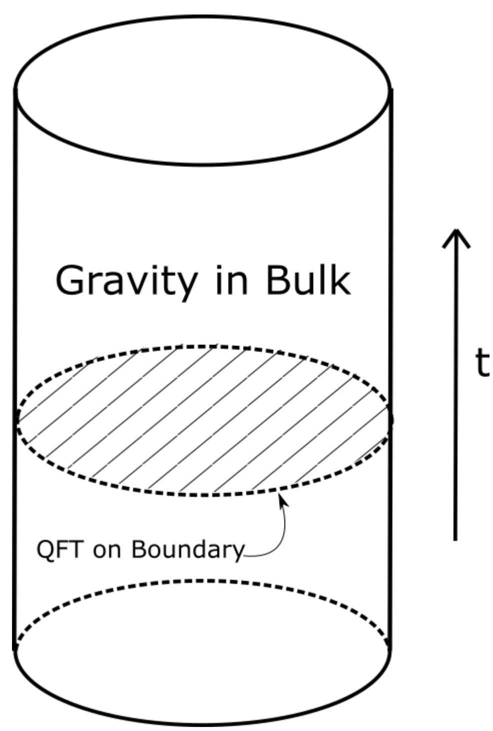

Figure 2: AdS/CFT in a picture

One should note that this is a strange form of emergence, because normally in condensed matter systems the dynamics of electrons and atoms conspire in a certain way that can be described by a spatially varying order parameter. This order parameter field usually lives on top of the existing spacetime that the atoms live in.

In the case of AdS/CFT, the dynamics of the QFT degrees of freedom conspire to be described by a theory in one more dimension and the degrees of freedom in this new theory act like they live on a dynamical curved spacetime! This is very strange and we would like to understand how exactly this happens. One idea is that the entanglement between these QFT degrees of freedom somehow generates the gravitational degrees of freedom.

A simple example of this is the Ryu - Takayanagi conjecture, where the entanglement entropy of the quantum fields in the boundary (a measure of entanglement in quantum field theories) can be calculated in the bulk theory by computing the area of a minimal surface whose boundary is the entanglement cut [6].

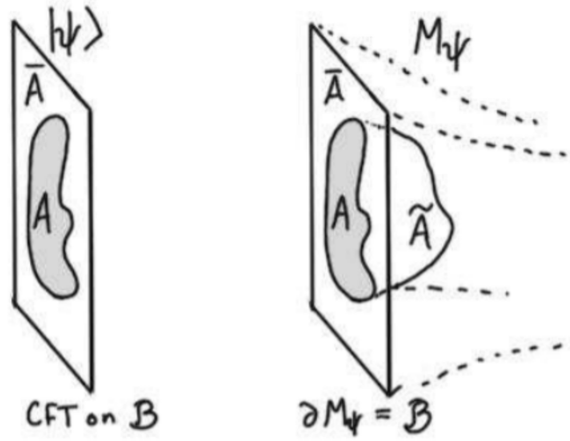

Figure 3: The bulk surface $\tilde{A}$ has a boundary that coincides with the boundary of the sub-region A in the CFT.

$$S(A) = \frac{1}{4G_N} Area(\tilde{A}) \tag{9}$$

The left-hand side of this formula is the entanglement entropy of the quantum field theory restricted to the subregion A. It measures the amount of entanglement between degrees of freedom inside A and the complement $\bar{A}$.

One could imagine varying the region A on the QFT by deforming the entanglement cut, which would imply a slightly deformed minimal surface. If the Ryu-Takyanagi conjecture holds, the change in the entanglement entropy of the QFT living on the boudary should exactly reproduce the change in the area of the minimal surface. If this is the case, one could map out all the bulk simply by changing the enclosed area on the boundary.

The most important facet of this story is that the QFT has certain operators (or quantum fields) that have dual descriptions in the bulk theory. We expect that for a theory of quantum gravity, there should be a particle that mediates the gravitational force. This is known as a graviton. This is an important concept because, in some sense spacetime itself can be considered a condensate of gravitons. These gravitons have a dual operator in the CFT: the stress energy tensor. This should always exist in any reasonable CFT.

If we can motivate a general mapping between the operators on the boundary and operators in the bulk this suggests that there is an isomorphism between the Hilbert spaces of the two theories. For a particular CFT, there might exist a certain sector that can actually be described by a gravitational theory. To check this, we use the mapping we have already established to define a dual bulk operator to the stress-energy tensor:

$$T_{\mu\nu}^{CFT} \Longleftrightarrow g_{\mu\nu}\,. \tag{10}$$

4 Comparison with Experiment

There are no experiments that can definitely disprove any of these theories, but we can put constraints at which scale an emergent spacetime should breakdown. However a breakdown of spacetime implies that Lorentz-invariance breaks down at some scale. At the level of a photon, Lorentz invariance implies that the dispersion relation of a photon should be proportional to the frequency by a constant. If there is any energy dependence at all in this constant, we cannot say that Lorentz invariance holds for all length scales. This test has been done repeatedly and the most recent experiment suggests that the photon dispersion relation holds up to energy scales above the Planck scale [7]!

5 Conclusion

It might be the case that gravitational degrees of freedom are actually lurking in quantum field theories. Several directions of research suggest theoretical ideas of why this is the case, but as of today no one can say for certain whether gravity and spacetime are actually emergent in our universe.

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