Honors General Exam

Solutions

Harvard University
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Part 1: Microeconomics

Question 1

The inverse demand function for apples is defined by the equation $p = 214 - 5q$, where q is the number of units sold. The inverse supply function is defined by $p = 7 + 4q$.

(a) How many apples will be sold in equilibrium?

Solution: In equilibrium, the price paid by buyers is equal to the price received by sellers. Thus, to find the number of apples sold, we equate p^D with p^S and solve for q.

$$\begin{aligned} 214 - 5q &= 7 + 4q \\ 207 &= 9q \\ q &= 23 \end{aligned}$$

(b) Suppose the government decides to provide a subsidy of \$36 to suppliers for each unit of apples that they sell. How many apples will be sold after the subsidy is provided?

Solution: In this case, the price received by the suppliers is \$36 higher than the price paid by the consumers. We account for this and approach the problem the same way as in (a):

$$\begin{aligned} p^D + 36 &= p^S \\ 214 - 5q + 36 &= 7 + 4q \\ 243 &= 9q \\ q &= 27 \end{aligned}$$

(c) Now, suppose that the government decides to provide the subsidy of \$36 to consumers instead of suppliers for each unit of apples that they buy. How many apples will be sold after the subsidy is provided?

Solution: In this case, the price actually paid by the consumers is \$36 lower than the price received by the suppliers, but the method of solving is analogous to parts (a) and (b). Note that, as parts (b) and (c) illustrate, the equilibrium quantity is the same regardless of whether producers or consumers receive the subsidy.

$$\begin{aligned} p^D &= p^S - 36 \\ 214 - 5q &= 7 + 4q - 36 \\ 243 &= 9q \end{aligned}$$

$$q = 27$$

Question 2

Two players are engaged in a game of Matching Pennies. There are two possible actions, Heads and Tails. If both players choose Heads or if both players choose Tails, then the row player receives a payoff of 1 and the column player receives a payoff of -1. If either player chooses Heads and the other player chooses Tails, then the row player receives a payoff of -1 and the column player receives a payoff of 1.

(a) Draw the payoff matrix.

Solution: The payoff matrix is:

		Player B	
		Heads	Tails
Player A	Heads	1,-1	-1,1
	Tails	-1,1	1,-1

(b) Identify all pure strategy equilibria, if any exist.

Solution: In order for a pure strategy Nash equilibrium to exist, there must be no incentive for either player to deviate when in that outcome. It is easy to see that in any of the four squares, one player will always have an incentive to deviate, as doing so would result in a higher payoff. Thus, there are no pure strategy Nash equilibria in this game.

(c) Identify all mixed strategy equilibria, if any exist.

Solution: We find the mixed strategy equilibrium by doing the following: we let Player A play Heads with probability p and Tails with probability $1 - p$, and Player B play Heads with probability q and Tails with probability $1 - q$.

For Player A to be indifferent between playing Heads and Tails, the expected outcomes of each must be identical. Thus,

$$q(1) + (1 - q)(-1) = q(-1) + (1 - q)(1)$$
$$q = \frac{1}{2}$$

Similarly, the following must be true for Player B to be indifferent between playing Heads and Tails:

$$p(-1) + (1-p)(1) = p(1) + (1-p)(-1)$$
$$P = \frac{1}{2}$$

Therefore, the mixed strategy Nash equilibrium is that each player plays Heads with probability $\frac{1}{2}$ and Tails with probability $\frac{1}{2}$.

Question 3

Pete and Dud live in a two-commodity world. Pete's utility function is $U^P(a_P, b_P) = a_P^{\frac{1}{3}} b_P^{\frac{2}{3}}$. Dud's utility function is $U^D(a_D, b_D) = a_D^{\frac{1}{3}} b_D^{\frac{2}{3}}$. Pete is initially endowed with 3 units of commodity a and 2 units of commodity b. Dud is initially endowed with 3 units of commodity a and 7 units of commodity b. What is the competitive equilibrium (i.e. price and allocation)?

Solution: In the competitive equilibrium, the marginal rates of substitution for each individual must be equal, and both must also equal the price ratio. Allow a to be the numeraire (with price 1), and the price of b to be p. Thus,

$$MRS_{a,b}^P = MRS_{a,b}^D = \frac{P_a}{P_b}$$

$$\frac{MU_a^P}{MU_b^P} = \frac{MU_a^D}{MU_b^D} = \frac{P_a}{P_b}$$

$$\frac{\frac{1}{3} a_P^{-\frac{2}{3}} b_P^{\frac{2}{3}}}{\frac{2}{3} a_P^{\frac{1}{3}} b_P^{-\frac{1}{3}}} = \frac{\frac{1}{3} a_D^{-\frac{2}{3}} b_D^{\frac{2}{3}}}{\frac{2}{3} a_D^{\frac{1}{3}} b_D^{-\frac{1}{3}}} = \frac{1}{p}$$

$$\frac{b_P}{2a_P} = \frac{b_D}{2a_D} = \frac{1}{p}$$

Hence,

$$2a_P = pb_P \qquad (1)$$
$$2a_D = pb_D \qquad (2)$$

And thus, through vertical summation,

$$2(a_P + a_D) = p(b_P + b_D) \tag{3}$$

Now, based on the initial endowments, we know that

$$a_P + a_D = 6$$
$$b_P + b_D = 9$$

Therefore, plugging into equation (3),

$$2(6) = p(9)$$
$$p = \frac{4}{3}$$

(This is not the only way to solve for p. Others will lead to the same answer.)

And we know that each person must fulfill his budget constraint. That is, given the prices of goods a and b, the values of their initial endowments must equal the value of their final allocations.

$$a_P + \frac{4}{3} b_P = 3 + \frac{4}{3}(2)$$
$$a_D + \frac{4}{3} b_D = 3 + \frac{4}{3}(7)$$

We can use these budget constraints, the fact that $p = \frac{4}{3}$, and equations (1) and (2) to solve for the competitive equilibrium. (Plug $p = \frac{4}{3}$ into equations (1) and (2), and then for each person, solve for a or b and plug into the budget constraint for that person. Then use that to solve for the equilibrium quantity of the other good.) This yields the following equilibrium allocation of goods:

$$a_P = \frac{17}{9} \qquad b_P = \frac{17}{6}$$
$$a_P = \frac{37}{9} \qquad b_P = \frac{37}{6}$$

Question 4

Xavier, Yvette, and Zachary share the same collection of songs downloaded from iTunes (they share one computer). Each song costs \$1. If Xavier downloads x

songs, Yvette y songs, and Zachary z songs, their collection will contain $S = x + y + z$ songs.

The utility functions of Xavier, Yvette, and Zachary are given by:

$$u_x(x, S) = 100ln(x + S) - x$$
$$u_y(y, S) = 100ln(y + S) - y$$
$$u_z(z, S) = 100ln(z + S) - z$$

(a) Find the optimal number of downloads x by Xavier (his best response) for any choice of y by Yvette and z by Zachary.

Solution: Xavier's optimal number of downloads can be determined by taking the first-order condition of his utility function with respect to x, and solving for x:

$$u_x(x, S) = 100ln(x + S) - x$$
$$u_x(x, S) = 100ln(2x + y + z) - x$$
$$\frac{\partial u_x}{\partial x} = \frac{200}{2x+y+z} - 1 = 0$$
$$x^* = \frac{200 - y - z}{2}$$

This gives the optimal number of downloads by Xavier for any choices by Yvette and Zachary.

(b) Find the number of downloads by Xavier, Yvette, and Zachary in the Nash equilibrium.

Solution: Following the results of part (a), since Xavier, Yvette and Zachary have identical utility functions, the best response functions will be:

$$x^* = \frac{200 - y - z}{2}$$
$$y^* = \frac{200 - x - z}{2}$$
$$z^* = \frac{200 - x - y}{2}$$

Summing $x^* + y^* + z^*$ leaves:

$$x + y + z = \frac{600 - 2x - 2y - 2z}{2}$$

$$x + y + z = 300 - x - y - z$$

This simplifies to:

$$x + y + z = 150 \tag{1}$$

Plugging into the equations for x^*, y^* and z^* leaves:

$x^* = \frac{200-y-z}{2} = \frac{200-(150-x)}{2}$, which simplifies to $x = 50$
$y^* = \frac{200-x-z}{2} = \frac{200-(150-y)}{2}$, which simplifies to $y = 50$
$z^* = \frac{200-x-y}{2} = \frac{200-(150-z)}{2}$, which simplifies to $z = 50$

(One can also argue that, given equation (1) and the fact that all three have identical utility functions, $x = y = z$. However, such an argument must include a clear and thorough explanation as to why this is true.)

Thus, in the Nash equilibrium, Xavier, Yvette, and Zachary each download 50 songs.