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Quantification of non-homogeneous interval uncertainty based on scatter in modal properties

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Abstract

The objective of this work is to develop and validate a methodology for the identification of intra-variability based on the Interval Field concept, with a specific focus on dynamic FE models. The principal idea is to find a solution to an inverse problem, where the variability on the output side of the model (i.e., the eigenfrequencies) is known from measurement data, but the spatial uncertainty on the input parameters is unknown. Specifically, focus is placed on the identification of the topology of the non-homogeneous interval uncertainty, modelled as an interval field. This identification is performed by comparing the gradients of the halfspaces bounding the uncertain set resulting from the propagation of the interval field and the measurement data set. It is shown that an accurate identification is feasible following the proposed methodology.

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1. Introduction

Continuing advances in computing power and the upswing of high-performance computing (HPC) possibilities allow for making highly detailed numerical models to approximate the partial differential equations that govern most physical processes in our everyday live. This allows for making informed design choices, even in very early stages in a product development cycle. Despite the high resolution of the thus achieved results, the deterministic analysis may not be sufficient to assess the quality of a design, as depending on the design stage, some physical properties are unknown or not yet determined. Moreover, even in a final design stage production tolerances, uncertainty in the loading situation and scatter in material properties induce non-determinism in the response of the design in a functional situation. Recent approaches in the field of computational engineering aim therefore to incorporate model parameter uncertainty into numerical models. As such, a realistic assessment of the reliability of the design, including the uncertainty is attained. Moreover, robustness of the design with respect to these variations can also be ensured. In this context, two complementary philosophies exist: probabilistic and interval numerical analysis. Both techniques have their own field of applicability, depending on the amount of information is available to the designer.

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As an alternative to the well-established framework of probabilistic uncertainty representation, interval techniques such as Interval FE (IFE) or Fuzzy FE (FFE) were introduced. Following these techniques, the non-determinism is respectively depicted as an interval or fuzzy set and thus propagated through the numerical model. These concepts eliminate the need for the identification of a full probabilistic data description, which may be very cumbersome. Moreover, less expensive numerical procedures are necessary for the description of the variability [1–3], which makes these techniques highly suitable for early design stages. Interval fields were only recently introduced as an extension to this concept to account for non-homogeneous non-determinism, and can be regarded as a interval counterpart to the established framework of Random Fields [4]. The description of an interval field is based on the superposition of n_b base vectors ψ_i, scaled by independent interval scalars α_i^I. The base vectors ψ_i represent a set of uncertainty patterns and describe the spatial nature of the uncertainty that is modelled by the interval field. The interval scalars α_i^I capture the uncertainty at the input of the model. An interval field is formally expressed as:

$$\gamma_F^I(\mathbf{r}) = \sum_{i=1}^{n_b} \psi_i(\mathbf{r})\alpha_i^I \tag{1}$$

Application of these powerful interval techniques however requires the identification of their driving parameters. The authors proposed a generic methodology for this identification in [5–8], including a generic scheme for the reduction of the corresponding problem dimensionality, based on the computation of the convex hulls over the computed realisations of the input non-determinism and the set of measurement data. However, the definition and identification of the base vectors ψ_i of the interval field as yet remains unclear. This paper therefore applies the inverse distance weighting interpolation scheme for the identification of a possible set of base vectors (as proposed by the authors in [9]). Based on this scheme, a generic procedure for the identification of the subset of base vetors that results in the optimal overlap with the experimental data is presented. The technique is introduced and illustrated based on a veritual test example using simulated modal data.

2. The interval field finite element method

2.1. Interval finite elements

The interval field FE method comes down to finding the solution set $\tilde{\mathbf{y}}$, when the model parameter uncertainty is depicted as an interval field $\boldsymbol{\gamma}_F^I(\mathbf{r}) \in \mathbb{IR}^k$ over the geometrical model domain Ω, with $\mathbb{IR}^k$ the k-dimensional space of interval scalars and $\mathbf{r} \in \Omega \subset \mathbb{R}^t$. t is the physical dimensionality of the problem at hand (e.g. $t = 4$ for a time dependent simulation in three physical dimensions). $\tilde{\mathbf{y}}$ usually spans a multidimensional region in $\mathbb{R}^d$. This region is in general not convex, which makes an exact computation of this set numerically very hard. $\tilde{\mathbf{y}}$ is therefore commonly approximated by the construction of an uncertain realization set $\tilde{\mathbf{y}}_s$, which is defined as:

$$\tilde{\mathbf{y}}_s = \left\{\mathbf{y}_{sj} \mid \mathbf{y}_{sj} = f(\boldsymbol{\gamma}_{F,j}(\mathbf{r})); \boldsymbol{\gamma}_{F,j}(\mathbf{r}) \in \boldsymbol{\gamma}_F^I(\mathbf{r})\right\} \tag{2}$$

and is obtained by propagating q deterministic realizations $\mathbf{y}_{sj}$ of the interval field $\boldsymbol{\gamma}_F^I(\mathbf{r})$. Herein, $\mathbf{y}_{sj}$ is a vector containing the d output responses of the deterministic solution of the propagation of the j^{th} input uncertainty realization, with $j \in [1, q]$:

$$\mathbf{y}_{sj} = [y_{s1},\ y_{s2},\ ..,\ y_{sd}] \tag{3}$$

These q deterministic propagations should represent the solution set $\tilde{\mathbf{y}}$ as close as possible and may stem from sampling techniques such as e.g. uniform Monte Carlo sampling, DOE or Latin Hypercube sampling. In the case of strict monotonicity of $f()$, also the transformation method [3] may be used.

2.2. Definition of base vectors

The definition of the base vectors $\boldsymbol{\psi}_i$ in (1) in order to construct realistic realisations of the interval field $\boldsymbol{\gamma}_{\mathbf{F}}^{\mathbf{I}}(\mathbf{r})$, proves to be a non-trivial task. These base vectors should be able to translate expert knowledge of the analyst on the spatial nature of the uncertainty to a mathematical formulation in an intuitive way, while delivering a realistic representation of this uncertainty.

Specifically in this paper, $\boldsymbol{\psi}_i$ is constructed according to an inverse distance weighting (IDW) scheme, as proposed by the authors in [9]. This scheme controls the complexity of the field realisations by choosing an appropriate number of control points on predefined locations $\mathbf{r}_i$ inside the topology Ω of the FE model. At each $\mathbf{r}_i$, an independent interval scalar α_i^I determines the local value of the uncertain parameter. The uncertain field realisation is then constructed by extrapolating these local values towards neighbouring elements according to a weighting scheme inversely proportional to the Euclidean distance. Based on this approach, the number of control points directly affects the spatial unevenness, which as such can be tuned by the analyst. Formally, the base vectors $\boldsymbol{\psi}_i$ that constitute the interval field $\boldsymbol{\gamma}_F^I(\mathbf{r})$ are constructed according to:

$$\boldsymbol{\psi}_i(\mathbf{r}) = \frac{w_i(\mathbf{r})}{\sum_{j=1}^{n_b} w_j(\mathbf{r})} \tag{4}$$

with $w_i(\mathbf{r})$:

$$w_i(\mathbf{r}) = \frac{1}{[d(\mathbf{r}_i, \mathbf{r})]^p} \tag{5}$$

The weights $w_i(\mathbf{r})$ are calculated as the inverse of $d(\mathbf{r}_i, \mathbf{r})$ to the power of a strictly positive scalar $p \in \mathbb{R}^+$. $d(\mathbf{r}_i, \mathbf{r})$ is a distance measure between the location $\mathbf{r}_i$ where the interval scalar is defined and any other location $\mathbf{r}$ in the model. By constructing these base vectors via the Inverse Distance Weighting technique, the locally defined interval scalars remain perfectly decoupled, whereas the uncertainty in the remainder of the model is a weighted sum of their influences. Moreover, the identification of the spatial topology of the uncertainty is as such reduced to finding the correct control points of the base vectors.

3. Identification of interval field uncertainty

The application of the interval field concept for the representation of spatial uncertainty requires the identification of the uncertainty on model parameters, which is captured by the interval field $\boldsymbol{\gamma}_F^I(\mathbf{r})$, which in its turn is a function of $\boldsymbol{\alpha}^I$,n_b and $\boldsymbol{\psi}_i(\mathbf{r})$).Specifically for the identification of interval fields for the representation of spatial uncertainty in an interval context, a truthfull estimation of the the base vectors $\boldsymbol{\psi}_i(\mathbf{r})$ is necessary.

In a first step of the identification, the base vectors $\boldsymbol{\psi}_i(\mathbf{r})$ are identified based on the gradients of the convex hulls of the uncertain realisation set and the set of measurement data. Herefore, an initial estimate of the corresponding interval scalars α_i^I and field dimensionality is used. Once the correct base vectors are identified, the corresponding interval uncertainty is quantified using the method that was presented by the authors in [5,6,8]. Section 3.1 presents the novel methodology for the identification of the base vectors. The method for quantification of the interval scalar uncertainty is given in section 3.2

3.1. Base vector identification

For the identification of the base vectors $\boldsymbol{\psi}_i(\mathbf{r})$, the convex hull of the uncertain realisation set C_s and the convex hull of the measurement data set C_m are represented as a set of h_s half-spaces in $\mathbb{R}^d$, following Minkowski-Weyl's theorem:

$$C_s \equiv \mathbf{A}_s \mathbf{y}^T - \mathbf{b}_s \geq 0 \tag{6}$$

with $\mathbf{A}_s \in \mathbb{R}^{d \times h_s}$, $\mathbf{b}_s \in \mathbb{R}^d$ defining the set of halfspaces and $\mathbf{y} \in \mathbb{R}^d$ a vector of model responses. Analogous definitions are used for C_m. The number of halfspaces h_s that comprise the convex hull of the uncertain realisation set C_s is purely based on the elements of the set and their respective topology in $\mathbb{R}^d$. These sets of halfspaces represent the inequalities that describe boundaries of the corresponding convex hull and are in fact a set of vector-valued functions $\mathbf{f}_s(\boldsymbol{\alpha}^I, \boldsymbol{\psi}(\mathbf{r}))$, which depend on the interval field that is defined on the parameters at the input of $g()$. When considering only the boundary of C_s, this leads to:

$$\mathbf{f}_s(\alpha^I, \boldsymbol{\psi}(\mathbf{r})) = [f_1, f_2, ..., f_{h_s}]^T = \mathbf{A}_s\mathbf{y}^T - \mathbf{b}_s = \mathbf{0} \tag{7}$$

with $\forall f_i, i = 1, ..., h_s : \mathbb{R}^d \mapsto 0$. These functions are analogously defined for the measurement data set.

The correct base vectors that constitute the interval field that underlies the measured system responses in C_m are identified by minimising the discrepancy between the gradients of the halfspaces that bound C_m and C_s, given that the model under consideration is linear and modelled correctly with respect to the system at hand. The latter corresponds to selecting a correct model $\mathcal{M}$ for the deterministic representation of the response of the physical behaviour of the model under consideration. The correct set of basevectors $\boldsymbol{\psi}^*(\mathbf{r})$ is selected as:

$$\boldsymbol{\psi}^*(\mathbf{r}) = \text{argmin}\left(\left\|\nabla\mathbf{f}_s\left(\alpha^I, \boldsymbol{\psi}(\mathbf{r})\right)\big|_{\alpha^I=\alpha^I_0} - \nabla\mathbf{f}_m\right\|_2^2\right) \tag{8}$$

This objective function is specifically constructed, based on the idea that the gradient of the halfspaces contains information on the *dependency* of the considered responses, which in its turn is determined by the base vectors of the interval field. As such, the discrepancy in the gradients of the halfspaces of the convex hulls of respectively the measurement data set and the uncertain realisation set serves as a measure for the correctness of the base vectors. In the specific case when the base vectors are constructed using inverse distance weighting, this problem reduces to finding the correct locations $\mathbf{r}$ where the locally defined intervals are defined. As such, the optimisation problem can be solved using integer-valued optimisation algorithms such as integer genetic algorithms or the hill-climbing algorithm.

3.2. Identification of interval scalars

Identification of the vector of interval scalars $\boldsymbol{\alpha}^I$ is performed by minimizing a cost function $\delta(\boldsymbol{\alpha}^I)$, which describes the discrepancy between $\tilde{\mathbf{y}}_s$ and $\tilde{\mathbf{y}}_m$, as introduced in [8]. $\delta(\boldsymbol{\alpha}^I)$ is based on the computation of the convex hulls of $\tilde{\mathbf{y}}_m$ and $\tilde{\mathbf{y}}_s$, respectively C_m and C_s and corresponding d-dimensional volumes $\mathcal{V}_s$ of C_s:

$$\delta(\boldsymbol{\alpha}^I) = \left((1 - \frac{\mathcal{V}_s(\boldsymbol{\alpha}^I)}{\mathcal{V}_m})^2 + (1 - \frac{\mathcal{V}_o(\boldsymbol{\alpha}^I)}{\mathcal{V}_m})^2 + \left\|\mathbf{c}_m - \mathbf{c}_s(\boldsymbol{\alpha}^I)\right\|_2^2\right) \tag{9}$$

with $\mathbf{c}_m$ and $\mathbf{c}_s$ the geometrical centres of mass of respectively $\tilde{\mathbf{y}}_m$ and $\tilde{\mathbf{y}}_s$. $\mathcal{V}_o$ is the multidimensional volume of the overlap $\tilde{\mathbf{y}}_o$ between $\tilde{\mathbf{y}}_m$ and $\tilde{\mathbf{y}}_s$. The interval vector $\boldsymbol{\alpha}^{I,*}$, used for the construction of $\boldsymbol{\gamma}^I_F(\mathbf{r})$, is finally determined as :

$$\boldsymbol{\alpha}^{I,*} = \text{argmin}\left(\delta(\boldsymbol{\alpha}^I)\right) \tag{10}$$

4. Illustration of the base vector identification

4.1. Case presentation

An illustration of the developed methodology is showed using a dynamic numerical model of a one-dimensional cantilever beam, as indicated in figure 1. For illustration purposes, Young's modulus of the beam is subjected to uncertainty with a continuous spatial component over the model domain Ω, which is modelled using the interval field concept. The considered beam has a length of 300 *mm* and a constant rectangular cross section of 40 *mm* × 2 *mm*. The validation shows how to quantify the interval field uncertainty in such a part based on scattered eigenfrequencies coming from experimental modal analysis.

The governing second order ordinary differential equation is discretised and solved for the first 10 eigenfrequencies using a finite element (FE) model containing 10 linear shell elements with 4 nodes. Both in-plane and out-of-plane bending modes, as well as torsional modes are considered. For benchmarking purposes, measurement data are generated by sampling a two dimensional *a priori* defined interval field (later also referred to as goal interval field). Interval scalars $\alpha^I_1 = [1.35; 1.45]\ GPa$ and $\alpha^I_2 = [1.55; 1.65]\ GPa$ are defined at predefined locations $\mathbf{r}_1 = 1$ and $\mathbf{r}_2 = 9$ in the model domain Ω. The 10-dimensional measurement set $\tilde{\mathbf{y}}_m$ is obtained by solving the FE model for the first 10 eigenfrequencies, based on 500 deterministic realisations of this interval field. In analogy to [8], the measurement set $\tilde{\mathbf{y}}_m$

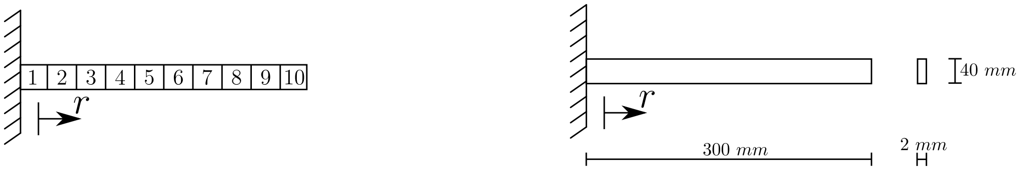

Fig. 1: Left: Illustration of the Finite-Element model of the cantilever beam on which the methodology is validated. **r** indicates the positive direction along the beam, the numbers in the finite elements indicate the element numbers. Right: Geometrical properties of the cantilever beam.

is built using the 5^{th} and 6^{th} eigenfrequency of the model. The uncertain realisation set $\tilde{\mathbf{y}}_s$ is constructed analogously. As an initial estimate, the base vectors are constructed with $\mathbf{r}_1 = 4$ and $\mathbf{r}_2 = 5$, with the corresponding interval scalars $\alpha_1^I = [1.0; 2.0]\ GPa$ and $\alpha_2^I = [1.2; 2.2]\ GPa$. Using the presented methodology, the entire interval field is identified.

4.2. Results

The result of the interval field identification (i.e. base vectors and interval scalar) is shown in figure 2. Figure 3 also shows a two-dimensional cross-section of the ten-dimensional convex hulls of the uncertain realisation set before and after identification, as well as the measurement data set. From this figure, the performance of the combined methodology in both identifying the base vectors and interval scalars is clear. In fact, a close to exact identification of the constituting interval field is obtained, as can be noted by the fact that the updated interval field and goal interval field are perfectly coincident.

The first step of the identification consists of matching the base vectors, illustrated in the bottom plot of figure 2, by solving the minimsation problem introduced in eq. (8). Specifically, an integer implementation of the hill-climbing algorithm was used. For this specific case, the optimisation found the correct base vectors in only 4 iterations, needing 36 deterministic calls to the NASTRAN solver. The corresponding interval field $\boldsymbol{\gamma}_F^I(\mathbf{r})$ with correct base vectors, but incorrect interval scalars is also shown in figure 2. The result of propagating this interval field is also shown in figure 3. Note how the gradients of the convex hull of the initial estimate are corrected with respect to the measurement data set. The second step of the identification consists of quantifying the corresponding interval uncertainty. This is done by solving the optimisation problem, as introduced in eq. (9) by means of a sequential quadratic programming algorithm, as explained in [8], which converged after 25 iterations.

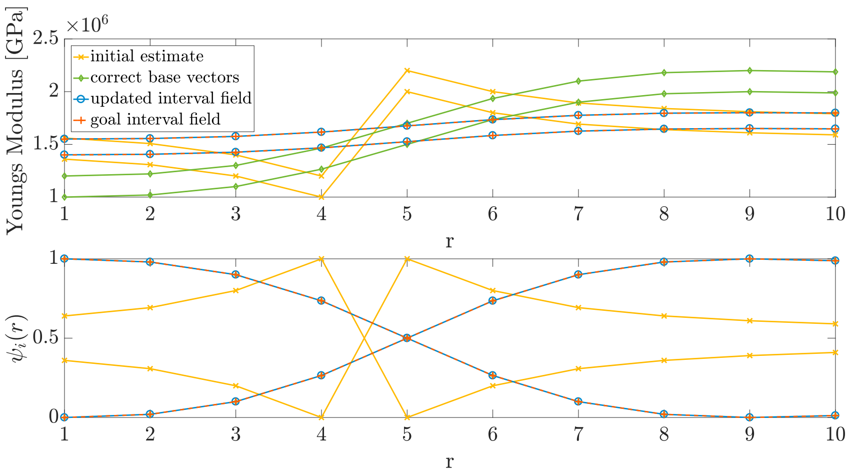

Fig. 2: Top: extreme realisations of the initial estimate on the interval field, the identified interval field and the interval field that was used for the construction of the measurement data (the goal interval field) for case 1. Bottom: the initially estimated base vectors, the identified (updated) base vectors and the base vectors that were used for the construction of the measurement data (goal base vectors).

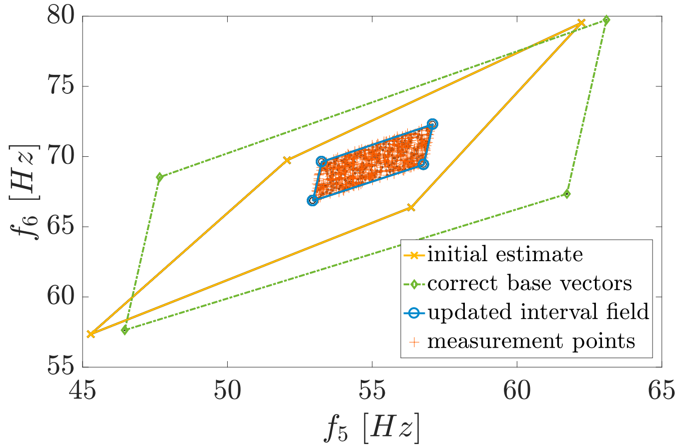

Fig. 3: Two-dimensional cross-section of the ten-dimensional convex hulls of the uncertain realisation set before and after identification, as well as the measurement data set. Note that also the gradients of the convex hulls are changed due to the identification of the correct base vectors.

5. Conclusions

In order to represent spatial uncertainty in the physical model parameters, constituting the input of a Finite Element (FE) model, the interval field concept proves to deliver accurate results with limited data. The use of this concept however, requires the identification of its driving parameters: n_b, $\boldsymbol{\psi}_i(\mathbf{r})$ and $\boldsymbol{\alpha}^I$. A methodology for this identification, based on the computation of the convex hull over the uncertain realisation set and a set of measurement data, was proposed by the authors in previous work [5,6,8].

This paper introduces an extension to this identification methodology by allowing for the joint identification of the base vectors of the interval field and the interval scalars that constitute the interval field. Specifically, this is obtained by incorporating information on the gradients of the halfspaces that denote the limits of both the uncertain realisation set and measurement data set. The performance of the technique is illustrated using a simple academic example, and it is shown that an accurate identification of the base interval field is obtained.

Future work will include an extension of the methodlogy to higher dimensional spaces, both in terms of interval fields at the input and output of the model. Also, validational tests using real-live measurement data are planned.

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