Problem Set 2
14.02
Due Sept. 27, 1995

1. Consider a closed economy in which government expenditure and taxation are exogenously given. The economy is characterized by the following equations:

1. $C = c_0 + c_1 Y_D$, $Y_D = Y - T$, with $0 < c_1 < 1$
2. $I = d_0 + d_1 \cdot Y$, with $0 < d_1 < 1$
3. $G = G$
4. $Z = C + I + G$
5. $Y = Z$

a) Explain the second equation. (Please give some economic intuition, not just a mathematical description.)

b) Solve for the equilibrium level of output. What happens in this model if $c_1 + d_1 > 1$? Show the geometric series of the reaction of output to an increase in government spending.

c) Assume that $c_1 + d_1 < 1$ for the remainder of the question. Solve for the effect of an increase in government spending on output. Compare the multiplier in this model to the multiplier when investment is exogenous (as in the model in the text). Explain the difference.

d) Rewrite the equilibrium condition in terms of saving, investment and the budget deficit. Suppose the government increases spending by \$100. Does this equation imply that saving also increases by \$100? Why not?

e) Find the equilibrium level of saving. [Hint: Write saving in terms of income, consumption and taxes. Use equation 1 to replace consumption, and then replace income by the equilibrium level computed in part **b**.] Using this expression calculate the effect of the increased government spending on saving.

2. This problem will lead you through two exercises in calculating the dynamics of a simple economic model.

Consider a closed economy with no government. Production must be set one period in advance, and high costs prohibit any adjustment of production in the current period. Assume that in its desire to set production equal to expected demand, the firm decides to set production equal to the average demand in the two preceeding periods:

1. $Y_t = 0.5 \cdot Z_{t-1} + 0.5 \cdot Z_{t-2}$.

Individuals in this model take time to adjust their consumption, so that consumption is a function of both current income and income lagged one period. The consumption function has the following form:

2. $C_t = c_0 + c_1(0.5 \cdot Y_t + 0.5 \cdot Y_{t-1})$, with $0 < c_1 < 1$.

The economy is closed. There is no government in this economy, and investment is taken as exogenous. The remainder of the model is thus given by the following relationships:

3. $G_t = T_t = 0$
4. $I_t = I^*$
5. $Z_t = C_t + I_t + G_t$.

a) What is the marginal propensity to consume out of income?

b) Derive an equation describing the dynamics of output. That is, find an expression for Y_t in terms of its lagged values (namely, Y_{t-1}, Y_{t-2}, and Y_{t-3}) and exogenous variables.

c) The economy will settle to a constant level of output. Solve for the level of output at which output is no longer changing. How does this compare to the equilibrium level when $Y_t = Z_t$ replaces equation 1?

For the remainder of the problem, set the parameter values as follows:

$I^* = 1000$, $c_0 = 100$ and $c_1 = 0.4$.

d) A sudden wave of optimism sweeps the economy, resulting in a permanent jump in investment by \$100. To what level of production will the economy converge? How does it compare to the equilibrium output level before the rise in investment? Since we are interested in the dynamics of this economy, calculate the value of output and consumption for the period in which the shift occurs and the five following periods. Will these variables converge? Graph the response of these two variables in levels, assuming that these variables equalled their previous equilibrium values before the shift in investment. The graph should contain the time periods on the X-axis and the level of output and consumption on the Y-axis. Do the dynamics resemble the example given in Chapter 4?

[Hint: You may want to use a simple program to compute Y_t as a function of Y_{t-1}, Y_{t-2} and Y_{t-3}. A good candidate is a spreadsheet program called Xess on Athena. You can access this by choosing Numerical/Math from the menu bar and selecting Spreadsheets, Xess. As a general strategy, use one column for output and one column for investment. Begin each column with three rows that contain the old equilibrium values. Row 4 will correspond to period in which the jump in

investment occurs. Enter the new value of investment for all remaining periods. Use an equation in row 4 of the output column to calculate the new value. Copy this equation to the remaining rows of the output column. Consumption can be calculated using a third column and the computed values for output. A few notes about using Xess: To enter numerical values, simply type the algebraic expression. To enter formulas, type = and the expression. Use cell numbers in the expression to represent values from other cells (for example, in cell c1 you could type "=.4*b1+.1*a1"). To copy formulas, highlight the cell containing the formula you wish to copy and choose Edit, Copy, Copy Formulas. Then click, hold, and drag over the cells to which you are copying. To graph a column of cells, choose Graph, NewGraph, Line Graph. From the graph menu bar, choose Edit, Data Sets. Under Y-data, enter the series to be graphed (for example, enter "a1..a30" for all cells between a1 and a30). If you have trouble printing the spreadsheet directly, choose File, Export to create a text file that can be printed from the athena% prompt using the lpr command. If you have trouble printing the graph, save it by choosing File, Print As and choose a filename of the form "filename.ps". This file can then be printed using lpr.]

e) Suppose the optimism is instead short-lived. As before, investment increases to \$1100 during period t, but it then returns to its original value of \$1000 in period t+1 and remains at this level. Repeat all of the exercises from section **d**). Explain your results in words.

3. In Question 4e of Problem Set 1, you were asked whether government expenditure actually varies with the level of income in the economy. One possible reason for this to occur is that increases in income raise tax revenue, allowing the government to spend more. Suppose we wish to empirically investigate the relationship between the two variables. Suppose we estimate the following equation and get the following result:

$\Delta G_t = 0.38 + 0.19 \cdot \Delta Y_t + residual$, $R^2 = 0.20$.

Can you therefore conclude that increases in production cause increases in government spending? Explain why not. [Hint: Think about the issue of causality.]