ARTICLE

Energetic Consequences of Walking Like an Inverted Pendulum: Step-to-Step Transitions

Arthur D. Kuo,[1] J. Maxwell Donelan,[2] and Andy Ruina[3]

[1]Departments of Mechanical Engineering & Biomedical Engineering, University of Michigan, Ann Arbor, MI; [2]School of Kinesiology, Simon Fraser University, Burnaby, BC, Canada; and [3]Department of Theoretical & Applied Mechanics, Cornell University, Ithaca, NY

KUO, A.D, J.M. DONELAN, and A. RUINA. Energetic consequences of walking like an inverted pendulum: Step-to-step transitions. *Exerc. Sport Sci. Rev.*, Vol. 33, No. 2, pp. 88–97, 2005. *Walking like an inverted pendulum reduces muscle-force and work demands during single support, but it also unavoidably requires mechanical work to redirect the body's center of mass in the transition between steps, when one pendular motion is substituted by the next. Production of this work exacts a proportional metabolic cost that is a major determinant of the overall cost of walking.* **Key Words:** human locomotion, bipedal walking, center of mass, mechanical work, oxygen consumption, metabolic cost

INTRODUCTION

It costs several times the metabolic energy for a human to walk as it does to bicycle the same distance. Walking is often likened to the motion of two coupled pendula, because the stance leg behaves like an inverted pendulum moving about the stance foot, and the swing leg like a regular pendulum swinging about the hip. This analogy is used to explain conservation of energy during walking. In the absence of a dissipative load, the sustained periodic motion of a pendulum (or any jointed linkage of body segments) requires no net mechanical work. The pendulum analogy is attractive for its simplicity, but it also presents a paradox, because there is no reason why friction or other dissipation within the joints should be greater for walking than for bicycling.

Why, then, is walking so much more costly? It is not that humans have little concern for energy. Walking appears to be highly optimized with respect to metabolic cost. Humans prefer to walk at the particular combination of step length, step frequency, and even step width that is energetically optimal (2,5,11). Walking is less costly than other human gaits, even though it still falls far short of wheeled transport. The relatively high cost of walking would surely be better understood if there were a more complete physiological model of the energetics of work and force production by muscle than the one that presently exists. But because the same physiology is shared between walking and bicycling, part of the answer may lie in physics.

The mechanics of walking require a transition between pendulum-like phases. In contrast to a wheel, a single inverted pendulum can only transport the body's center of mass (COM) a limited distance. Humans continue movement by transferring from one pendulum-like stance leg to the next. We propose that significant metabolic energy is expended as a consequence of this transition. Positive and negative mechanical work must be performed on the COM to accomplish these step-to-step transitions, and this work exacts a proportional metabolic cost. This cost is an unavoidable consequence of inverted pendulum behavior, and comprises a substantial fraction of the overall cost of walking.

The concept of step-to-step transitions in terms of the mechanics of an ideal, simple inverted pendulum is reviewed. We formulate a power law describing the major factors that contribute to transition costs, yielding predicted trends that can be tested independently from the model. We present experimental tests of the mechanical work predicted by step-to-step transitions, as well as tests of a metabolic cost proportional to this work. We then consider additional factors that might contribute to the overall metabolic cost of walking, followed by a refined interpretation of our results in the context of other data from the literature.

Address for correspondence: Arthur D. Kuo, Depts. of Mechanical Engineering & Biomedical Engineering, University of Michigan, 3200 G G Brown Building, 2350 Hayward St., Ann Arbor, MI 48109-2125 (E-mail: artkuo@umich.edu).

Accepted for publication: December 29, 2004.

0091-6331/3302/88–97
Exercise and Sport Sciences Reviews

Model of Inverted Pendulum with Step-to-Step Transitions

We begin with a simple pendulum model of walking (1) (Fig. 1), consisting of alternating single- and double-support phases, with one and two legs contacting the ground, respectively. We assume that the stance leg is straight, and that the dominant inertia is that of the body's COM, placed at the hip. We initially consider an ideal inverted pendulum gait, concentrating on the theoretical consequences of such a gait on the transition between inverted pendulum phases (Fig. 1). This model can be studied analytically, yet it captures major characteristics of walking. It is also easily augmented with more realistic anthropomorphic features.

A leg behaving like a rigid inverted pendulum during single support (Fig. 1a) will conserve mechanical energy. No work is therefore needed to move the COM. The leg can also be kept at full extension with minimal muscle force, so that all single support could, in principle, come at no cost in terms of work or muscle force. But single support only applies over the distance of a single step. Even if the succeeding step involves another energetically conservative inverted pendulum, the transition between steps, in which one pendulum stops and the next starts, must involve mechanical work.

The step-to-step transition (6,10) involves redirection of the COM velocity (1,9,13) (Fig. 1b) and simultaneous positive and negative work by the two legs (7). At the end of one step, the COM is moving forward, but with a downward velocity component, as prescribed by the pendular arc. By the beginning of the next step, the COM must be redirected to move upward, to follow the arc prescribed by the leading leg. To maintain steady walking speed, the magnitude of the COM velocity should be the same at the beginning and end of single support. Still, changing the direction of COM velocity requires force, produced separately by the trailing and leading legs, and directed along each leg. The trailing leg will perform positive work on the COM, and the leading leg will perform negative work (Fig. 1c). This is best illustrated by allowing the legs to be nonrigid during double support, so that their lengths can change by a small amount. The rate of work is equal to the dot product of the force and velocity vectors, and the trailing leg's force will be directed at an acute angle with the COM velocity, yielding a positive dot product. The leading leg's force will be directed at an obtuse angle, yielding a negative dot product. Thus, even if no net work is performed on the COM over the course of the transition, positive and negative work must be performed by the separate legs.

The mathematical details of the step-to-step transition are as follows. The simple pendulum model constrains the COM velocity, $\vec{v}_{\mathrm{com}}$ (vectors denoted by the arrow symbol), to be perpendicular to the stance leg during single support. During the redirection interval (roughly double support), the acceleration is the rate of change of $\vec{v}_{\mathrm{com}}$, or from Newton's Law:

$$\dot{\vec{v}}_{\mathrm{com}} = \frac{1}{M}\left(\vec{F}_{\mathrm{lead}} + \vec{F}_{\mathrm{trail}}\right) + \vec{g} \qquad [1]$$

where $\vec{F}_{\mathrm{lead}}$ and $\vec{F}_{\mathrm{trail}}$ are the ground reaction forces from the leading and trailing legs, M is body mass, and $\vec{g}$ is the

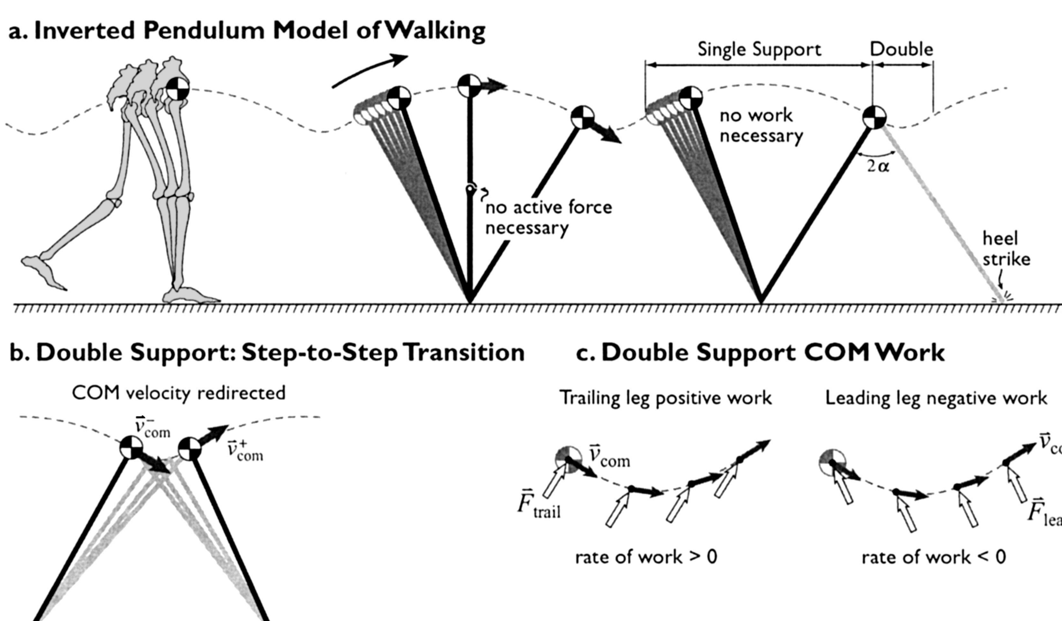

Figure 1. Schematic diagram of the simple inverted pendulum model of walking, which requires energy not for pendular motion but rather to redirect the body's center of mass (COM) between steps. (a) During single support (when one leg contacts the ground), the rigid pendulum conserves mechanical energy, and the COM can be supported with no muscle force. Consecutive single-support phases are separated by a double-support phase (commencing with heel strike) as one stance leg is replaced with the next. (b) This is referred to as the step-to-step transition, in which the COM velocity is redirected to a new pendular arc, from $\vec{v}^-_{\mathrm{com}}$ before transition to $\vec{v}^+_{\mathrm{com}}$ after. (c) During double support, the trailing and leading legs perform positive and negative work on the COM, respectively. Here, the legs may be considered nonrigid to allow for a U-shaped displacement of the COM. The trailing and leading leg forces, $\vec{F}_{\mathrm{trail}}$ and $\vec{F}_{\mathrm{lead}}$, are assumed to be directed along the legs, and are shown separately for the same COM trajectory. The theoretical rate of work is represented by the relative directions of force and velocity, where an angle less than 90° denotes positive work, and an angle above 90° denotes negative work.

gravitational acceleration. This equation may be integrated to yield the change in velocity. Labeling the beginning and ending times $t-$ and $t+$, and the corresponding velocities $\vec{v}^{-}_{com}$ and $\vec{v}^{+}_{com}$, respectively, the integral is:

$$
\begin{aligned}
\vec{v}^{+}_{com} - \vec{v}^{-}_{com} &= \frac{1}{M}\int_{t-}^{t+} \vec{F}_{lead}\,dt + \frac{1}{M}\int_{t-}^{t+} \vec{F}_{trail}\,dt + \int_{t-}^{t+} \vec{g}\,dt \\
&= \hat{F}_{lead} + \hat{F}_{trail} \int_{t-}^{t+} \vec{g}\,dt
\end{aligned}
\qquad [2]
$$

where $\hat{F}_{lead}$ and $\hat{F}_{trail}$ are each leg's integrated and body mass–normalized contribution to redirecting the COM. A convenient simplification is to treat the duration as short enough that the configuration of the legs is roughly unchanged, even though there may be nonzero displacement of the COM. The short duration also means that the gravitational (last) term in equation 2 will be negligible.

The work performed by the legs contributes to the COM kinetic energy. We begin with a simplified case in which $\hat{F}_{trail}$ and $\hat{F}_{lead}$ have equal magnitude and are produced both impulsively and in immediate succession, with the trailing leg pushing off first (Fig. 2). The short duration causes the COM displacement during double support to approach zero. The COM kinetic energy before and after each impulse is proportional to the squared magnitude of $\vec{v}_{com}$ at each instant, and the work performed by each impulse is equal to the change in kinetic energy it produces. Each leg's work is therefore proportional to the difference in squared velocities. In this simple model, with pushoff preceding heel strike, the Pythagorean theorem may be applied to yield the trailing and leading leg work:

$$
\begin{aligned}
W_{trail} &= \frac{1}{2}M(v^{-}_{com}\tan\alpha)^2 \\
W_{lead} &= -\frac{1}{2}M(v^{-}_{com}\tan\alpha)^2
\end{aligned}
\qquad [3]
$$

where v_{com} is the (scalar) COM speed, and α is the half angle between the legs. The action of the leading leg may be regarded as a collision, with the force directed along the leg and performing negative work on the COM (9,10). An equal magnitude of positive work is performed by first pushing off with the trailing leg, anticipating the need to restore the energy lost in the subsequent collision.

The step-to-step transition is optimal when pushoff and then collision are of equal magnitude and are performed for short durations. It is theoretically more costly if the legs do not perform equal amounts of work during double support, because of additional work needed during single support to maintain a steady walking speed. For example, a collision exceeding the pushoff (Fig. 2c) will cause the inverted pendulum to begin the next step more slowly than the previous one ended, and positive work must then be performed during single support to make up the difference. This work may be performed by gravity when walking down a slope (13), or by active hip torques for level ground (10). In contrast, a pushoff exceeding the collision (Fig. 2d) will cause the pendulum to have additional energy, which could be expended in going up a slope. However, it is the case of equal work contributions (equation 3) that yields the least overall positive work (10). It is advantageous not to perform net positive work on the COM during single support, lest that increase the collision loss. It is also ideal to minimize the time and displacement of the step-to-step transition. Greater COM displacement during the redirection means more work by each leg against gravity. More realistic pushoff and collision could occur over finite time and displacement, and overlap in time, with some energetic penalty compared with the ideal.

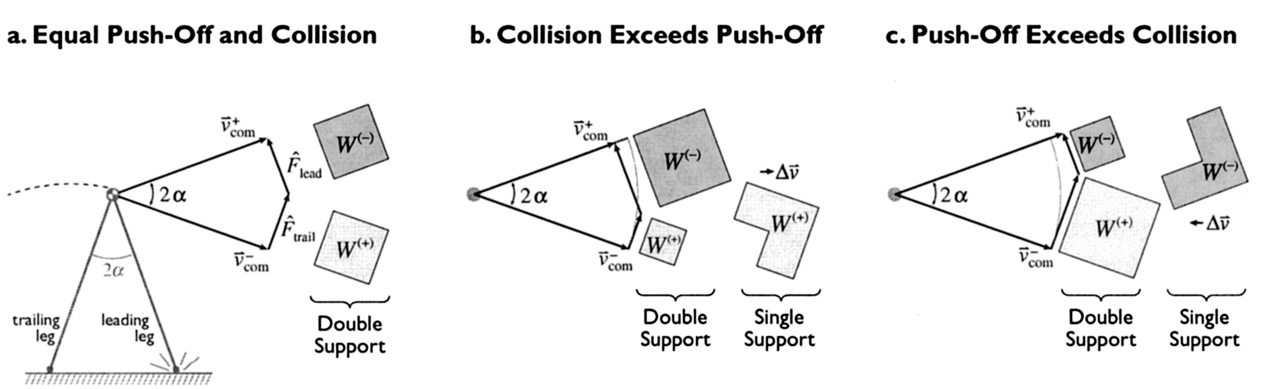

Figure 2. Geometric diagram of COM velocity redirection by trailing and leading legs. The velocity vector $\vec{v}_{com}$ changes during double support because of the forces applied along the legs, separated by angle 2α. The theoretical work performed by each leg per step is proportional to the square of the integrated and mass-normalized forces $\hat{F}_{trail}$ and $\hat{F}_{lead}$ (see equation 2), with positive pushoff work $W^{(+)}$ by the trailing leg and negative collision work $W^{(-)}$ by the leading leg (*shaded regions*). (a) If pushoff is equal to collision magnitude, the total amount of positive and negative work is minimized, and no work need be performed during single support. (b) If the collision exceeds the pushoff, the next step begins with smaller velocity. To maintain steady walking speed, velocity can be increased (by an amount $\Delta\vec{v}$) by performing additional positive work during single support, or by walking downhill. (c) If pushoff exceeds collision, additional negative work is needed during single support to slow the pendulum (*e.g.,* by walking uphill. In both of the latter cases, more overall positive or negative work must be performed than with equal pushoff and collision magnitudes, shown here by the smaller areas of the shaded regions in (a), compared with those in parts (b) and (c).

More realistic models, and even physical machines, have dynamics similar to the simple model presented. This includes models with a trunk, knees, different foot configurations, and anthropomorphic mass distribution (13). Some of these models have been applied to design of physical walking machines (4,13), demonstrating that the theoretical mechanics and energetics described here also apply to the real world. The models' collision dynamics can also be scaled for a wide variety of mass and length scales (9,10). Knees have little effect on step-to-step transitions (9), although they are helpful for gaining ground clearance and speeding the swing phase (13). A rigid, curved foot that rolls along the ground and causes the center-of-pressure point to translate forward during single support can reduce the coefficient of proportionality in equation 3, but not its form. Inclusion of frontal plane dynamics, with an anthropomorphic pelvis width, also causes step-to-step transition work to depend on step width (12). Even with all of these features, the predicted work from each leg is still well approximated by:

$$\begin{aligned} W_{\text{trail}} &\propto M(v_{\text{com}}^{-}\tan\alpha)^2 \\ W_{\text{lead}} &\propto M(v_{\text{com}}^{-}\tan\alpha)^2 \end{aligned} \quad . \qquad [4]$$

The robustness of equation 4 to model features and parameters makes it well suited to experimental testing. We test for trends in rates of work as a function of an independent variable, rather than testing for a coefficient of proportionality that is highly sensitive to model variations.

Measurement of Step-to-Step Transitions with Center-of-Mass Work

The work performed on the COM in step-to-step transitions can be estimated experimentally (7). COM work is equal to the work performed by external (*i.e.*, ground reaction) forces as if they moved through displacements of the body's COM (3,7), even though it is the muscles that actually perform the work. A drawback is that COM work (also called external work (3,10)) does not capture work for motions relative to the COM (often referred to as internal work), which we assume to be small during double support. It might therefore be preferable to use the actual work performed by muscle fibers as a more direct indicator of metabolic cost. However, the external forces $\bar{F}_{\text{lead}}$ and $\bar{F}_{\text{trail}}$ are far simpler to measure than the muscle forces and lengths that are needed to evaluate muscle fiber work. COM work also has an advantage. Because the mechanical energy of the COM is influenced only by COM work (10), changes in energy of a simple pendulum may be examined through this measure alone.

Measured COM work can quantify the separate contributions of the two legs. If leading and trailing leg work on the COM is performed simultaneously, it is possible for the net COM work to be zero, even though metabolic energy may be expended within each leg. We therefore define the individual-limb method (ILM) (7) for quantifying the individual limbs' contributions to net COM work W_{com}:

$$\begin{aligned} W_{\text{com}} &= \int \vec{F}_{\text{lead}} \cdot \vec{v}_{\text{com}}\, dt + \int \vec{F}_{\text{trail}} \cdot \vec{v}_{\text{com}}\, dt \\ &= \int P_{\text{lead}}\, dt + \int P_{\text{trail}}\, dt \\ &= W_{\text{lead}} + W_{\text{trail}} \end{aligned} \qquad [5]$$

where P_{lead} and P_{trail} are the instantaneous power or rate of COM mechanical work (Fig. 3a), and W_{lead} and W_{trail} are the

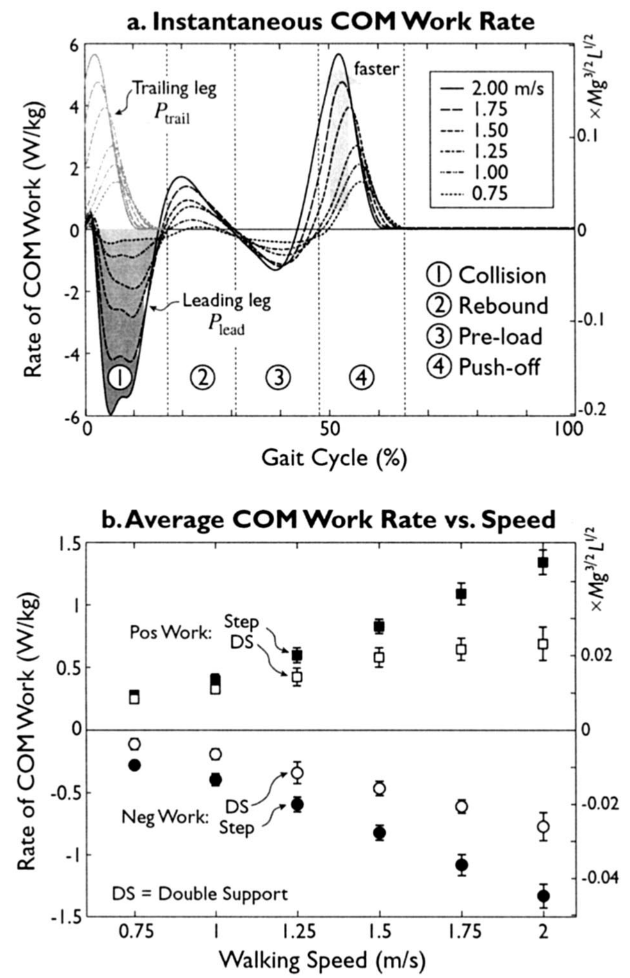

Figure 3. The individual limbs perform simultaneous positive and negative work on the body's center of mass (COM). (a) Measurements of instantaneous COM work rate over a gait cycle show positive "pushoff" work performed by the trailing leg starting just before heel strike and continuing over double support. Negative "collision" work is performed by the leading limb over slightly more than double support, in amounts increasing with walking speed (*shaded areas*). Simultaneous work redirects the COM between successive single-support phases. Some COM work is also performed during single support (6), in phases labeled "rebound" and "preload". (b) The average rates of positive and negative work over a full step (*filled symbols*) and over double support alone (DS, *unfilled symbols*) all increase with walking speed. The amounts are computed by integrating positive or negative instantaneous power over these intervals and then multiplying by step frequency. For example, DS negative work is equal to the area of the shaded region of part (a). Data shown are from 10 normal human subjects (7), and are averaged and plotted in dimensionless units (right axes), with more familiar units (W·kg^{-1}) shown for convenience. Gait cycle consists of a full stride (two steps) commencing with heel strike. Double support occurs over approximately the first 10% of the gait cycle, and opposite-leg heel strike occurs at 50%.

work per step for each leg. The integrals in equation 5 can be evaluated only for times of positive or negative work. They can additionally be restricted to just the double-support phase (when most of the redirection is expected to occur), or to an entire step. We divide these measures by step period to yield average rates of positive or negative work, for either double support or entire steps.

Empirical data (Fig. 3b) show that the legs' separate work contributions during the double-support phase of normal human walking are quite substantial, and nearly equal in magnitude (7). At a typical walking speed of 1.25 $m \cdot s^{-1}$, approximately 15.4 $\pm$ 2.6 J (mean $\pm$ SD for 10 normal subjects) of positive COM work is performed by the trailing leg during double support for each step (pushoff in Fig. 3a). Approximately 12.4 $\pm$ 3.1 J of negative work is simultaneously performed by the leading leg (collision). Both of these contributions increase with walking speed, as would be expected from equation 4. At any speed, the separate contributions nearly cancel each other, meaning that the body's COM experiences a relatively small change in mechanical energy compared with the work exertions of each leg. A closer observation reveals that the trailing leg pushoff begins slightly before double support, and the leading leg collision extends slightly beyond double support. Judging from COM mechanical power, redirection of the COM might occur over an additional 24 ms before (2.2% of gait cycle) and 40 ms after (3.6%) the typical double-support interval of approximately 150 ms (13.6%), for a walking speed of 1.25 $m \cdot s^{-1}$. As speed increases, pushoff commences earlier in the stride, whereas the collision becomes slightly shorter. The overlap between the two is least at 2.00 $m \cdot s^{-1}$, and greatest at 0.75 $m \cdot s^{-1}$.

Not only individual limbs' COM work measured over double support increases with speed; the COM work evaluated over the entire step also increases (7) (Fig. 3). The leading leg collision is followed by a single-support phase, which begins with a brief period of positive work as the leg extends and the COM appears to rebound (Fig. 3a). Single support then concludes with a similar period of negative work, labeled preload, that precedes the positive work of pushoff. The work of the rebound and preload intervals increases with speed along with the collision and pushoff, and both may be consequences of step-to-step transitions, even though the simple model did not anticipate these intervals (discussed later).

Overall positive COM work per step is approximately 21.7 $\pm$ 2.2 J at 1.25 $m \cdot s^{-1}$, with an equal magnitude of negative work at steady speed. In dimensionless terms, the mechanical cost of transport (positive COM work over a step, divided by body weight and distance traveled) is 0.049 $\pm$ 0.005.

Comparisons of Center-of-Mass Work Measurements and Metabolic Cost

The theory of step-to-step transitions makes testable predictions regarding mechanical work and metabolic cost. The overall energetic cost of walking may contain other components, such as for supporting body weight or moving the legs with respect to the body (11). However, if these components can be controlled, then step-to-step transition work can be isolated under experimental conditions. This work also would be expected to exact a proportional metabolic cost. As discussed, the trends in equation (4) are robust to a variety of model features and parameters. Here, we examine two methods for testing these trends by varying step length and step width. These yield four predictions: the rate of mechanical work performed on the COM will increase with 1) the fourth power of step length and 2) the second power of step width, and 3) and 4) metabolic rate will increase proportionately with mechanical rate for both of these cases.

Varying step length alone, COM work rate and metabolic rate are predicted to increase roughly with the fourth power of step length. Keeping step frequency f fixed, walking speed $v = f \times l$ will increase proportional to step length l. The average rate of negative work, denoted $\dot{W}^{(-)}$, is then predicted to be the work per step from equation 4, multiplied by step frequency f. (We use the term "rate of work" rather than the equivalent "mechanical power" to avoid confusion with terms such as "fourth power of step length.") In terms of step length and frequency, the prediction is:

$$\text{varying step length: } \dot{W}^{(-)} \propto M f^3 l^4 , \qquad [6]$$

assuming that $v \approx v^{-}_{\text{com}}$. The rate of positive work $\dot{W}^{(+)}$ is predicted to be of the same magnitude as $\dot{W}^{(-)}$. The metabolic rate $\dot{E}$ is predicted to be proportional to both:

$$\text{varying step length: } \dot{E} \propto M f^3 l^4 , \qquad [7]$$

with the exception of a small additional term in work rate over a step, increasing with the square of step length (6). This term is from the swing leg's contribution to COM work during single support, and is not included in double-support $\dot{W}^{(-)}$ or in $\dot{E}$ (11). Other costs, such as for supporting body weight or moving the legs, would be expected to add an offset term to equation 6 or 7, which changes very little with l increasing and f fixed.

Another means for manipulating step-to-step transitions is to vary step width while keeping step length and frequency, and thus speed, fixed (5). This yields predictions that COM work rate and metabolic rate will increase with the square of step width. In the frontal plane, the pushoff and collision mechanics remain the same as described in equation 4, except that step width w, rather than length, is interpreted as proportional to α. The velocity $\vec{v}^{-}_{\text{com}}$ is dominated by forward walking speed, and bears much less dependence on step width (5). Keeping step length and frequency fixed, the predicted dominant terms for rate of work and metabolic cost are:

$$\text{varying step width: } \dot{W}^{(-)} \propto M v^2 f \cdot w^2 \qquad [8]$$

$$\text{varying step width: } \dot{E} \propto M v^2 f \cdot w^2 \qquad [9]$$

where f and l are kept fixed. As with equations 6 and 7, constant offsets may be added to the proportionalities, reflecting other potential costs that are presumed not to vary with experimental conditions.

Experimental results from normal humans walking under these varying conditions (Fig. 5) are consistent with these predictions. The measured rate of COM work performed during double support increases approximately with $\dot{W} \propto l^4$ when speed increases proportional to step length and step frequency is kept fixed ($R^2 = 0.96$; R^2 indicates degree of variability about each curve fit), and approximately with $\dot{W} \propto w^2$ when only step width is varied ($R^2 = 0.86$). The rate of work over an entire step also increases with similar relationships for $\dot{W} \propto l^4$ ($R^2 = 0.97$) and $\dot{W} \propto w^2$ ($R^2 = 0.83$), but with different proportionalities. This confirms the observation that step-to-step transition work is not confined to double support alone. And, as discussed, it also might not be confined to pushoff and collision intervals.

Each leg's COM work appears to be performed at a metabolic cost. For variations of both step length and step width, the measured metabolic rate is approximately proportional to the rate of COM work (Fig. 5). We report here the net metabolic rate, measured from total oxygen consumption rate during walking, subtracting the rate for quiet standing. As with work, net metabolic cost increases approximately with $\dot{E} \propto l^4$ ($R^2 = 0.95$) when varying step length, and $\dot{E} \propto w^2$ ($R^2 = 0.91$) for step width. Over the step lengths shown, metabolic cost ranges from 78 to 276% of the nominal rate at $1.25\ m \cdot s^{-1}$. With changing step width, metabolic cost increases up to 43%, ranging from 2.4 to $3.4\ W \cdot kg^{-1}$. For reference, the nominal net metabolic rate at $1.25\ m \cdot s^{-1}$ is $2.3 \pm 0.3\ W \cdot kg^{-1}$, for a dimensionless net metabolic cost of transport (energy per weight and distance traveled) of 0.19 ± 0.02.

These combined results are interlinked. The mechanical work curves appear individually (Fig. 5), but should be considered to be alternate views of a single surface (Fig. 4c). It may not be surprising for any single measure of gait to change when an independent variable is manipulated. However, the results shown here vary in different ways with step length and step width and, more importantly, with trends predicted from a single model of step-to-step transitions. The metabolic rates vary in a similar manner, approximately proportional to the mechanical work rates.

We use these results to estimate the contribution of step-to-step transitions to the net rate of metabolic energy expenditure during walking at $1.25\ m \cdot s^{-1}$. We assume that double support is a suitable interval for evaluating step-to-step transitions, and that all of the positive work is performed actively with an efficiency of 25% (6). The negative work could be performed passively at no cost, or actively at an efficiency of −120%. Adding these contributions yields a crude estimate for the cost of step-to-step transitions, of approximately 60 to 70% of the net metabolic cost of walking at $1.25\ m \cdot s^{-1}$.

It may also cost energy to move the legs back and forth relative to the body. We previously hypothesized that such a cost might act as a tradeoff against step-to-step transitions

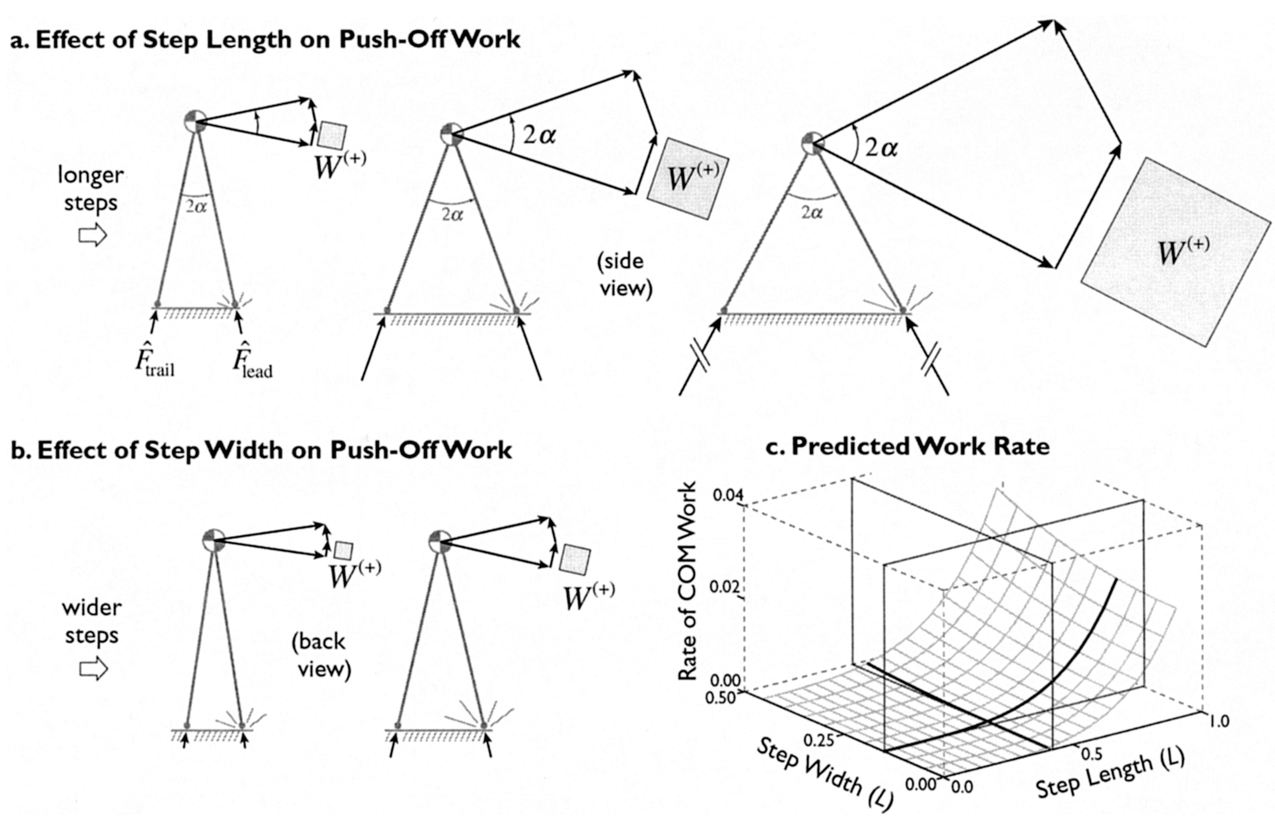

Figure 4. Predictions of step-to-step transitions for changing step length and width. (a) Walking with increasing step length and fixed step frequency theoretically requires pushoff work to increase sharply (6), because the body's center-of-mass (COM) velocity increases in magnitude, and the angle through which it must be redirected also increases. (b) Walking with increasing step width and fixed step length and frequency also requires increasing pushoff (5), but to a lesser degree because the magnitude of collision velocity is dominated by the (fixed) walking speed, and therefore changes little. Step width does, however, contribute to the change in direction of COM velocity, so that the rate of work performed on the COM is predicted to increase approximately with the square of step width. (c) Predicted work rate is actually a function of both step width and step length, represented as a surface. Variations of step width alone and step length alone yield separate curves along the surface equations 6 and 8, with a larger predicted cost for step length (increasing with the fourth power). Step length and step width are shown as a fraction of leg length *L*.

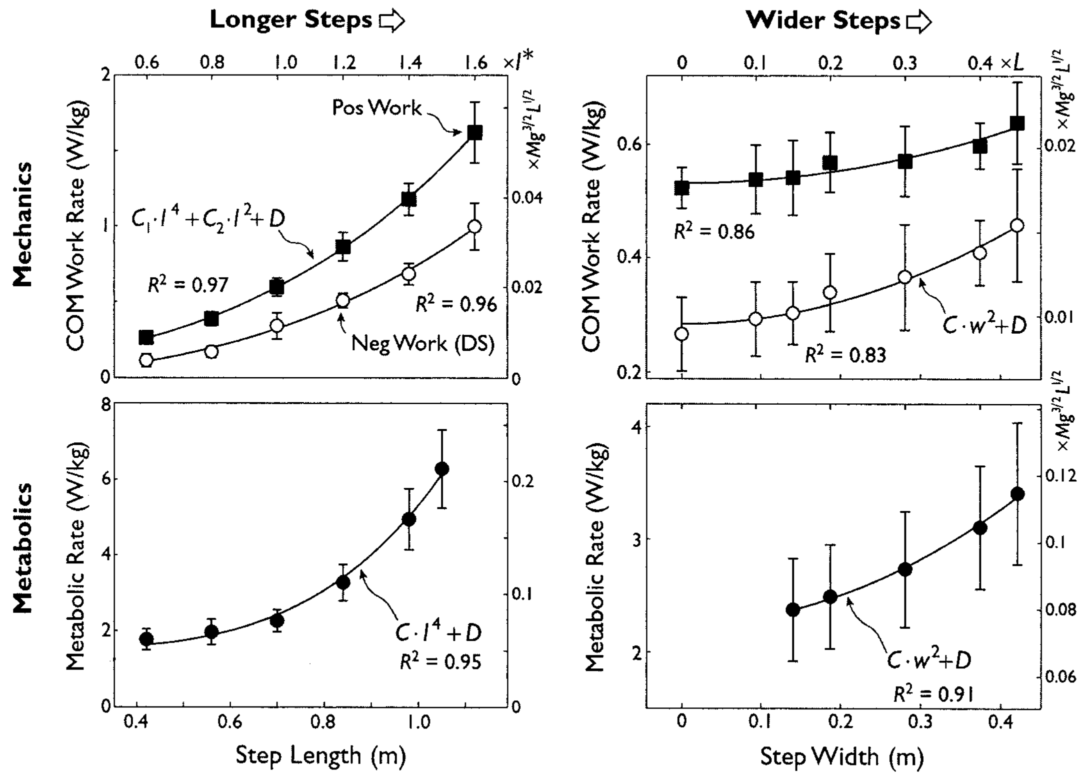

Figure 5. Experimental results comparing measured mechanics (top) and metabolics of walking (bottom) vs increasing step lengths (left) and step widths (right), against predictions for step-to-step transitions. Upper left shows that average rate of mechanical work performed on the body's center of mass (COM), $\dot{W}$, increases approximately with fourth power of step length (accounting for leg motion in an additional squared term (6)), keeping step frequency fixed. Upper right shows that work rate also increases approximately with the square of step width as predicted (Fig. 4), keeping step length and frequency fixed. Both total work measured over a step (*filled symbols*) and negative work measured over double support (DS, *unfilled circles*) change with condition, indicating that work related to step-to-step transitions is not limited to double support alone. Bottom row shows that net metabolic rates (subtracting the cost of quiet standing) also increase approximately proportional to mechanical work rates, indicating a proportional cost for step-to-step transitions. All data are plotted in dimensionless units (right and top axes), but scaled to more familiar dimensional units (left and bottom axes, scaled by the mean nondimensionalizing factor) for convenience. The curve fits are derived from the model predictions of equations 6–9. Increasing step lengths were applied as fractions of each subject's preferred step length l^* at 1.25 $m \cdot s^{-1}$, and increasing step widths as fractions of each subject's leg length, L. The parameters C and D are coefficients, different for each curve, whose values were derived from curve fit rather than from models. Data are from two separate studies (5,6) with 9 and 10 subjects, respectively, recomputed here using a single consistent procedure.

(10,11). The high cost of step-to-step transitions alone would favor walking at very short steps and high step frequencies. Periodic actuation of the opposing hips for this purpose could thereby reduce step-to-step transition costs. Walking could then resemble the rolling of a wheel (13), except for the difficulty of moving the legs quickly. In humans, the metabolically optimum step length does not occur at very short steps, indicating a cost that increases sharply with step frequency (11) and that trades off against step-to-step transitions. We hypothesized that forcing the legs to move quickly would exact a substantial metabolic cost increasing sharply with frequency (specifically, a rate proportional to $f^4 \times l$). The activity of the hips can be interpreted as having two components. One is analogous to springlike actuation caused by a combination of muscle and tendon, most likely exacting a metabolic cost for moving the legs relative to the body. The second is nonspringlike, net positive work performed by the hips over a stride, which may ultimately contribute to push-off, and would therefore be included in step-to-step transition costs.

There are surely other metabolic costs for walking not considered here. Lateral leg motion and step variability associated with stabilizing balance might also exact a metabolic cost (12). They appear to act in concert with the step width dependency (Fig. 5) to explain why the preferred step width is energetically optimal (6). Other possible costs might be for supporting body weight, balancing the trunk, or actively moving the arms during walking. These contributions are by no means exhaustive, nor will they necessarily sum linearly. However, they also appear not to change significantly under the experimental conditions considered here.

Refined Conceptual Model

The measures used thus far show when mechanical work is performed on the COM, but not which joints or muscles perform this work, nor when or whether this work is performed actively by muscle fibers. In human walking, COM redirection occurs over a period longer than double support and with nonzero displacement, and the single-support phase never behaves exactly as an inverted pendulum. Insight into these behaviors, as well as the possible muscular sources of work, can be gained by comparing our results against joint-power data published by Winter (15).

Here, we examine joint power (resultant joint moment multiplied by joint angular velocity) for normal walking, in the context of the four intervals of the stance phase (Figs. 3a and 6), demarcated by sign changes in the instantaneous rate of individual-limb COM work (6). We propose interpretations of a few aspects of joint power that can be related directly to our simple model, cautioning that these interpretations are necessarily descriptive rather than prescriptive. For convenience, we take "muscle" and "tendon" to mean the active contractile component of muscle and series elastic components, respectively. We also define "midstance" to mean the instance between rebound and preload, when the stance leg is approximately vertical and the knee is near its maximum extension.

Collision

The collision phase refers to the interval after heel strike when substantial negative COM work is performed by the leading leg. Negative work is actively performed first at the ankle joint, and then at the knee, but in summed amount insufficient to account for work performed on the COM. Much of the negative COM work is therefore likely not attributable to joints or muscles of the leading leg.

Some of the negative work might be performed elsewhere, most obviously by the shoe, heel pad, and plantar ligament, but also in the damped motion of fat, viscera, and muscle. Nonrigid or wobbling mass accounts for the majority of body mass. It plays a major role in passively dissipating energy during running and jumping, one that might also apply to walking. Unfortunately, this dissipation is difficult to quantify theoretically or empirically. Rigid body inverse dynamics methods can only assign dissipation to modeled degrees of freedom, and the addition of more degrees of freedom, especially for soft tissues, demands parameters and data that are exceedingly difficult to determine or verify. Here, the nonparametric nature of COM work measurements comes to advantage, because they require only knowledge of external forces, and the integrated forces can capture soft tissue contributions to COM motion. Measured COM work does not quantify total energy change, but it may quantify the negative work associated with collisions better than the present alternatives.

Rebound

Rebound is characterized by positive COM work, as the stance leg extends before midstance. Some of this work can be attributed to extension of the stance knee. Quadriceps muscles are active during this interval, indicating extensor force and quite possibly work. But the loading conditions and timing also admit the possibility of some elastic rebound at the knee, to an unknown degree. Rebound of the knee, whether elastic or not, can be considered a direct consequence of the collision, because the amount of extension will largely be dependent on the amount of flexion occurring during collision. This may explain why the rebound work rate increases with the collision work rate (Fig. 3).

There may be tradeoffs in the amount of rebound extension desirable. A fully extended knee minimizes the force needed to support body weight at midstance, but reaching that state likely requires work. This tradeoff means that metabolic cost is likely minimized with less than full extension at midstance.

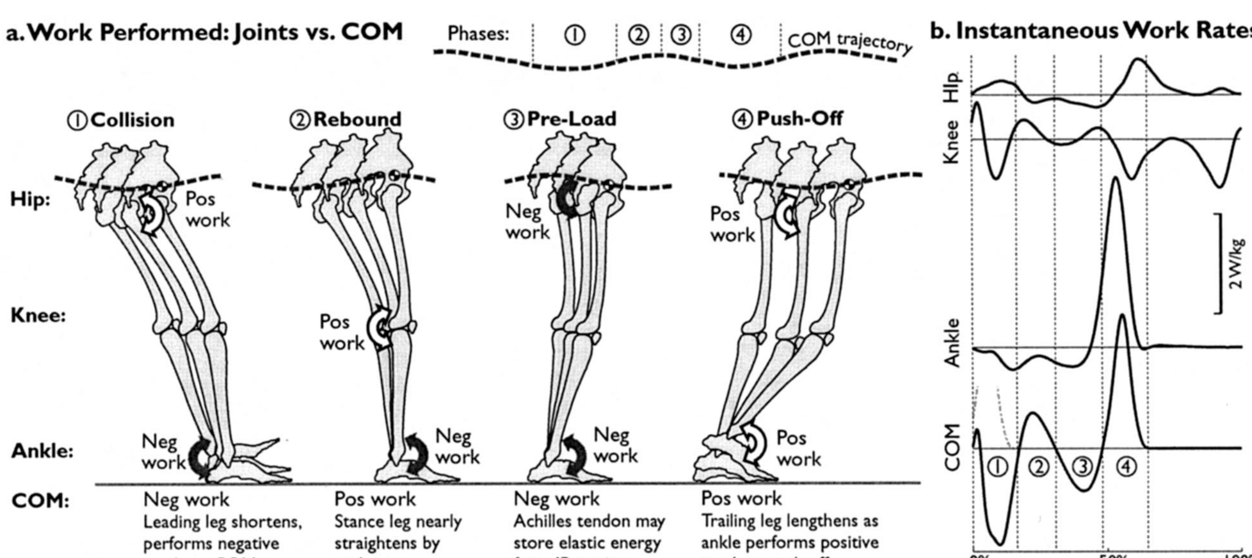

Figure 6. Refined conceptual diagram for a stance phase, divided into four subphases. (a) Major instances of work for the joints and the body's center of mass (COM) are shown along with the sagittal plane COM trajectory (1). During the collision, the COM undergoes displacement as negative work is performed actively at the ankle and then the knee, and passively throughout the body (2). The stance leg then rebounds slightly before midstance, with some positive work performed at the knee (3). Preload is characterized by continuing negative work at the ankle, possibly slowing pendular motion and storing elastic energy for the ensuing pushoff (4). This pushoff is mostly powered by the ankle joint, with elastic energy potentially playing a large role. Arched arrows denote direction of joint torque when substantial joint power occurs. (b) Also shown are typical joint-power trajectories (after Winter (15)), as well as the instantaneous rate of COM work from Figure 3.

Not all of the COM work observed during rebound occurs at the knee. Some work may instead be attributable to the hips (6). They perform net positive work that moves the swing leg with respect to the body and accelerates the inverted pendulum. The term "rebound" therefore roughly refers to a time interval during which the knee extends after being flexed, where the amount of COM work performed during this interval is not all performed at the knee, and not necessarily elastically.

Preload

After midstance, preload is characterized by negative COM work, which can largely be attributed to the ankle joint. Substantial work is likely performed on the Achilles tendon (8), such that the muscle fibers may actually be isometric, or even perform positive work. Elastic energy storage provides three potential advantages. First, it may allow the work for pushoff to be performed over a long duration including both rebound and preload, rather than during pushoff alone. Second, it allows pushoff energy to be derived not only from ankle muscles but also from the inverted pendulum motion. Temporal and spatial distribution of pushoff work might allow muscle to perform at optimum efficiency, avoiding the need to produce high forces for short durations and low efficiency. Finally, slowing of the inverted pendulum not only stores energy, but also reduces the COM velocity so that less energy is lost at collision. Preloading may ultimately allow the net positive work generated by the hips over a stride to contribute substantially to pushoff. These various mechanisms may explain why the preload work rate increases with the pushoff work rate (Fig. 3).

Pushoff

Positive COM work during pushoff is almost entirely attributable to the ankle joint. The knee and hip joints perform little net work over this interval, whereas the stance-leg ankle produces the largest single burst of positive work in the entire stride. As stated, some or even all of this positive work may result from elastic energy stored in tendon. But even if the tendon performs most of pushoff, there are several reasons why muscles might actively perform work to store that energy elastically. First, the energy lost at collision cannot be regenerated by muscle, and only a fraction is likely stored and returned elastically, meaning that active work must restore that energy. Second, the proportionality between step-to-step transition work and metabolic energy also indicates that much of pushoff is actively powered. Elastic energy storage can obscure the timing and even the source, but it is nevertheless likely that some muscles, not necessarily the ankle extensors alone, perform the work for pushoff, and at substantial metabolic cost.

Other Considerations

We briefly consider the swing phase, which is dominated by pendulum dynamics (14), but with significant muscle activity at both the beginning and end of swing. Some of the active hip torque may be springlike in the sense of speeding the pendulum motion (11) without performing much net work over a stride. Elastic tendon may contribute to this motion, reducing the muscle work (but not force) needed to speed and slow the limb. But whether work or force dominates, actively moving the legs back and forth must cost metabolic energy. Examination of hip power also reveals that the hip performs net positive work over a stride, possibly contributing to COM motion, and ultimately to pushoff through the energy storage described.

Reexamining these interpretations, it is apparent that work performed at a joint does not necessarily indicate which muscles perform work, or when. Pushoff might even be partially powered by the positive work performed by hip flexor muscles during collision and rebound, directly accelerating the inverted pendulum. The COM energy might then be stored in Achilles tendon as the ankle extensors activate during preload phase, culminating with release of that energy at pushoff. Performing positive work by multiple muscles, and for relatively long durations, might reduce the demands for peak muscle force and power, perhaps allowing the muscles to operate at higher efficiency and to avoid fatigue.

Given these refinements, the reality of human walking might differ in many details from inverted pendulum arcs. Almost the entire gait cycle is spent in some combination of either redirecting the COM velocity, or recovering from or preparing for it. This may reflect competing mechanical and metabolic demands. Step-to-step transitions are mechanically the least costly if pushoff is performed impulsively at high forces and short durations, whereas muscle is most metabolically efficient at moderate forces exerted over moderate durations. Such tradeoffs imply that the simple pendulum is not quite a biological ideal. Energy is expended, not in the vain attempt to emulate the ideal, but rather to reconcile the unavoidable step-to-step transitions associated with a pendulum against presumably higher force and work requirements for nonpendular motion.

CONCLUSION

The simple pendulum model predicts energy expenditure not for pendulum motion itself, but rather for the transition between steps. Work is required to redirect the COM between pendular arcs, with positive work performed by the trailing leg just before or simultaneous with negative work by the leading leg. Experimental results indicate that most of this work occurs during double support, but with pushoff beginning before this interval and collision continuing beyond it, all with proportional metabolic cost. COM work is also performed before and after midstance, some of it a consequence of collision and pushoff. Both the rates of this work and of metabolic energy expenditure increase approximately with the fourth power of step length and the second power of step width. Step-to-step transitions appear to be significant not only in their direct contributions to the energetics of walking, but also in explaining why humans prefer certain gait parameters for step length, width, and frequency.

Despite its simplistic nature, this model provides useful insight into human walking. Step-to-step transitions explain why mechanical energy must be dissipated in the periodic motion of the limbs, and this dissipation requires that positive work must then be performed to restore the energy lost. This hardly constitutes a full explanation of the metabolic cost of walking, but it offers quantitative predictions supported by simple accessible models and experimental tests made through relatively simple measurements. These measurements suggest a substantial metabolic cost associated with step-to-step transitions as a major consequence of walking like an inverted pendulum.

Acknowledgments

The authors thank R. Kram for his collaboration. This work supported in part by NIH R21 DC006466 and NSF 0320308.

References

1. Alexander, R.M. Simple models of human motion. *Appl. Mech. Rev.* 48:461–469, 1995.
2. Bertram, J.E., and A. Ruina. Multiple walking speed-frequency relations are predicted by constrained optimization. *J. Theor. Biol.* 209:445–453, 2001.
3. Cavagna, G.A. Force platforms as ergometers. *J. Appl. Physiol.* 39:174–179, 1975.
4. Collins, S.H., M. Wisse, and A. Ruina. A three-dimensional passive-dynamic walking robot with two legs and knees. *Int. J. Robot. Res.* 20:607–615, 2001.
5. Donelan, J.M., R. Kram, and A.D. Kuo. Mechanical and metabolic determinants of the preferred step width in human walking. *Proc. R. Soc. Lond. B.* 268:1985–1992, 2001.
6. Donelan, J.M., R. Kram, and A.D. Kuo. Mechanical work for step-to-step transitions is a major determinant of the metabolic cost of human walking. *J. Exp. Biol.* 205:3717–3727, 2002.
7. Donelan, J.M., R. Kram, and A.D. Kuo. Simultaneous positive and negative external mechanical work in human walking. *J. Biomech.* 35:117–124, 2002.
8. Fukunaga, T., K. Kubo, Y. Kawakami, S. Fukashiro, H. Kanehisa, and C.N. Maganaris. *In vivo* behaviour of human muscle tendon during walking. *Proc. R. Soc. Lond. B.* 268:229–233, 2001.
9. Garcia, M., A. Chatterjee, and A. Ruina. Efficiency, speed, and scaling of passive dynamic walking. *Dyn. Stabil. Syst.* 15:75–100, 2000.
10. Kuo, A.D. Energetics of actively powered locomotion using the simplest walking model. *J. Biomech. Eng.* 124:113–120, 2002.
11. Kuo, A.D. A simple model of bipedal walking predicts the preferred speed-step length relationship. *J. Biomech. Eng.* 123:264–269, 2001.
12. Kuo, A.D. Stabilization of lateral motion in passive dynamic walking. *Int. J. Robot. Res.* 18:917–930, 1999.
13. McGeer, T. Passive dynamic walking. *Int. J. Robot. Res.* 9:62–82, 1990.
14. Mochon, S., and T.A. McMahon. Ballistic walking. *J. Biomech.* 13:49–57, 1980.
15. Winter, D.A. *The Biomechanics and Motor Control of Human Gait: Normal, Elderly and Pathological.* 2nd ed. Waterloo, Ontario: Waterloo Biomechanics, 1991, 143.