PROBLEM SET THREE SOLUTIONS

1. Applied Econometrics

This table reports coefficient estimates and t-statistics for each of the four estimated models.

Model	**A**	**B**	**C**	**D**
Intercept	-317.21	-319.42	-1.10	0.04
	-8.49	-9.535	-3.62	1.18
K	0.863	0.887		
	448.71	175.98		
N	2.59	2.72		
	8.94	10.42		
T		-0.80		0.01
		-4.96		146.08
Ln(K)			0.95	0.58
			305.72	235.17
Ln(N)			0.28	0.40
			4.42	93.49

Coefficient estimates are so different between models A and B, and also between models C and D due to an omitted variables bias. As time has been excluded from the model, and time is correlated with both K and N, which are growing over time, models A and C estimate garbage. See *the Handout on Basic Econometrics* for a full discussion if you are interested.

It is difficult to choose between models B and D just looking at the numbers. The R-square is the percentage variance of the dependent variable explained by the right hand-side variables, and has a maximum at 1. By this measure both models do very well in explaining the data.

The time trend is meant to capture technological progress. If the production function is the following:
$Y_t = K_t^{\alpha}(A_tN_t)^{1-\alpha}$
Take natural logs on both sides of this equation so it follows that:
$\ln(Y_t) = \alpha\ln(K_t)+(1-\alpha)\ln(N_t)+(1-\alpha)\ln(A_t)$
Ideally, we would estimate this equation, but unfortunately we don't observe technology A_t. If technology is growing at a constant rate, note the following is true:
$A_t=A_1(1+g_A)^t$
Again taking natural logs we have the following:
$\ln(A_t) = \ln(A_1)+t*\ln(1+g_A)$
Use the log approximation $\ln(1+x) = x$ for small x to rewrite this as the following:
$\ln(A_t) = \ln(A_1)+t*g_A$
This implies we can rewrite our equation above, substituting in for A_t, as follows:
$\ln(Y_t) = \alpha\ln(K_t)+(1-\alpha)\ln(N_t)+(1-\alpha)\ln(A_1)+(1-\alpha)t*g_A$
If we estimate the OLS equation
$\ln(Y_t) = \beta_0+\beta_1\ln(K_t)+\beta_2\ln(N_t)+\beta_3*t$
We can make the following interpretations of the OLS estimated coefficients:
$\beta_0=(1-\alpha)\ln(A_1)$
$\beta_1=\alpha$
$\beta_2=(1-\alpha)$
$\beta_3=(1-\alpha)g_A$
OLS thus implies $\alpha=0.584$, $g_A=3.054\%$, and $A_1=1.04$. This implies our production function is simply $Y=K^{0.584}(AN)^{0.416}$.

You are interested in model E because it should return to you population growth. The coefficient estimate should be interpreted as the percentage change in the population driven by a one unit change in time. From the model we have $\beta_0=4.85$ with a t-ratio of 501.47 and $\beta_1=0.000$ with a t-ratio of 1.96. It looks like it is

pretty close to zero. This is how the data was generated, so we are fine. Proof that the coefficient is what you want follows.
$\ln(N_t)=\beta_0+\beta_1 t$
$N_t=\exp(\beta_0+\beta_1 t)$
$dN_t/dt = \beta_1\exp(\beta_0+\beta_1 t)= \beta_1 N_t$
$\%\Delta N_t=n=(dN_t/dt)/N_t=\beta_1$
QED

The final equation of interest is for capital accumulation. Recall from the Solow model the following
$K_{t+1}=K_t(1-\delta)+sY_t$
If we estimate the OLS equation
$K_{t+1}=\beta_0+\beta_1 K_t+\beta_2 Y_t$
We can make the following associations
$\beta_0=0$
$\beta_1=1-\delta$
$\beta_2=s$
In the code which was given to you there was a mistake. Instead of regressing capital on lagged capital and lagged output, I regressed on lagged capital and current output. This creates a problem, and explains why the estimates are so different from how the data were generated. Notice the difference in t-ratios.

This table reports coefficient estimates and t-statistics for each of the four estimated models.

Model	**Incorrect F**	**Corrected F**
Intercept	1.64	-0.00
	2.78	-1.087
Lag(K)	0.998	0.90
	77.632	987437
Y	0.035	
	2.42	
Lag(Y)		0.15
		142734

Note to graders, please do not take of points for the incorrect estimation of model F.

For the incorrect specification, this yields a steady-state capital per effective worker of $[0.035/(0+0+0.03054)]^{1/0.416}=1.392$ and steady-state output per effective worker of $1.392^{0.584}=1.213$.

For the correct specification, this yields a steady-state capital per effect worker of $[0.15/(0.10+0+0.03054]^{1/0.416}=1.397$ and steady-state output per effective worker of $1.397^{0.584}=1.216$.

Both answers are pretty close, but don't be fooled, as they are estimated very differently.

We have $Y_{100}=2982.48$ and $K_{100}=3456$ and $N_{100}=129.06$. Construct technology using our estimates above as follows:
$A_{100}=A_1(1+g_A)^{100}=1.04(1.03054)^{100}=21.06$.
This implies $(K/AN)_{100}=(3456/(129.06*21.06))=1.272$ and $(Y/AN)_{100}=(2982.48/(129.06*21.06))=1.097$. Output per effective worker is a little less than 90% of its steady-state value.

2. Portfolio Choice

Things to keep in the back of your mind for this problem. Think of wealth being allocated between money and bonds. The level of wealth is fixed, and the supplies of money and bonds are exogenous (do not depend on the interest rate). The interest rate adjusts so supply equals demand.

$W = B^s+M^s = B^d+M^d$

The first equality is true as wealth can only be allocated between money and bonds by assumption, so the value of wealth must be equal to the value of money and bonds. The second equality is true as people allocate their wealth between money and bonds, so the sum of their demands must be equal to their wealth as a budget constraint in the choice between money and bonds.

This implies $B^s-B^d=M^s-M^d$
And then implies $B^s=B^d$ iff $M^s=M^d$.
If the bond market clears, the money market clears, and vice versa.

Also useful to note changes in the money supply only can come about by changes in bond supply
$\Delta W=\Delta M^s+\Delta B^s$ but $\Delta W=0$ (wealth is fixed).
$\Delta M^s=-\Delta B^s$

a. $M^d=5*10,000(0.5-0.05)=247,500$

b. $M^s=M^d$ (note $M^s=W-B^s=500,000-475,000=25,000$)
25,000 = 5*10,000(0.05-i) implies i=0.
$B^s=B^d$ (note $B^d=W-M^d$)
475,000 = 500,000-5*10,000(0.05-i) implies i=0.

c. 25,000(0.9)=50,000(.5-i) implies i=0.05
475,000+0.1*25,000=500,000-50,000(0.5-i) implies i=0.05
At the old interest rate, reducing the supply of money (and thus increasing the supply of bonds) creates an excess demand for money (and excess supply of bonds). Agents want to shift funds from bonds to money, so sell bonds, driving down their price and increasing the interest rate on bonds in the process. Higher interest rates reduce money demand and increases bond demand, and the process continues until markets clear.

d. 25,000=50,000(0.9)(0.5-i) implies i=-0.05
475,000=500,000-50,000(0.9)(0.5-i) implies i=-0.05
At the old interest rate, reducing the demand for money (and increasing the demand for bonds) creates an excess supply of money (and excess demand for bonds). Agents want to shift funds from money to bonds, so buy bonds, driving up their price and reducing the interest rate on bonds in the process. Lower interest rates increase money demand and reduce bond demand, and the process continues until markets clear. But in this case there is a problem, that being a lower bound on nominal interest rates at zero. As money pays a zero nominal interest rate, bonds can never pay less than zero else nobody will want to hold any bonds. This does not mean that this case is uninteresting. In the last problem set we saw that changes in interest rates could change investment and equilibrium output. The central bank often engages in open market purchases of bonds (reducing the private bond supply and increasing the money supply) to reduce the interest rate and help promote high levels of output. When the nominal interest rate approaches zero, the central bank can no longer do this. Many economists think Japan is in this situation, called a liquidity trap.