An interest rate is the price at which we exchange through time. It is the answer to the question: How much will it cost me to anticipate the consumption of a certain good?

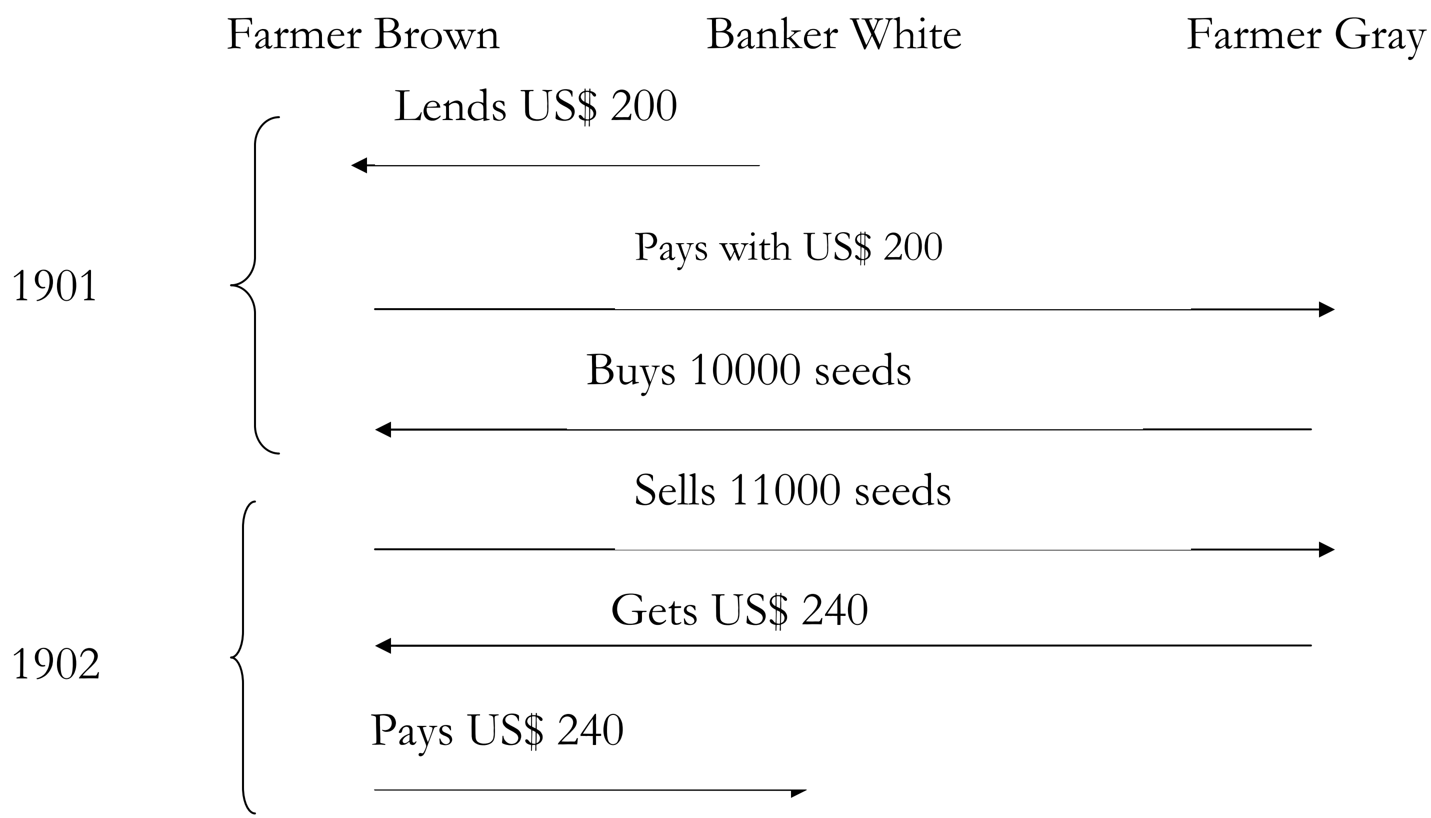
Example: Farmer Brown has arable land but no seeds.

	Farmer Brown		Farmer Gray
1901		$\xleftarrow{\text{Lends 100 packs of seeds}}$	
1902		$\xrightarrow{\text{Returns 110 packs of seeds}}$	

so the REAL INTEREST RATE on this operation is 10%. We could also say that the ANNUAL RETURN for FG is 10%. The interest rate on seeds is then 10%.

$$(1+r) = \frac{\text{Seeds Sold}_{1902}}{\text{Seeds Bought}_{1901}}$$

The farmer who is trying to change the time profile of his production is trying to change consumption or use of a commodity in the future for consumption of that commodity in the present. He will sometimes have to exchange back and from that commodity to satisfy that necessity.



Now the interest rate on money is 20%, but the exchange in terms of seeds has a rate of 10%. The difference between the two is explained by the change in prices. The interest rate on the money loan, THE NOMINAL INTEREST RATE is

$$(1+i) = \frac{\text{Payment on the Loan}}{\text{Value of the Loan}}$$

since

$$\text{Payment on the Loan} = \text{Price of Seeds}_{1902} * \text{Seeds Sold}_{1902}$$

and

$$\text{Value of the Loan} = \text{Price of Seeds}_{1901} * \text{Seeds Bought}_{1901}$$

hence

$$(1+i) = \frac{\text{Price of Seeds}_{1902} * \text{Seeds Sold}_{1902}}{\text{Price of Seeds}_{1901} * \text{Seeds Bought}_{1901}} = (1+\pi_s)(1+r)$$

where, in this case π_s just represents "seed inflation".

1. COMPLICATION 1: private returns and the interest rate

Lets think of why FB is doing this. Presumably he has some excess return in terms of seeds, such that, even paying the 10% real interest rate his crops will yield a return in consumption and in seeds that will enable him to get on with business. Lets suppose that the seeds can be eaten or planted.

Lets call these returns R_t where t=1902, 1903………etc. Remember that he had to plant 10000 seeds to produce lets call this "Investing" or I_t. He does this every period to get a return the next one.

What can he consume every period?

$$C_t = R_t - I_t \text{ if } t>1902 \quad \text{and} \quad C_t = R_t - I_t - (\text{Seeds used to pay loan and interest})_t \text{ if } t=1902$$

So there is a big private return for FB in excess of the interest he has to pay to the bank just to get started. There is a difference between the return he gets on HIS capital (the land) and the return that BW gets on his (the money he lent to FB).

Only if the REAL INTEREST (seeds) increase too much will FB decide not to get the loan and produce but rather to sell the land and look for another job. How much?

COMPLICATION 2: RISK

Now, in the aggregate economy there are many people trying to do these types of INTERTEMPORAL TRADING. Some people are trading seeds through time (FB), other people are simply trading consumption (loan for vacations), other investors are trading other commodities.

1) The fact that Farmer Brown could do this intertemporal trading increased the GDP flows of the economy in the future. It made it possible to activate a resource (his land) that would otherwise not be used.
2) All of these trades are possible because there is trust between the people that are making the transactions. This trust can be produced by the existence of custom, but it is mostly produced by the existence of a law and credible institutions that enforce the law.
3) If this trust was not there it would make intertemporal trading harder. Suppose that Farmer Gray does not know if Farmer Brown will be arround next year. (He has no way of catching him if he runs away, the Apaches….etc.) If he is going to lend to farmer Brown, he is going to ask for a slightly higher interest rate just because of the risk he is facing in lending to farmer Brown.

$$(1+r)=(1+r_{\text{risk free}})(1+\rho)$$

ρ is usually called a RISK PREMIUM.

COMPLICATION 3: SOURCES OF FLUCTUATION OF i

Now think for a second about poor Farmer Gray. He has some excess seeds that he is providing (loaning or selling) to Farmer Brown. Why? Because he thinks that the 10% return he is going to get is higher than what he could get on his land. Lets suppose that he already produces all the seed he needs for his top quality land, but there is a border that is of lower quality. It is so bad, that it will produce only 9% return each year.

If this real return on the marginal investment he can do suddenly increases to 11% he will demand a higher risk free return on the loan or a higher price on the seeds.

Without any discussion of inflation expectations there are already three sources of fluctuation for the nominal interest rate.

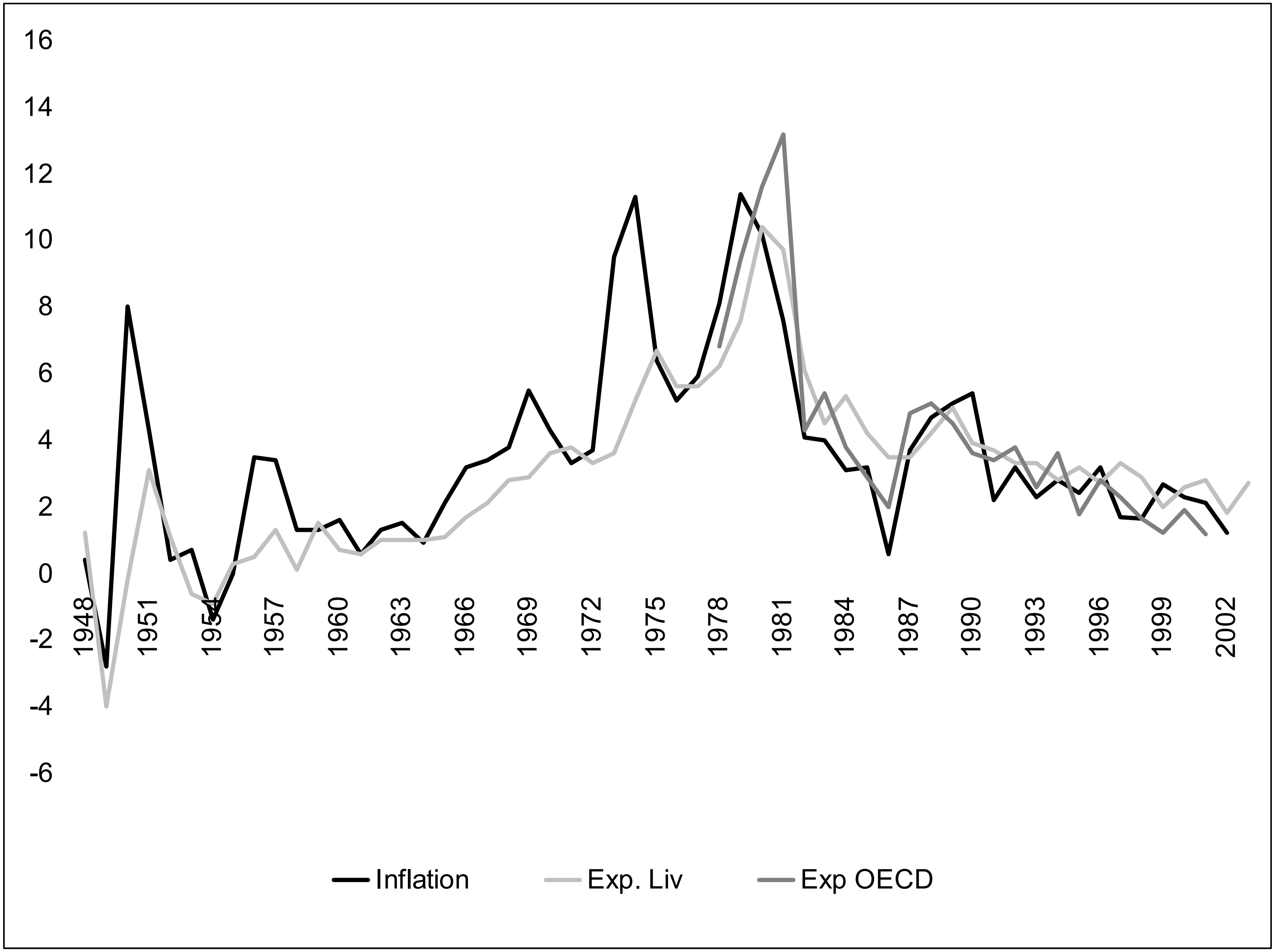
$$(1+i) = (1+\pi_s)(1+r_{rf})(1+\rho)$$

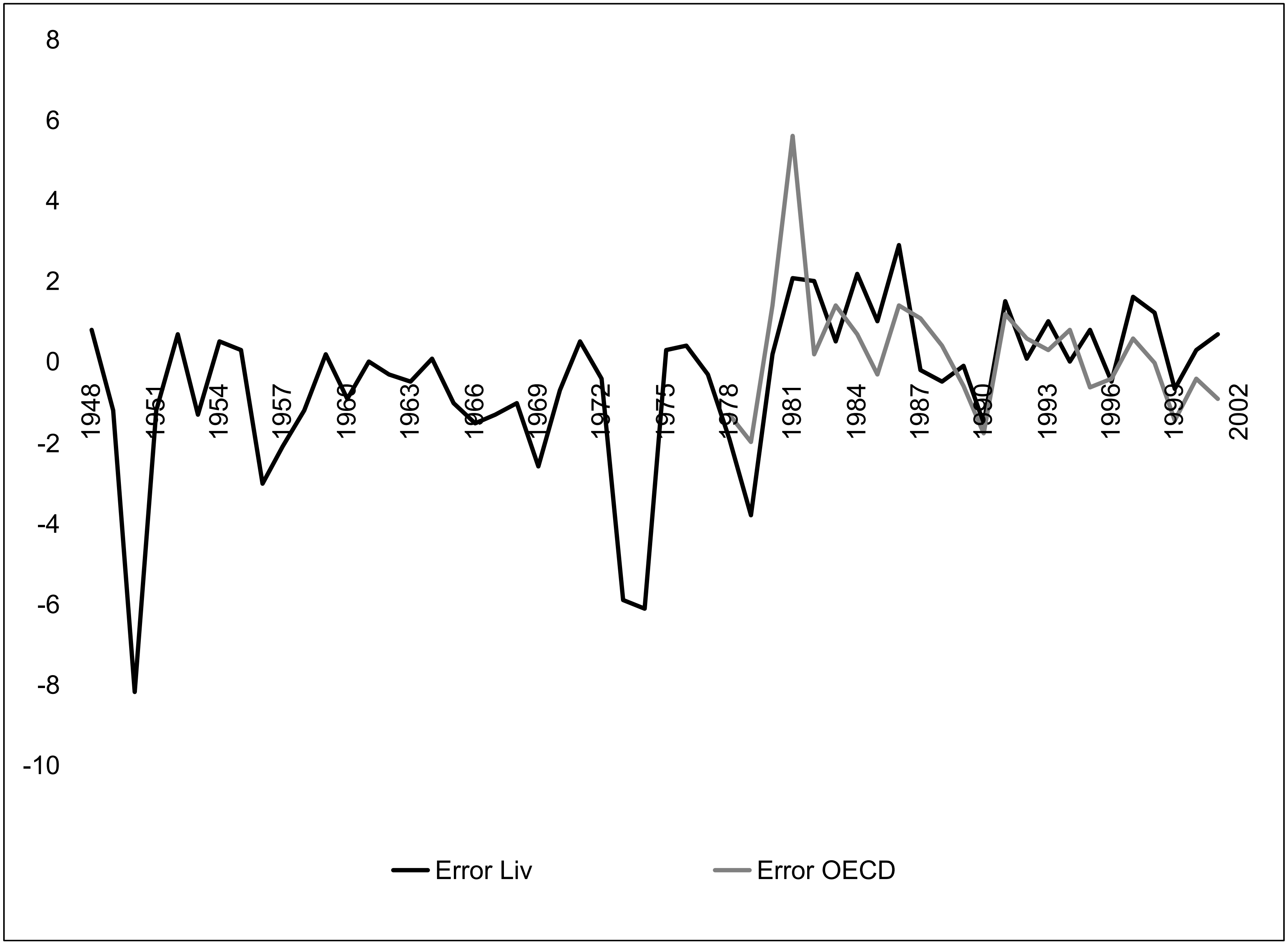
But for the rest of the class lets ignore the existence of ρ.

But people don't know what is going to happen to inflation when they contract the credit. So they form an expectation and use that as the basis of their financial desitions, (without ρ) the relevant real interest rate that FB is going to use to decide if investing in his farm is worthwhile or not is going to be.

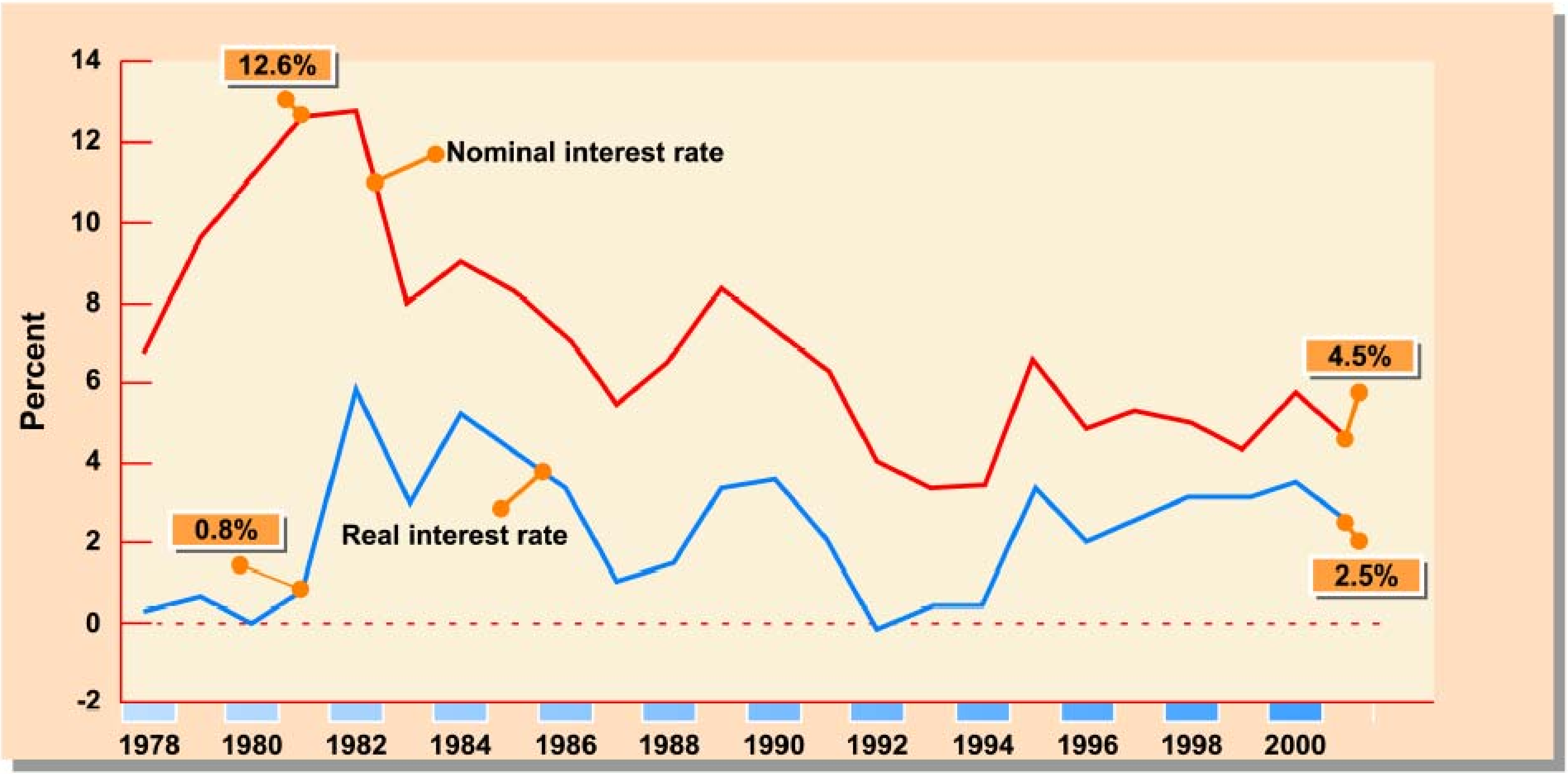
$$(1+r)=\frac{(1+i)}{(1+\pi^e)}$$

So how much do inflation expectations and actual inflation differ? It seems that it is less so. It seems that monetary policy used to surprise the private sector much more than today.


16
14
12
10
8
6
4
2
0
-2
-4
-6
1948
1951
1954
1957
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
Inflation
Exp. Liv
Exp OECD



8
6
4
2
0
-2
-4
-6
-8
-10
1948
1951
1954
1957
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
Error Liv
Error OECD



14
12
10
8
6
4
2
0
-2
Percent
12.6%
Nominal interest rate
4.5%
0.8%
Real interest rate
2.5%
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000

For countries with low inflation and real interest rates it is approximately true that

$$r \approx i - \pi^e$$

we have shown how Farmer Brown's real production decisions are affected by the real interest rate while the money market (where the banker works) clears with the nominal interest rate.

1. Equilibrium in the goods market was represented by the IS curve.

   $$Y = A(Y, r) + \overline{A}$$

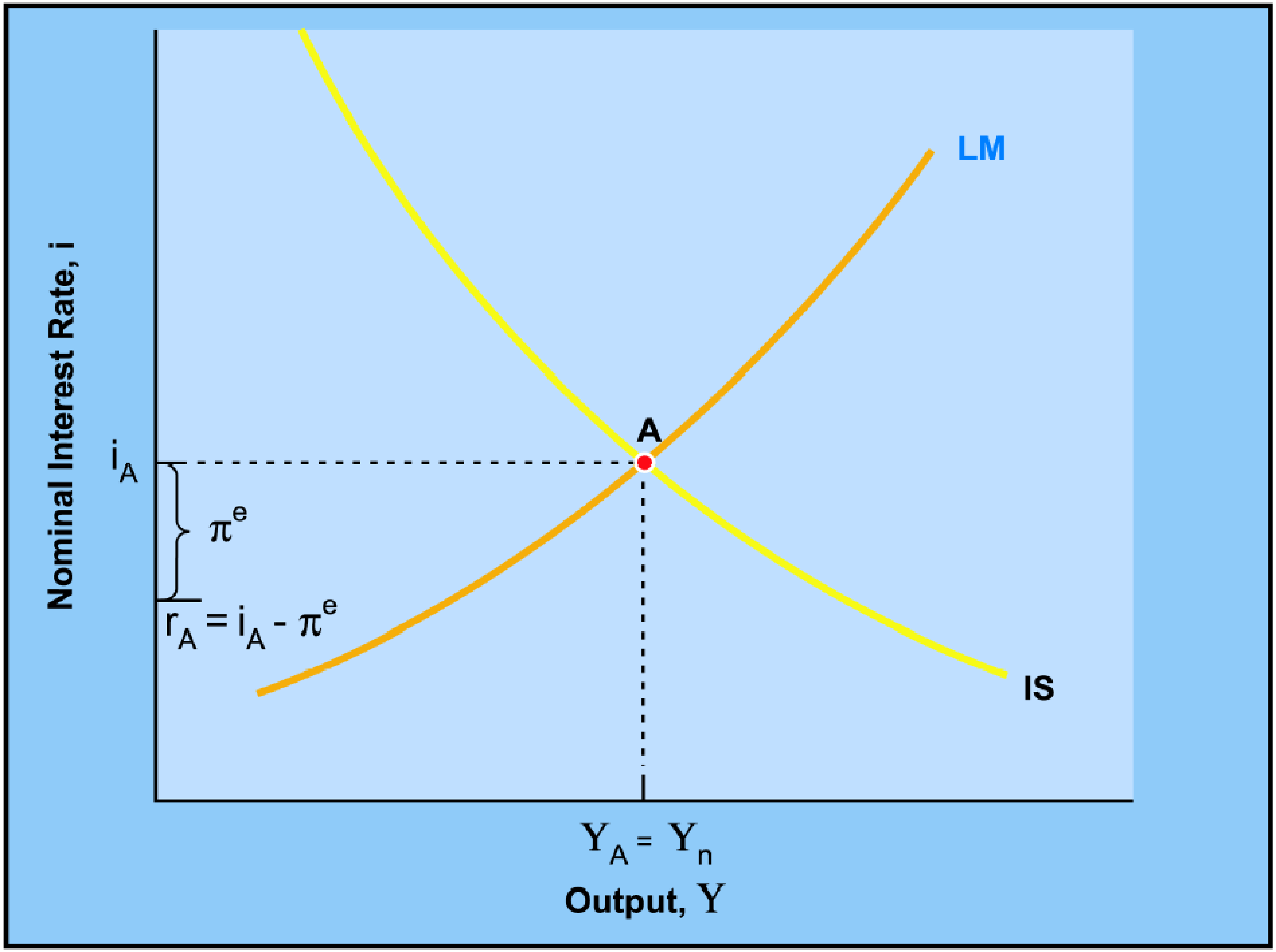
   and in the medium term $Y=Y_n$, hence there is, for every set of parameters of function A (marginal propensity to consume, sensitivity of investment to the interest rate) and for every level of autonomous spending (government spending) a NATURAL REAL INTEREST RATE r_n .
2. In the short run, we have shown that expectations can be cheated a bit but not in the long run. In the long run inflation is equal to the growth rate of money minus the normal growth rate.

Hence, in the medium term

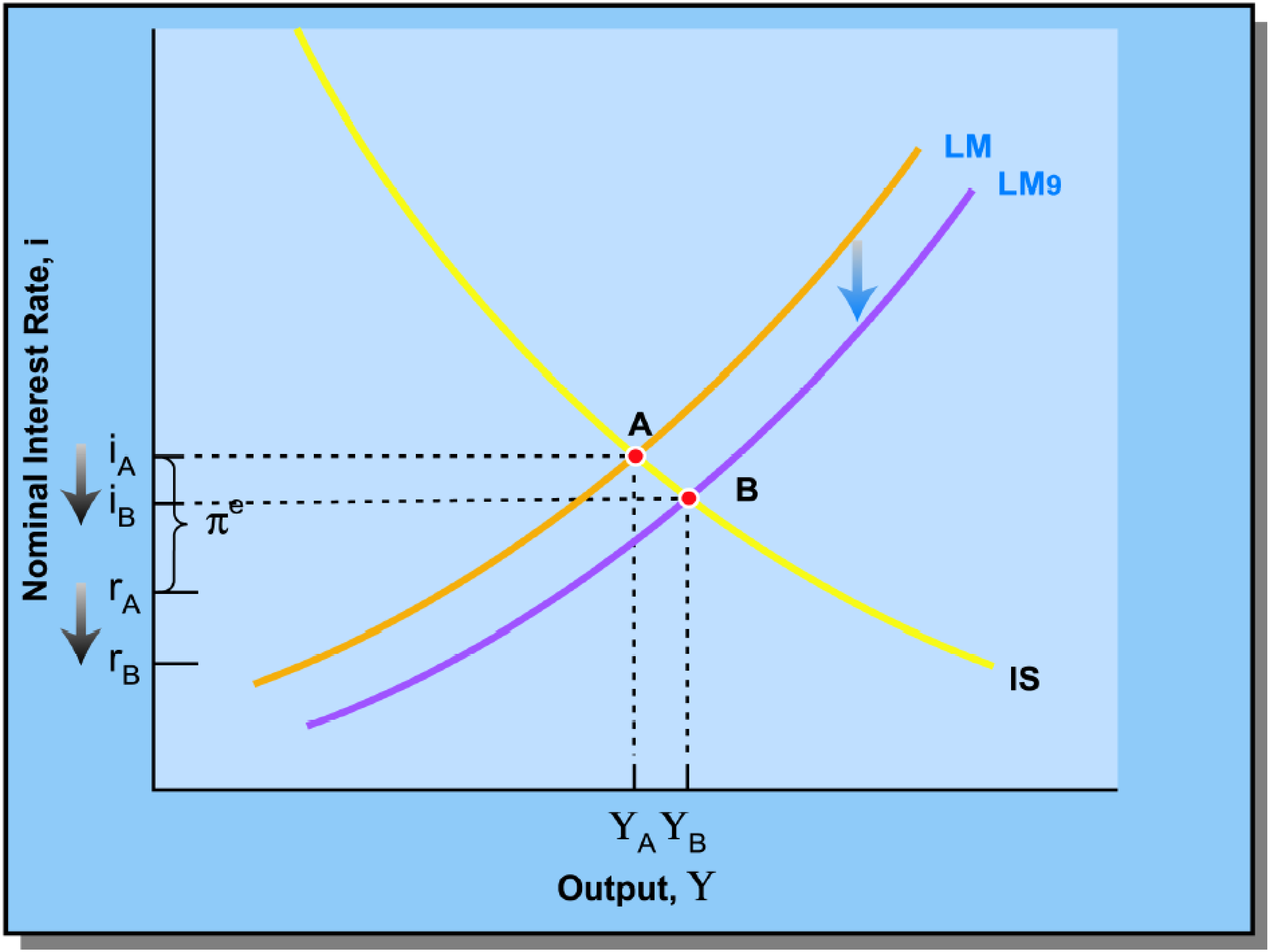
$$i \approx r_n + g_m - \overline{g}_y$$

but in the short term

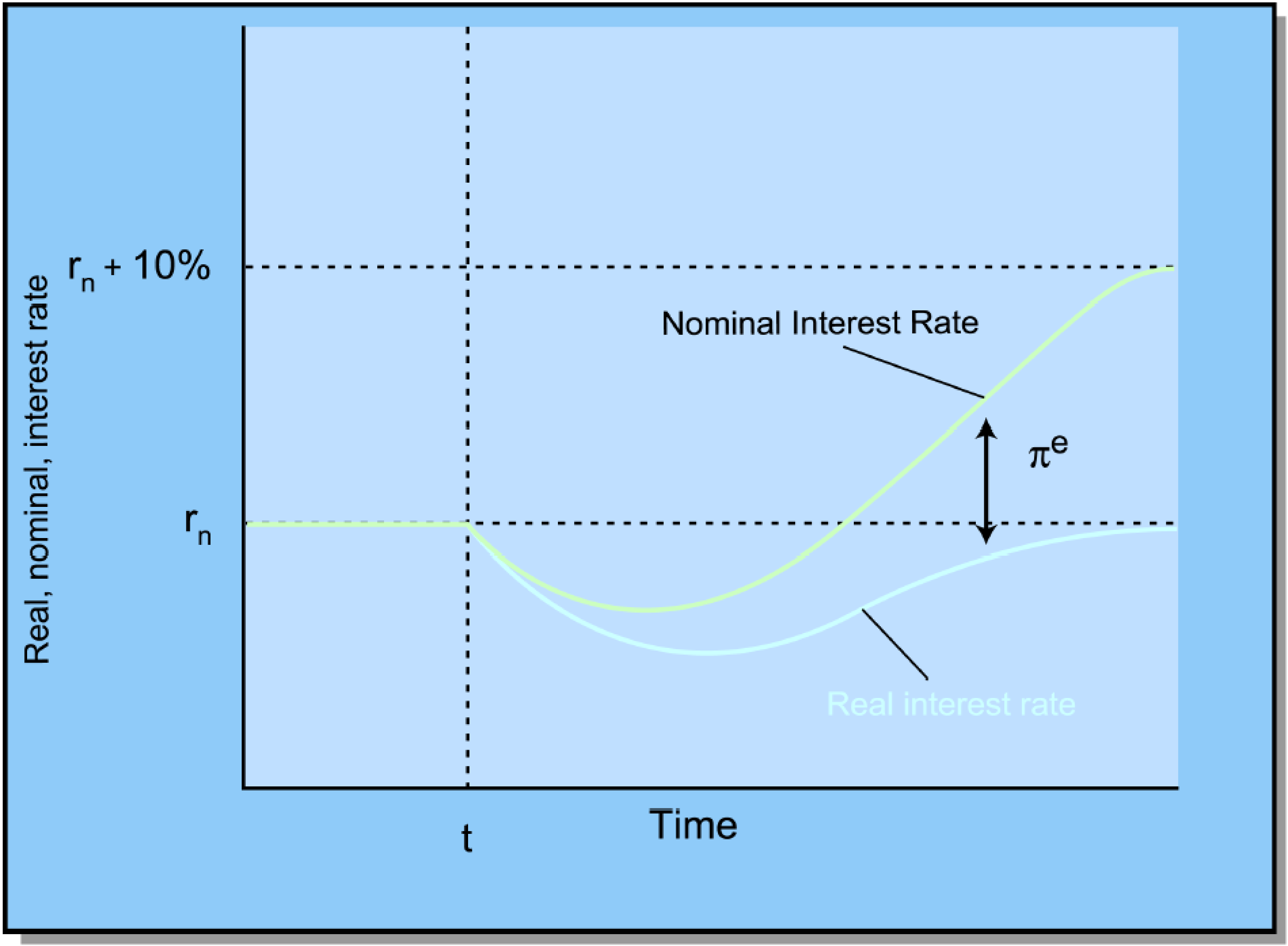
$$i \approx r + \pi^e$$



Nominal Interest Rate, i
LM
A
i_A
π^e
$r_A = i_A - \pi^e$
IS
$Y_A = Y_n$
Output, Y



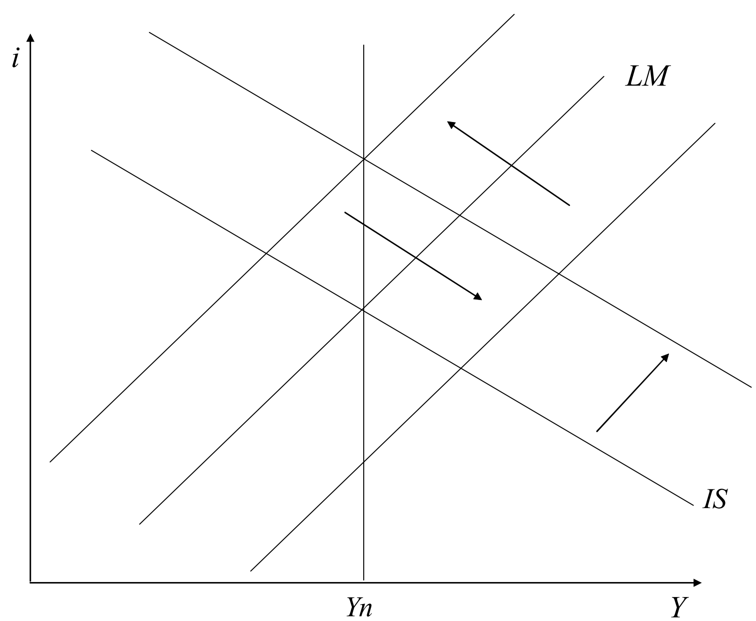
LM
LM9
A
B
IS
Nominal Interest Rate, i
i_A
i_B
π^e
r_A
r_B
Y_A Y_B
Output, Y



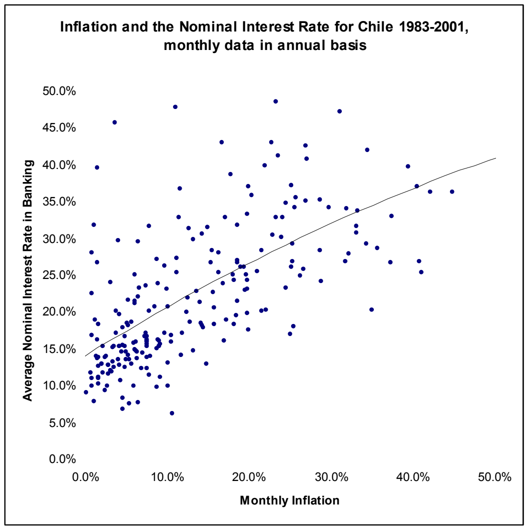
Real, nominal, interest rate
$r_n + 10\%$
r_n
Nominal Interest Rate
π^e
Real interest rate
t
Time

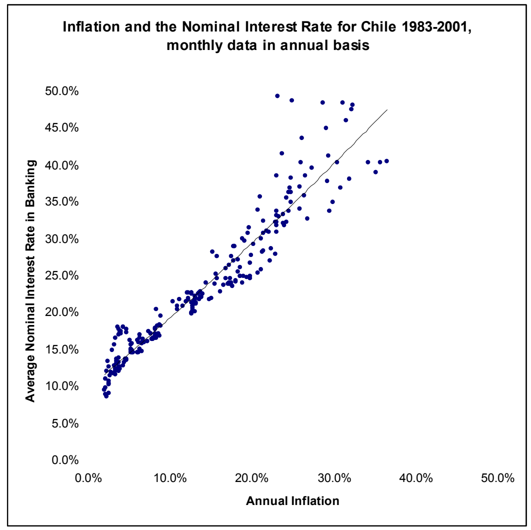
If the central bank increases the growth rate of money it will surprise some agents. This means that both the nominal and the real interest rate will fall. If the real interest rate falls, unemployment will fall bellow its natural rate.

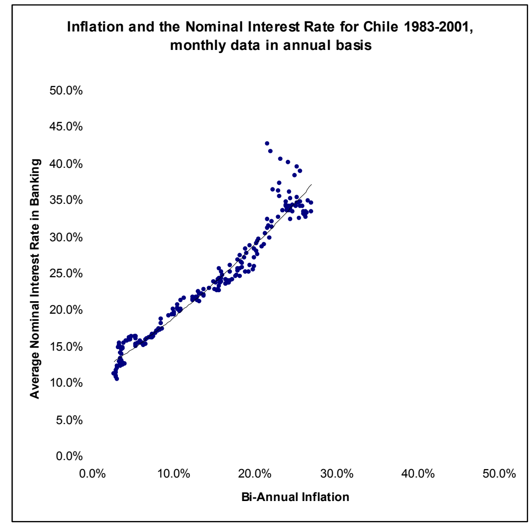
2. As long as some agents can react by adjusting inflation expectations, real interest rates will have to fall more than the nominal interest rate.
3. When the economy adapts its inflation expectations to the new reality it ends up with higher nominal interest rates, the same real interest rate and higher inflation.



The hypothesis that in the long term, the nominal interest rate increases with the inflation rate is known as the FISHER EFFECT.







So, there seems to be evidence that the Fisher effect holds in the medium term, but that there are some months in which expectations can be cheated.

Now lets rethink Farmer Brown's decision to borrow in order to activate his land. Lets suppose that Farmer Brown actually has the seeds to start production.

We must think of what are the actual choices of Farmer Brown. Lets suppose he has two choices. He can invest in his land or sell it, lend the money to the bank for an interest rate each period and use the proceeds to consume.

	If he lends $1 to the bank in 1901 he will get	So getting a $1 in year …. will cost in 1901
In 1902	$(1+i_{02})$	$(1+i_{02})^{-1}$
In 1903	$(1+i_{02})(1+i_{03})$	$(1+i_{02})^{-1}(1+i_{03})^{-1}$
In 1904	$(1+i_{02})(1+i_{03})(1+i_{04})$	$(1+i_{02})^{-1}(1+i_{03})^{-1}(1+i_{04})^{-1}$

So whatever return he gets in any given year hust be deflated by the SACRIFICE he has made to get it. His returns were R_t-I_t, every year, if he didn't have to take the loan.

	If he decides not to sell his land and to produce he will get	Which is worth in 1901
In 1902	R_{02}-I	$(R_{02}-I)/\ (1+i_{02})^{-1}$
In 1903	R_{03}-I	$(R_{03}-I)/\ (1+i_{02})^{-1}(1+i_{03})^{-1}$
In 1904	R_{04}-I	$(R_{04}-I)/\ (1+i_{02})^{-1}(1+i_{03})^{-1}(1+i_{04})^{-1}$

Now if we add up all of the flows valued at what it would cost to produce them if we gave the money to the bank we have PRESENT DISCOUNTED VALUE (PDV). In particular the PDV for a project that generates a return flow $\{x_t | t=0\ldots..n\}$ and requires an investment of K in the first period, is

$$PDV = x_0 + \sum_{t=1}^{n} \frac{x_t}{\prod_{s=1}^{t}(1+i_s)} - K$$

Since flows and interest rates are actually expected rather than know, usually we deal with EXPECTED PDVs

USEFUL STUFF 1:

If interest rates were constant (if the bank offered n years of a certain nominal return) and the return is the same every year: x_t=x.

$$PDV = \sum_{t=0}^{n} \frac{x}{(1+i)^t} - K$$

$$PDV = \sum_{t=0}^{\infty} \frac{x}{(1+i)^t} - \sum_{t=n+1}^{\infty} \frac{x}{(1+i)^t} - K$$

since 1/(1+i) is lower than 1.

$$PDV = \sum_{t=0}^{\infty} \frac{x}{(1+i)^t} \left(1 - \frac{1}{(1+i)^{n+1}}\right) - K$$

$$PDV = \frac{x\left(1 - \frac{1}{(1+i)^{n+1}}\right)}{\left(1 - \frac{1}{(1+i)}\right)} - K = x\left(\frac{(1+i)^{n+1} - 1}{i(1+i)^n}\right) - K$$

USEFUL STUFF 2:

Suppose the flow was infinite (consol, where K would be its price), then

$$PDV = \sum_{t=0}^{\infty} \frac{x}{(1+i)^t} - K$$

$$PDV = x + \sum_{t=1}^{\infty} \frac{x}{(1+i)^t} - K$$

$$PDV = x + \left(\frac{1}{(1+i)}\right)\left(\sum_{t=0}^{\infty} \frac{x}{(1+i)^t}\right) - K$$

$$PDV = x + x\left(\frac{1}{(1+i)}\right)\left(\frac{1}{1-\frac{1}{(1+i)}}\right) - K$$

$$PDV = x + \frac{x}{i} - K$$

so that x/i is the PV of a perpetuity that starts paying next period.

We know that the price of a one period bond is

$$P_{1,t} = \frac{\$1}{(1+i_{1,t})}$$

this is an example of a present value.

Now consider a two period bond. If you hold it for a year it becomes a one period bond. In the second period it will return the expected price of a one period bond in the future. Since it costs the price of a two period bond right now. Its return is

$$\frac{P^e_{1,t+1}}{P_{2,t}}$$

now in the first period, the market will make the return to holding a two period bond for one period, equal to the return of holding a one period bond for one period. The fact that the market will always have individuals moving money to make this happen is called ARBITRAGE. To arbitrage prices is to exploit differences in prices to make a profit. Arbitrageurs (middle-men) are a fundamental part of the well-functioning of markets. They also cause problems when they gain monopolistic power or when they become involved in bubbles, panics and manias.

So we have that by arbitrage,

$$1+i_{1,t}=\frac{P^e_{1,t+1}}{P_{2,t}} \quad \text{hence} \quad P_{2,t}=\frac{P^e_{1,t+1}}{1+i_{1,t}}$$

the value of a two-period bond is the PV of a one period bond held in the next period. So that,

$$P_{2,t}=\frac{\$1}{\left(1+i^e_{1,t+1}\right)\left(1+i_{1,t}\right)}$$

when we were calculating NPV for flows that had many periods we were multiplying each periods flow times the present price of money in the future. Remember that $P_{2,t}$ is the price today of a dollar in two periods in the future. Our PDV formula was

$$PDV=x_0+\sum_{t=1}^{n}\frac{x_t}{\prod_{s=1}^{t}\left(1+i_s\right)}-K \quad \text{but could be} \quad PDV=x_0+\sum_{t=1}^{n}x_tP_{t,0}-K$$

which in the end is

$$PDV=\{\text{Flow of money today}\}+\sum_{t=1}^{n}\{\text{Flow of money in period t}\}\{\text{Price today of money in period t}\}-\text{Cost today}$$

The period yield of a two period bond can be calculated considering that

$$\frac{\$1}{\left(1+i_{2,t}\right)^2} = P_{2,t} = \frac{\$1}{\left(1+i^e_{1,t+1}\right)\left(1+i_{1,t}\right)}$$

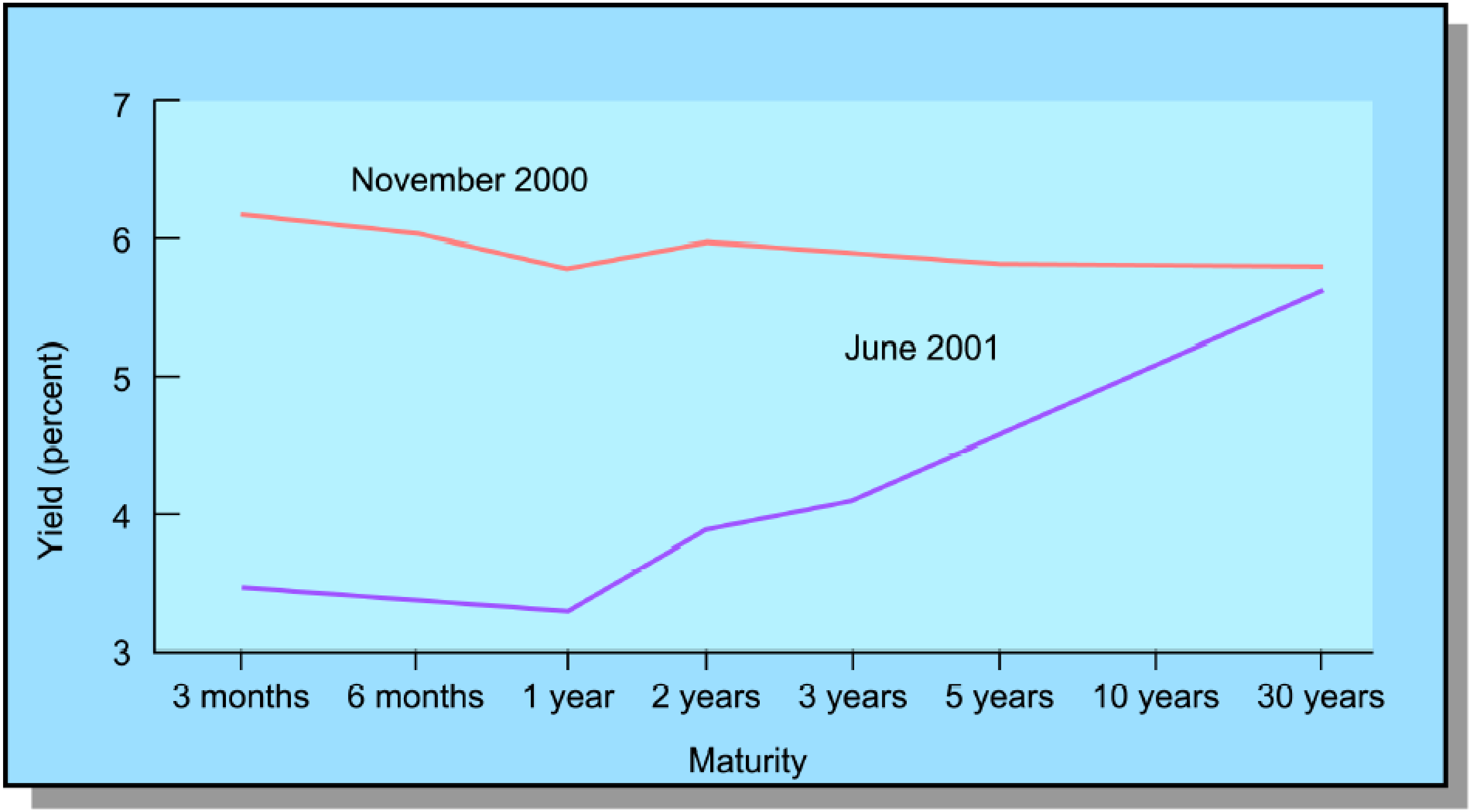
the question being asked is: What is the annual return I am getting on this two period bond.

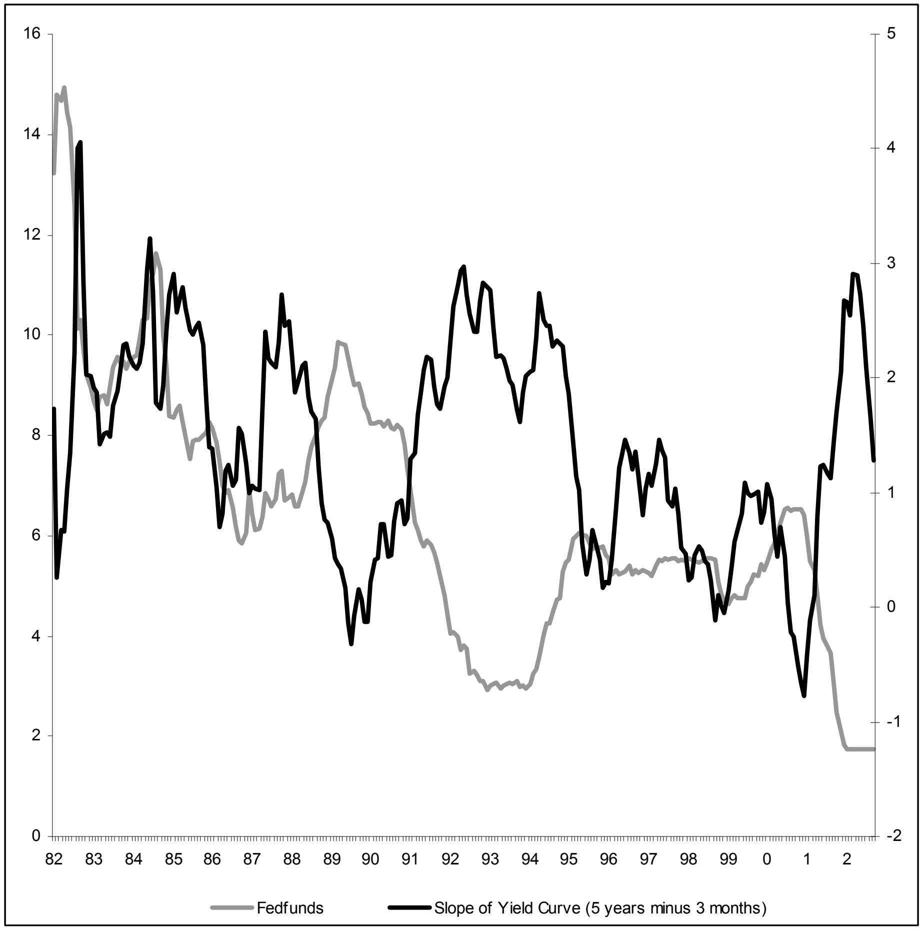
$$\left(1+i_{2,t}\right)^2 = \left(1+i^e_{1,t+1}\right)\left(1+i_{1,t}\right)$$

or approximately (when interest rates are low)

$$i_{2,t} \approx \frac{\left(i_{1,t} + i^e_{1,t+1}\right)}{2}$$

An ordering of the yields of different maturities of bonds is called the YIELD CURVE


7
6
5
4
3
Yield (percent)
November 2000
June 2001
3 months
6 months
1 year
2 years
3 years
5 years
10 years
30 years
Maturity



1. Notice the anticipation of rate cuts in 89 and 00. Notice anticipation of increases in 91-94.
2. Expectations on inflation as well as on the real interest rate.
3. Mostly there is a positive slope. There is a risk premium in the future of about 1%.

Another way in which firms raise money is through stocks. Stocks are participations in the property of a firm. They are different in at least two fundamental ways from stocks.

1. Holding a stock means bearing the risk of the business much more directly than for a bond-holder. This implies that there are participation rights for stock-holders. Bond holder only have participation when there is bankruptcy.
2. Stocks pay dividends which are decided by the firm. They come out of profits, when profits are not payed out they are called RETAINED PROFITS.

The price of a stock will be the PV of its dividends:

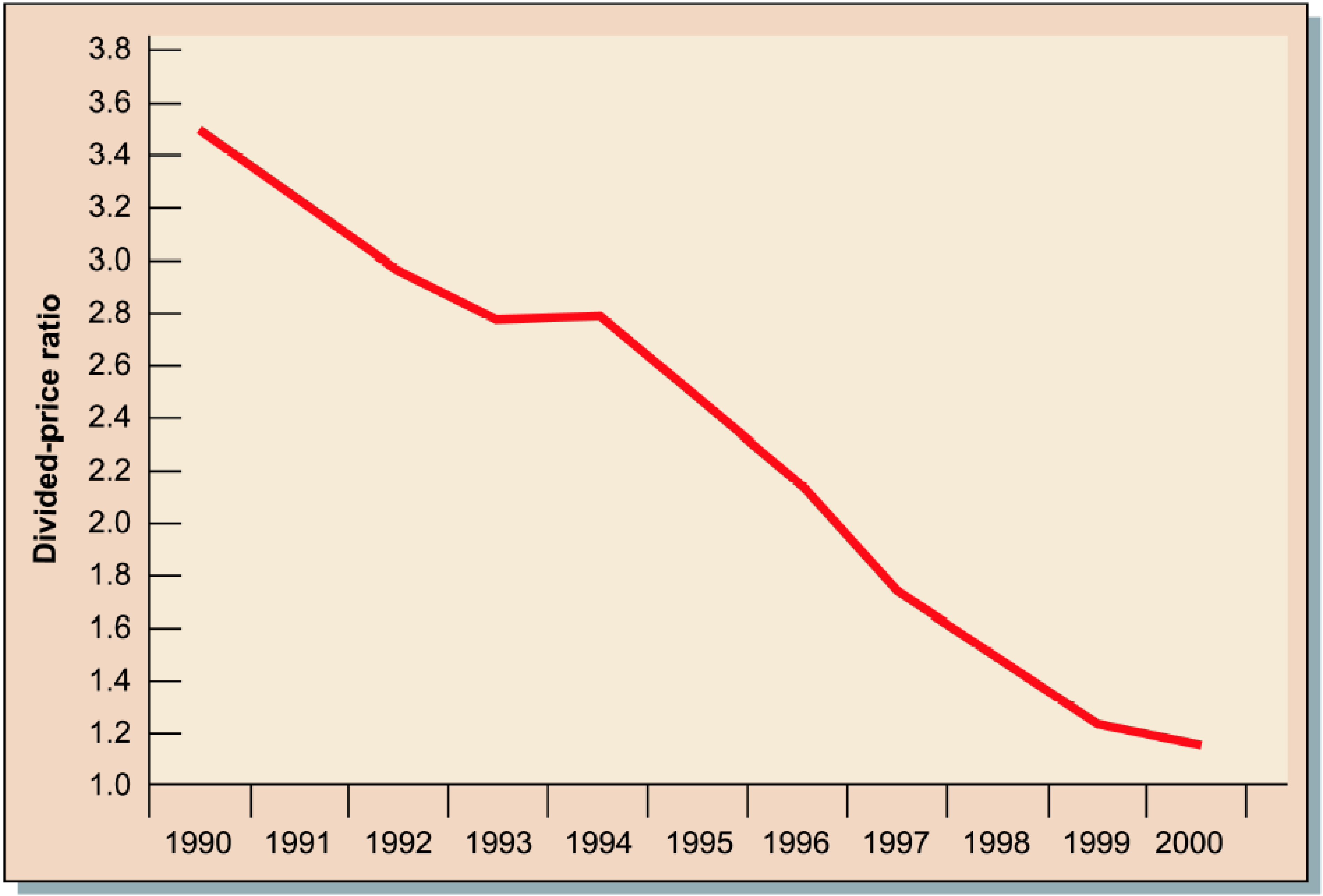
$$Q_t = \sum_{t=1}^{\infty} \frac{D_t}{\prod_{s=1}^{t} (1 + i_s)}$$

So, in theory the price of stocks should be a good signal of how a company is doing (what the outlook for it should be). Since the people who hold the stocks have a right and interest to participate in the firm and be informed, the price should reflect the information. This mechanism does not always work so well. Stockholders are not always as well informed as we would expect and there are individuals with an incentive and position for speculating and manipulating the price by distorting information.

A common way of looking at how expensive or cheap stocks are is the PRICE/DIVIDEND ratio. Imagine a stock that produces a flow of dividends during its history. The price dividend ratio is

$$PD_t = \frac{\displaystyle\sum_{t=1}^{\infty} \frac{D_t}{\prod_{s=1}^{t}(1+i_s)}}{\displaystyle\sum_{t=-1}^{-T} \frac{D_t}{\prod_{s=1}^{t}(1+i_s)}}$$

where T is some period toward the past that we choose to calculate the dividends. There is always the argument against this measure that if a company has not produced in the past it will in the future because it is investing at the beginning (that was the argument arround most dotcoms). So we must choose a sufficiently long T that it is a good representation of how the firms usually behave in the economy.

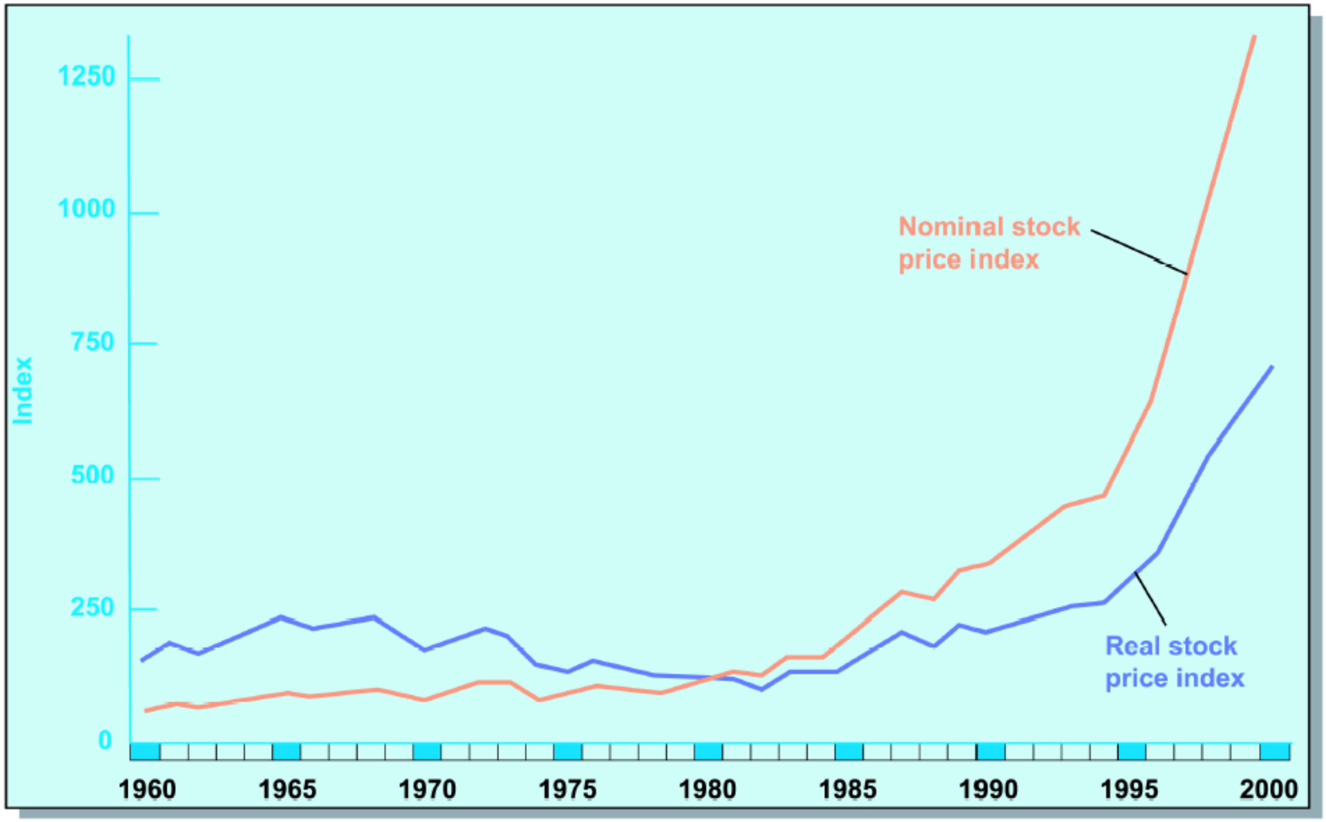


Divided-price ratio
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000


20-year rate of real stock price (percent)
200
150
100
50
0
-50
-100
2
4
6
8
10
Dividend-price ratio (percent)

GENERAL POINTS ON STOCKS:

1. They are very hard to predict. There are many speculators trying to predict and arbitrage so it is hard to anticipate something that the market has not priced in already.
2. In the long term they are more profitable than stocks.



3. It is better for the uninformed investor to buy indexes than to speculate….alas!

4. **An unanticipated** monetary expansion should increase stock prices because it produces a fall in interest rates and an increase in potential dividends.

5. A shock to production (consumer optimism) increases dividends but also increases interest rates. It can also anticipate some anti-inflationary measure by the central bank.