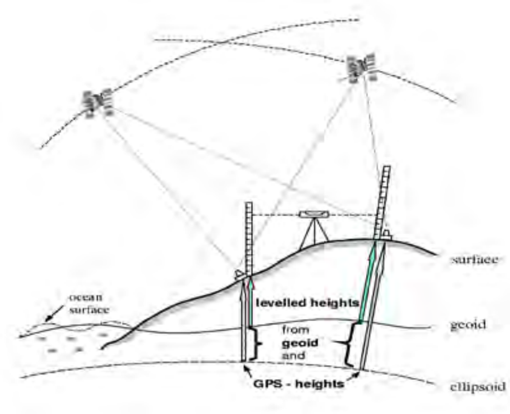
Airborne Gravimetry for Geodesy summer school

DTU Space National Space Institute

Geoid modelling with GRAVSOFT

Rene Forsberg, DTU-Space, Denmark

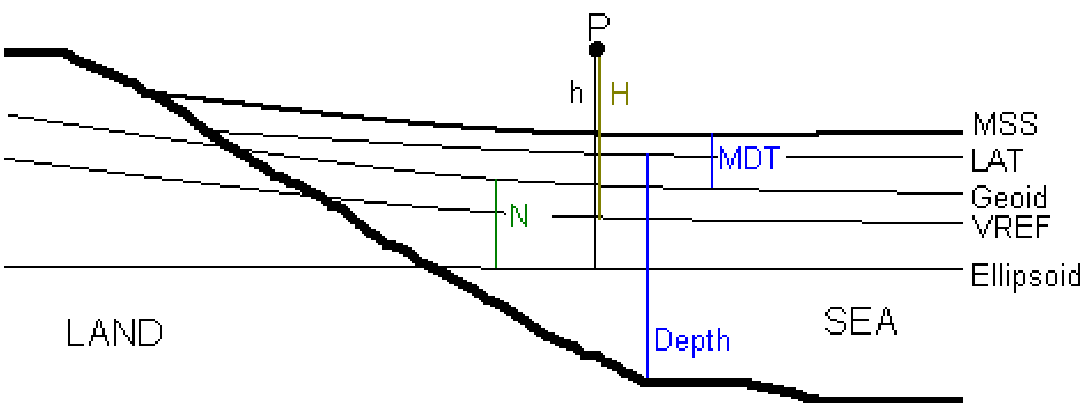
The geoid use



Heights from GPS:

$H = h^{ellipsoidal} - N$

The 1 cm-geoid is within reach in countries with good gravity coverage or for special projects like large bridges ..



Marine areas:

$MDT = MSS - N$

MSS: mean sea surface (Altimetry or GPS)
Several "geoids" in practical use, need for improved datums

Geoid determination

Anomalous potential T = $W_{physical} - U_{normal}$

$$N = \frac{T}{\gamma}(on\, geoid);$$

The anomalous gravity potential *T* is split into 3 parts:

$$T = T_{EGM} + T_{RTM} + T_{res}$$

T_{EGM} – Global spherical harmonic model (EGM08/GOCE to degree 360 or 2190)

T_{RTM} – residual terrain effect (RTM) .. Computed by *prism integration*

T_{res} – residual (i.e. unmodelled) local gravity effect

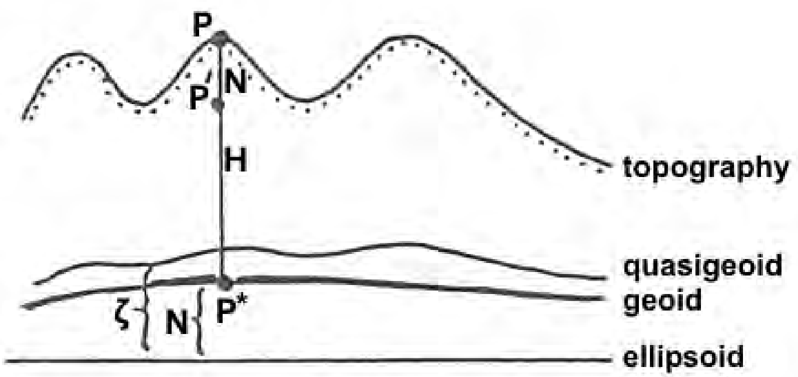
Principle used much in gravimetric geoid determination: “remove-restore”

Stokes function usually implemented by
Fast Fourier Techniques and/or least-squares collocation

Geoid and quasigeoid

Non-level surface => Molodenskys formula: ζ is quasi-geoid

$$\zeta = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g + g_1) S(\psi)\, d\sigma$$



Definition of gravity anomaly:
Refers to surface of topography!

$$\Delta g_P = g_P^{observed} - \gamma_{P'} \approx g_P^{observed} - \gamma_o + \frac{\partial \gamma}{\partial h} H$$

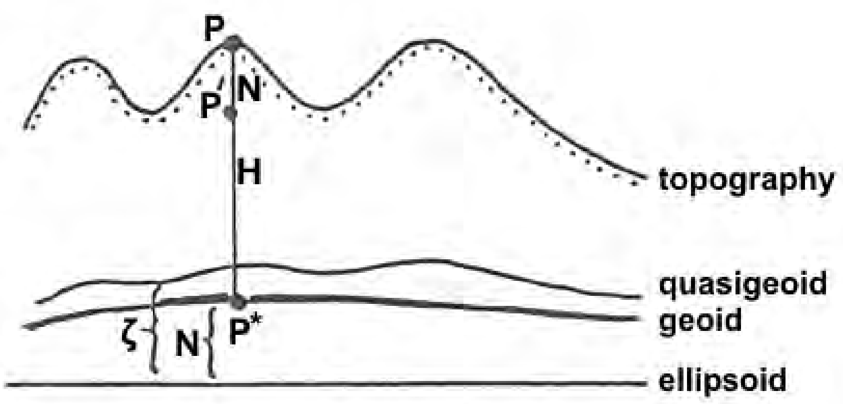
Height systems

Normal and orthometric (Helmert) heights <-> quasigeoid and geoid

$$H_{Helmert} = \frac{C}{g} \approx \frac{C}{g_P + 0.0424[mgal/m]H}$$

$$H^* = \frac{C}{\gamma} \approx \frac{C}{\gamma_o - 0.1543[mgal/m]\, H^*}$$

$$\zeta - N = H_P - H_P^* \approx -\frac{g_P - \gamma_o + 0.1967[mgal/m]H}{\gamma_o} \cdot H \approx -\frac{\Delta g_B}{\gamma_o} H$$



Normal heights: quasigeoid
Orthometric heights: geoid

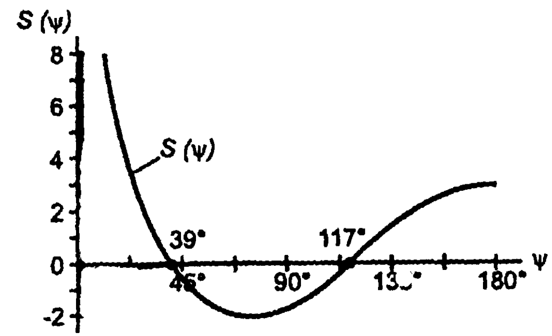
Theoretical problem: Density of Earth must be known to define geoid
- *"New Theory" approx. 1960 by Molodensky*

Integral Formulas – space domain

Stokes Formula

- Relating N with gravity observations.

$$T = \gamma N = \frac{R}{4\pi} \iint_{\sigma} \Delta g S(\psi) d\sigma$$



- Stokes Kernel

$$S(\psi) = \sum_{2}^{\infty} \frac{2l+1}{l-1} w_1 P_l(\cos\psi) =$$

$$\frac{1}{\sin(\psi/2)} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi \ln(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2})$$

$$\sin^2\frac{\psi}{2} = \sin^2\frac{(\varphi_p - \varphi)}{2} + \sin^2\frac{(\lambda_p - \lambda)}{2}\cos\varphi_p \cos\varphi$$

- w is weight function … used to limit influence of low harmonics

Fourier transf. and Stokes integral

- Stokes integral can conveniently be evaluated using FFT methods (Strang van Hess, 1990).

$$N = \frac{R}{4\gamma\pi} \iint_{\sigma} \Delta g(\varphi, \lambda) S(\varphi_p, \lambda_p, \varphi, \lambda) \cos\varphi d\varphi d\lambda$$

- This is convolution form if cosφ is considered constant ("simple spherical FFT") and the sin-formula is used for ψ

…

$$\sin^2 \frac{\psi}{2} = \sin^2 \frac{(\varphi_p - \varphi)}{2} + \sin^2 \frac{(\lambda_p - \lambda)}{2} \cos\varphi_p \cos\varphi$$

- Stokes formula in planar approximation gives:

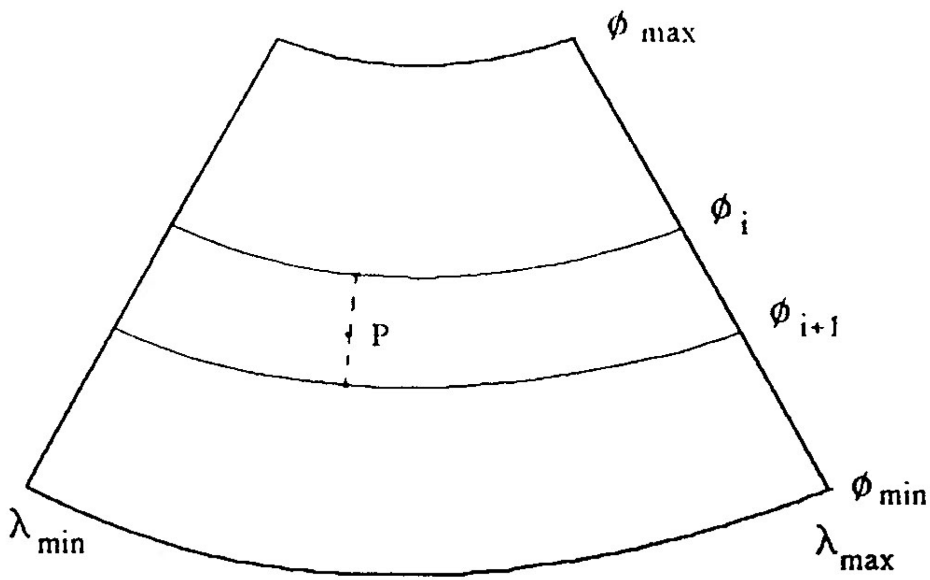
$$N(x_p, y_p) = \frac{1}{2\pi\gamma} \iint_A \frac{\Delta g(x,y)}{\sqrt{(x_p - x)^2 + (y_p - y)}} dxdy = \frac{1}{2\pi\gamma} \Delta g(x_p, y_p) * \frac{1}{s}(x_p, y_p)$$

$$N(x_p, y_p) = \frac{1}{2\pi\gamma} F^{-1}[F(\Delta g(x_p, y_p))) F(\frac{1}{s}(x_p, y_p))]$$

Improved method: bandwise FFT

$$\sin^2\frac{\psi}{2} \approx \sin^2\frac{\varphi_p-\varphi}{2} + \sin^2\frac{\lambda_p-\lambda}{2}\cos\overline{\varphi}_\iota\cos[\overline{\varphi}_\iota-(\overline{\varphi}_\iota-\varphi)]$$

$$\approx \sin^2\frac{\varphi_p-\varphi}{2} + \sin^2\frac{\lambda_p-\lambda}{2}[\cos^2\overline{\varphi}_\iota\cos(\overline{\varphi}_\iota-\varphi)+\cos\overline{\varphi}_\iota\sin\overline{\varphi}_\iota\sin(\overline{\varphi}_\iota-\varphi)]$$



Method is exact along borders between bands, linear interpolation in between (Forsberg and Sideris, 1987)

1-D FFT – Van Hees, 1990

Rigorous Spherical Kernel

$$N(\varphi_l,\lambda_k) = \frac{R}{4\pi\gamma}\sum_{j=0}^{N-1}\left[\sum_{i=0}^{M-1}\Delta g(\varphi_j,\lambda_i)\cos\varphi_j S(\varphi_l,\varphi_j,\lambda_k-\lambda_i)\Delta\lambda\right]\Delta\varphi, \qquad \varphi_l=\varphi_1,\varphi_2,\ldots,\varphi_N$$

Addition Theorem of DFT

$$N(\varphi_l,\lambda_k) = \frac{R}{4\pi\gamma}\boldsymbol{F_1^{-1}}\{\sum_{j=0}^{N-1}\boldsymbol{F_1}\{\Delta g(\varphi_j,\lambda_k)\cos\varphi_j\}\boldsymbol{F_1}\{S(\varphi_l,\varphi_j,\lambda_k\}\}, \qquad \varphi_l=\varphi_1,\varphi_2,\ldots,\varphi_N$$

The advantage of the 1D spherical FFT approach: it gives exactly the same results as those obtained by direct numerical integration; it only needs to deal with one one-dimensional complex array each time, resulting in a considerable saving in computer memory as compared to the 2D FFT technique discussed before .. But it is slower than bandwise FFT

Modified kernels needed

Takes into account the influence of possible (likely!) terrestrial+airborne biases

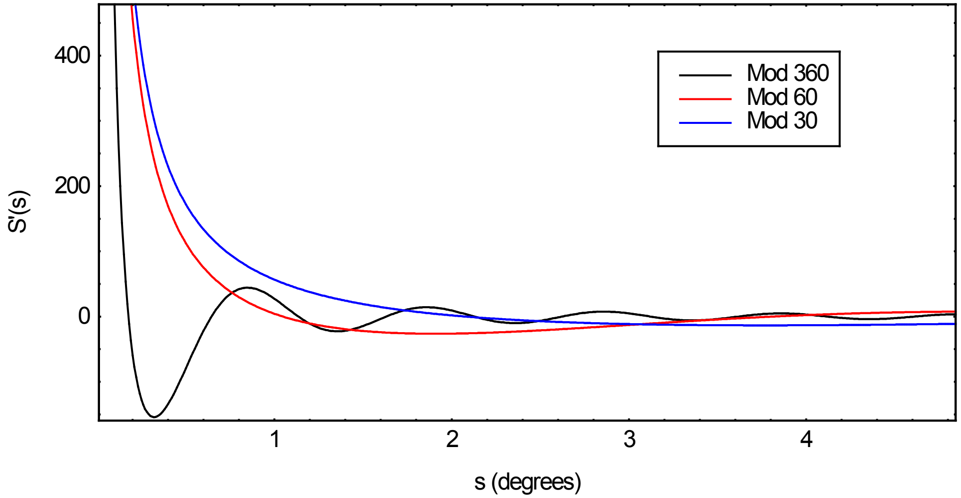
$$S_{\text{mod}}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n+1}{n-1} P_n \cos\psi$$

- where

$$\alpha(n) = \begin{cases} 1 & \text{for} \quad 2 \le n \le N_1 \\ \dfrac{N_2 - n}{N_2 - N_1} & \text{for} \quad N_1 \le n \le N_2, \qquad n = 2,\ldots,N \\ 0 & \text{for} \quad N \ge n > N_2 \end{cases}$$

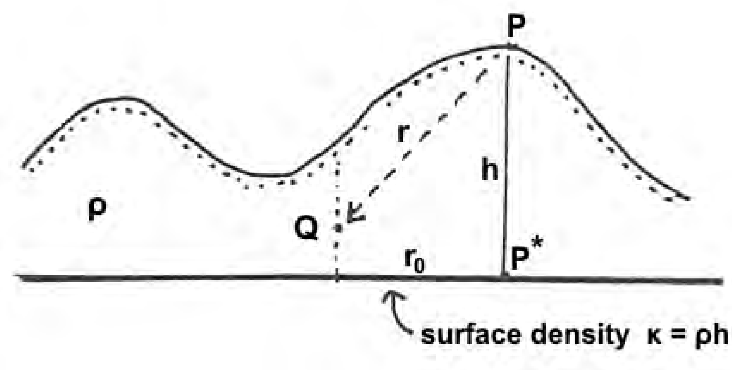
Choice of degree of modification for GOCE still an open problem! (requires good GPS-levelling)

Wong-Gore modified Stokes function



Helmert condensation methods

1. *Remove complete Bouguer effect* $\Delta g^{B} = \Delta g - (2\pi G \rho h - c)$
2. *Downward continue* $\Delta g^{B*} \approx \Delta g^{B} \left(assume\ \ T_{zz}^{B} = 0\right)$
3. *Restore condensed topography* $\Delta g^{*} = \Delta g + 2\pi G \rho h$



Terrain mass condensed into a surface layer

The outcome of this process is the Faye anomaly $(\Delta g + c)$, which is not smoothe "

Geoid formulas involve "indirect" or "terrain inflation" corrections … all approximative (also)

$$N = S\left(\Delta g + c - 2\frac{T_m}{\gamma r}\right) + \frac{T_m}{\gamma}$$

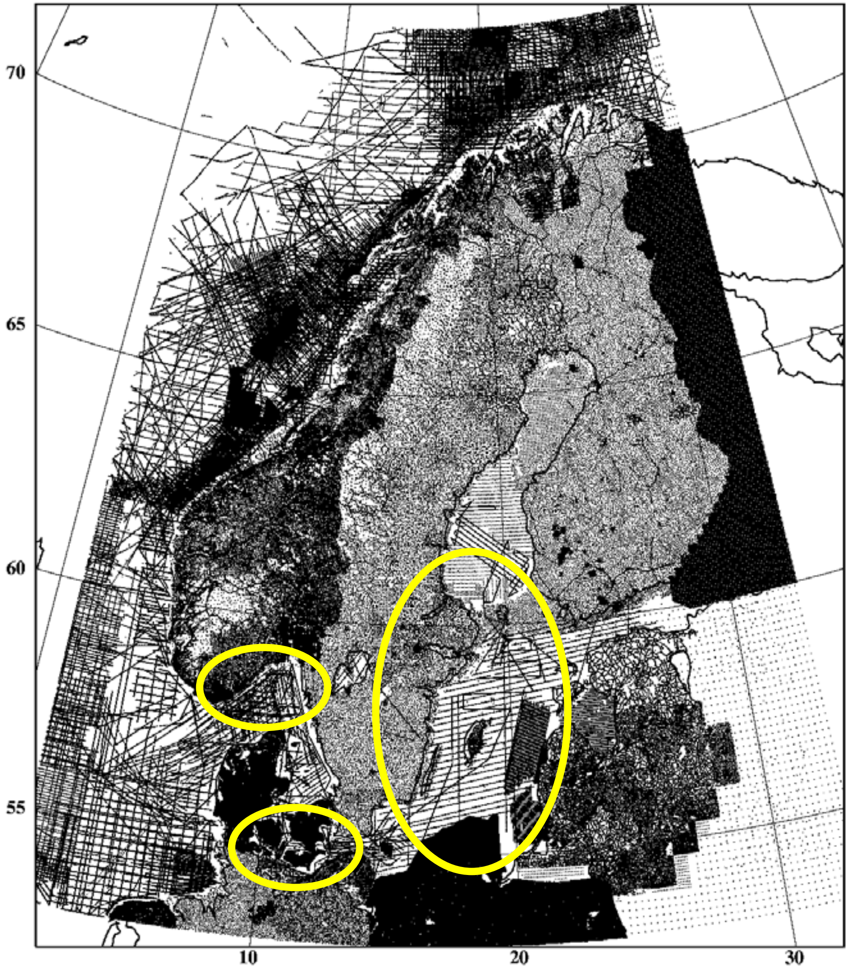
$$T_m(P^*) = G\rho \int_{-\infty}^{\infty}\int\int_{0}^{h} \frac{1}{r} dx\,dy\,dz - G\rho h \int_{-\infty}^{\infty}\int \frac{1}{r_0} dxdy \approx -\pi G \rho h_P^2$$

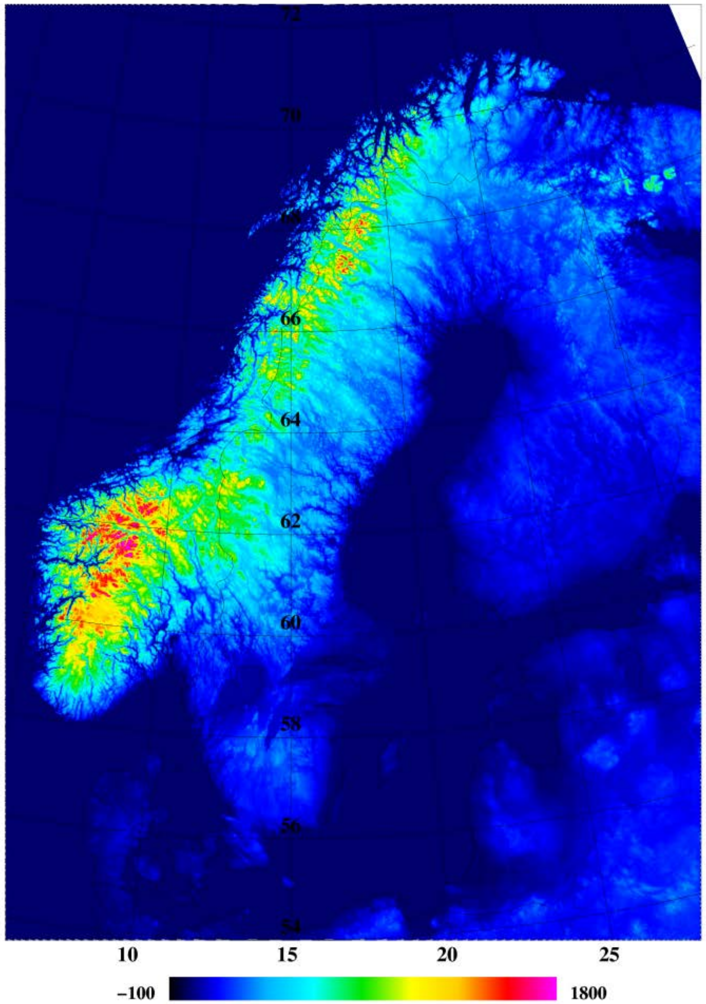
$$r = \sqrt{(x - x_P)^2 + (y - y_P)^2}, \quad r = \sqrt{r_0^2 + z^2}$$

Airborne Gravimetry for Geodesy summer school

Nordic geoid example

DTU Space National Space Institute





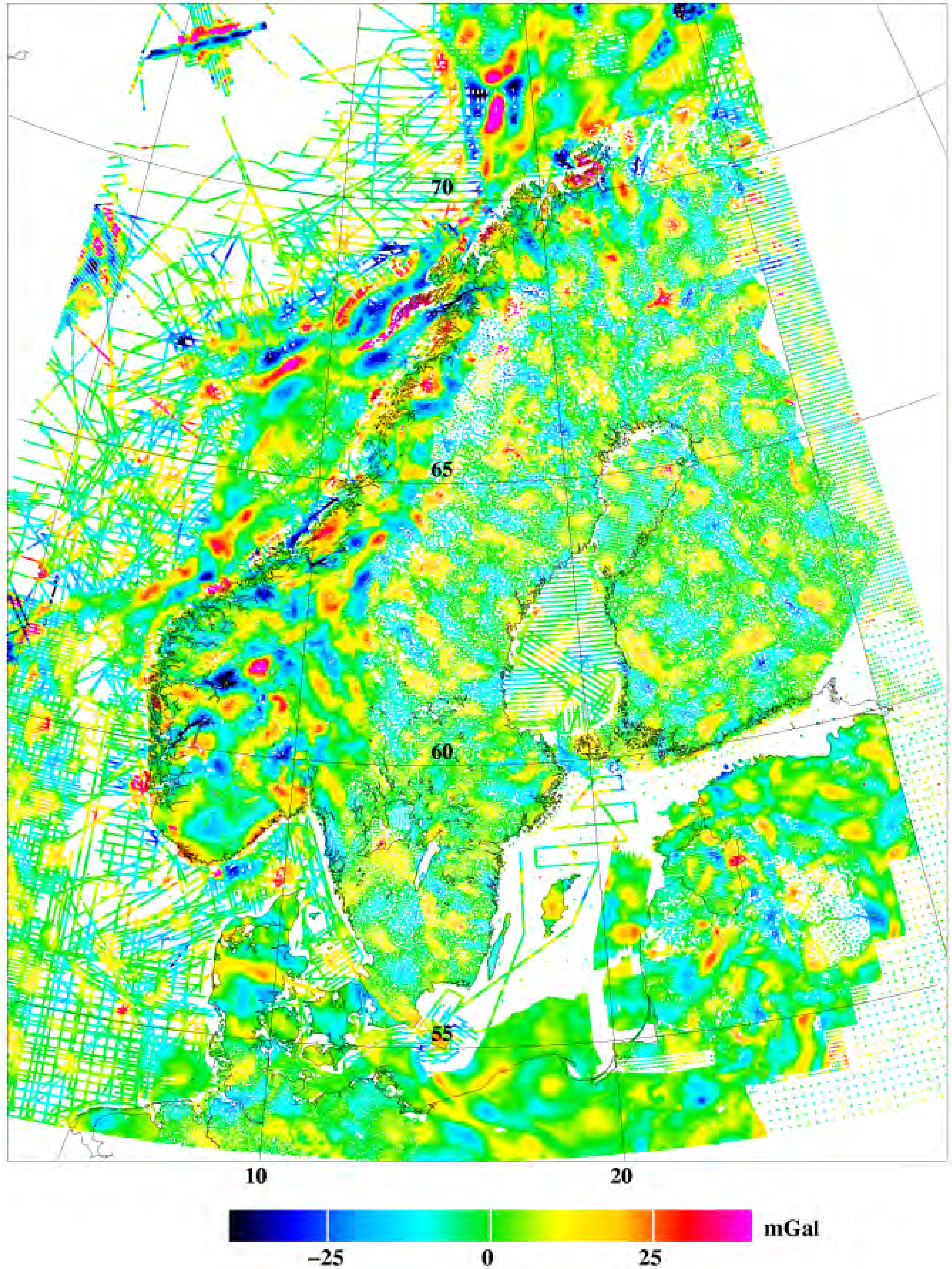
***NKG geoids**: Denmark,Sweden, Norway, Finland, Estonia, Latvia, Lithuania .. Joint geoid*
Next: NKG-2015 – goal: 1 cm geoid (2 cm in the mountains), airborne gravity fill-in (yellow)

Airborne Gravimetry for Geodesy summer school

Data reduction: GOCE R5+EGM08 composite model / terrain effects

GRAVSOFT:
HARMEXP, TC and *SPFOUR*

Free-air gravity Data	Land gravity (434925)	Airborne gravity (17238)
Original Mean	1.0	-8.7
Std.dev.	25.2	23.3
-**GOCE/EGM** Mean	-1.1	1.2
Std.dev.	16.0	15.4
-**RTM effect** Mean	0.3	1.3
Std.dev.	10.8	15.4

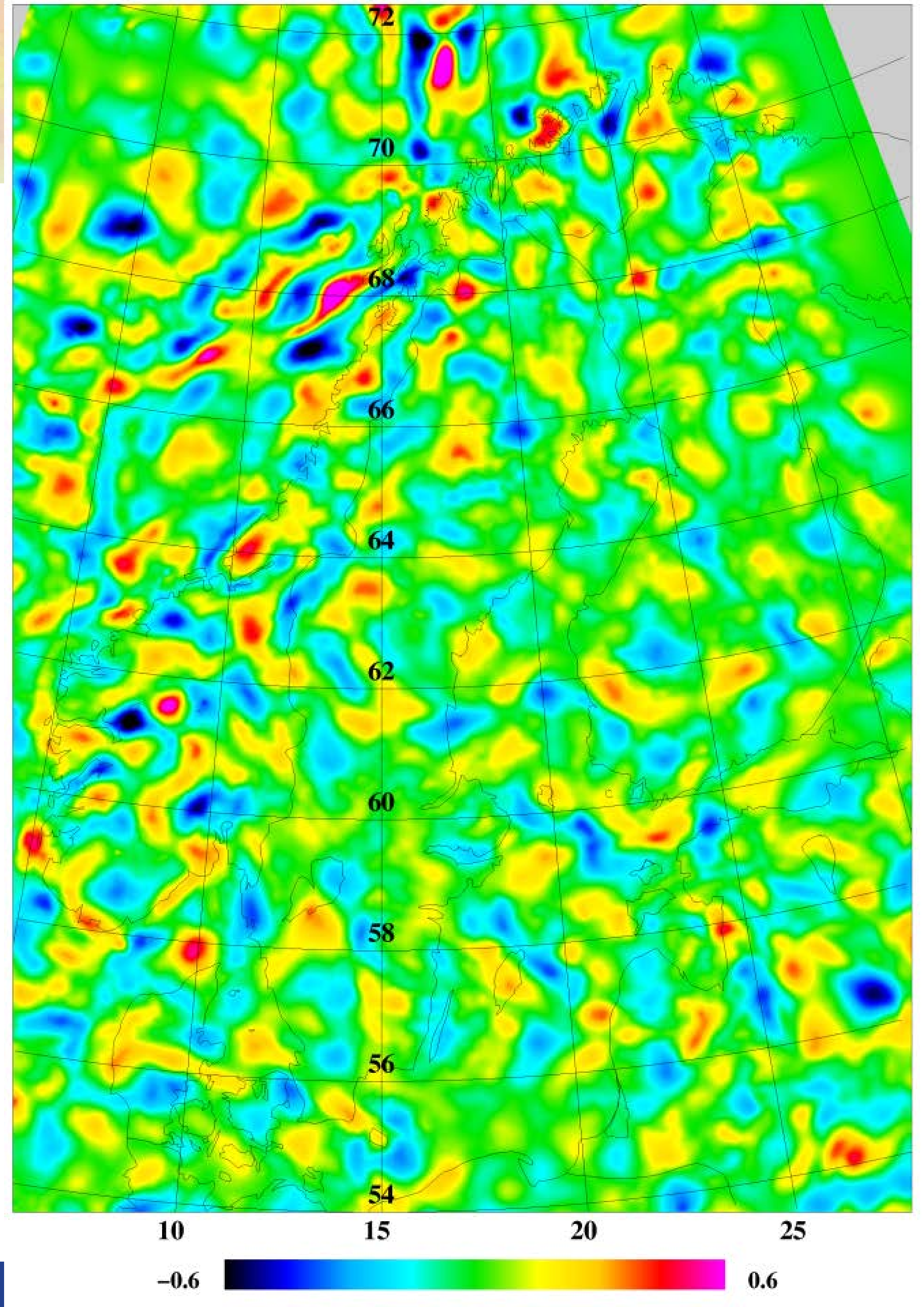


Airborne Gravimetry for Geodesy summer school

Fourier conversion to geoid
– *SPFOUR*

5 reference bands
in spherical FFT

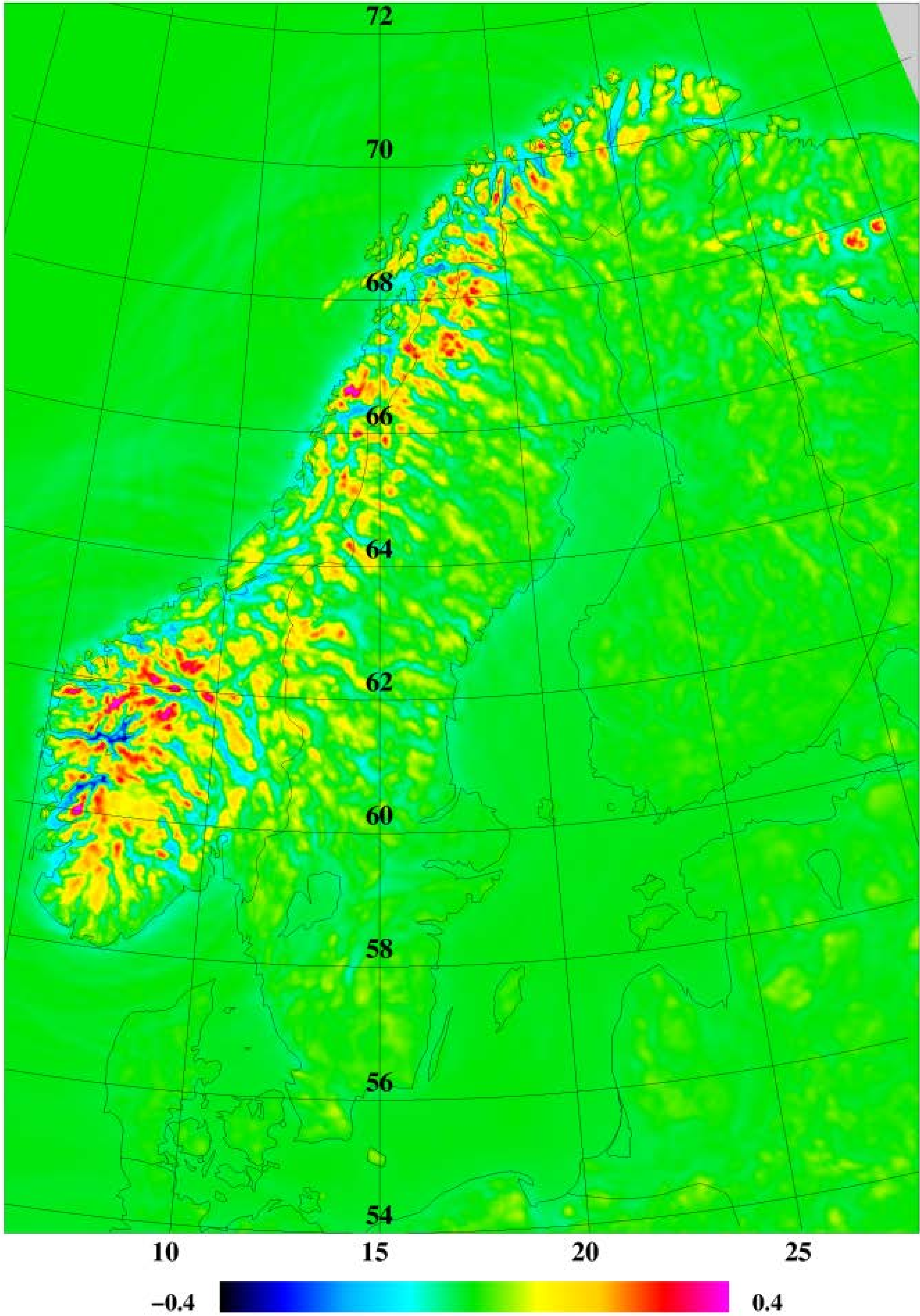
Various Stokes modifications
(here deg 180-190)



Airborne Gravimetry for Geodesy summer school

Terrain effects

Computed by FFT



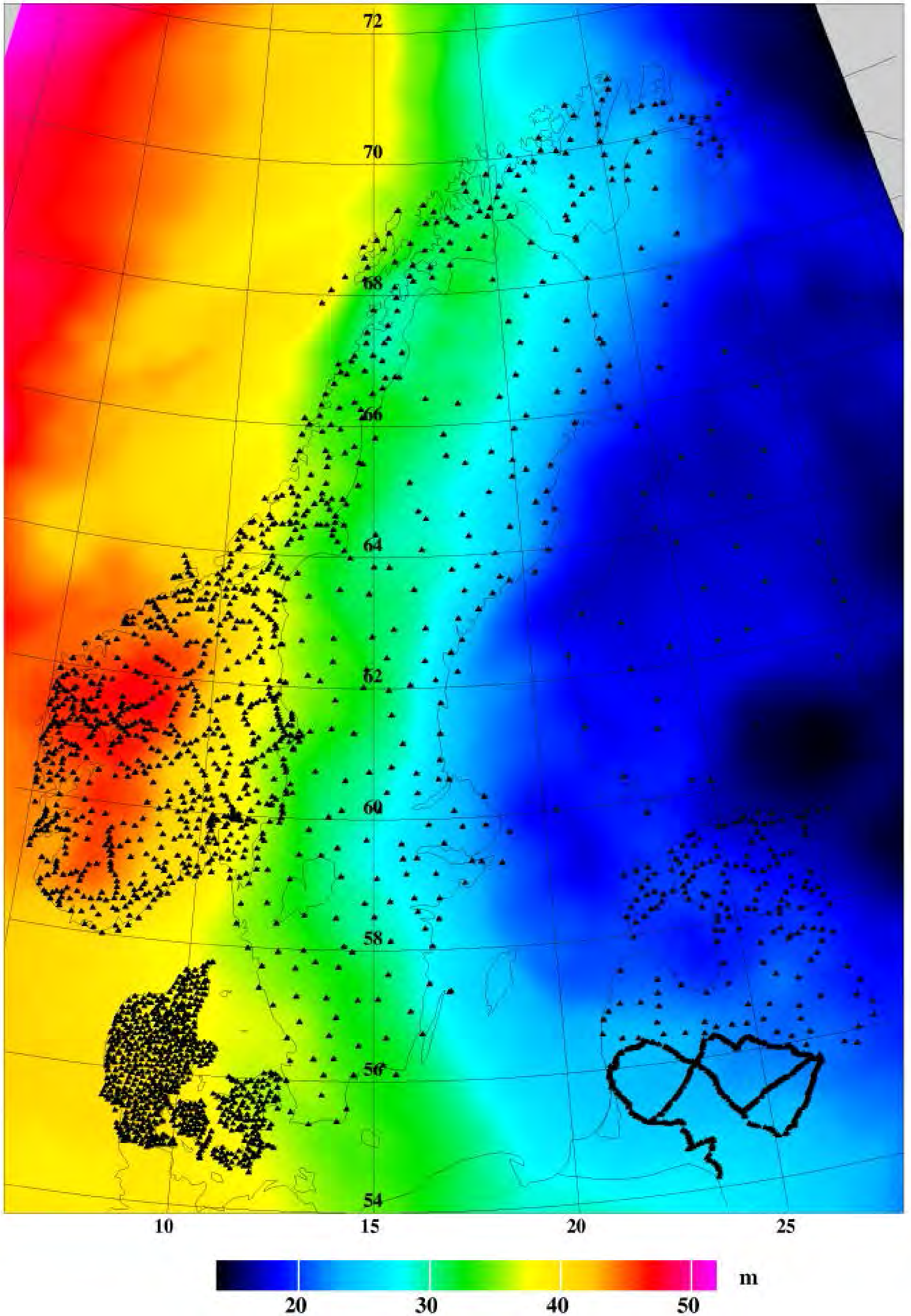
Airborne Gravimetry for Geodesy summer school

Comparison of final geoid to GPS levelling

National GPS/levelling networks

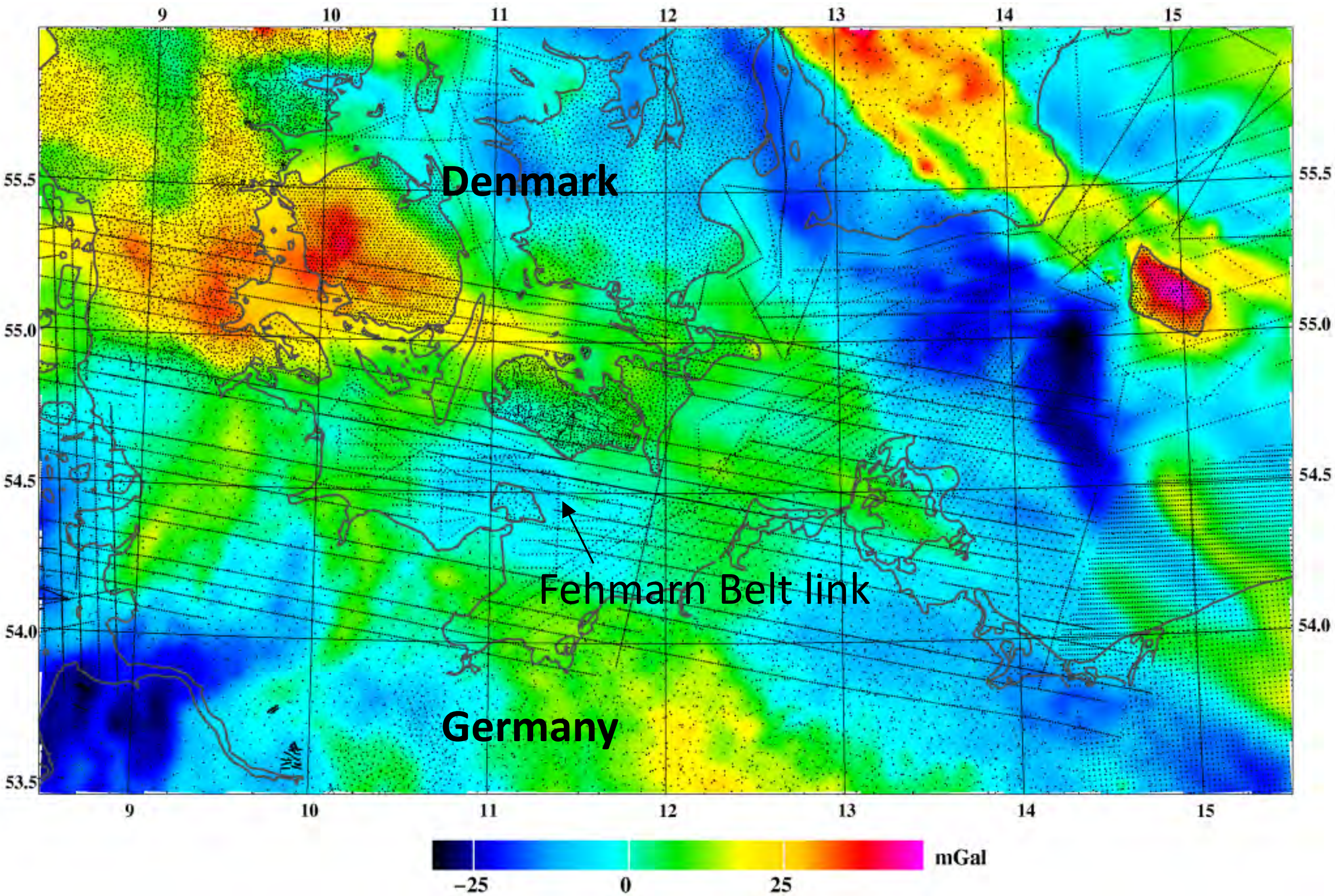
Comp for different kernel modifications
No clear answer what is best!

	mean	stddev	mean	stddev
DK:	0.209	0.025	0.222	0.026
SE:	0.249	0.030	0.245	0.028
NO:	0.190	0.042	0.196	0.052
FI:	0.241	0.017	0.238	0.025
EST:	0.243	0.016	0.247	0.022
LAT:	0.215	0.026	0.224	0.033
LITH:	0.309	0.028	0.315	0.036
	(mod 95-105)		**(mod 180-190)**	



Gravity anomalies – Fehmarn Belt tunnel project

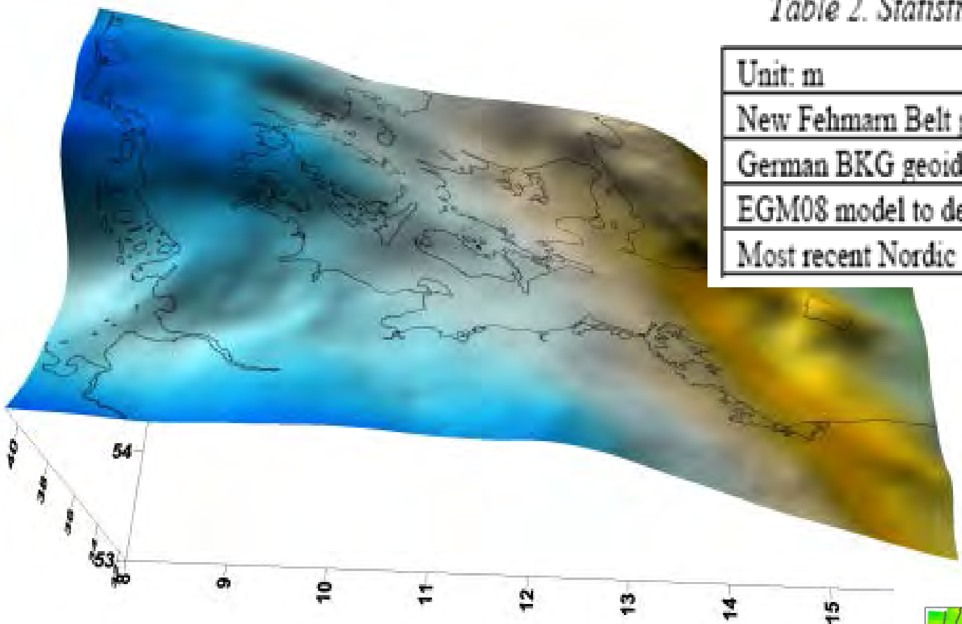
Land, marine and airborne (DTU/BKG - COWI acft)



Geoid connections across Fehmarn Belt

Table 2. Statistics of fit to REFDK/DVR90 geoid heights (66 points in southern Denmark)

Unit: m	Mean	Std.dev.	Min.	Max.
New Fehmarn Belt geoid	0.276	0.013	0.246	0.299
German BKG geoid	(-0.007)	0.015	-0.038	0.022
EGM08 model to degree 2160	0.285	0.027	0.213	0.339
Most recent Nordic geoid (NKG2004)	(0.011)	0.014	-0.028	0.043



Operational geoid <1 cm to fixed link (tunnel project)
… common elevation reference

Difference DTU-BKG geoid

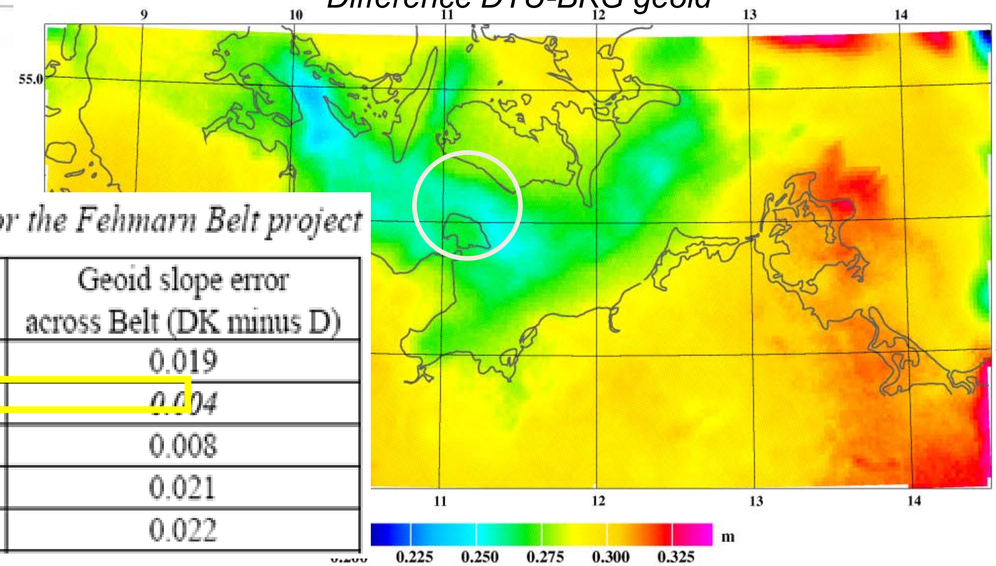
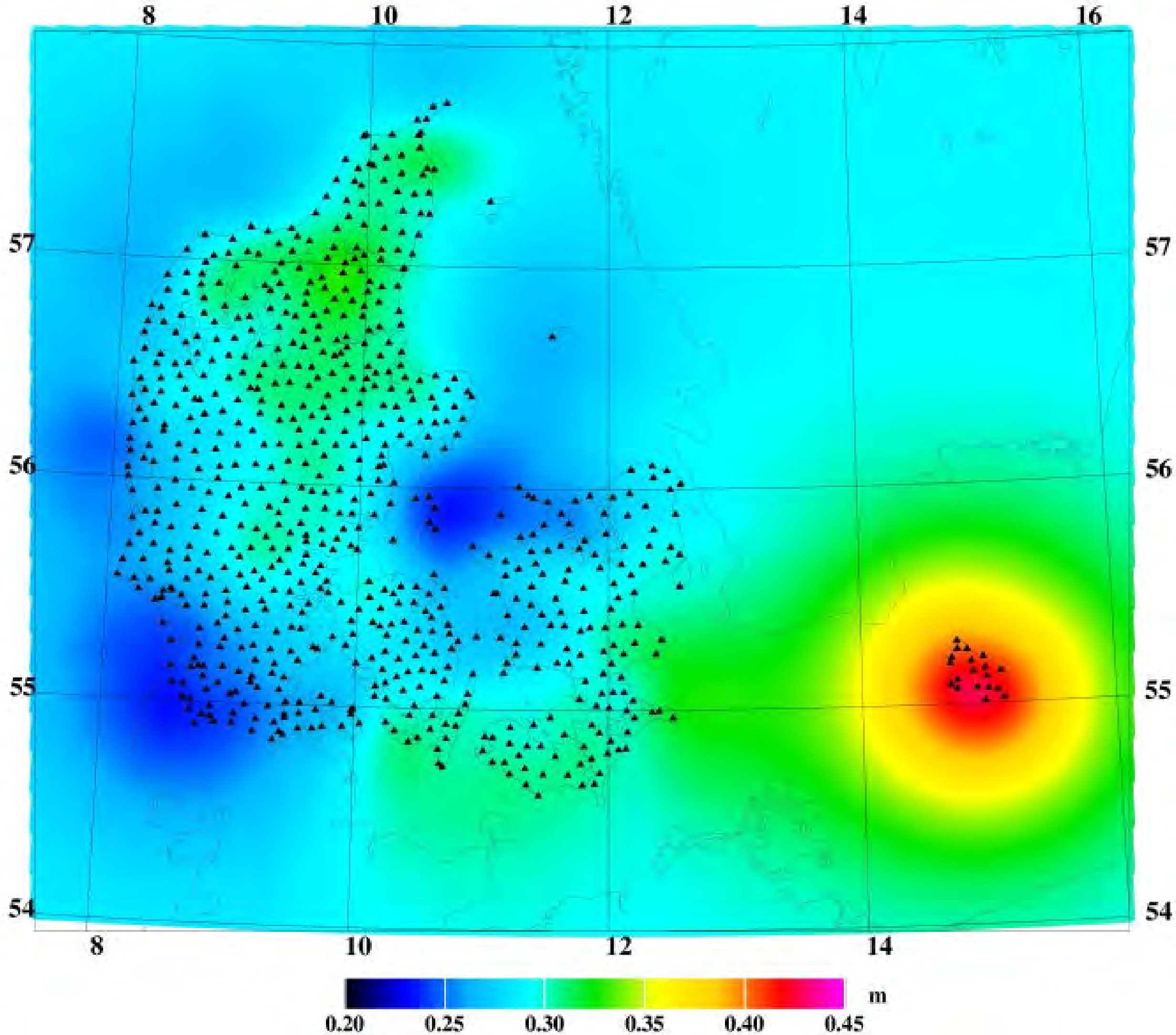


Table 4. Statistics of fit of 4 defining GPS-levelling points for the Fehmarn Belt project

Unit: m	Mean	Std.dev.	Geoid slope error across Belt (DK minus D)
Fehmarn Belt geoid	0.335	0.012	0.019
Fehmarn Belt fitted geoid	*0.000*	0.005	0.004
German BKG geoid	(0.065)	0.006	0.008
EGM08 model to degree 2160	0.340	0.023	0.021
Most recent Nordic geoid (NKG2004)	(0.064)	0.014	0.022

Geoid fits to GPS – Denmark

Fitted to DVR90 – 3rd national levelling (with geoid used for datum on small islands)



REFDK – national
10 km GPS net

Geoid correction signal:
- *Bornholm in own datum*
- *Small islands in geoid-determined datum*

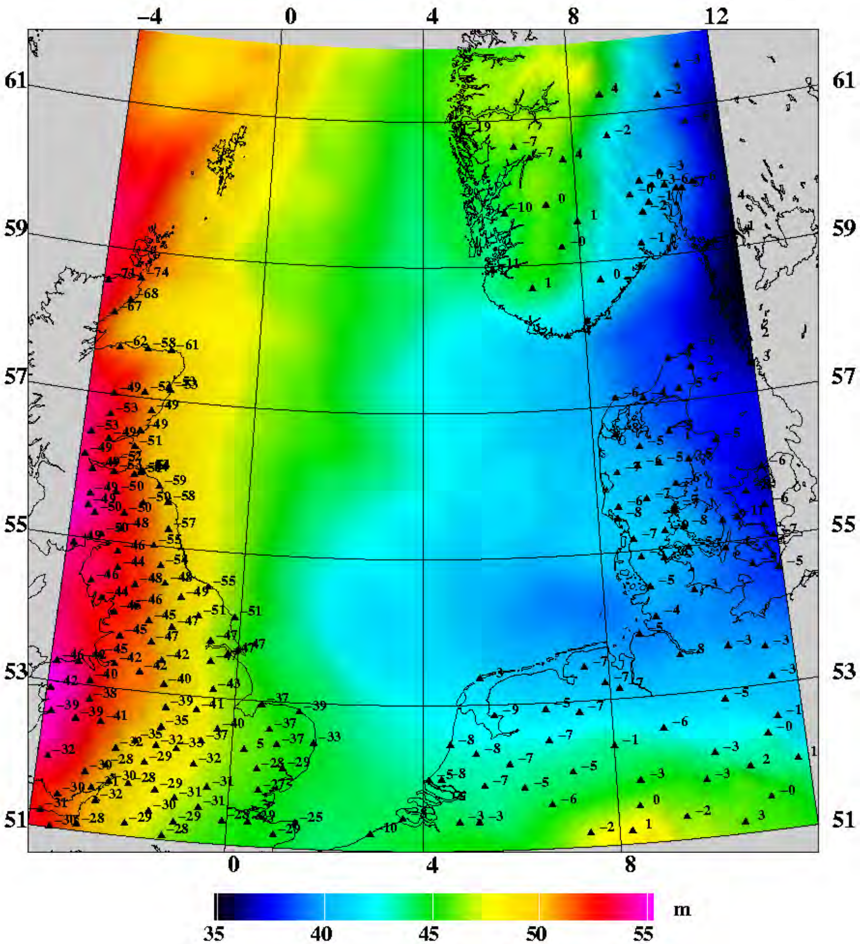
Denmark geoid compared to GPS

Geoid data – Unit m	*All GPS except small islands (677 pts)*		*Bornholm (20 pts)*	
	Mean	*Std.dev.*	*Mean*	*Std.dev.*
NKG04 geoid	-0.004	0.027	0.107	0.158
EGM2008	0.298	0.026	0.439	0.016
DKGEOID (GOCE mod. degree 150-160)	0.288	0.021	0.414	0.013
Fitted geoid	0.000	0.006	0.001	0.006

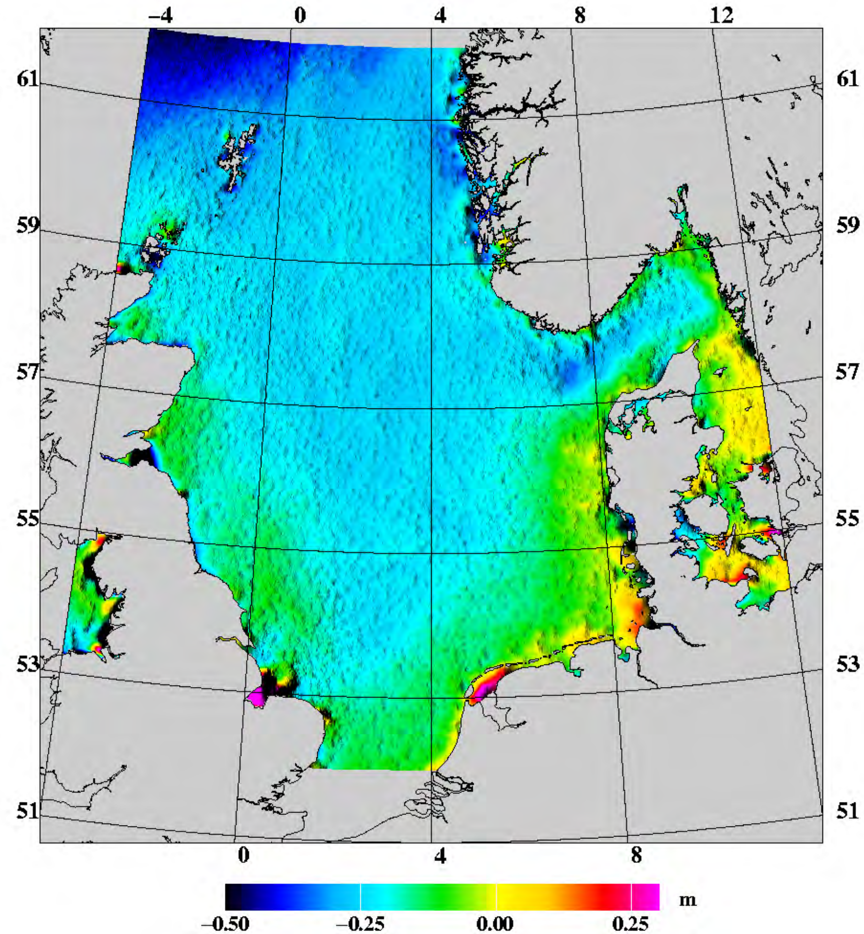
- The 1-cm geoid is available – used externsively
- Current challenges: *marine geoid … how to fit?*

North Sea geoid surfaces

(EU Interreg project BLAST – Bringing Land And Sea Together)



Geoid (with 1.order GPS points)



MDT (mean sea surface from satellites - geoid)

Greenland geoid

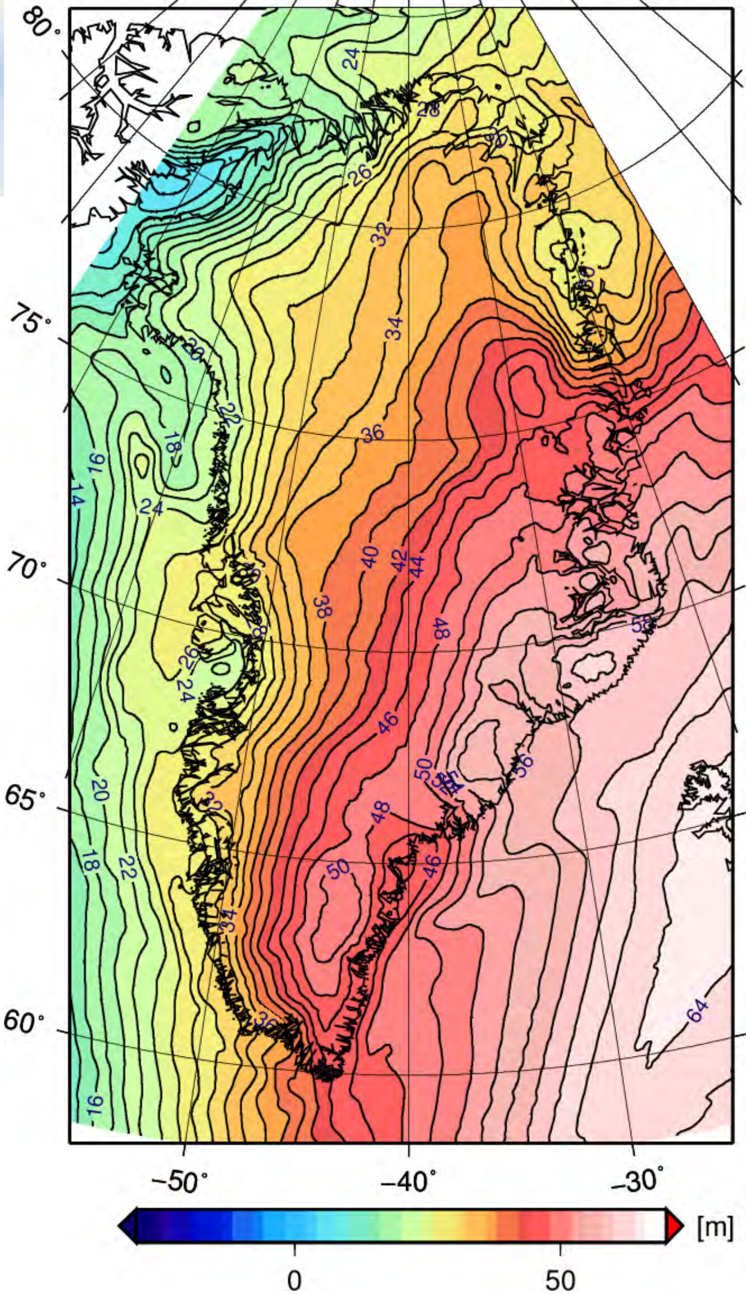
Greenland geoid model 2014

- Old one outdated – new data (DEM, GOCE, airborne)
- More coastal construction and engineering (hydropower)
- Need for joint vertical datum between towns
- Reference for ice sheet modelling and remote sensing
- North American geoid contribution (CDN 2013, US 2020)

Computed from airborne gravity measurements + GOCE

Challenges for geoid determination:

- Ice sheet and glaciers … *unknown thickness => errors*
- Deep fjords … *mostly unsurveyed (until OMG!)*
- Mountains, sparse gravimetry, several airborne sources

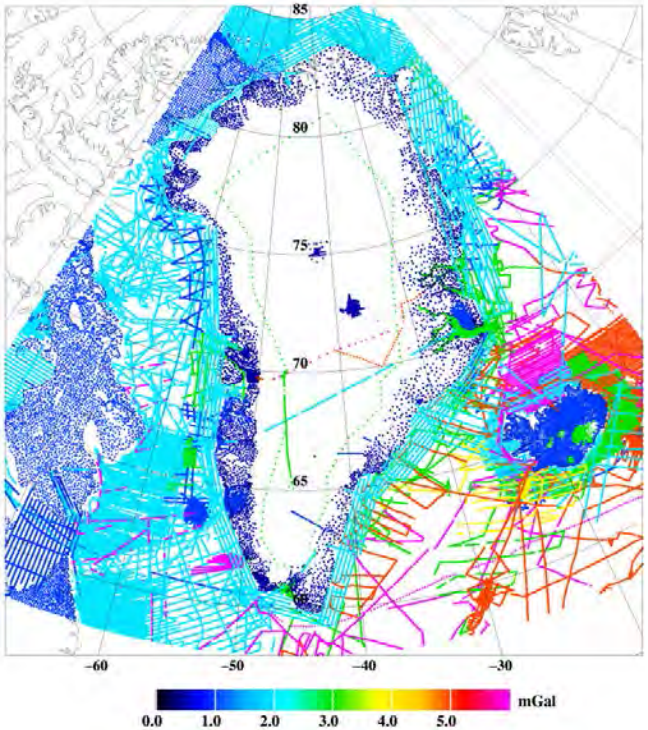


Greenland geoid 2014

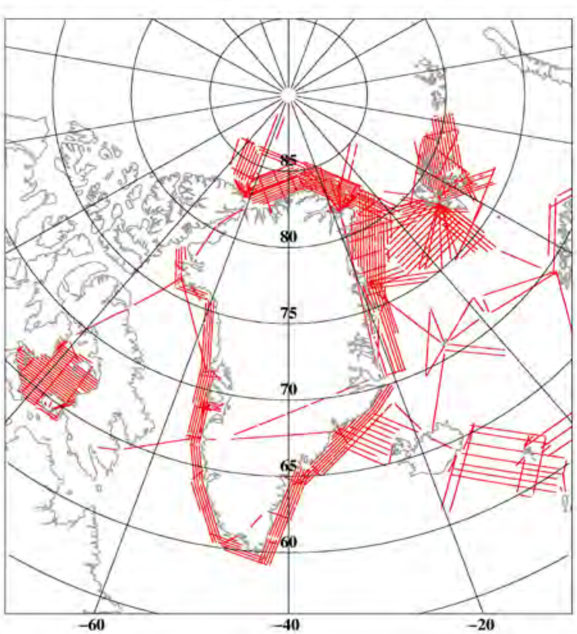
Gravimetry sources - older

- Helicopter gravity surveys
- Marine surveys (Nunaoil)
- Few ice sheet profiles
- Canada and Iceland data
- Low-level airborne (DTU+partners)
- High-level airborne (NRL 1991-92)

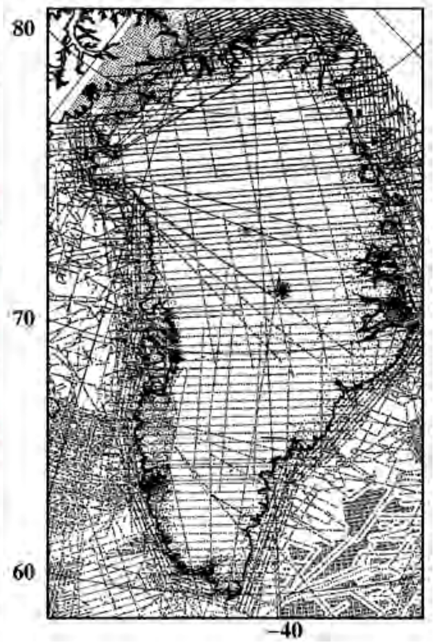
Helicopter surveys 1991-97



Airborne DTU-Space 1998-2003

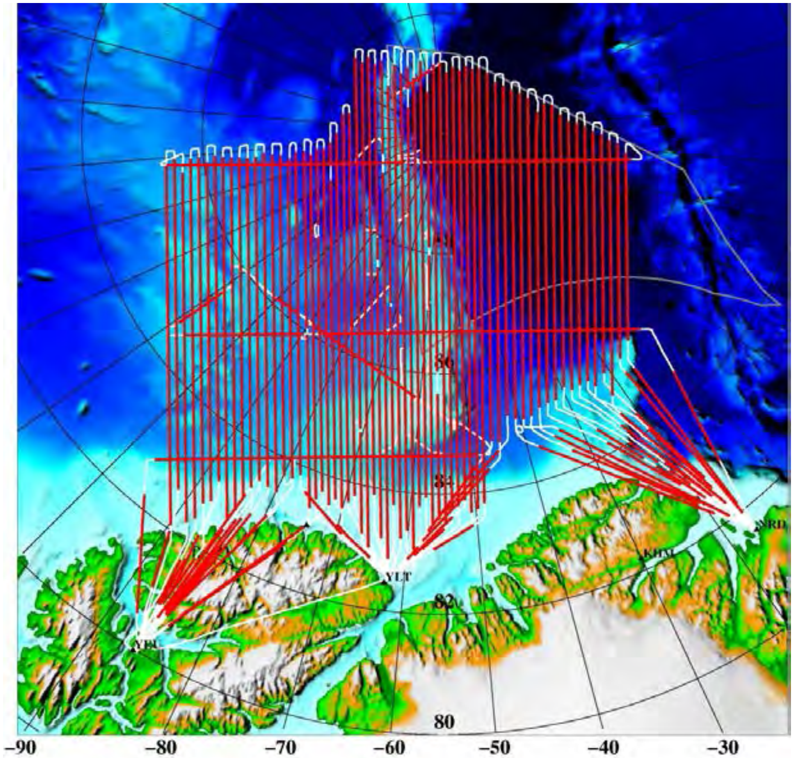


NRL 1991-92

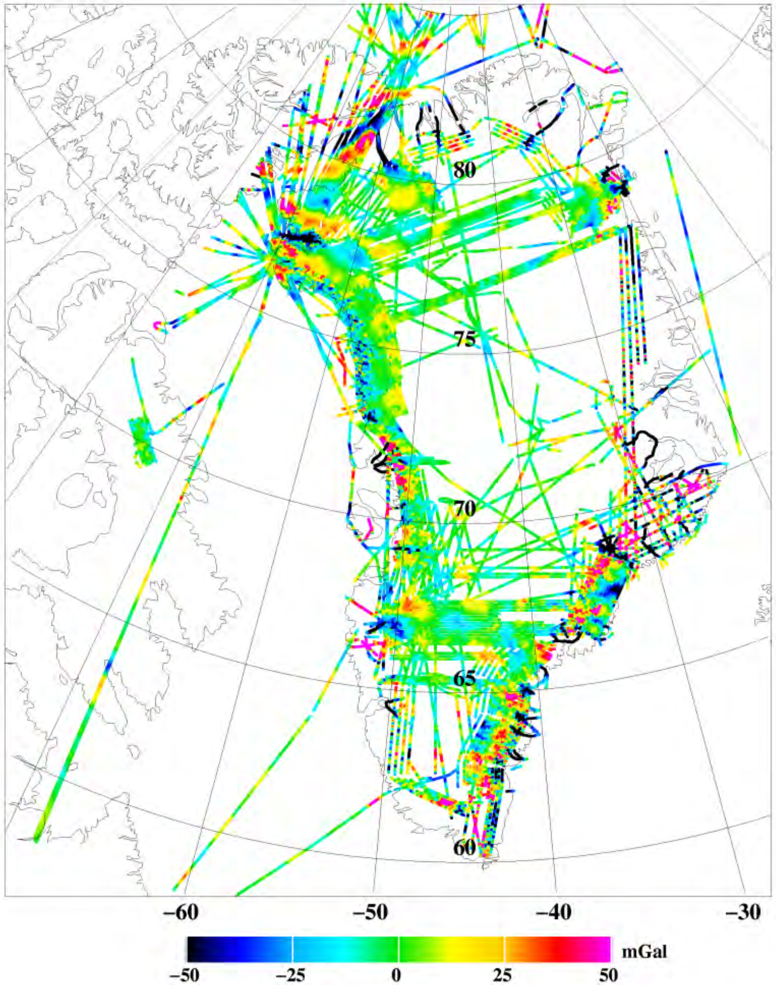


Newer gravimetry sources

- Marine/airborne UNCLOS
- NASA IceBridge airborne
- UNCLOS marine surveys in Arctic Ocean
- Satellite altimetry gravity (DTU13)



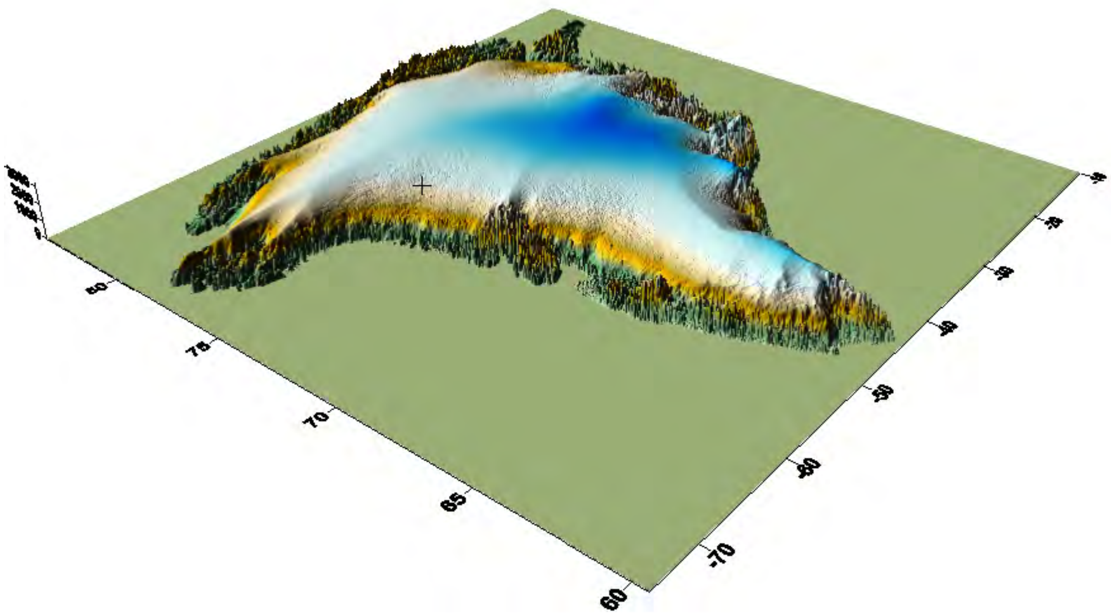
LOMGRAV2009 DTU-Space/NRCan 2009



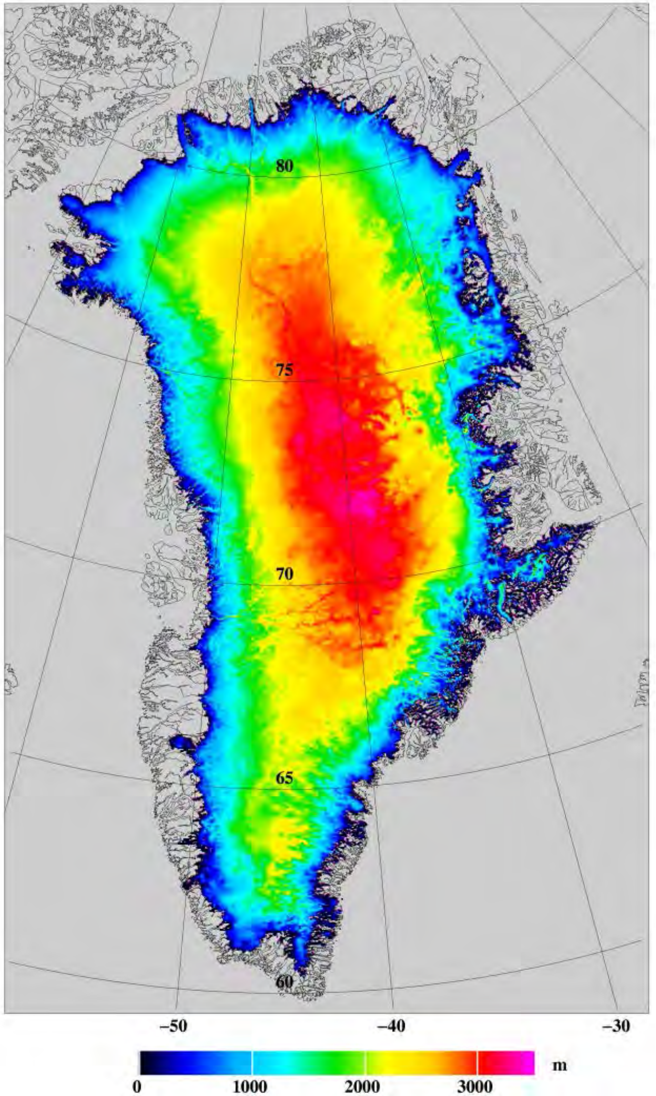
NASA IceBridge 2009-13
(Sandar Geophysics AIRGRAV instrument
.. draped flights along glaciers)

DEMs – surface and ice thickness

- New Greenland DEM (CryoSat/IceSat/Aster/Photogrammetry)
- Ice thickness DEM from radar measurements (IceBridge + DTU/AWI, Bamber et al)



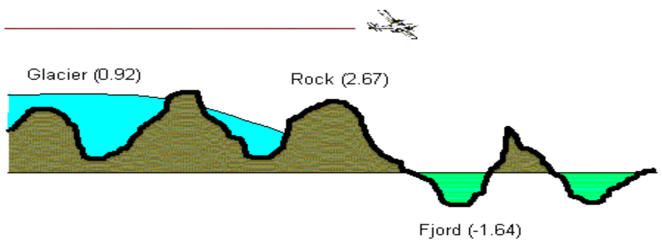
New Greenland DEM from CryoSat and ASTER



Ice thickness grid – outlying ice caps missing

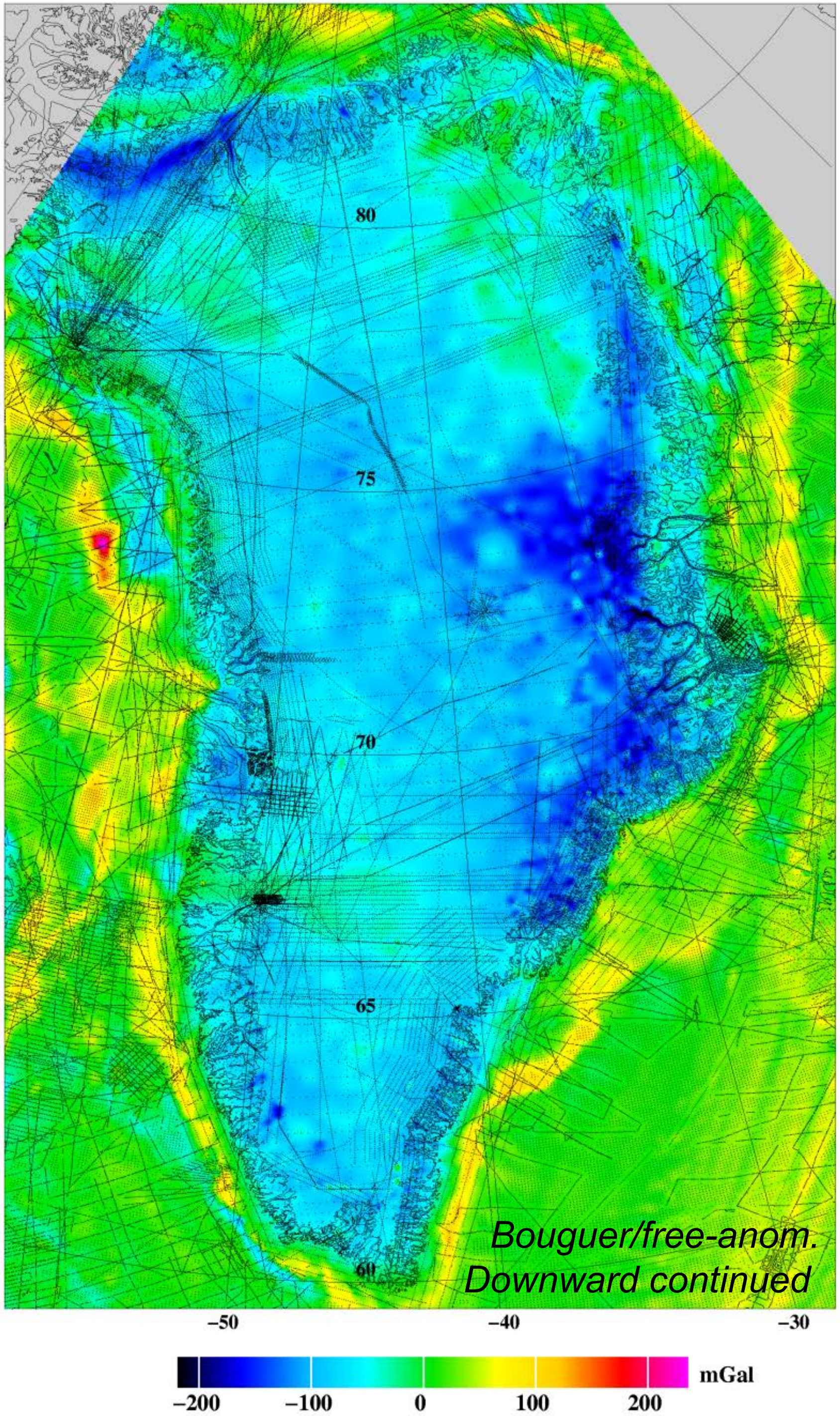
Downward continuation

Data over the fjords, outlet glacier, ice sheet marginal zones => major improvements
SGL OIB data high quality (~ 1 mGal)



Statistics of original andRTM- reduced gravity (mGal)

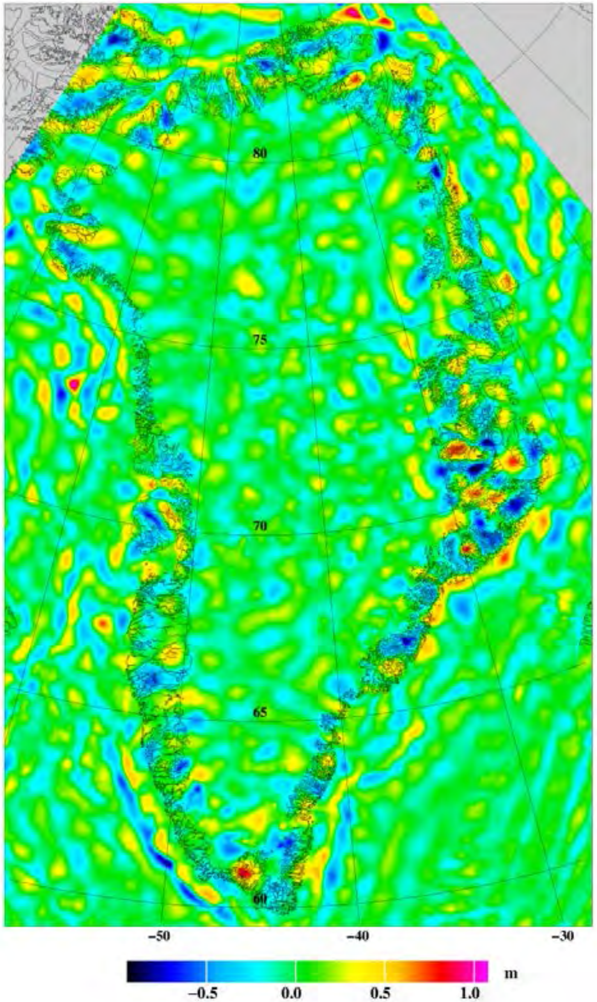
Data source	Orig. Mean	stdev	Red. mean	stdev
Land gravity	-16.8	44.8	-4.6	16.0
IceBridge airborne	10.5	44.3	-1.7	12.9
NRL airborne	16.4	38.0	0.1	11.5
DTU airborne	9.8	38.0	0.3	16.1



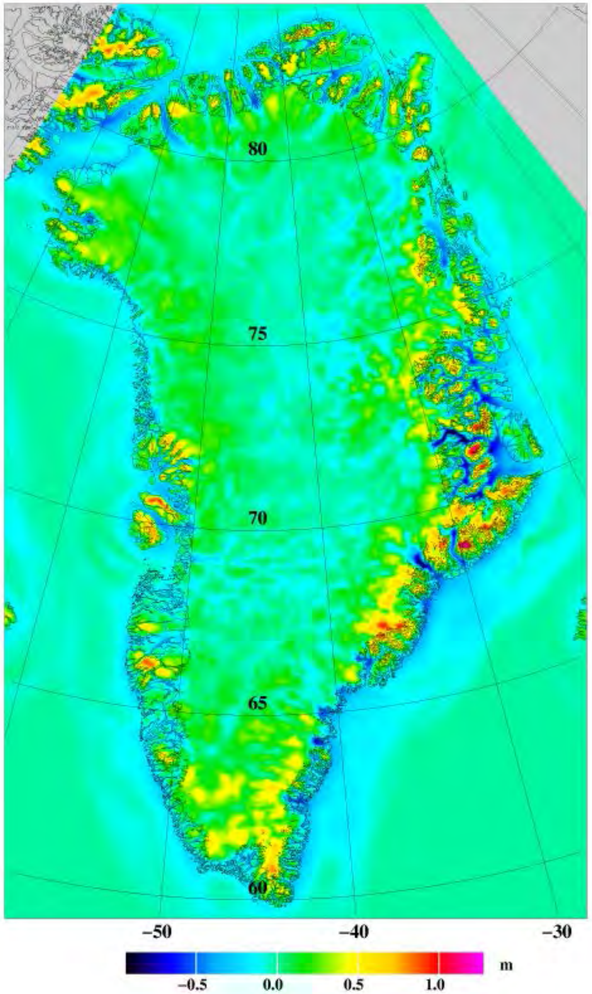
Gravimetric computation steps

1. Terrain reduce data + QC
2. Downward continuation and gridding of reduced data (lsc, 1° blocks with overlap)
3. FFT conversion gravity -> quasigeoid (Wong-Gore)
4. Restore RTM terrain + ice effects
5. Restore GOCE/EGM08 geoid grids

Geoid part from FFT



Geoid part from terrain+ice



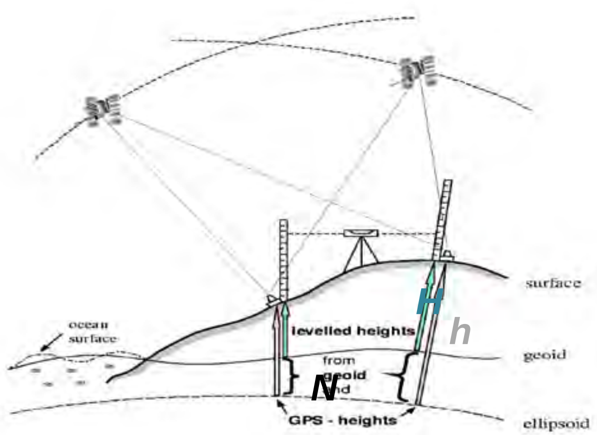
Geoid/height system validation

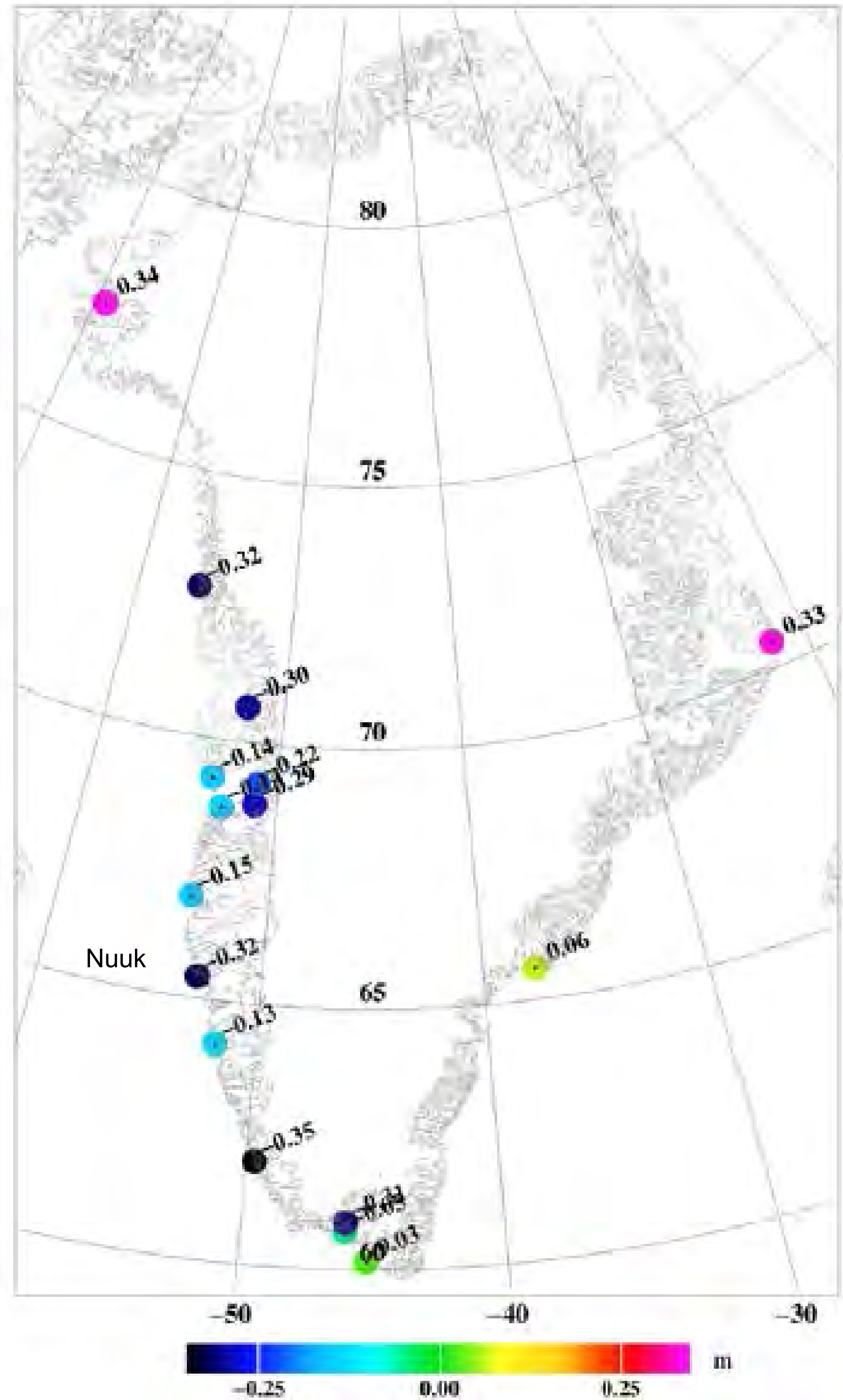
Fit to ”apparent geoid” from GPS and local survey benchmarks (ASIAQ)

$$N^{GPS} = h^{GPS} - H_{local}$$

Model offsets due to height system definition:

- *Height system from local tide gauges (Ocean not ”level” … dynamic topography)*
- *Land uplift due to ice melt*



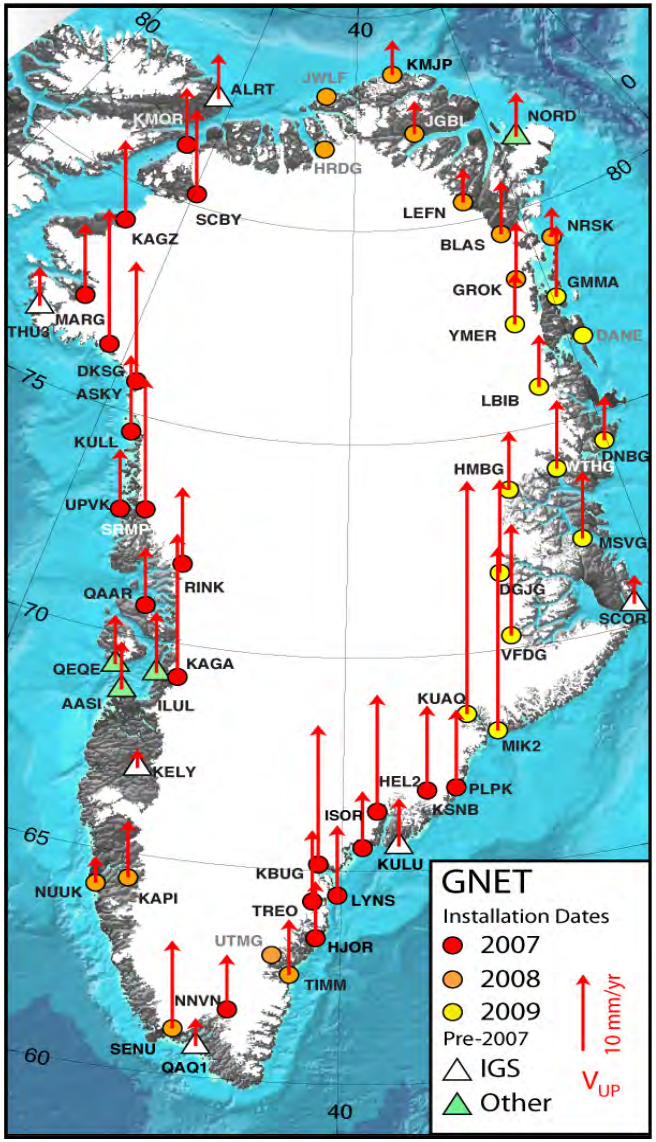


Height system issues

Land uplift (Bevis et al, 2012)
Height datum: tide gauges (1960's)

Are some "errors" in towns uplift?

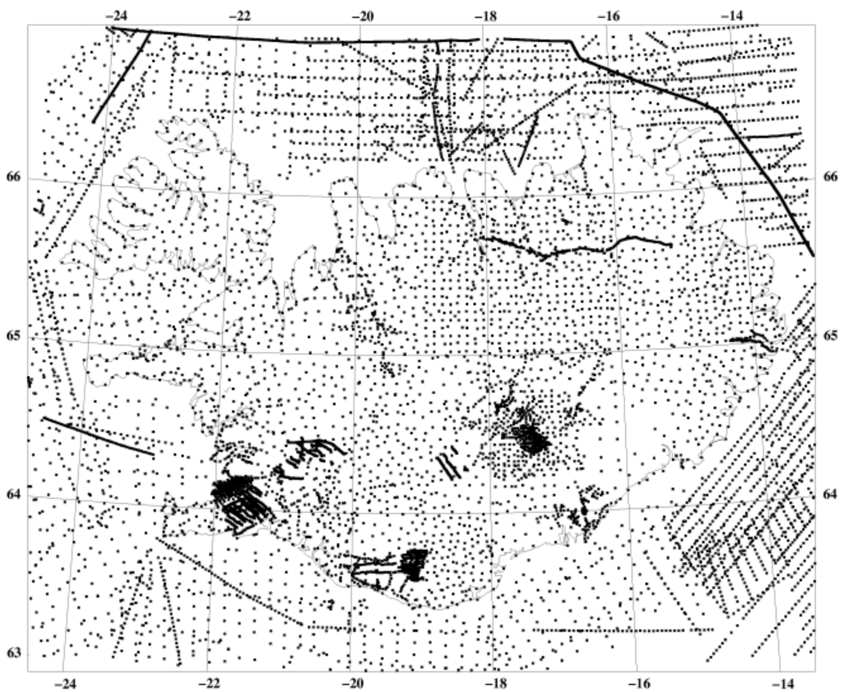
$$N = h^{GPS} - H + MDT + dh/dt * \Delta t$$



Geoid validation: Iceland

Iceland ”snapshot”relevelling 20089 (Landmalingar Islands) … Iceland in Greenland geoid grid

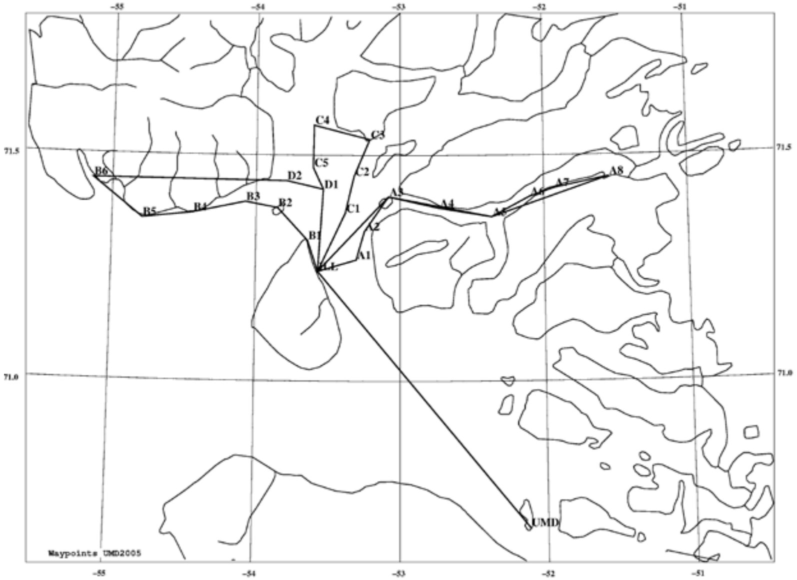
Gravity coverage



GPS-levelling data

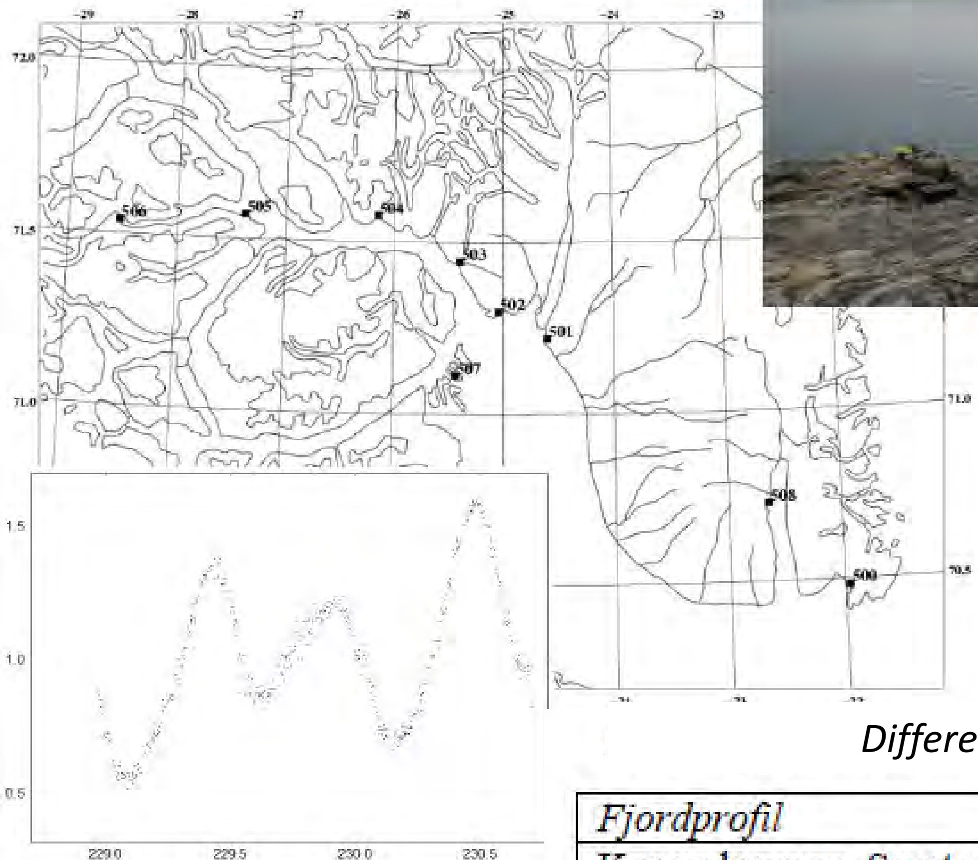
Comparison GPS/lev (m)	Mean	St.dev.
Original GPS/lev-data	65.54	0.794
Difference – new geoid	-0.10	**0.037**
Difference – EGM08	-0.12	0.113

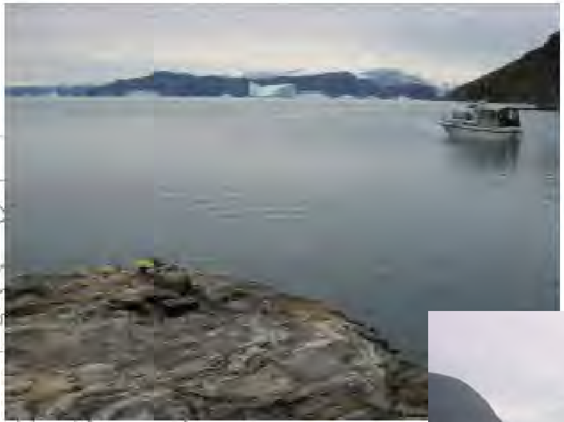
Greenland fjord tide gauge profiles



Sea level in fjords ”proxy” for geoid + MDT (small) – relative tidal measurements on land, GPS on ice

Typical fjord profile results





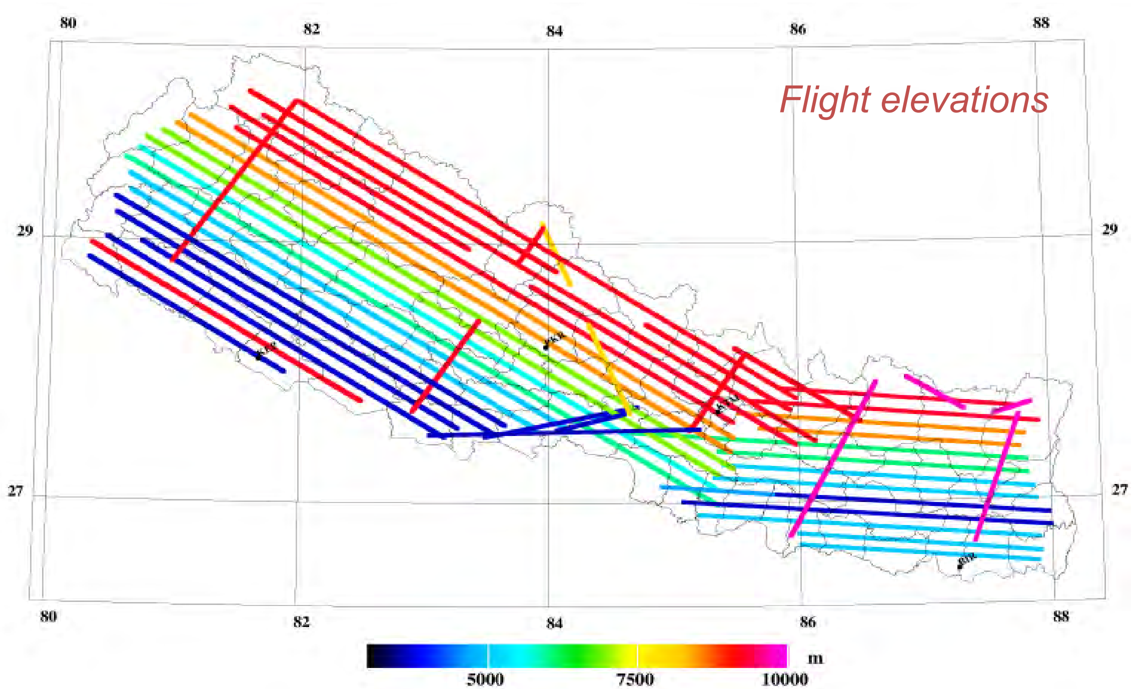
Difference between fjord N^{GPS} and new geoid

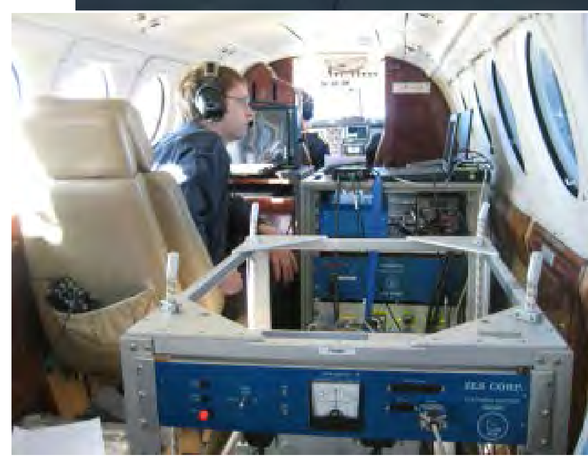
Fjordprofil	*Middel (cm)*	*Std.dev. (cm)*
Kangerlussuaq- Svartenhuk (Uummannaq)	-0.240	-0.073
Ameralik (Nuuk)	-0.083	0.117
Tasermiut (Nanortalik)	0.054	0.127
Ammassalliip Kangertiva (Tasiilaq)	-0.140	0.089

A ”worst-case” geoid example - Nepal

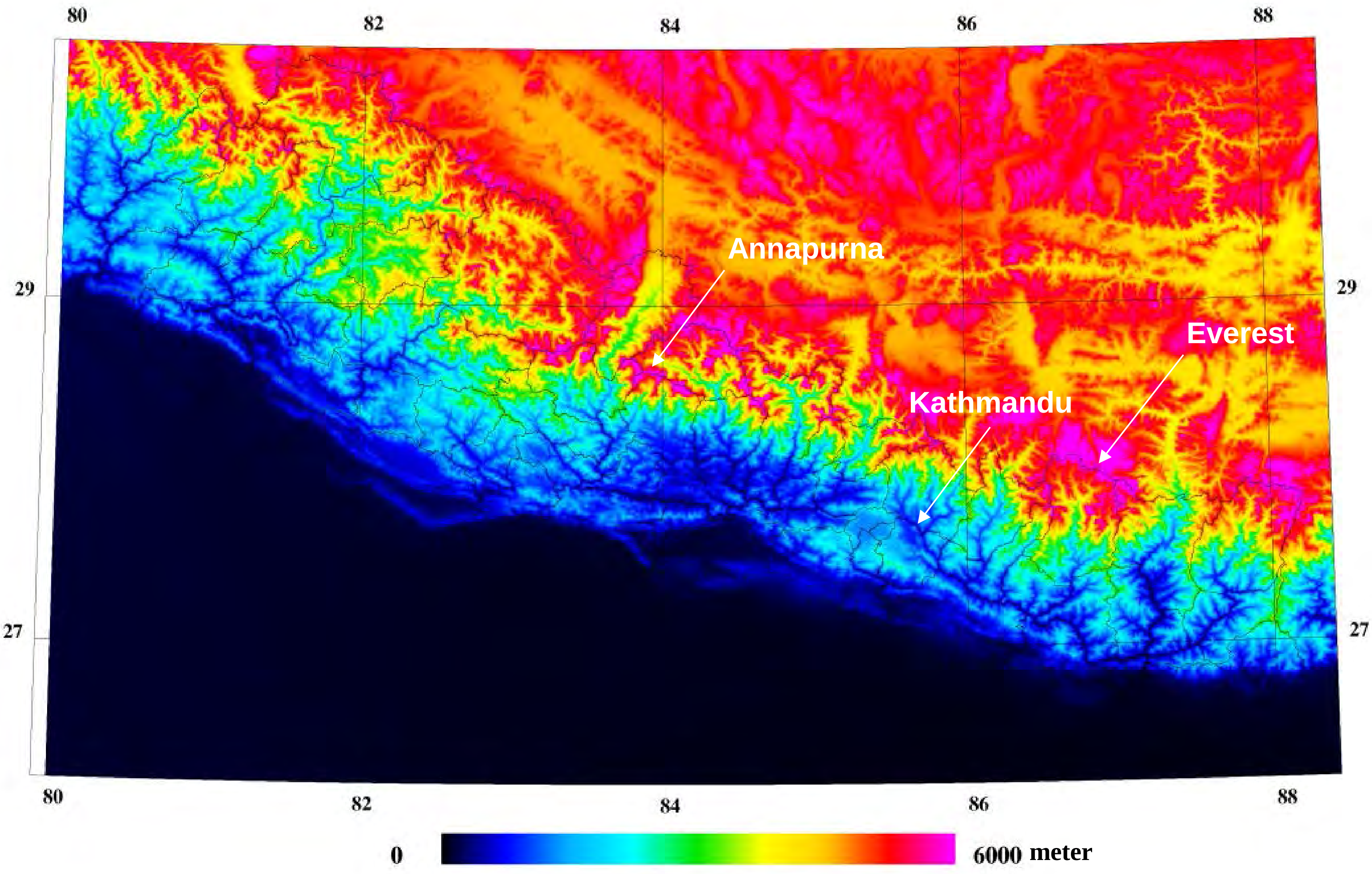
Nepal aerogravity survey 2010

- DTU Airborne gravity survey of Nepal 2010 – Beech King Air 200
- Geoid project – Nepal Department of Survey + NGA
- 57 flight hr excl. ferry flights to/from DK
- Base GPS and gravity ties at Kathmandu Airport
- LaCoste and Romberg gravimeter, Chekan-AM
- Auxillary: Honeywell IMU, numerous GPS's



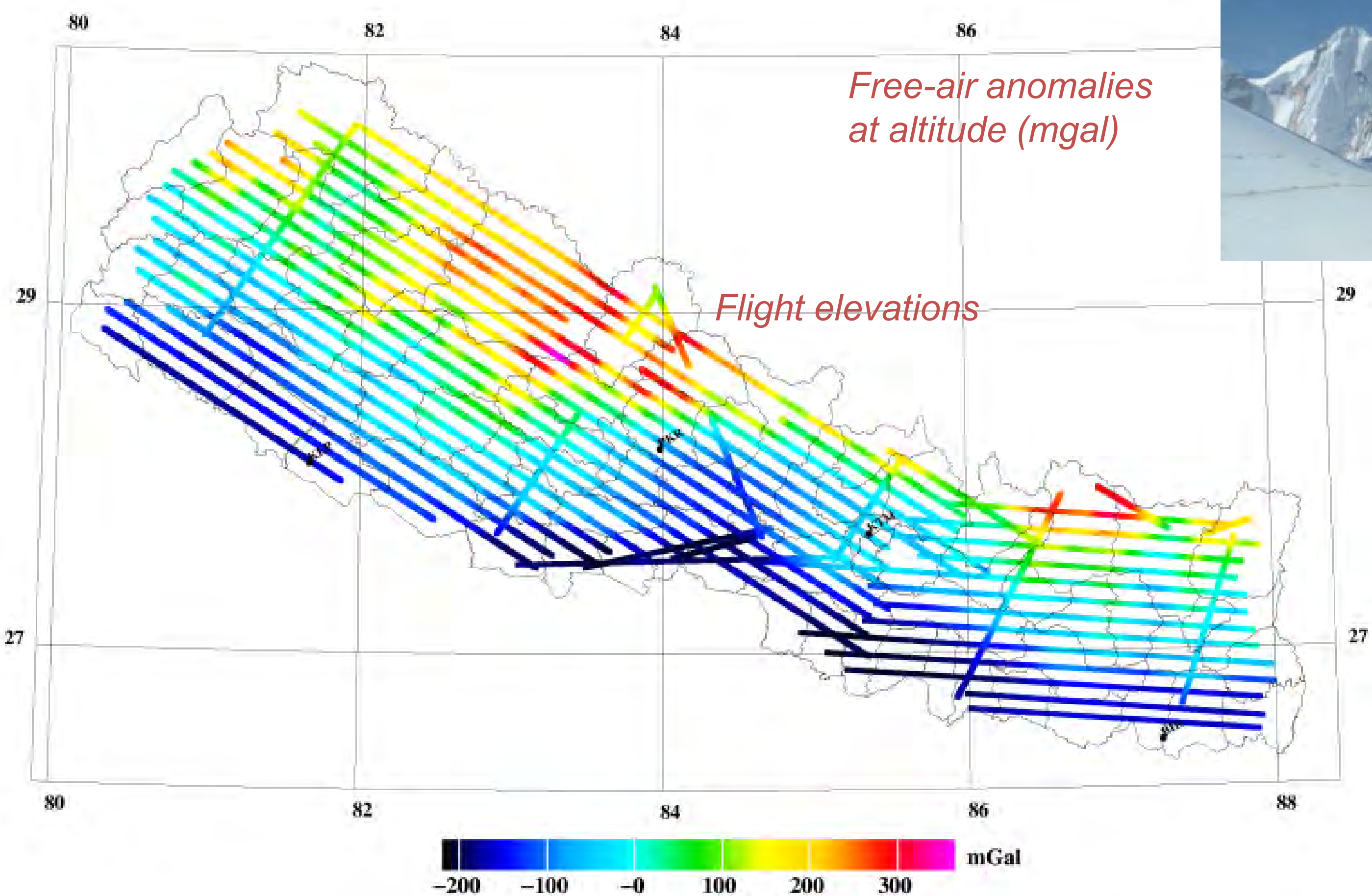


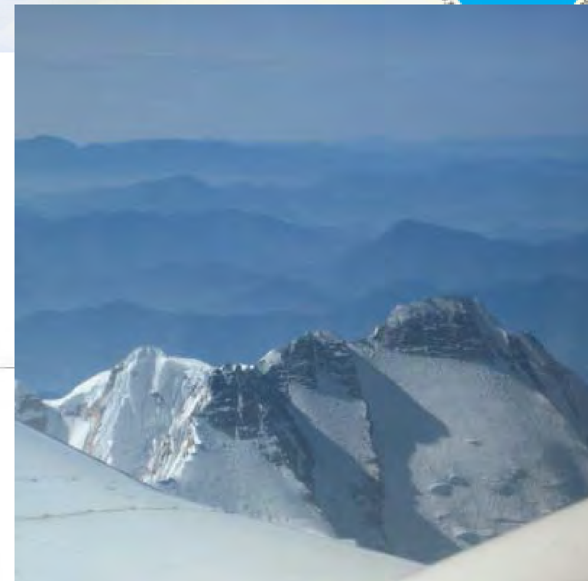
Nepal SRTM terrain model



AG Results

- Airborne processing with DTU-Space system – LCR and Chekan AM
  … *cross-overs not reliable due to differing heights*





- *Challenges:*
- *- mountain waves*
- *- jet streams*
- *- turbulence*

… especially Annapurna region

AG Results

- Chekan-AM (Russia) and LCR S-34 gravimeter processed independently

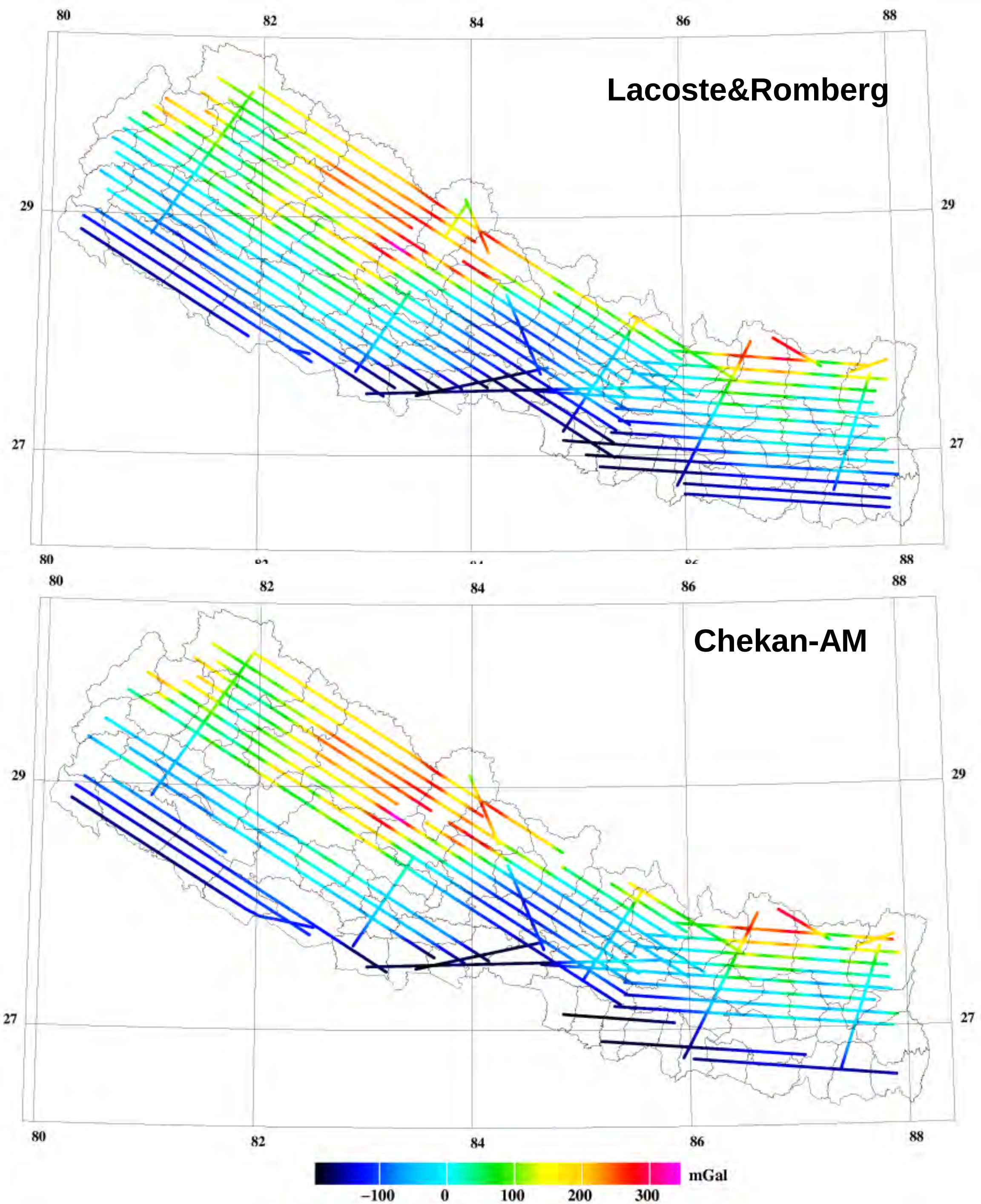
Agreement between gravimeters: 4.3 mGal rms

Merged set made by averaging Chekan-AM and LCR at common points

- X-over errors:
  ~ 10 mgal rms

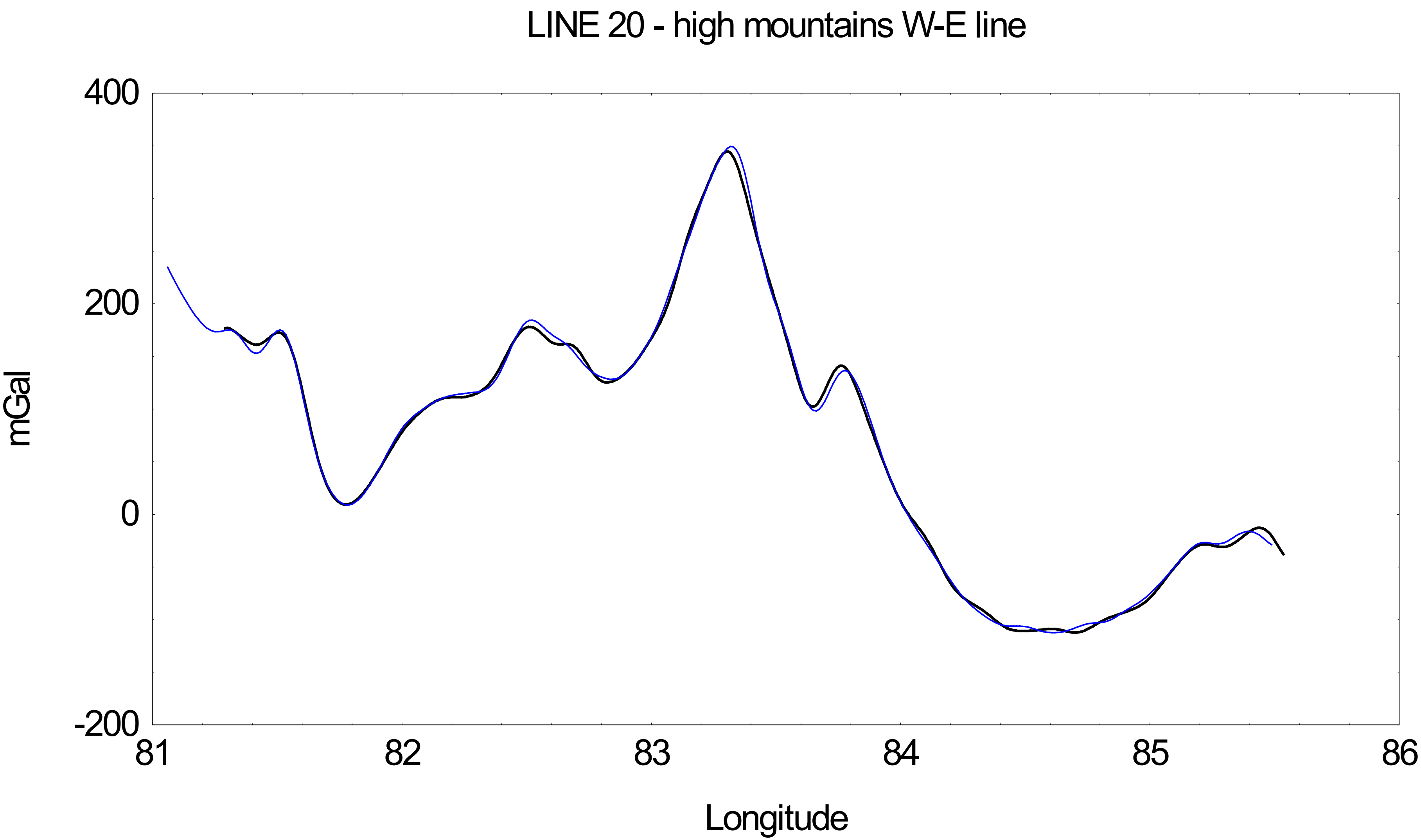
.. cross-overs not useful for QC due to differing flight heights.
After continuation to 6600 m:

LCR: 4.6 mgal rms
Chekan: 5.1 -
Merged: 3.9 mgal rms

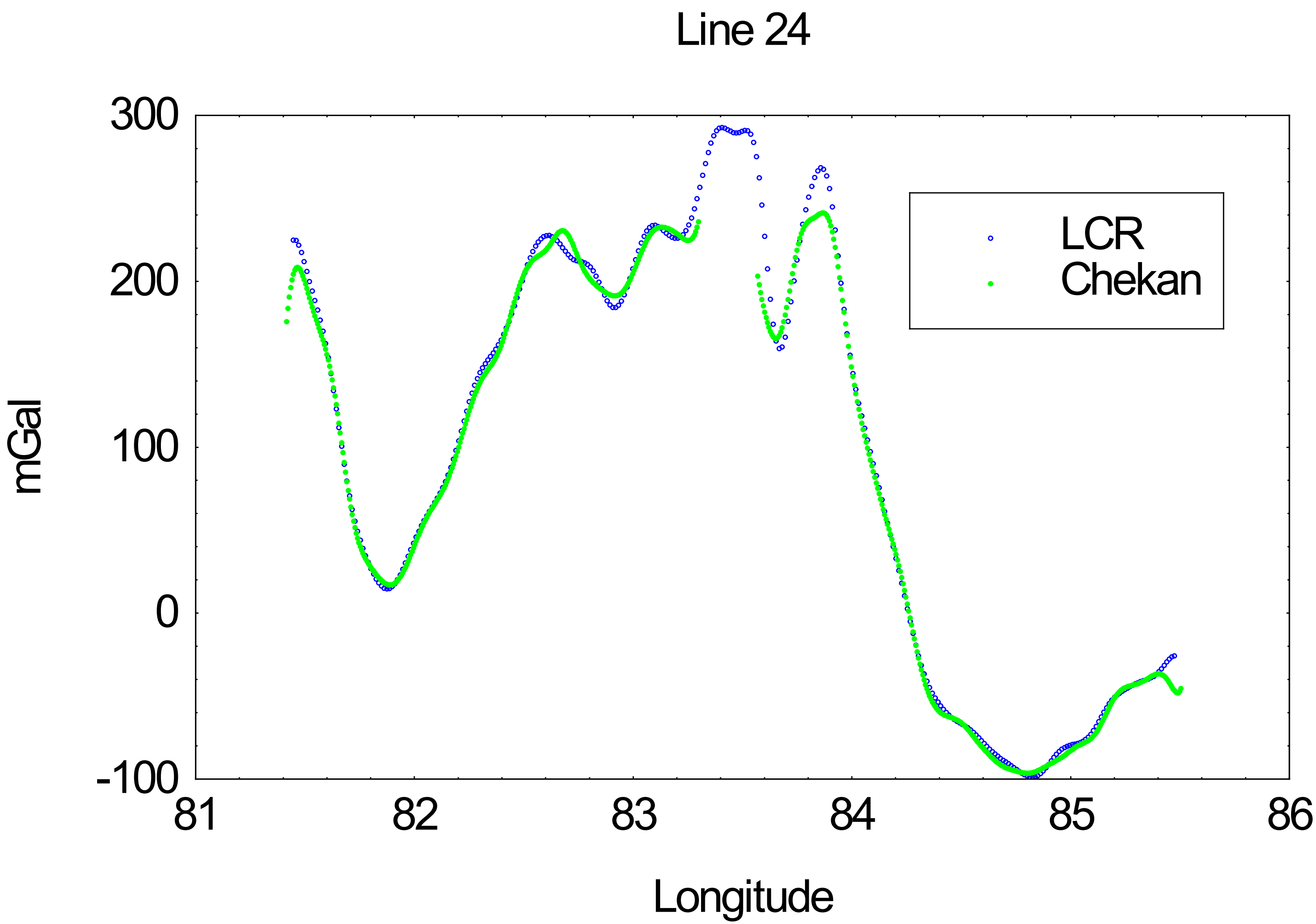


Free-air anomalies at altitude (mgal)

LCR and Chekan data differences

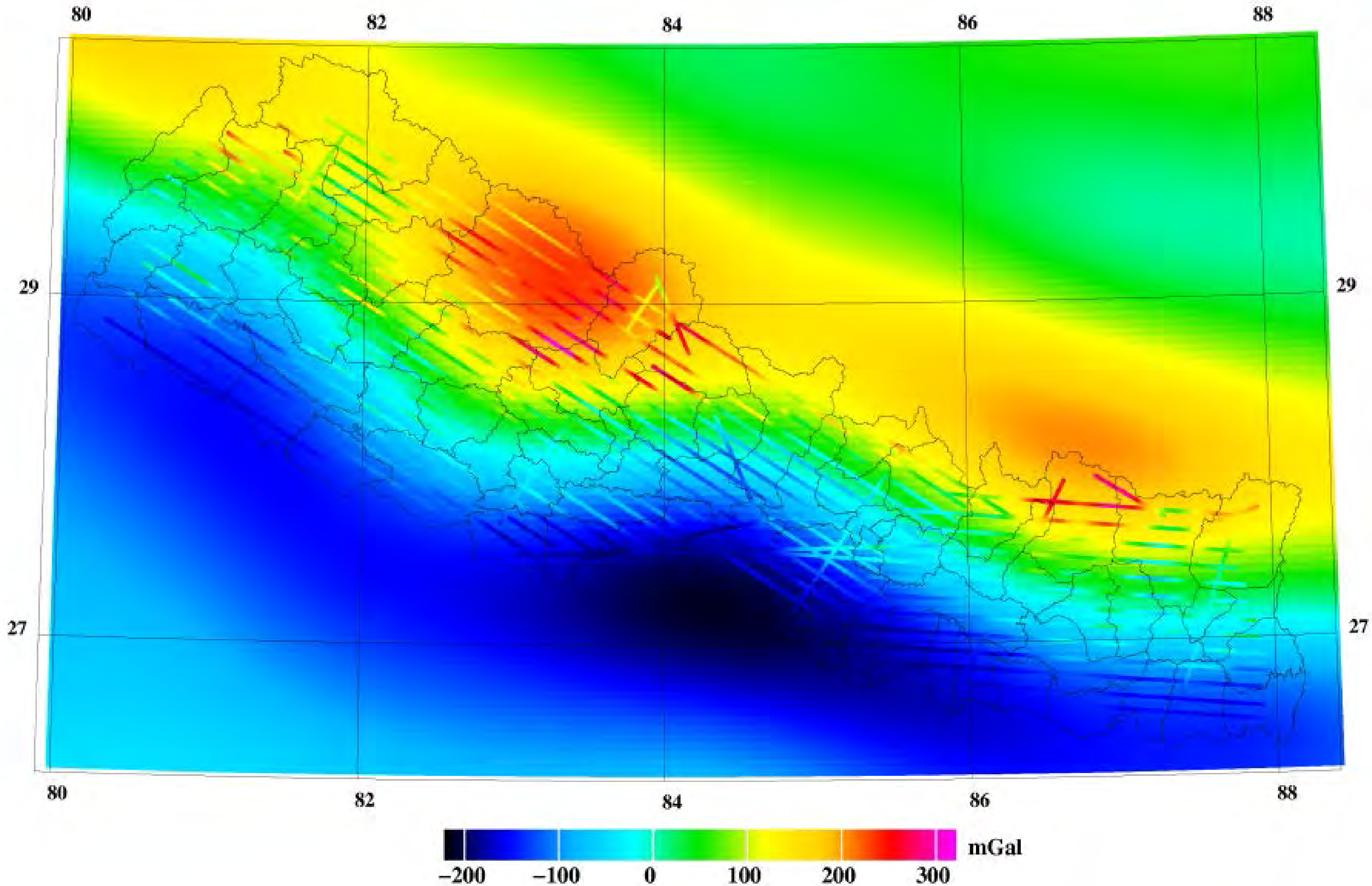


LCR and Chekan – example #2



LCR airborne data and GOCE

GOCE satellite gravity @ 100 km resolution confirmed



Geoid determination – reduce steps

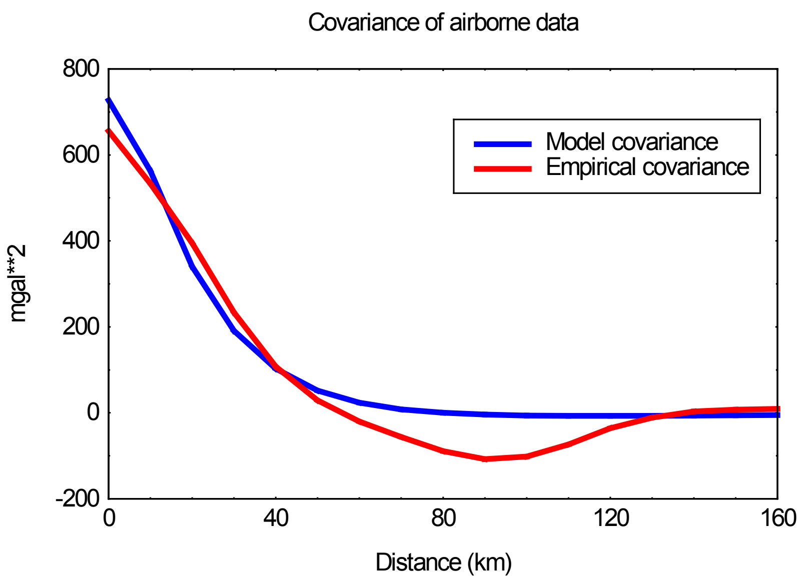
Reference field: EGM2008 – augmented with GOCE
(linear merging at degrees 80-90 and 180-190 with GOCE in middle band)

• Some special effects on airborne gravity: Terrain effects must be *filtered* with along-track filter corresponding to airborne gravity filter (forward/backward Butterworth filter ~ 90 sec time constant)

Statistics of data reductions (unit: mgal)

Data	Mean	Std.dev.
Airborne data	-14.2	119.4
Airborne – GOCE (360)	1.4	36.5
Reduced airborne data	2.9	21.2
Surface data	-87.3	103.0
Surface – GOCE	-24.1	71.2
Reduced surface data	-1.2	26.0

• Downward continuation of both surface and airborne data by least-squares collocation

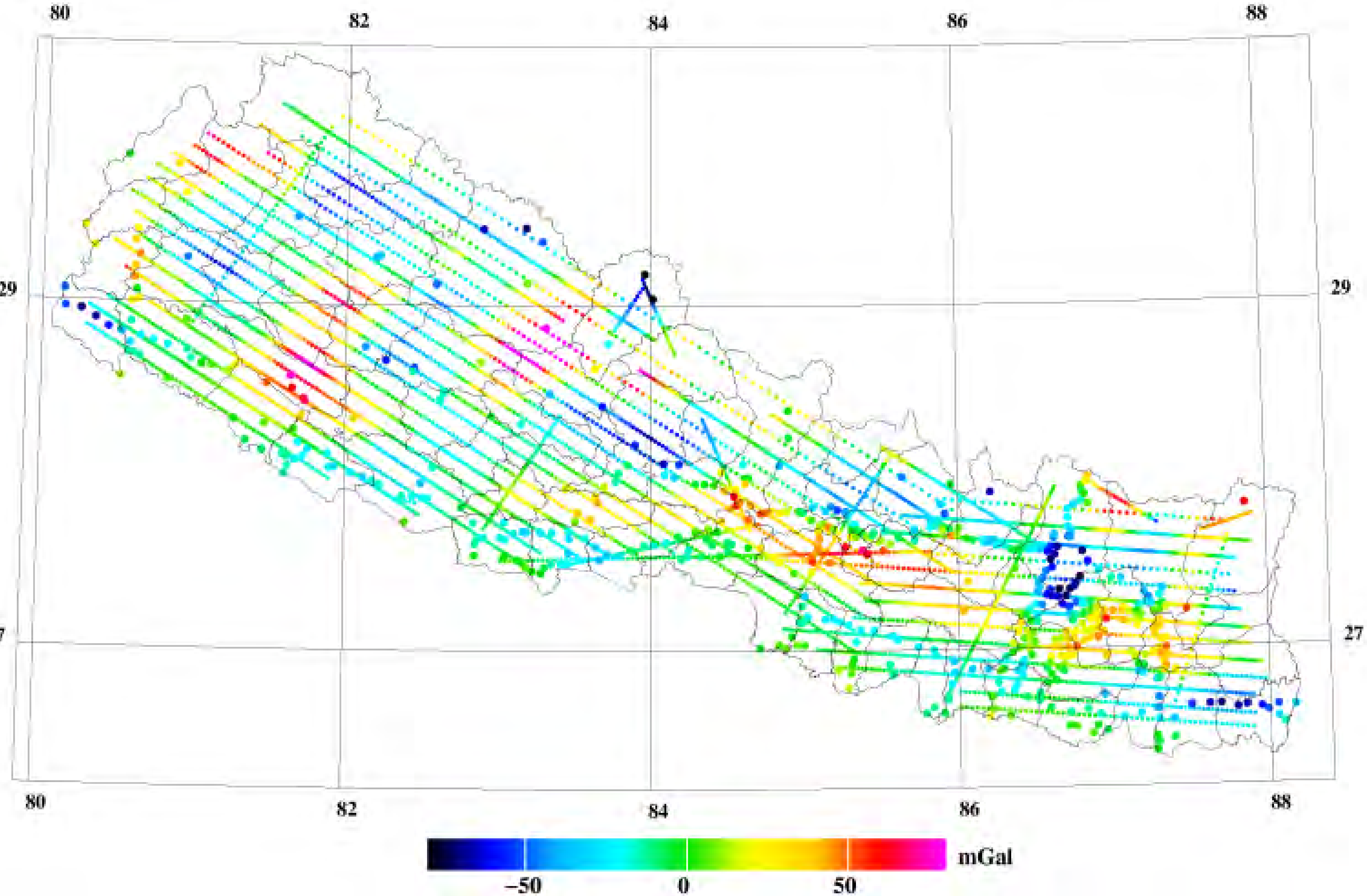


Airborne Gravimetry for Geodesy summer school

Reduced airborne and surface gravity

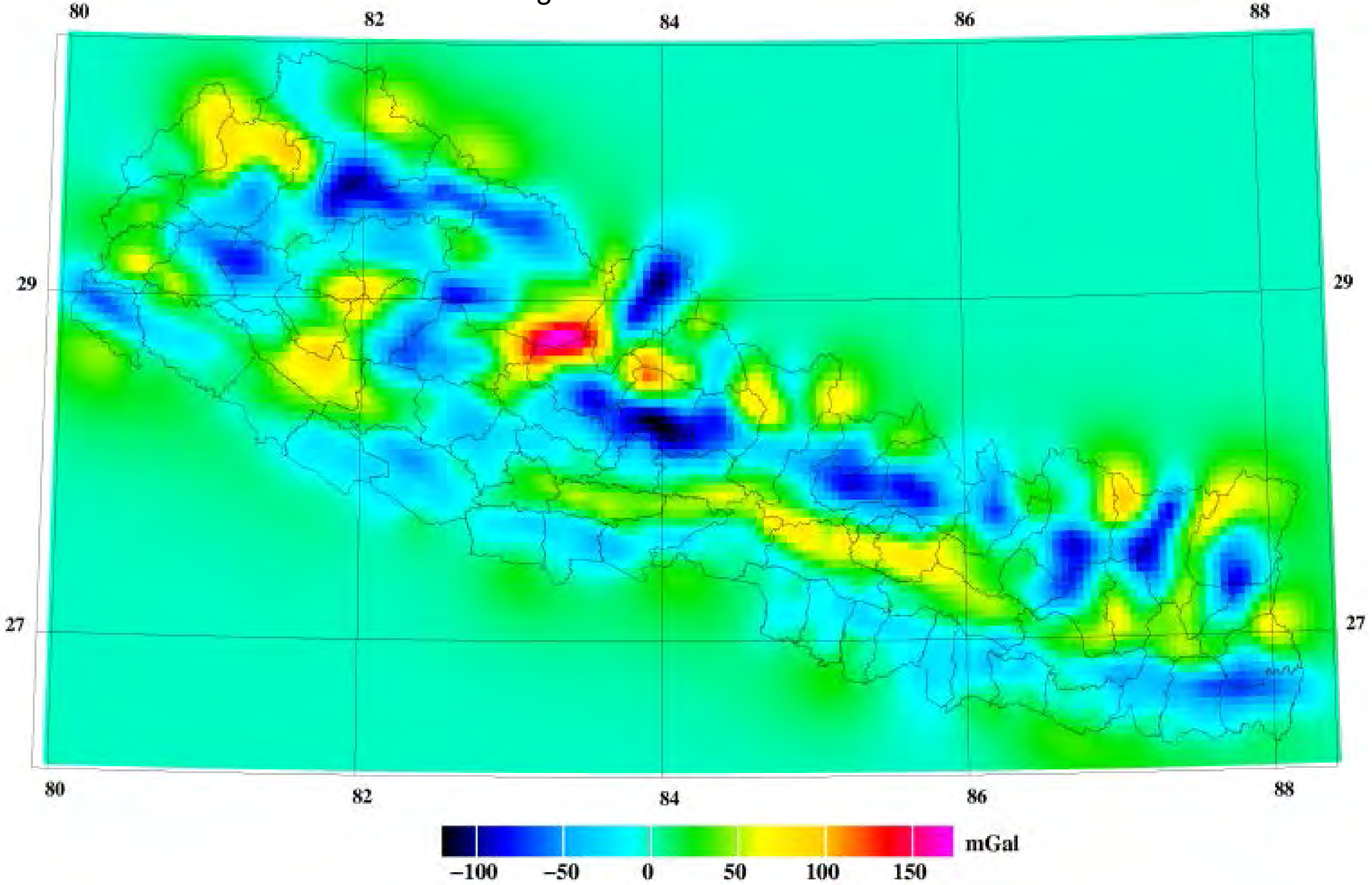
DTU Space National Space Institute

Reduced gravity data showing contribution from airborne data and surface data (changes in spacing on airborne lines due to headwind and tailwind from jet stream)



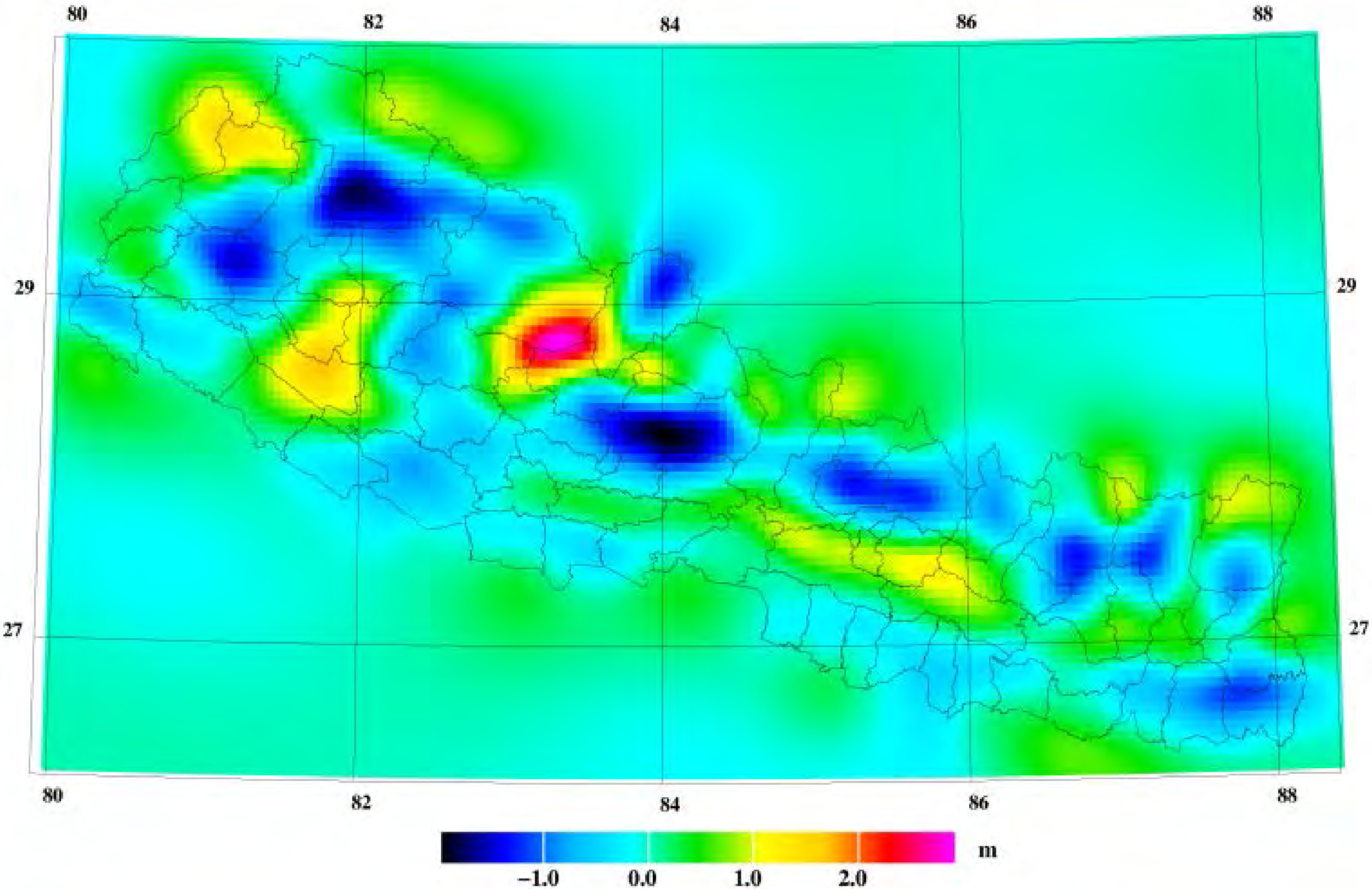
Downward continuation of all data

Deg 720 reference field



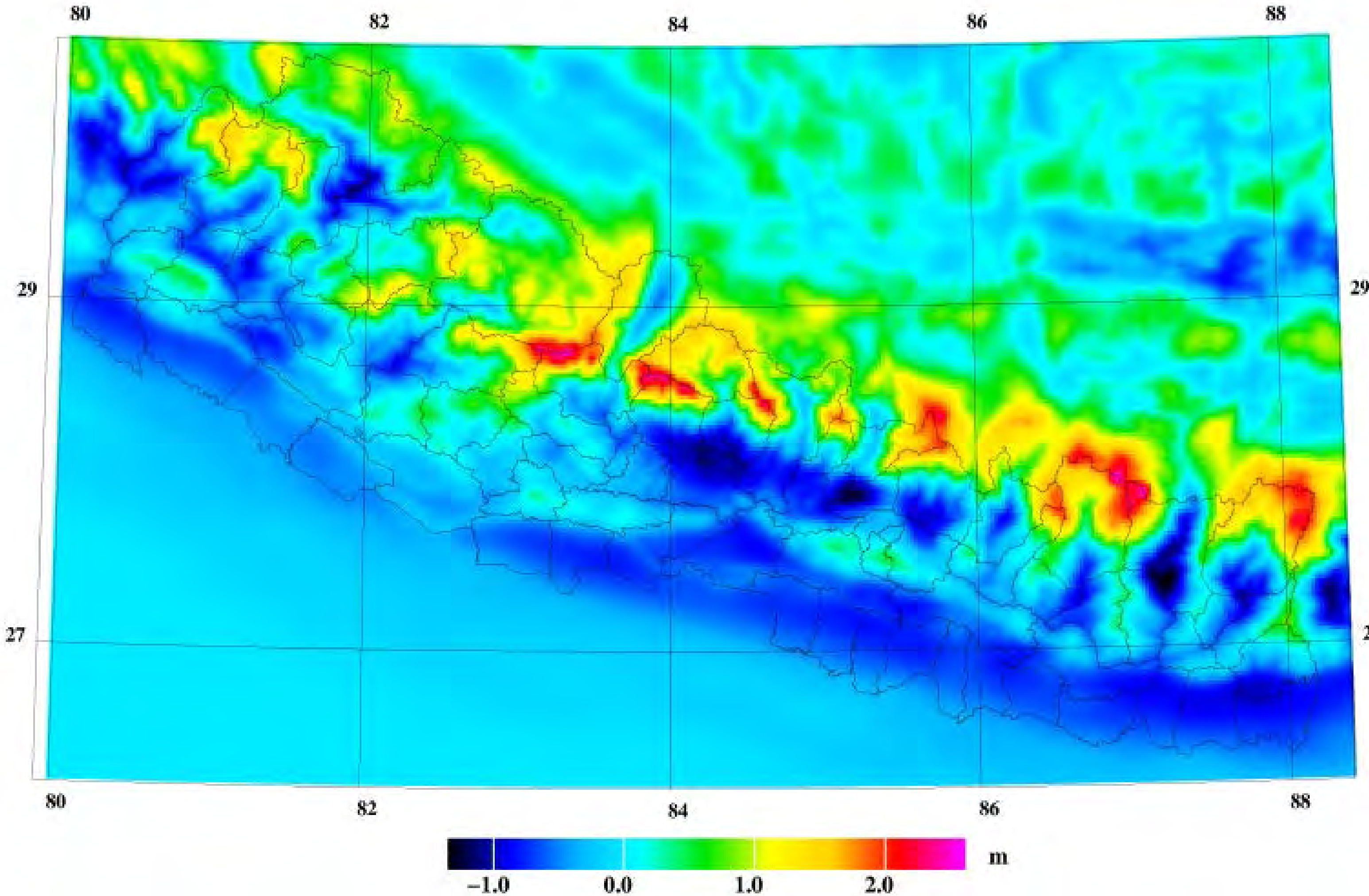
Geoid (reduced quasigeoid)

Quasigeoid contribution from spherical FFT relative to ref field



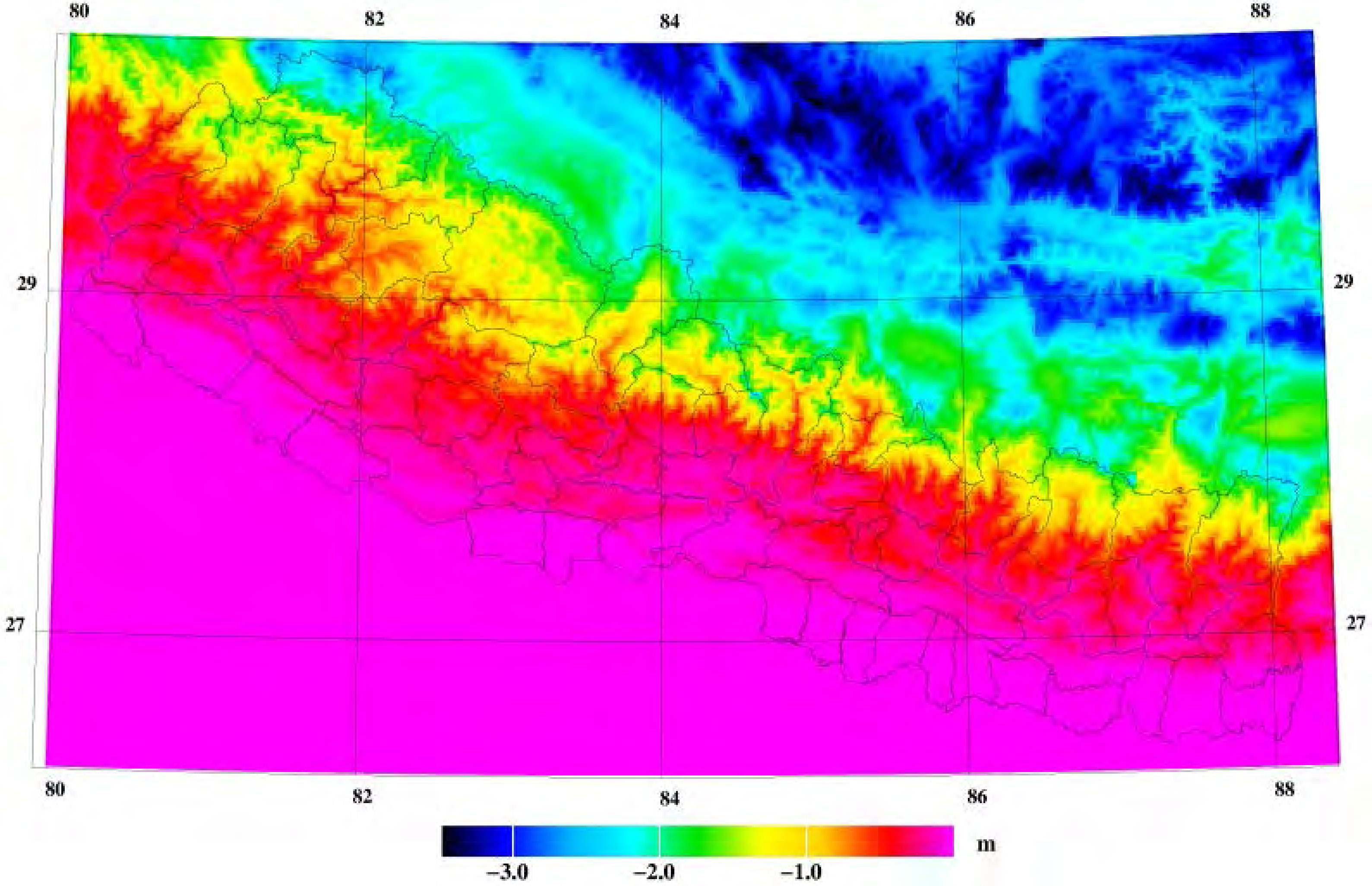
Geoid (terrain restore part)

Quasigeoid contribution from SRTM terrain 30” resolution



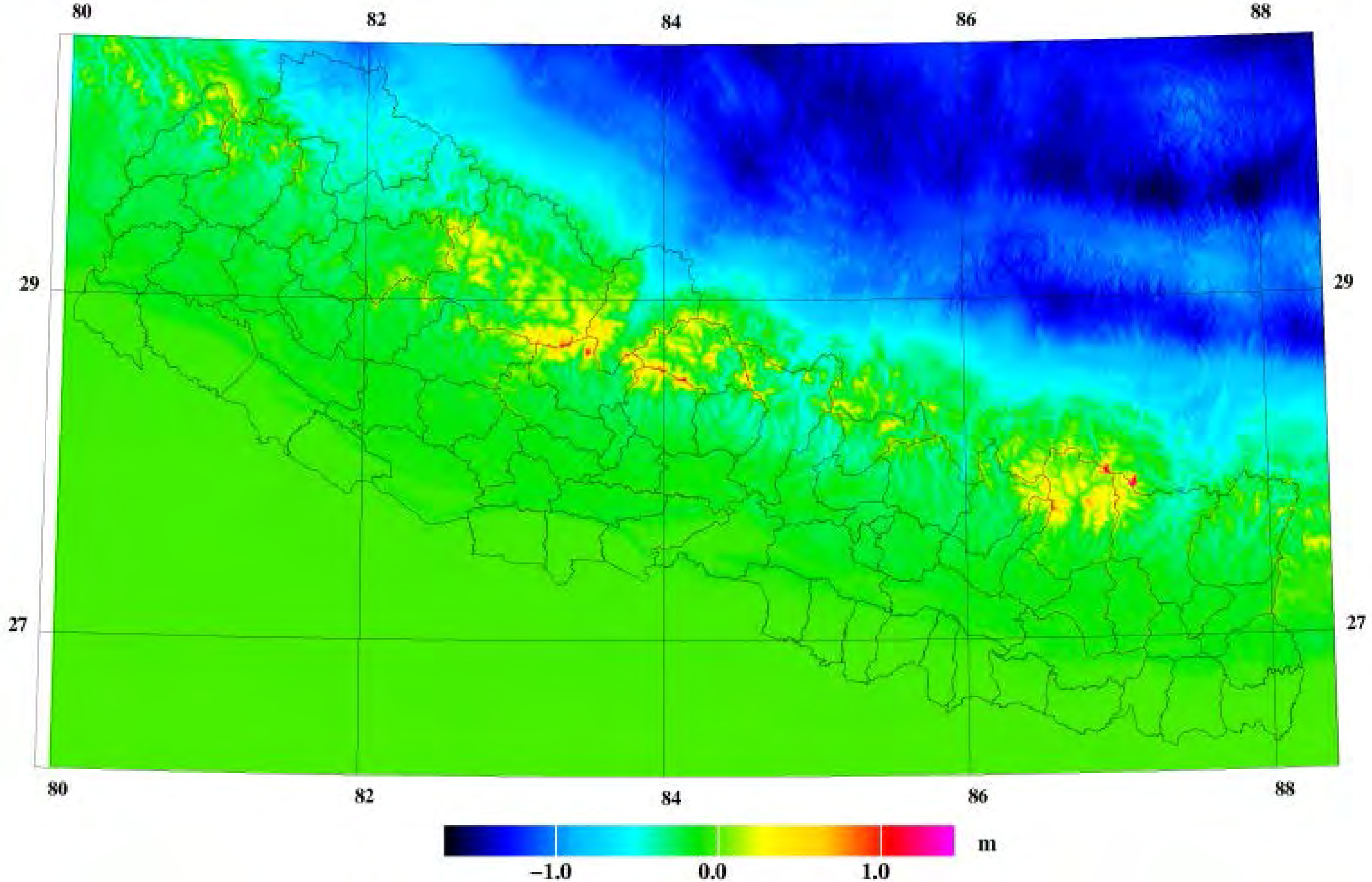
Difference N - ζ

Huge separation geoid-quasigeoid due to Tibetan Plateau

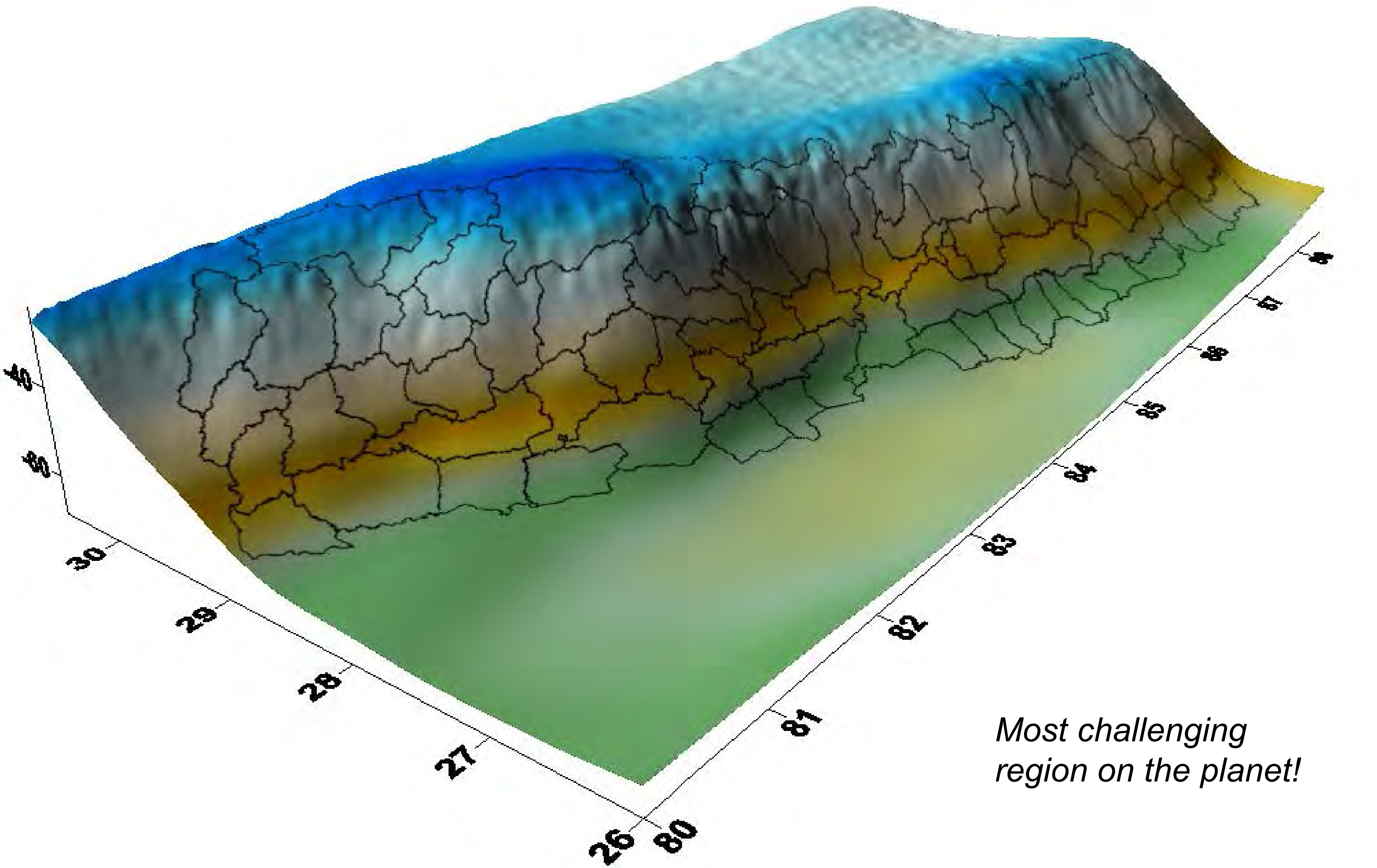


Difference Molodensky-Helmert geoids

Molodensky: $\zeta - N \approx -\frac{\Delta g_B}{\gamma_o} H$ Helmert: $N = N^* - \pi G\rho/\gamma\, h2$ + *higher-order*



Final geoid



Most challenging region on the planet!

Geoid determination – GPS levelling comparison

Very limited data set –
8 points in Kathmandu Valley
(large bias due to datum error
.. GPS height bias 18.5 m!)

Comparison of geoid in Kathmandu Valley

Geoid model	Mean	R.m.s. (m)
Geoid model (Molodensky)	19.32	0.07
Geoid model (Helmert)	19.19	0.08
GOCE	19.75	0.51
EGM08	18.51	0.30

Bias of GPS => Heights in Nepal datum lower than in (EGM08) World Height System

$\delta W_0 \sim 70$ cm

Height of Mt Everest

- Classical determination:

Triangulation from Himalaya foothills, Survey of India mid-1800's (Bengal Bay datum)
Chinese determination by levelling and triangulation from Yellow Sea (1970's onwards)

- GPS measurements: China and Italy – 1995, 1999, 2005 ..

Heights of Mt Everest (meter)

Survey of Nepal Official value	**8848**
China (Y. Chen, 2005)	**8847.93**
Nat. Geographic Society (Washburn et al, 1999 - EGM96 based)	**8850**
Ellipsoidal height ITRF (Chen, 2005; Poretti et al, 2004)	8821.40 ±.03
Geoid height (Mol.meth)	-25.71
Geoid-quasigeoid sep.	-2.47
Height from Molodensky	**8847.11**
Geoid height (Helmert)	-26.66
Height from Helmert	**8848.06**

Mt Everest summit from south, Dec 17, 2010


GRAVSOFT demo

GRAVSOFT

Set of DTU-Space / UCPH (late C C Tscherning) Fortran programs … sharing common format
Gridding, selection, interpolation, FFT, collocation, satellite altimetry, terrain models

GEOCOL - least-squares collocation and computation of reference fields.
GPCOL – least-squares collocation, especially good for downward continuation
EMPCOV, COVFIT, GPFIT - empirical covariance function estimation and fitting.
STOKES - Stokes' formula integration by grids.
GEOFOUR, SPFOUR, SP1D - FFT gravity field modelling (planar or spherical).
GEOGRID - rapid gridding by collocation or weighted means.
TC - terrain effects by prism integration.
TCFOUR - terrain effects by FFT methods.
COVFFT - covariance functions by FFT.

GEOIP - interpolation from grids to points or another grid.
SELECT - thin data or make average grids.
FCOMP - add/subtract data files
GCOMB - add/subtract and merge grid data
TCGRID - average grids and make reference topography
POINTMASS - make grid or data list of point-mass effects.
GEOID, GBIN - interpolation and conversion of fast binary grid format

GRAVSOFT is *not* freeware … *made available for non-commercial scientific work*

GRAVSOFT standard formats

Point data format: Data list in free format, with lines

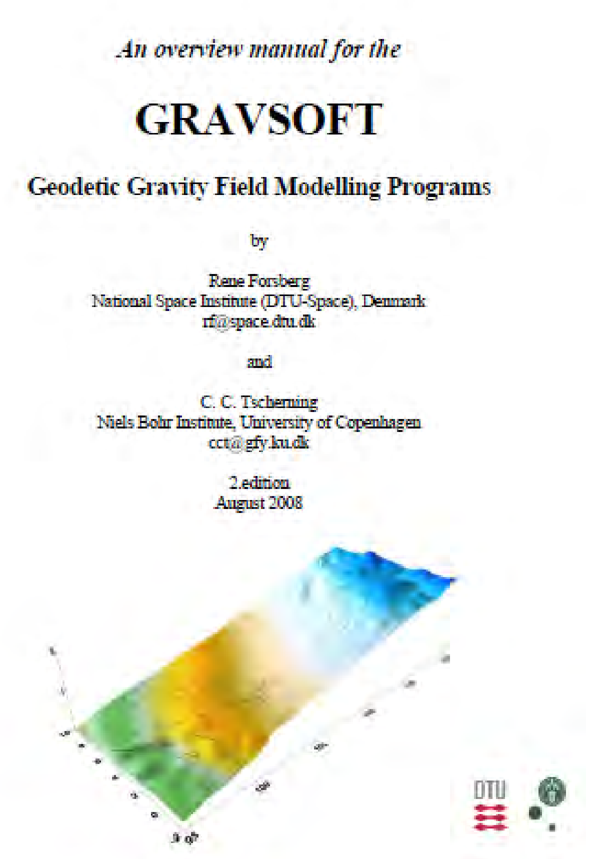
id, φ, λ (degrees), h, data1, data2, ...

Grid data: Data stored rowwise from N to S, initiated with label:

φ_1 , φ_2 , λ_1 , λ_2 , $\Delta\varphi$, $\Delta\lambda$

d_{n1} d_{n2} ... d_{nm}
......
......
d_{11} d_{12} .... d_{1m}

Unknown data are signalled by "9999". Grids may be in UTM projection.


An overview manual for the

GRAVSOFT

Geodetic Gravity Field Modelling Programs

by

Rene Forsberg
National Space Institute (DTU-Space), Denmark
rf@space.dtu.dk

and

C. C. Tscherning
Niels Bohr Institute, University of Copenhagen
cct@gfy.ku.dk

2.edition
August 2008

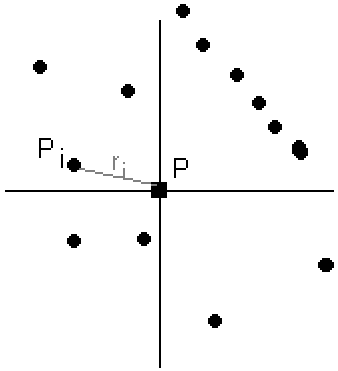
GRAVSOFT gridding

- GEOGRID central program to perform interpolation using collocation (mode 1) or weighted means (mode 2)
- *Collocation: second order Gauss-Markov covariance function*:

$$\hat{s} = C_{sx} C_{xx}^{-1} \underline{x} \qquad C(s) = C_0 (1 + \frac{s}{\alpha}) e^{(-s/\alpha)}$$

- *s* is the distance, C_0 is the signal variance, α is the correlation length …
- *Weighted means prediction (power 2) – "quick and dirty"*:

$$\hat{s} = \frac{\sum_i \frac{x_i}{r_i^2}}{\sum_i \frac{1}{r_i^2}}$$



select 5 closest pr quadrant

- In practice: select closest neighbours (e.g. 5/quadrant)
- *New 2008 module: "fit_geoid" … does all steps of geoid fitting in one go*

Useful software for PC applications

EF-commander – makes file navigation easy ..
VI or EDITPLUS – essential with a good text editor ..
MINGW32 – GNU fortran compiler
SURFER – powerful graphics software (Golden Software)

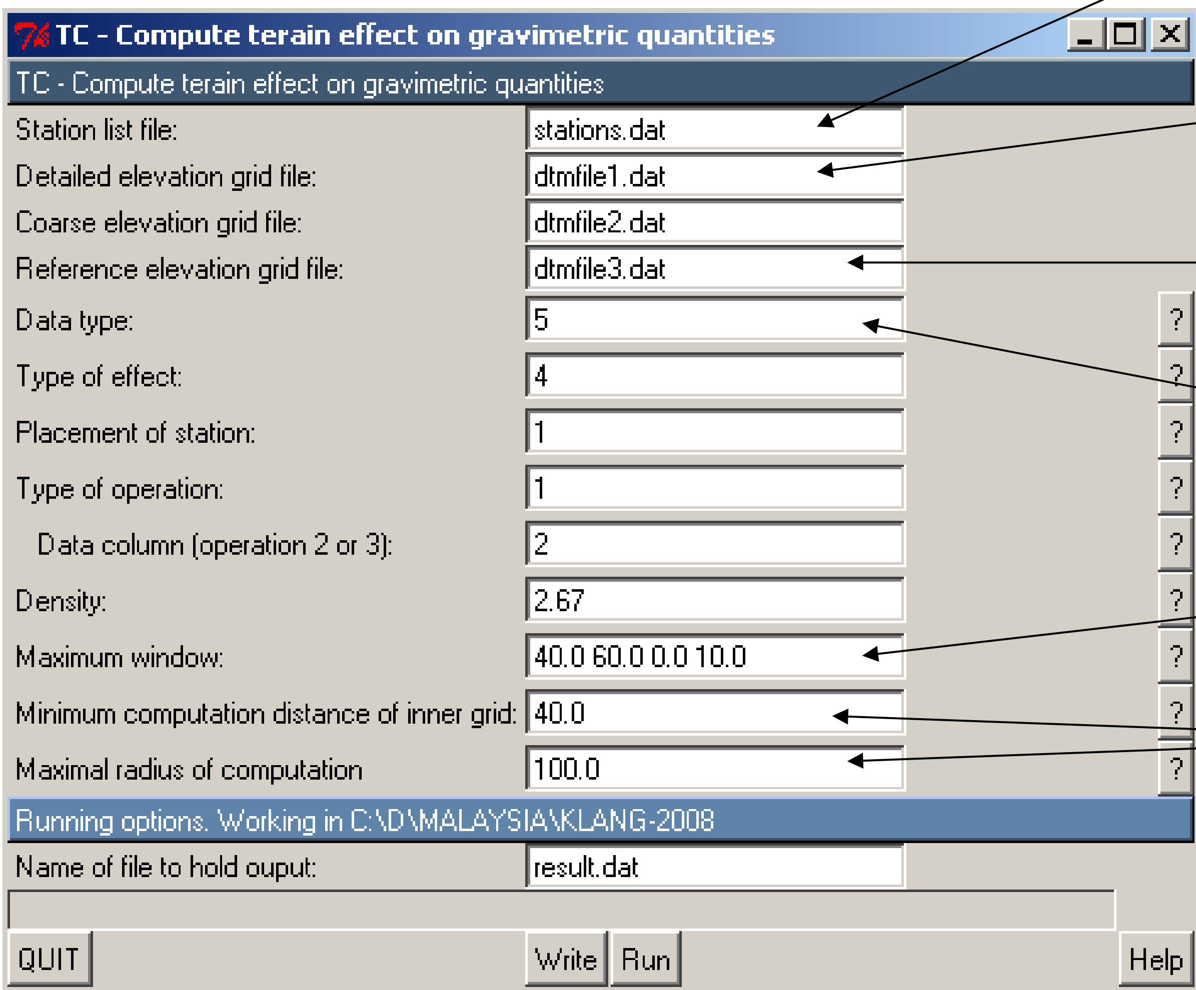
…. Gravsoft grid format -> surfer grid: G2SUR
…. Coastline files: .BNA
…. Allows posting (point plots), colour and contour plots, 3-D views ..

Supplementary tool: ”job.bat” - allows UNIX-like job operations in windows (big help)!

```
tcgrid <<!
nmdtm5
nmdtmref
0 0 0 0 0          ! dummy values, may be used to select smaller area
2 2 9 9            ! average 2 x 2 cells, then do 9 x 9 moving window
!
```

To run properly set up ”path” parameter in windows – also needed for Python to run
(start – settings – control panel – system – advanced – environment variables)

Python interface to TC



Data points to be computed

DEM file, must be larger than the data area. 2nd DEM optional

RTM reference DEM, filter with TCGRID or GFILT

1=δg, 3=geoid,
5 = Δg, 7 = T_{zz} ..

Important: prisms only used in this "fixed" area

Define r_1 and r_2

Reference to TC:
Ohio State University report 355, 1984