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Longitudinal data modelling

Peterhouse College, Cambridge

6th to 8th April 2011

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This course is prepared by

Anna Brown, PhD ab936@medschl.cam.ac.uk

Research Associate, Department of Psychiatry

Tim Croudace, PhD tjc39@cam.ac.uk

Senior Lecturer, Department of Psychiatry

Jon Heron, PhD Jon.Heron@bristol.ac.uk

Research Fellow, School of Social and Community Medicine

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E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

RDI Researcher Development Initiative

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Timetable

- Wednesday 6th April
  13:00 sandwich lunch
  13:30 start
  18:00 finish

- Thursday 7th April
  09:00 start
  13:00 sandwich lunch
  18:00 finish

- Friday 8th April
  09:00 start
  13:00 finish, sandwich lunch

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Programme

- Day 1

  Longitudinal designs
  Models for change
  Autoregressive models

- Day 2

  Growth curve models
  Sequential cohort design

- Day 3

  Growth mixture models
  Measurement invariance in longitudinal studies

Introduction

LONGITUDINAL DATA AND DESIGNS

Basic explanations of change

- Imagine we measured religiousness
  - Cross-sectionally with in 20, 40 and 60 year-olds
  - Longitudinally (3 repeated measures with 20 year-olds)
- Possible explanations for any differences?
  - Age effect (people change as they grow older)
  - Cohort effect (people differ depending on the time when they were born)
  - Period effect (overall change in the population during the course of longitudinal study)
- These alternative explanations are linearly dependent

Cohort + Age = Period

- Assumptions need to be made

Basic longitudinal designs

- Simultaneous cross-sectional studies
- Trend studies
- Time series studies
- Intervention studies

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Simultaneous cross-sectional studies

Age Group	Sample	Occasion	Observed Variables
A1	S1	T1	X1,X2, X3,…,Xm
A2	S2	T1	X1,X2, X3,…,Xm
…	…	…	…
AG	SG	T1	X1,X2, X3,…,Xm

- Example: educational progress in mathematics achievement

- Different age groups are sampled on the same occasion
- Any "change" assumes there is no cohort effect
- Any changes can be identified at the aggregate level only

Trend studies

Age Group	Sample	Occasion	Observed Variables
A1	S1	T1	X1,X2, X3,…,Xm
A1	S2	T2	X1,X2, X3,…,Xm
…	…	…	…
A1	ST	TT	X1,X2, X3,…,Xm

- Example: research on trends in crime prevalence among youth

- Random sample is drawn from the same population on different occasions
- Any changes can be identified at the aggregate level only

Time series studies

Age Group	Sample	Occasion	Observed Variables
A1	S1	T1	X1,X2,X3,…,Xm
A2	S1	T2	X1,X2,X3,…,Xm
…	…	…	…
AT	S1	TT	X1,X2, X3,…,Xm

- Example: relationship between personality characteristics, life events and disease

- The same subjects are followed at successive time points
- Possible to investigate intra-individual change
- Prospective versus retrospective designs

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Intervention studies

Age Group	Treatment group	No treatment group	Sample	Occasion	Observed Variables
A1	E1	C1	S1	T1	X1,X2,X3,…,Xm
A2	E1	C1	S1	T2	X1,X2,X3,…,Xm
…	…	…	…	…	…
AT	E1	C1	S1	TT	X1,X2, X3,…,Xm

- Example: effects of intervention or treatment

- The intervention or treatment affects only the subjects in experimental group
- Pre- and post-intervention data is collected from all subjects at successive time points

What you can research with only 2 time points

CHANGE MODELS

Why interest in change?

- Even basic two-measurement occasion design can answer developmental questions
- Change after an intervention (“before” and “after”)
  - Training or educational programme effectiveness
  - Drug effectiveness
  - Therapy effectiveness
- Focus of interest can be on
  - Mean change
  - Variation of change
  - Relationship between change and the baseline measure
  - Individual change as predicted by external variables

What is wrong with y2-y1?

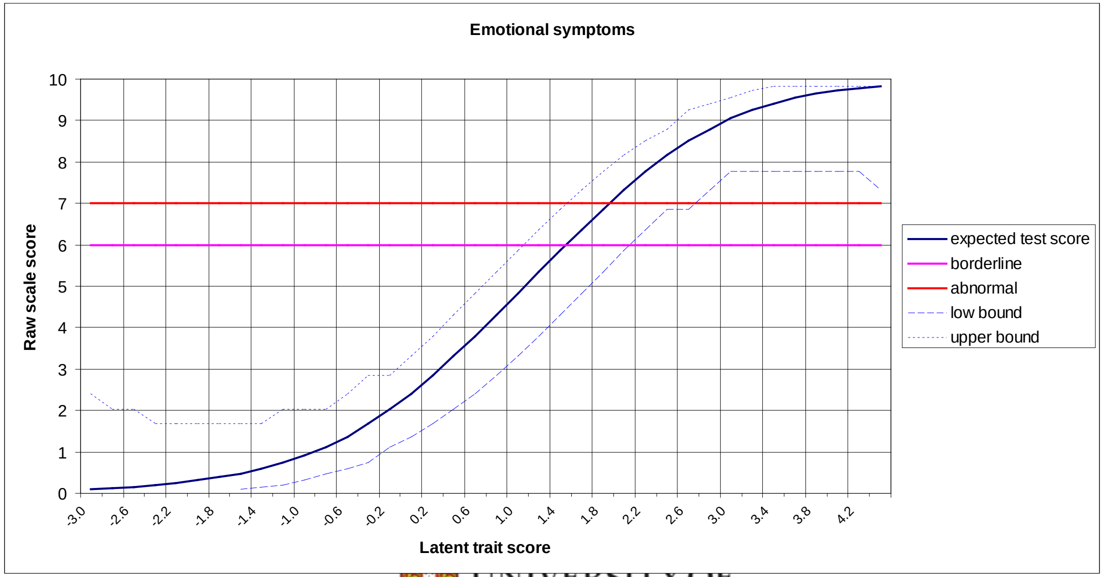
- When summed score is used, computing simple difference leads to spurious effects (Bereiter, 1963)
  - reliability of the difference score is inversely related to the test-retest correlation;
  - change may be not measured on the same scale for persons with different scores on the original measure;
  - spurious negative correlations between the baseline and the change score.
- These psychometric problems led to suggestions to abandon change measurement (Cronbach & Furby, 1970)

Sum scores are on ordinal scale

- Many psychometric instruments are scored by summing item responses
  - They are on ordinal, not interval, scale (e.g. Reise & Haviland, 2005)
  - Such scores typically preserve ordering of people well
  - But they can distort the distances between people, particularly at the extremes of the trait
- The meaning of the simple difference score of the same magnitude might be different depending on the baseline score.
  - smaller underlying difference for average baseline scores
  - larger difference for extreme baseline scores.
- It might not be possible to detect change due to floor and ceiling effects in the raw test score.

Latent modelling approach

- Latent trait modelling (also with categorical variables - IRT) can resolve these problems
- Item responses are indicators of underlying (latent) traits
- Latent traits are modelled so that they are unbounded, on the interval scale, and free of error
- Error of measurement is modelled (in IRT depends on the trait)



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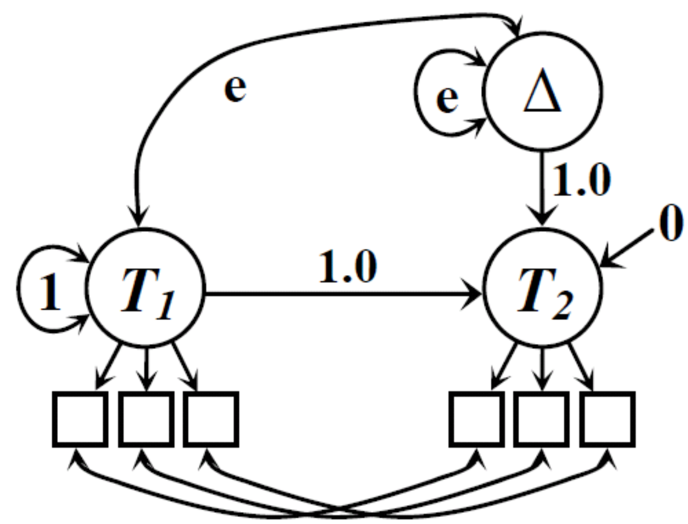
Latent change models

- Model latent traits at T1 and T2
  - Assume measurement invariance, i.e. item discriminations and intercepts stay the same over time (otherwise construct meaning changes)
  - Set the scale at T1 (for instance by setting mean=0, var=1)
  - Leave the scale at T2 freely estimated
  - Residual variances for each item are correlated over time (item-specific variance is likely to be dependent across time points)
- Now latent change can be defined
  - Little, T., Bovaird, J. & Slegers, D. (2005). Methods of the analysis of change. In Mroczek, D. & Little, T. (Eds.). Handbook of personality Development. Mahwah, NJ: Erlbaum.

Latent Difference score

T2 = T1 + Delta var(T2)=var(T1)+var(Delta)+2cov(T1,Delta)

- Variance of T2 is decomposed into
  - Variance associated wit one's absolute standing at Time 1
  - Variance associated with the absolute difference from Time 1
  - Covariance between baseline and difference
- LD score is useful to model
  - Mean change over time
  - Individual differences around that mean change
- Means
  - T1 is set to 0
  - Delta is estimated



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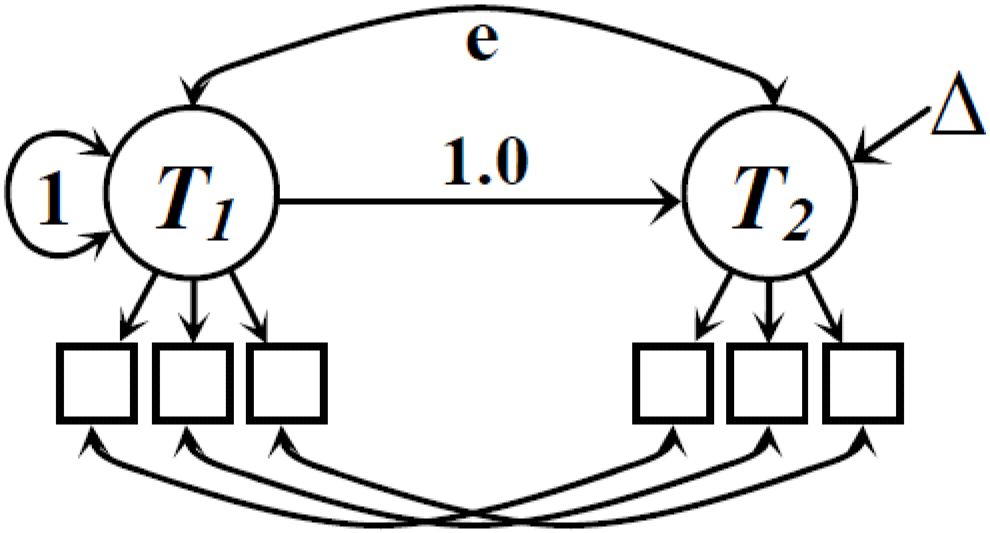
Latent Difference score (2)

- Simplified representation



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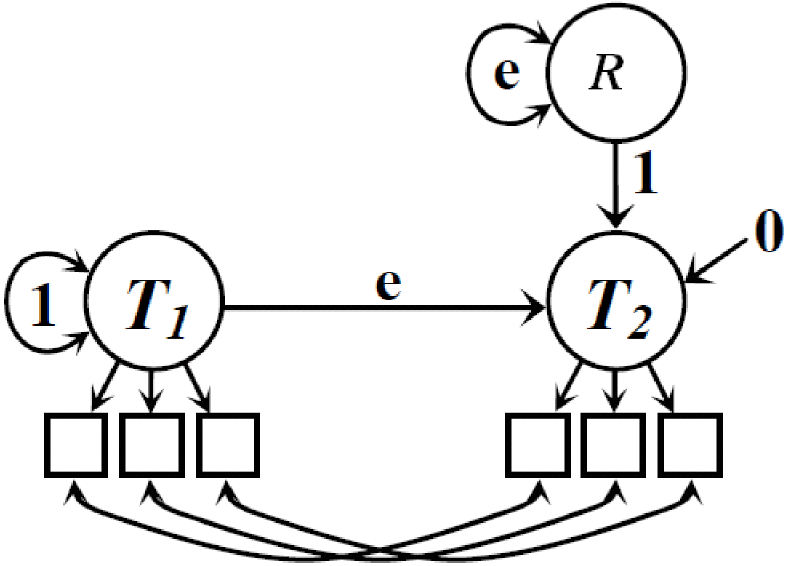
Latent Residual score

T2 = ρ*T1 + Res

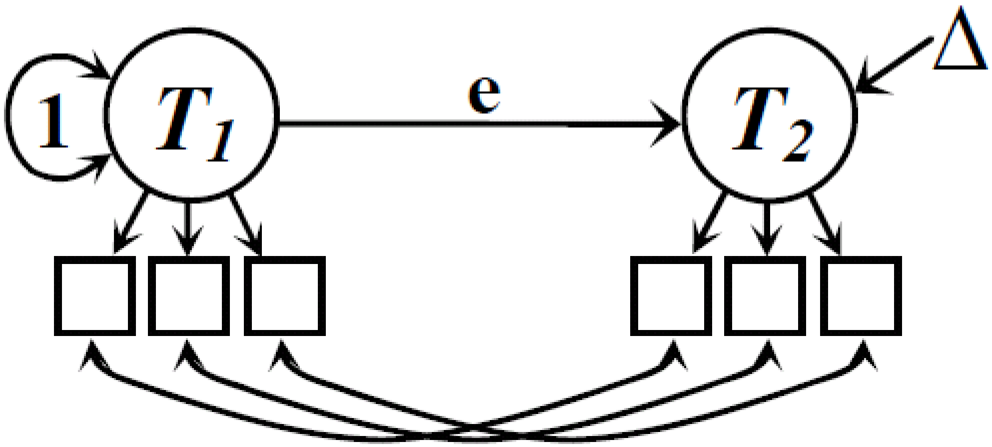
Var(T2) = ρ^2var(T1) + var(Res)

- Variance of T2 is decomposed into
  - one's relative standing at Time 1 (i.e. the degree of correlation)
  - the change in relative standing at Time 2
- LR score is most useful to examine
  - Stability of individual differences
  - The individual differences around that stability
- Mean structure
  - T1 is set to 0; Residual is 0
  - Intercept at T2 is estimated



Latent Residual score (2)

- Simplified representation



- Both LD and LR models are equivalent in terms of their ability to reproduce the observed variance-covariance matrix, and will have exactly the same fit

Example: patient-reported change using SDQ

- Strength and Difficulties Questionnaire (SDQ; Goodman); designed to screen children with mental health problems
- 5 subscales (4 describing "problems" and 1 "strength")
  - Hyperactivity, Emotional diff., Conduct problems, Peer problems, Pro-social (-)
- Total Difficulties is a sum of 4 "problem" subscales
  - Assumes the subscales measure one general factor
  - In fact the subscales form 2 broader factors

SDQ Externalising latent difference

- Parent-reported SDQ for N=2010 children who attended CAMHS providers
- Reports were completed on 2 occasions
  - On referral
  - After about 6 months (4 to 8 months) receiving treatment
- We will look at "Externalising" dimension
  - Hyperactivity (+), Conduct problems (+), Pro-social (-)
- We only have available subscale scores ranging from 0 to 10. We will treat them as "testlet" scores (ordinal data)
  - For simplicity, collapse every 2 categories into 1 (6 categories)
- We measure symptoms at T1 and T2; therefore Latent Difference score will also conceptualise symptoms
  - We expect mean difference to be negative (reduction)

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Measurement model setup

```
ANALYSIS:  ESTIMATOR IS WLSMV;      PARAMETERIZATION=THETA;
MODEL:
 T1 BY HYP_T1* (1)         !Time 1 Externalising
   COND_T1 (3)
   PROS_T1 (5);
 T1@1;   !Set the factor metric at T1, mean is automatically 0 and variance is set to 1
   T2 BY HYP_T2* (1)        !Time 2  Externalising
   COND_T2 (3)
   PROS_T2 (5);
 !thresholds to be the same across 2 time points
[HYP_T1$1  HYP_T2$1] (h1);
[COND_T1$1  COND_T2$1] (c1);
[PROS_T1$1  PROS_T2$1] (s1);
……  !more thresholds go here
!common specific variances across T1 and T2
HYP_T1 WITH HYP_T2*;
COND_T1 WITH COND_T2*;
PROS_T1 WITH PROS_T2*;
```

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Model 1: Latent difference

```
!score at T2 is determined by T1 plus DIFF, so disturbance is 0
T2@0; ! to compute factor scores, set the disturbance to small non-
  !zero value, e.g. 0.001

!Regression with fixed effect
T2 ON T1@1;

! Latent Difference score
DIFF BY T2@1;  ! to introduce a new latent variable use BY statement
DIFF*;  !variance of difference score is estimated
[DIFF*];  !mean of difference score is estimated
DIFF WITH T1*; !covariance of T1 and difference score is estimated
```

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Results: Measurement model

	Estimate	S.E.	Est./S.E.	P-Value
T1 BY				
HYP_T1	0.904	0.039	23.358	0.000
COND_T1	2.548	0.314	8.107	0.000
PROS_T1	-0.753	0.032	-23.710	0.000
T2 BY				
HYP_T2	0.904	0.039	23.358	0.000
COND_T2	2.548	0.314	8.107	0.000
PROS_T2	-0.753	0.032	-23.710	0.000
Thresholds				
HYP_T1$1	-1.790	0.051	-35.036	0.000
HYP_T2$1	-1.790	0.051	-35.036	0.000
HYP_T1$2	-0.909	0.039	-23.164	0.000
HYP_T2$2	-0.909	0.039	-23.164	0.000

….. Etc.

Factor loadings are invariant across time

Thresholds are invariant too (appear in Mplus output in different order)

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Results: Structural model

	Estimate	S.E.	Est./S.E.	P-Value
DIFF WITH				
T1	-0.136	0.023	-5.917	0.000
Means				
DIFF	-0.288	0.021	-13.592	0.000
Variances				
T1	1.000	0.000	999.000	999.000
DIFF	0.408	0.031	13.267	0.000

1) Externalising problems reduced on average;
2) those with higher baseline improved MORE (less increase in symptoms)

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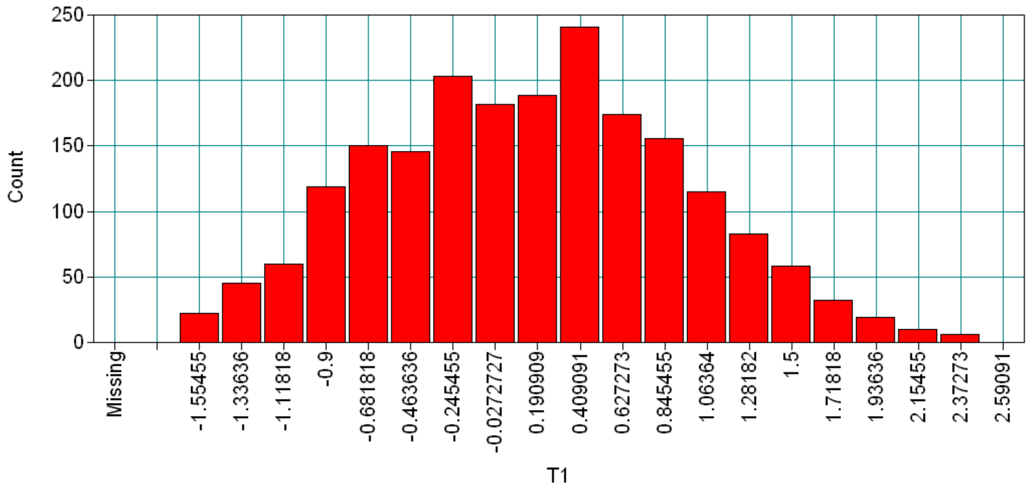
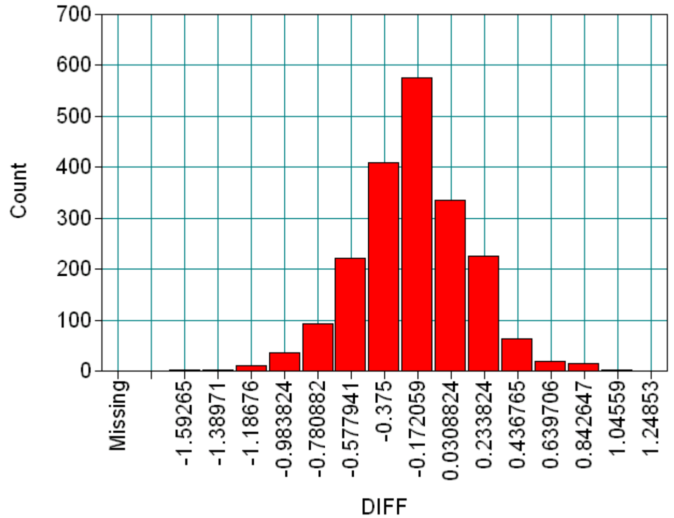
Estimated scores

- Externalising problems at Time 1



- Difference score



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Model 2: Latent residual

```
!score at T2 is determined by T1 and RES, so disturbance is 0
T2@0;       ! to compute factor scores, set to small non-zero value
[T2*];      !intercept at T2 is estimated

!Regression effect estimated
T2 ON T1*1;

!Latent residual
RES BY T2@1;
RES*1;  !variance of residual score is estimated
!Latent Residual is orthogonal to T1
RES WITH T1@0;
```

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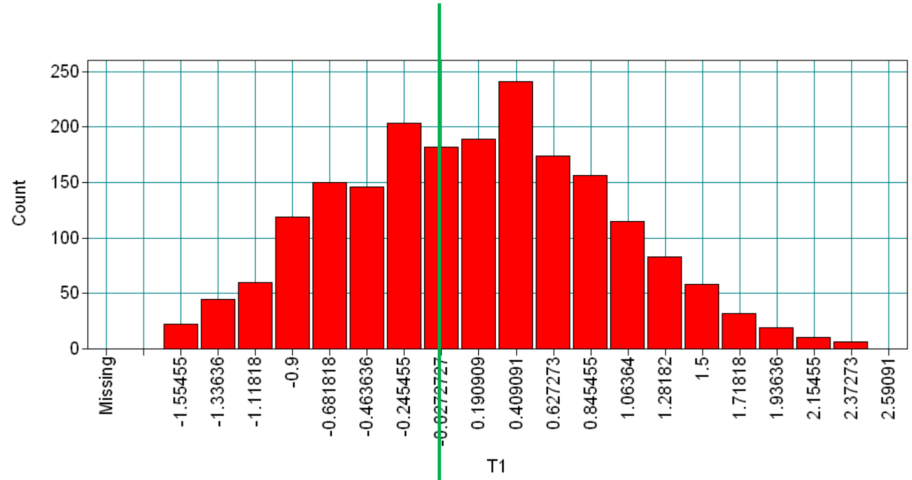
Results: Structural model

	Estimate	S.E.	Est./S.E.	P-Value
T2 ON				
T1	0.865	0.023	37.745	0.000
RES WITH				
T1	0.000	0.000	999.000	999.000
Intercepts				
T2	-0.288	0.021	-13.591	0.000
Variances				
T1	1.000	0.000	999.000	999.000
RES	0.389	0.030	13.048	0.000

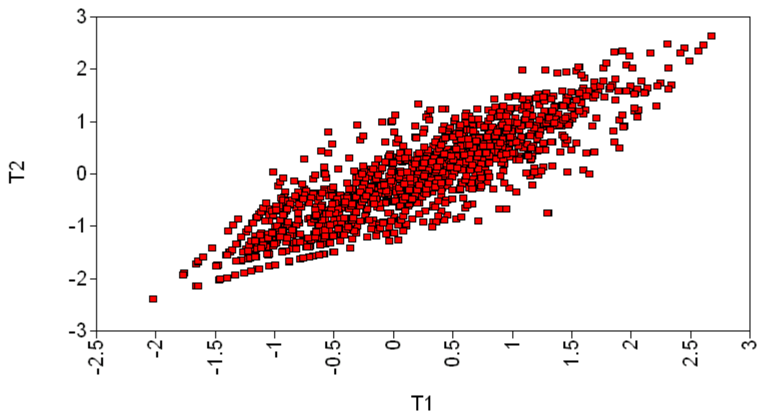
1. Externalising problems reduced on average;
2. Problems at T2 strongly relate to baseline

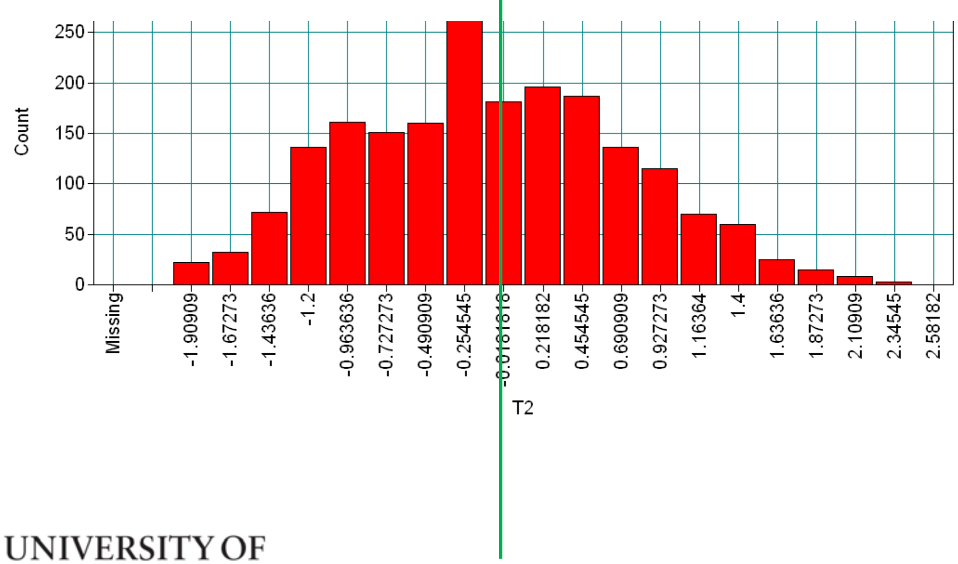
Estimated Scores

- Problems at Time 1



- Problems at Time 2





Practical 1

- SDQ for a community sample (pupils year 7) with 1 year interval
- Emotional Difficulties measured with five items
  1. I get a lot of headaches, stomach-aches or sickness
  2. I worry a lot
  3. I am often unhappy, down-hearted or tearful
  4. I am nervous in new situations. I easily lose confidence
  5. I have many fears, I am easily scared
- Data can be found in “PupilSDQEmot.dat” file
- Variables are YGROUP, GENDER (coded 0=male, 1=female),
  T1_i1-T1_i5 and T2_i1-T2_i5
- Tasks:
  - Specify and test the latent difference score model
  - Have the Emotional Difficulties increased or decreased?
  - Test genders separately. Any observations?

AUTOREGRESSIVE MODELS

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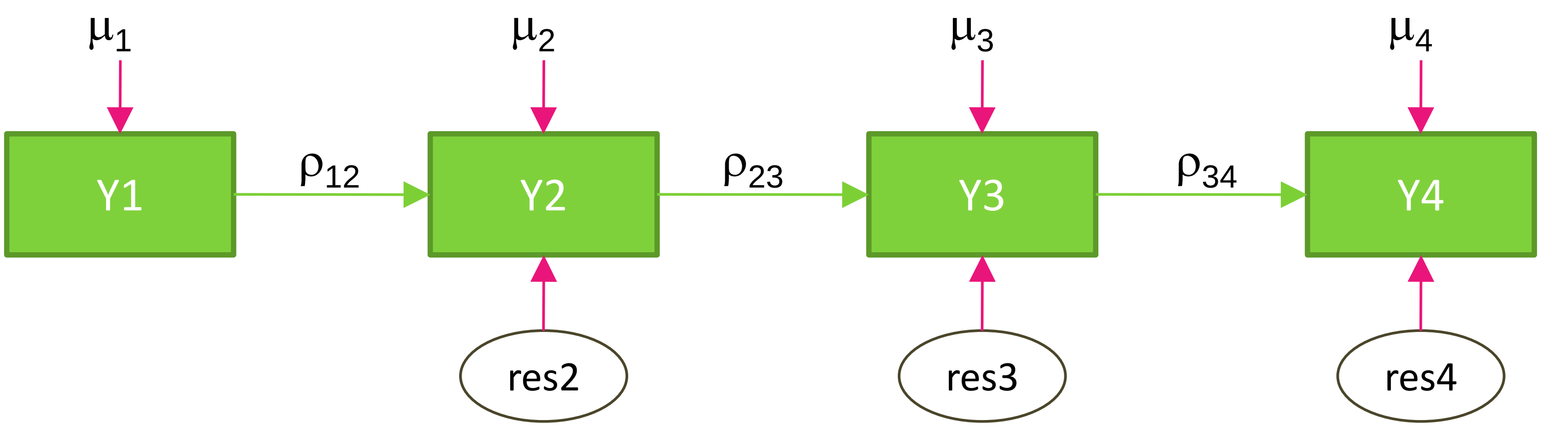
Univariate autoregressive model

- We have a panel of univariate data (i.e. test score) taken at consecutive time points
- Each subsequent measure is a function of the immediately preceding measure plus random disturbance (autoregressive)
  - Can include covariance structure only or mean and covariance
- The key feature of such data is that correlations with initial measure become progressively lower as time increases (simplex structure)



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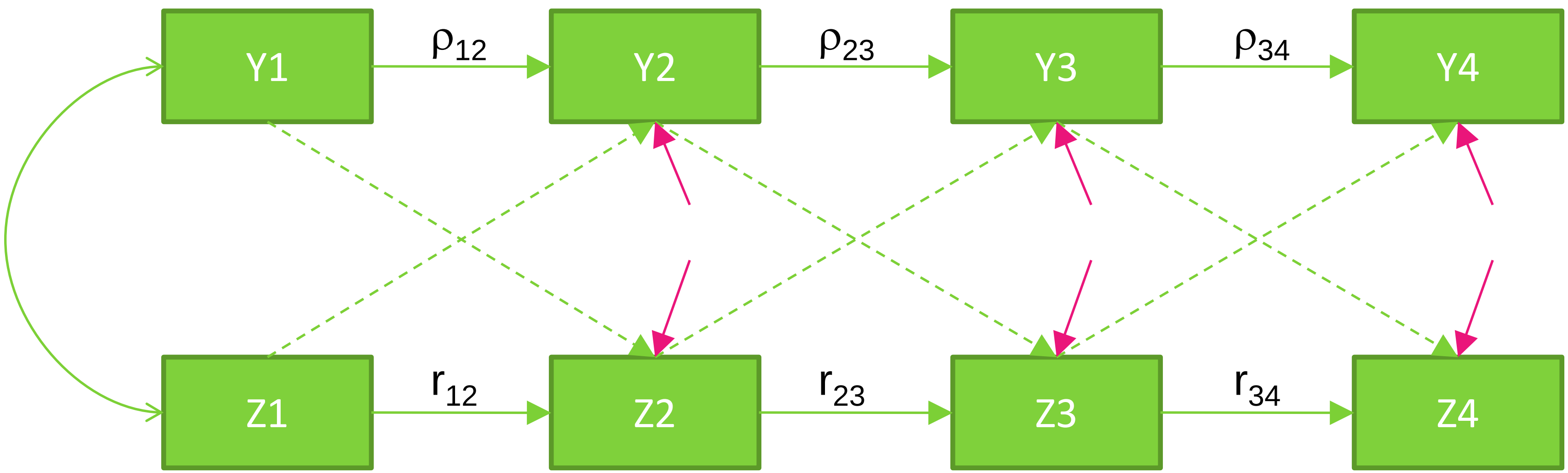
Crosslagged autoregressive model

- The univariate case can be easily extended to multivariate
- Variables are allowed to predict other variables at subsequent time periods (crosslagged)
- Error terms for the same time point may be allowed to correlate (unexplained fluctuations in performance on the day that is common to both tests)



Example – WISC data

- Wechsler Intelligence Test for Children, study by Osborne and Suddick (1972)
- Two subtests (Verbal and Non-verbal) – formed from 5 subscales each
- N=204 children took the tests in at ages 6, 7, 9 and 11

	V1	V2	V3	V4	N1	N2	N3	N4
M	19.58	25.41	32.60	43.74	18.00	27.68	39.35	50.92
SD	5.83	6.13	7.34	10.70	8.37	10.02	10.31	12.52

1							
.717	1						
.726	.756	1					
.653	.727	.797	1				
.609	.584	.622	.617	1			
.517	.600	.591	.631	.779	1		
.467	.530	.544	.593	.732	.793	1	
.476	.511	.529	.609	.695	.785	.811	1

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Simplex model: Syntax

```
TITLE: Autoregressive Simplex model with Wechsler Intelligence Scale
DATA: FILE IS WISC.dat;
TYPE IS CORRELATION MEANS STDEVIATIONS;
NOBSERVATIONS = 204;   !Sample size is rather small

VARIABLE: NAMES ARE V1-V4 NV1-NV4;
    USEVARIABLES ARE V1-V4; !Only verbal test

MODEL:
     V2  ON  V1;  V3 ON V2;   V4 ON V3;

OUTPUT: RES; STDYX;  !Ask for residuals
```

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Simplex model: Results

Unstandardized

		Estimate	S.E.	Est./S.E.	P-
V2	ON				
V1		0.754	0.051	14.691	0.000
V3	ON				
V2		0.905	0.055	16.496	0.000
V4	ON				
V3		1.162	0.062	18.847	0.000
Intercepts					
V2		10.649	1.048	10.159	0.000
V3		9.598	1.434	6.692	0.000
V4		5.864	2.060	2.847	0.004
Residual Variances					
V2		18.170	1.799	10.100	0.000
V3		22.971	2.274	10.100	0.000
V4		41.560	4.115	10.100	0.000

Standardized

		Estimate	S.E.	Est./S.E.	P-Value
V2	ON				
V1		0.717	0.034	21.075	0.000
V3	ON				
V2		0.756	0.030	25.201	0.000
V4	ON				
V3		0.797	0.026	31.205	0.000
Intercepts					
V2		1.741	0.238	7.304	0.000
V3		1.311	0.245	5.357	0.000
V4		0.549	0.212	2.591	0.010
Residual Variances					
V2		0.486	0.049	9.960	0.000
V3		0.428	0.045	9.446	0.000
V4		0.365	0.041	8.960	0.000

Simplex model: Fit

The fit is rather bad…

Chi-Square Test of Model Fit		
Value	58.473	
Degrees of Freedom	3	
P-Value	0.0000	
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.301	
90 Percent C.I.	0.237	0.371
Probability RMSEA <= .05	0.000	
CFI/TLI		
CFI	0.904	
TLI	0.808	

Examining residuals

Normalized Residuals for Means/Intercepts/Thresholds

V1	V2	V3	V4
0.000	0.000	0.000	0.000

Normalized Residuals for Covariances/ Correlations/ Residual Correlations

	V2	V3	V4	V1
V2	0.000			
V3	0.000	0.000		
V4	1.438	0.000	0.000	
V1	0.000	2.126	2.643	0.000

Correlations between non-adjacent time points are not explained

Multivariate Simplex model: Syntax

```
MODEL:
!Autoregressive part
V2  ON  V1;  V3 ON V2;   V4 ON V3;
NV2  ON  NV1;  NV3 ON NV2;   NV4 ON NV3;

!Crosslagged part
NV2 ON V1; V2 ON NV1;
NV3 ON V2; V3 ON NV2;
NV4 ON V3; V4 ON NV3;

!Correlated Residuals
V2 WITH NV2;
V3 WITH NV3;
V4 WITH NV4;
```

Multivariate Simplex model: Results

Unstandardized

	Estimate	S.E.	Est./S.E.	P-Value
V2 ON				
V1	0.604	0.062	9.685	0.000
NV1	0.172	0.043	3.949	0.000
V3 ON				
V2	0.751	0.066	11.346	0.000
NV2	0.157	0.040	3.884	0.000
V4 ON				
V3	0.982	0.070	14.085	0.000
NV3	0.235	0.050	4.734	0.000
NV2 ON				
NV1	0.883	0.066	13.379	0.000
V1	0.116	0.095	1.228	0.220
NV3 ON				
NV2	0.764	0.055	14.007	0.000
V2	0.142	0.089	1.598	0.110
NV4 ON				
NV3	0.902	0.058	15.473	0.000
V3	0.213	0.082	2.597	0.009

Standardized

	Estimate	S.E.	Est./S.E.	P-Value
V2 ON				
V1	0.574	0.053	10.772	0.000
NV1	0.234	0.059	3.987	0.000
V3 ON				
V2	0.627	0.048	12.956	0.000
NV2	0.215	0.055	3.905	0.000
V4 ON				
V3	0.674	0.040	16.652	0.000
NV3	0.226	0.048	4.736	0.000
NV2 ON				
NV1	0.738	0.045	16.467	0.000
V1	0.068	0.055	1.228	0.220
NV3 ON				
NV2	0.742	0.043	17.349	0.000
V2	0.085	0.053	1.598	0.110
NV4 ON				
NV3	0.743	0.038	19.587	0.000
V3	0.125	0.048	2.594	0.009

Multivariate Simplex model: Fit

Chi-Square Test of Model Fit	
Value	96.779
Degrees of Freedom	12
P-Value	0.0000
RMSEA (Root Mean Square Error Of Approximation)	
Estimate	0.186
90 Percent C.I.	0.153 0.221
Probability RMSEA <= .05	0.000
CFI/TLI	
CFI	0.935
TLI	0.853

The fit is not good

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Multivariate Simplex model: Residuals

Normalized Residuals for Covariances/ Correlations/ Residual Correlations

	V2	V3	V4	NV2	NV3	NV4	V1	NV1
V2	0.000							
V3	0.000	0.000						
V4	6.369	0.000	0.000					
NV2	0.000	0.000	5.676	0.000				
NV3	0.000	0.000	0.000	0.000	0.000			
NV4	1.744	0.000	0.000	15.226	0.000			
V1	0.000	7.039	10.836	0.000	1.350	5.505	0.000	
NV1	0.000	5.409	10.283	0.000	8.963	16.897	0.000	0.000

Correlations between non-adjacent time points are not explained

Simplex models: Discussion

- Simplex models clearly do not fit our data, and this is common with other data of this sort
- Correlations between the first and the subsequent occasions fail to decrease to the extent the model predicts:

  $\rho 13 = \rho 12 * \rho 23$

- Is the model wrong?

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Measurement error

- Psychological constructs are not measured perfectly
- Our observed variables contain the true value and the measurement error:

  $y = t + e$; $var(y) = var(t) + var(e)$
- Covariance between y1 and y2 therefore is:

  $cov(y1,y2) = cov(t1 + e1, t2 + e2) = cov(t1, t2)$
- Observed covariances are actually covariances of true scores
- But observed variances are sums of true and error variances!
- We are interested in true scores and their relationships so need to separate the error variances from our observed scores

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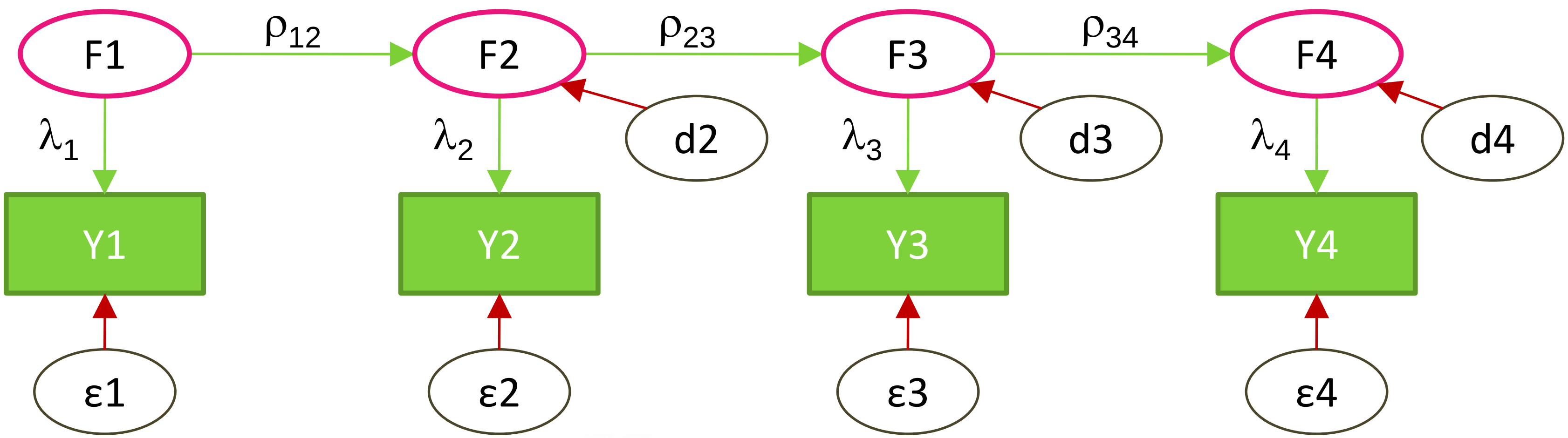
Latent Autoregressive model

- The autoregressive relationships are between latent constructs
- Errors of measurement are separated
- But there are many more parameters to estimate
- Additional constraints are needed for identification



Identification constraints

- Factor variances should be fixed
- If we use the same or parallel tests to measure a construct, loadings λ_i should be constrained equal
  - To make sure the latent construct stays the same across measurement occasions
- Measurement error terms can be constrained equal
  - The same reliability across measurement occasions
- With only 3 time points, such model is just identified – cannot be tested
  - Additional constraint of equal regression coefficients might be imposed

Latent Autoregressive model: Syntax

```
MODEL:
WISC1 BY V1; ! each factor loading =1
WISC2 BY V2; ! so that
WISC3 BY V3; ! construct has the same meaning
WISC4 BY V4;

!Reliability is the same
V1 V2 V3 V4 (1);

!Autoregressive part between latent variables
WISC2 ON WISC1;
WISC3 ON WISC2;
WISC4 ON WISC3;
```

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Latent Autoregressive model: Results

Unstandardized

	Estimate	S.E.	Est./S.E.	P-Value
WISC2 ON				
WISC1	1.010	0.082	12.356	0.000
WISC3 ON				
WISC2	1.186	0.074	15.983	0.000
WISC4 ON				
WISC3	1.375	0.077	17.940	0.000

Standardized

	Estimate	S.E.	Est./S.E.	P-Value
WISC1 BY				
V1	0.865	0.024	36.334	0.000
WISC2 BY				
V2	0.879	0.020	43.552	0.000
WISC3 BY				
V3	0.918	0.014	66.496	0.000
WISC4 BY				
V4	0.962	0.006	151.242	0.000
WISC2 ON				
WISC1	0.947	0.037	25.758	0.000
WISC3 ON				
WISC2	0.946	0.024	39.543	0.000
WISC4 ON				
WISC3	0.900	0.023	39.078	0.000

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Latent Autoregressive model: Fit

Chi-Square Test of Model Fit

Value	2.115
Degrees of Freedom	2
P-Value	0.3472

Notice only 2 degrees of freedom – that is why we needed constraints

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.017
90 Percent C.I.	0.000 0.141
Probability RMSEA <= .05	0.513

CFI/TLI

CFI	1.000
TLI	0.999

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Examining residuals

Normalized Residuals for Covariances/ Correlations/ Residual Correlations

	V1	V2	V3	V4
V1	0.000			
V2	-0.025	0.028		
V3	0.160	-0.075	-0.019	
V4	-0.221	0.092	0.018	0.000

Residuals are small

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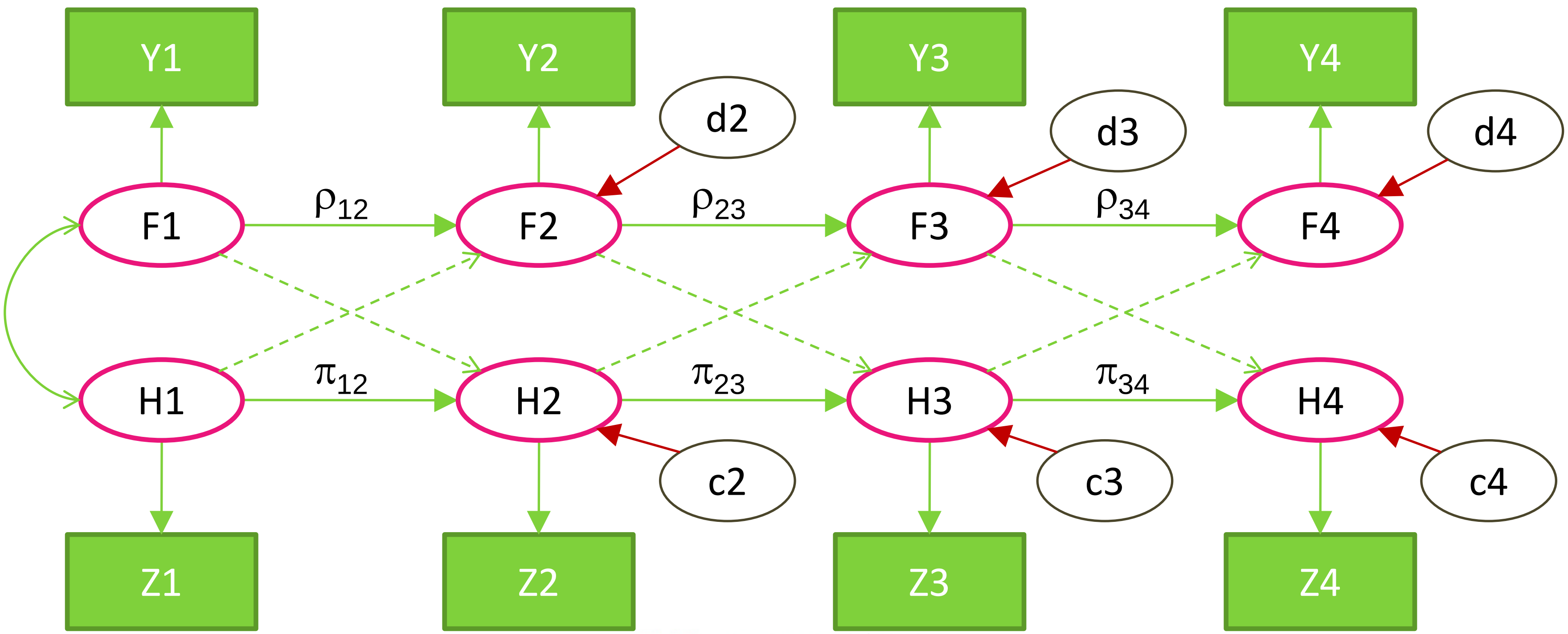
Crosslagged latent autoregressive model

- One latent construct might influence the other at next time point
- There also might be correlated disturbances



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Multivariate latent autoregressive: Syntax

```
MODEL:
VER1 BY V1; ! each factor loading =1
VER2 BY V2; ! so that
VER3 BY V3; ! construct has the same meaning
VER4 BY V4;
NONVER1 BY NV1;
NONVER2 BY NV2;
NONVER3 BY NV3;
NONVER4 BY NV4;
!Reliability is the same
V1 V2 V3 V4 (1);
NV1 NV2 NV3 NV4 (2);
!Autoregressive part between latent variables
VER4 ON VER3*1;   NONVER4 ON NONVER3*1;
VER3 ON VER2*1;   NONVER3 ON NONVER2*1;
VER2 ON VER1*1;   NONVER2 ON NONVER1*1;
```

Multivariate Latent Autoregressive model: Results

- Unstandardized

VER2 ON				
VER1	1.088	0.081	13.497	0.000
NONVER2 ON				
NONVER1	1.171	0.073	16.111	0.000
VER3 ON				
VER2	1.218	0.077	15.883	0.000
NONVER3 ON				
NONVER2	0.967	0.051	18.907	0.000
VER4 ON				
VER3	1.405	0.078	18.075	0.000
NONVER4 ON				
NONVER3	1.154	0.059	19.688	0.000
NONVER1 WITH				
VER1	28.280	3.806	7.430	0.000
NONVER4 WITH				
VER4	6.298	2.863	2.200	0.028

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Multivariate Latent Autoregressive model: Fit

Value	27.667
Degrees of Freedom	18
P-Value	0.0673

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.051
90 Percent C.I.	0.000 0.087
Probability RMSEA <= .05	0.438

CFI/TLI

CFI	0.993
TLI	0.989

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Multivariate Latent Autoregressive model: Residuals

Normalized Residuals for Covariances/ Correlations/ Residual Correlations

V1	0.222							
V2	0.139	-0.086						
V3	0.100	0.034	-0.081					
V4	-0.461	0.013	0.065	0.015				
NV1	0.325	-0.231	0.106	0.314	-0.091			
NV2	-0.667	0.129	-0.108	0.638	0.053	-0.026		
NV3	-0.887	-0.306	-0.250	0.606	0.047	-0.079	0.085	
NV4	-0.426	-0.201	-0.089	0.549	0.036	0.276	0.052	0.168

Adding mean structure

- Would be nice to get results on latent factor means
- Latent factors have no mean structure of their own; to set a scale one can
  - Set the mean of the first factor to 0
  - Constrain intercepts of y variables equal across time points
  - It is a mere reparameterization, so no change in fit
- In our univariate syntax

```
[V1* V2* V3* V4*] (2);
[WISC1@0 WISC2* WISC3* WISC4*];
```

- In multivariate syntax

```
[V1*19  V2* V3* V4*] (5);
[NV1*18  NV2* NV3* NV4*] (6);
[VER1@0  VER2* VER3* VER4*];
[NONVER1@0  NONVER2* NONVER3* NONVER4*];
```

Without starting values would not converge…

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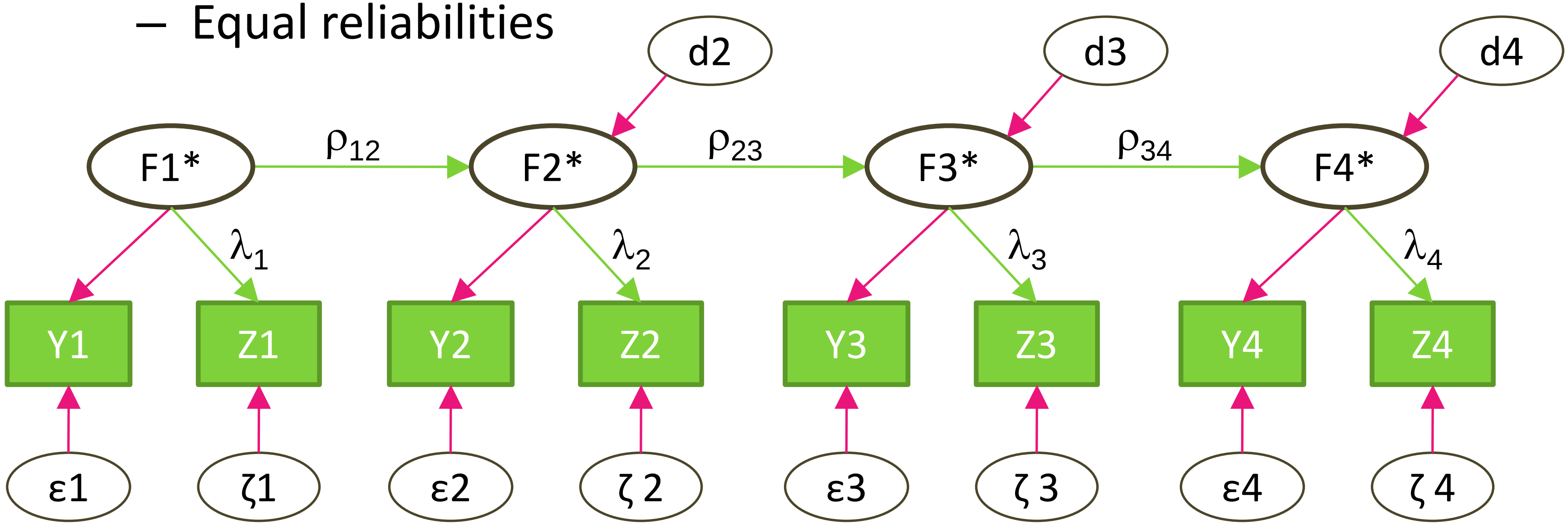
Discussion

- Our data complied to the simplex structure (when we took error of measurement to account)
- A non-simplex structure indicates that one or more factors have influenced the change process making the association between Time 1 and Time 3 either stronger or weaker than would be expected if the change process progressed at a constant linear rate.
- Cross-lagged effects in our data were rather weak, and we dropped them in the latent multivariate model
- These effects, when significant, indicate that change in one variable is related to prior status in the other variable.

Multivariate Latent Autoregressive

- When each construct has multiple indicators, the model becomes a factor model with time-related dependencies (dynamic factor model)
- Additional constraints are typically imposed
  - Equality of respective factor loadings across time
  - Equal reliabilities



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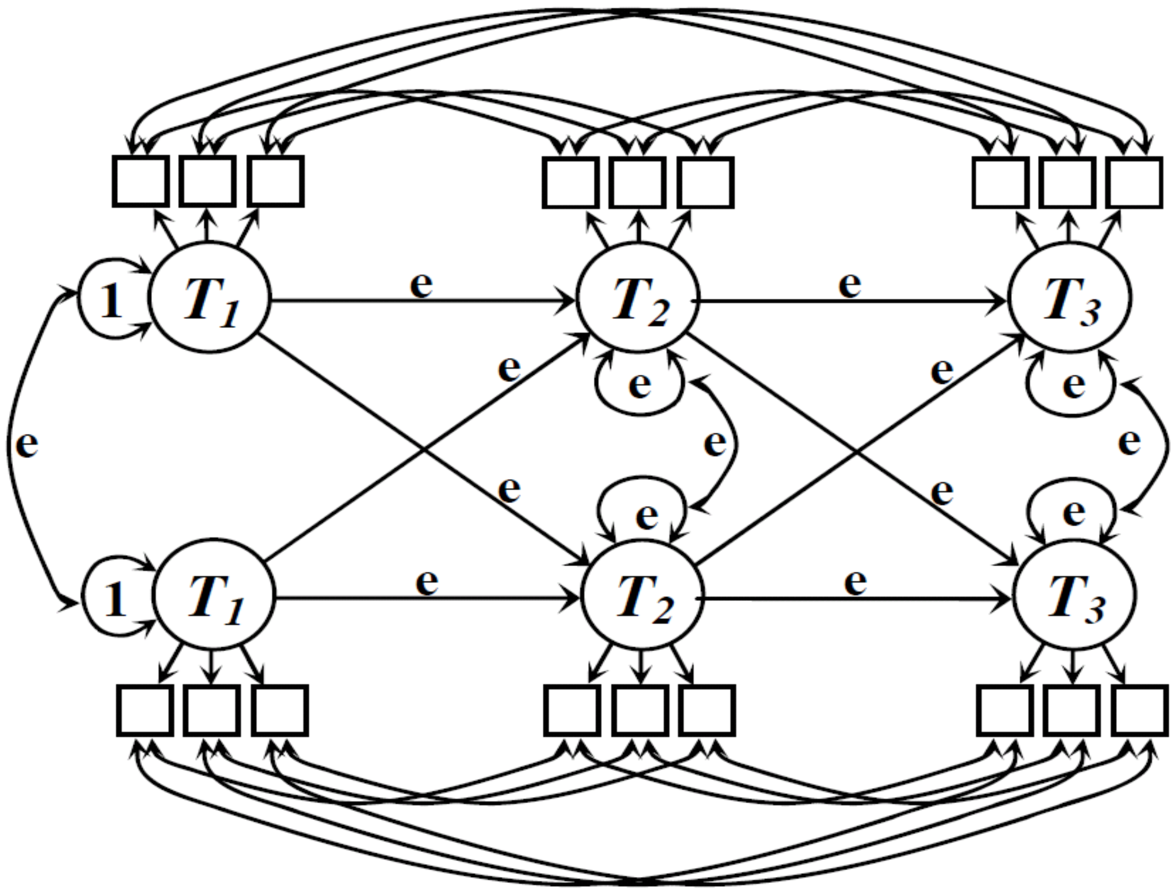
Dynamic factor model – several latent constructs



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Research questions for autoregressive models

- are the constructs measurement invariant over time (i.e., are the measures tapping in to the same thing at different points in time)?
- how stable are the constructs over the observed time span (i.e., to what degree do the individual differences standings get shuffled over the time intervals assessed)?
- what are the relative mean-level differences in the constructs over time?
- is the change process adequately captured by a simplex process (i.e., is the rate of change linearly constant and unaffected by other sources of influence)?
- is there any evidence of cross-lagged influences that are predictive of the cross-time changes?
- are the cross-time changes reciprocal or predominantly unidirectional?
- are the cross-time effects consistent between each adjacent time point?

Practical 2

- Math ability test taken by N=144 pupils in Grades 4, 5 and 6 (3 time points)
- “Concepts” and “Problems” subtest scores (data from Hanna & Lei, 1985)
- Means, SDs and correlations can be found in “MathAbility.dat” file
- The order of variables is C4 P4 C5 P5 C6 P6
- Tasks:
  - Test a cross-lagged model with observed scores
  - Test a cross-lagged model with latent factors
  - Test a multivariate latent autoregressive model
    - Reference: Hanna & Lei (1985). A longitudinal analysis using the Listrel model with structured means. Journal of Educational Statistics, 10, 161-169.