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# Motivations and history of some of my conjectures

Claude Berge

*CNRS CAMS, 54 Boulevard Raspail, 75270 Paris, France*

## 1. The path-partition conjectures

Let $G$ be a (directed) graph with vertex set $X$ and let $k$ be a positive integer. A *partial $k$-coloring* $(S_1, S_2, \ldots, S_k)$ is a family of $k$ disjoint stable sets of $G$. A vertex which belongs to $S_i$ is said to be 'of color $i$', but some vertices may be uncolored. We say that a partial $k$-coloring of $G$ *saturates* a subset $A$ of $X$ if the number of *different* colors represented in $A$ is exactly $B_k(A) = \min\{k; |A|\}$, i.e. the maximum possible. A partition of $X$ is *saturated* if each of its classes is saturated.

A *path* $\mu$ can denote either a sequence of distinct vertices (*directed elementary path*) or the subset of $X$ defined by these vertices. Consider a partition $M = \{\mu_1, \mu_2, \ldots\}$ of $X$ into paths; if $k$ is a positive integer, we write

$$B_k(M) = \sum_i \min\{k; |\mu_i|\}.$$

The partition $M$ is *$k$-optimal* if $M$ minimizes $B_k(M)$. These concepts permit to suggest two conjectures:

**The (weak) path-partition conjecture.** Let $G$ be a graph and let $k$ be a positive integer; there exists a path-partition which can be saturated by a partial $k$-coloring.

**The (strong) path-partition conjecture.** Let $G$ be a graph and let $k$ be a positive integer; for every $k$-optimal path-partition $M$ of $G$, there exists a partial $k$-coloring which saturates $M$.

These two conjectures, that I posed in 1981 at the Combinatorics Seminar of Paris and a few months later at the Silver Jubilee Conference of the University of Waterloo (see [13]), were an attempt to unify the Gallai–Milgram theorem [31] and a companion result proved independently by Gallai [30] and by Roy [48]. Clearly, for $k = 1$, the strong path-partition conjecture reduces to:

*For every path-partition $M$ with a minimum number of paths, there exists a stable set which meets all the paths of $M$.*


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This is the Gallai–Milgram theorem, or more precisely a strengthening of this famous result which can be obtained by the same proof (see [11]).

For $k \geqslant \max |\mu|$, the strong path-partition conjecture becomes:

*For every path-partition $M$, the graph $G$ can be colored with* $\max |\mu|$ *colors so that all the vertices which belong to the same path of $M$ have different colors.*

This is a slight and easy generalization of the classical result of Gallai and Roy [30, 48].

If the graph $G$ is transitive (and in particular if $G$ is the graph of a poset), the strong path-partition conjecture is valid because it is equivalent to the Greene–Kleitman theorem [34] (for a shorter proof, see [50]). In [11], we proved also the validity of the strong path-partition conjecture if $\max |\mu| = 3$, or if the skeleton of $G$ is bipartite. Sridharan [52] proved it for a connected graph with only one cycle. The weak path-partition conjecture has been proved for an acyclic graph by using linear programming [37], network flow theory, or other techniques [39, 50]. Also, a short proof of the weak path-partition conjecture when $G$ is symmetric is due to Payan [47].

More recently, we proved the weak path-partition conjecture for a large class of perfect graphs *with any orientation of the edges* and for specific values of $k$ [17, 18]. This class includes in particular:
— the comparability graphs,
— the co-comparability graphs (complement of comparability graphs),
— the balanced graphs ('with a balanced clique-incidence matrix'),
— the parity graphs (for $k = 2$ only), etc.

The statement of the weak path-partition conjecture is slightly stronger than an open problem proposed in 1981 by Linial [39]. For a graph $G$, let $\alpha_k(G)$ denote the maximum number of vertices which can be colored with a partial $k$-coloring of $G$. Then Linial asked if *there exists a path-partition $M$ such that $B_k(M) \leqslant \alpha_k(G)$?*

**Remark.** Later on, these conjectures suggested similar ones in which 'paths' and 'stable sets' are exchanging roles (see [1, 2, 37]). However, *it is not true that a graph $G$ admits necessarily a full coloring and $k$ disjoint paths whose union covers the maximum number of vertices, such that all the paths are saturated.* For $k = 1$, a counterexample is the graph $G$ defined by the arcs: $(a,b)$, $(a,e)$, $(c,b)$, $(c,d)$, $(d,e)$, $(f,g)$, $(f,j)$, $(h,g)$, $(h,i)$, $(i,j)$ together with all the arcs going from the set $\{a,b,c,d,e\}$ to the set $\{f,g,h,i,j\}$, except the arc $(a,f)$; a coloring with $\chi = 5$ colors has necessarily $C = \{a,f\}$ as a color class, but the (unique) longest path $\{c,d,e,h,i,j)$ does not meet $C$ and cannot be saturated. Also, for $k = 2$, the color class $C$ does not meet $\min\{|C|; 2\}$ different paths of the 2-optimal path-partition $\{cdehij, abfg\}$.

## 2. The perfect graph conjecture

In 1958–1959, I started to investigate new combinatorial properties of a graph $G$ with an emphasis for three invariants: $\alpha(G)$ (called first, with von Neumann, the 'coefficient of internal stability', then the 'stability number', or also the 'independence number'), $\theta(G)$ (the 'partition number', the minimum number of cliques needed to cover the vertex set), and $\phi(G)$ (the *zero-error capacity* introduced by Claude Shannon for a noisy channel). These investigations concerned the four following classes:

(1) a graph $G$ is in *Class* 1 if $\phi(G) = \alpha(G)$;

(2) a graph $G$ is in *Class* 2 if $\alpha(G') = \theta(G')$ (the Beautiful Property') for all induced subgraphs $G'$ of $G$;

(3) a graph $G$ is in *Class* 3 if $\gamma(G') = \omega(G')$ (the chromatic number is equal to the clique number) for every induced subgraph $G'$

(4) a graph $G$ is in *Class* 4 if $G$ contains no induced $C_{2k+1}$, the odd cycle of length $2k + 1 \geqslant 5$ (called *odd hole*) and no induced complement of an odd hole (called *odd antihole*).

It is easy to see that every graph in Class 2 is also in Class 4; the Perfect Graph Conjecture says that *Classes* 2 *and* 4 *are equivalent.*

To trace back the history of perfect graphs, we shall distinguish different steps:

*June 1957*: When he heard that I was writing a book on graph theory, my friend M.P. Schutzenberger drew my attention on an interesting paper of Shannon [51] which was presented at a meeting for engineers and statisticians, but which could have been missed by mathematicians working in algebra or combinatorics. In his paper, Shannon posed two problems:

(1) what are the minimal graphs which do not belong to Class 1? (He knew that $C_5$ was the smallest one);

(2) what is the zero-error capacity of the graph $C_5$?

The second problem was solved by Lovász [43] several years later. The first problem, completed by my young student Alain Ghouila-Houri (Shannon overlooked the antiholes), was discussed in January 1960 at the Seminar of Professor FORTET, where I asked:

*Is it true that every graph in Class* 4 *is also in Class* 1?

(see Ghouila-Houri [32]). This conjecture, somewhat weaker than the Perfect Graph Conjecture, was motivated by the remark that for the most usual channels, the graphs representing the possible confusions between a set of signals (in particular the interval graphs) have no odd holes and no odd antiholes, and are *optimal in the sense of Shannon.* I developed this idea at the General Assembly of U.R.S.I. (Information Theory) in Tokyo where my research paper [5] was distributed to all the participants; this paper appeared much later in a book edited by Caianello [6], but at that time I had the possibility to add in the galley proofs new references and an appendix with some results proved in [7], in order to make the conjecture more plausible and more interesting. In fact, at that time, no one really cared about such a problem except Ghouila-Houri; unfortunately, in 1966, this remarkable young mathematician committed suicide, and all the notes concerning his results about the zero-error capacity of the antiholes were definitively lost.

*October 1959*: Invited by T. Gallai, I attended the first international meeting on Graph Theory at Dobogokö (Hungary), with A. Stone, W. Tutte, A. Rényi, P. Erdős, G. Dirac, G. Hajós, H. Sachs, and others. At this meeting, Hajnal and Surányi presented an elegant result [35] which could be repharsed as follows: *Every triangulated graph belongs to Class* 2. (*a graph is 'triangulated', or 'chordal', if every cycle of length larger than* 3 *has a chord*).

*April 1960*: Invited by H. Sachs, I attended the second international meeting on Graph Theory at the Martin Luther University, Halle-Wittenberg, with G. Hajós, G. Dirac, A. Kotzig, and G. Ringel and I presented a new result:

*Every triangulated graph belongs to Class* 3.

At that time, I was trying to find all the minimal counterexamples to Class 3 (because *I suspected that the only ones were the holes and the antiholes*, conviction which appeared later to be equivalent to the Perfect Graph Conjecture). In the extended abstract of my talk in Halle published in German [4], as in a more developed text published simultaneously in French [3], I stressed the importance of the holes and of the antiholes for this problem.

A first remark was that all the graphs which were known to belong to Class 3 are *without odd holes*. In honor of T. Gallai, I proposed to call 'semi-Gallai' a graph which has no odd hole. However, a terminology change was imposed by the editor of the Proceedings of Halle-Wittenberg, who added to my paper the following footnote: 'The original title of the presentation given in Halle was *Coloring of Gallai, resp. semi-Gallai, graphs*. Gallai informed us, however, that this was an oversight since he had not concerned himself closely with these graphs. Therefore the title was changed with the author's agreement following a suggestion by Dirac'.

It is not true that every graph without holes belongs to Class 3, and the smallest counterexample, published in [3, 4], is the antihole of size 7. Clearly, no antihole belongs to Class 3, but we had also to check that the antiholes of size $\geqslant 7$ *do not contain a hole*, and are *minimal* with respect to the nonmembership in Class 3. At that time, we were pretty sure that there were no other minimal obstructions; for that reason, at the end of my talk in Halle, I proposed the following open problem: *If a graph G and its complement are semi-Gallai graphs, is it ture that* $\gamma(G) = \omega(G)$?

Clearly, this statement is equivalent to the Perfect Graph Conjecture.

*July–August 1961*: During a long symposium on Combinatorial Theory at Rand Corporation (with R.C. Bose, G. Dantzig, J. Edmonds, L. Ford, R. Fulkerson, A. Hoffman, N. Mendelsohn, Ph. Wolfe, and other), I presented a new result: *Every unimodular graph is in Class* 2 *and in Class* 3.
(I call '*totally unimodular*' a matrix which was called at that time '*matrix with the unimodularity property*', and a '*unimodular graph*' is a graph with a totally unimodular clique-incidence matrix). At this meeting, I met for the first time Alan Hoffman, who mentioned to me interesting new problems about comparability graphs. Also, the fruitful discussions we had together encouraged me to write a paper in English about all the graphs for which I could prove their membership in either Class 2 or Class

3 (with the obvious conclusion that each graph in Class 2 seems to belong to Class 3, and vice versa). When I came back to France, I sent my manuscript to Alan, at Yorktown Heights, for comments and hopefully for submission to some US journal.

*March–April 1964*: I attended a NATO Advanced Study Institute on Graph Theory organized by Dr. E. Aparo at the beautiful Villa Monastero in Frascati, Italy, with L.W. Beineke, P.R. Bryant, A.L. Dulmage, J. Groenveld, P.W. Kasteleyn, N.S. Mendelsohn, J.W. Moon, R.C. Read, W.T. Tutte and W.T. Youngs. During one week, I had the opportunity to present this new concept. I had received a few months earlier an answer from Alan who discussed the problem at the I.B.M. Research Center of Yorktown Heights with Paul Gilmore and Harry McAndrew and suggested some improvements to my paper; so, before the end of the meeting, I could handover to 'il Direttore' (Frank Harary) a final version, with proper acknowledgements to 'Dr. A.J. Hoffman and Dr. P. Gilmore for suggestions and helpful discussions' and to 'Dr. M.H. McAndrew for the proof of Theorem 5 which is shorter than our original version'. Unfortunately, my paper came out only three years later [7].

In fact, this approach led Gilmore to an attempt to axiomatize the relevant properties of cliques in graphs and to a rediscovery of the Halle open problem. This strengthened my conviction that the conjecture *in its strongest version* was valid, even if I was more interested in trying to prove that the graphs of Class 2 (the '$\alpha$-perfect' graphs) are the same as the graphs of Class 3 (the '$\gamma$-perfect' graphs). This became the 'weak' conjecture, which seemed easier to settle than the 'strong' conjecture. The weak conjecture was proved in 1971 by Lovász [41] who made this terminology obsolete: since '$\alpha$-perfect' and '$\gamma$-perfect' are synonymous, both of them may be replaced by 'perfect', and the 'strong conjecture' became the 'Perfect Graph Conjecture'.

*1965–1969*: During that period, I did not do much research in combinatorics: I was in Rome as elected Director of the International Computation Centre, and I was obliged to postpone the 'Seminar on Combinatorial Problems' of the University of Paris which we founded with M.P. Schutzenberger in 1961.

In July 1966, I organized in Rome an international symposium on graph theory with Andrásfai, Balas, Behzad, Dantzig, Dénes, Edmonds, Erdős, Hajós, Jewell, Kasteleyn, Kotzig, Lawler, Minty, Motzkin, Mycielski, Nash-Williams, Nivat, Raynaud, Rosa, Rosenstiehl, Sabidussi, Sachs, and others, and I invited Gilmore to be the Chairman. During the meeting, I worked with Hajós on some properties of the Gallai graphs that we presented together to the symposium. Gallai [27] had a generalization of the Hajnal–Surányi theorem: *If in a graph each of the odd cycles of length at least 5 has two noncrossing chords, then the graph belongs to Class 2.*

In fact, he proved more, but his proof was complicated and for that reason, Surányi published separately a shorter proof [55]. In a letter, Gallai told me that he knew also that his graphs belong to Class 3, but here again, he did not produce a short proof. Our proof was simple, but not as short and elegant as the proof produced by Meyniel [43] in 1972 for a stronger result (rediscovered nearly simultaneously in Armenia by Markosian and Karpetian [43]), which can be restated as follows: *If each odd cycle of length at least 5 has at least two chords, then the graph belongs to Class 3.*

In 1967, I gave several talks on perfect graphs, in particular at the Bose symposium of Chapel Hill where Mark Watkins published a short report (an addendum to [8]) which contributed to make the Perfect Graph Conjecture popular. It was not always so, and the first symposium lecture about perfect graphs from other mathematicians was delivered by Horst Sachs [49] at the Calgary conference in 1969. We learned from him that E. Olaru defended his doctoral dissertation on perfect graphs at Ilmenau in 1969; his thesis was the first one of this topic.

At the Waterloo conference in 1968, I proposed for the Perfect Graph Conjecture a completely different approach. A new idea at that time was to treat a general family of nonempty sets (called 'edges') the same way as the family of edges of a graph in order to obtain a theorem which reduces to *a graph theory* theorem when the 'edges' are 2-element subsets. In a paper of Lovász [40], this point of view was used to extend the concept of chromatic number, and this family was called a 'set-system'. In my paper [9], it was called 'graphoid', and this led me to discover a new class of perfect graphs, the 'balanced graphs', which generalize the line-graphs of bipartite graphs. We must add that it is because of the simplicity of this new point of view that L. Lovász found in 1971 a proof of the weak Perfect Graph Conjecture, published simultaneously in the context of hypergraph theory [41] and in the context of graph theory [42]. He gave later another equivalent formulation with the polyhedral point of view, and this was followed by a bunch of nice results (of e.g. R.G. Bland, V. Chvatal, R. Giles, R.L. Graham, H.C. Huang, M.W. Padberg, A.F. Perold, L.E. Trotter, A. Tucker, S.H. Whitesides).

Lovász's proof of the weak perfect graph conjecture was closely related to an earlier work of Ray Fulkerson on antiblocking pairs of polyhedra (especially his 'max–max inequality'). Ray proved that the conjecture was equivalent to another statement, which he found too strong to be true; when I sent him a postcard from Waterloo to inform him that the validity of the conjecture had just been established by Lovász, he was able to supply the missing link in only a few hours. Later, Ray invited me to publish the whole story in a volume that he was editing [27].

The most significant results obtained before 1980 have been assembled in [19], but many classes of perfect graphs have been introduced since then by different authors, using completely different arguments. Other papers deal with recognition algorithms for specific classes, complexity of optimization problems in perfect graphs (of e.g. Grötschel, Lovász, Schrijver). Other important results have been found in the last decase but, after more than 30 years, the Perfect Graph Conjecture remains open.

## 3. The first conjecture to extend Vizing's theorem

One of the most interesting combinatorial result in graph theory is the theorem of Vizing: If $G$ is a simple graph (no loops, no multiple edges) of maximum degree $\Delta(G)$, the smallest number of colors needed to color the edges so that no two edges of the same color have one endpoint in common, denoted $q(G)$ and called the *chromatic index*, is equal to $\Delta(G)$ or to $\Delta(G)+1$.

A *hypergraph* $H=(E_1,E_2,\dots,E_m)$ is a collection of nonempty subsets, called *edges*, of a finite set $X$, called the *vertex set*; if the edges have all cardinality $\leqslant 2$, $H$ is a graph. The name of *hypergraph* was suggested for the first time by an attendant of our seminar, J.M. Pla; by keeping the same terminology as in graph theory, we were trying to obtain for $H$ similar theorems which give *directly* some famous graph results when the $E_i$'s are all of cardinality 2. In 1972, during the hypergraph seminar that we organized with Ray-Chaudhuri at Colombus (Ohio), we failed to find an extension of Vizing's theorem in the spirit of the seminar (because hypergraphs are of a too general structure, and we found later that 'linear hypergraphs' are more appropriate to generalize graphs in this respect). For a hypergraph $H$, we studied the following parameters:

The *maximum degree* $\Delta(H)$: the maximum number of edges having one point in common, i.e. the maximum size of a 'star';

$\Delta_0(H)$: the maximum size of a family of edges which are pairwise intersecting;

the *chromatic index* $q(H)$: the smallest number of colors needed to color the edges so that no two edges of the same color intersect.

Clearly, we have $\Delta(H)\leqslant\Delta_0(H)\leqslant q(H)$. The *hereditary closure* of $H$, denoted $H^C$, is the collection of all the *different* nonempty subsets of the edges of $H$; the hypergraph $H$ is *linear* if no two edges intersect in more than one point. Thus, a graph $G$ is a linear hypergraph, and Vizing's theorem is equivalent to: $q(G^C)=\Delta(G^C)$. Consequently, during the third international conference sponsored by the New York Academy of Science. 10–14 June 1985, I posed the following conjecture:

**The first Vizing type conjecture:** Every linear hypergraph $H$ satisfies $q(H^C)=\Delta(H^C)$.

(for the proceedings of the conference, see [14].) For example, the lines of a finite projective plane with seven points $a,b,c,\dots,g$ constitute a linear hypergraph $H$ with $\Delta(H^C)=10$, and the edges of $H^C$ can be colored with 10 colors as follows:

(1) *abc, de, f, g*; (2) *adg, ef, bc*; (3) *aef, cd, bg*; (4) *bdf, cg, ae*; (5) *beg, ac, df*; (6) *cde, ab, fg*; (7) *cfg, ad, be*; (8) *ag, cf, bd, e*; (9) *af, ce, dg, b*; (10) *eg, bf, a, c, d*.

More generally, Gionfriddo and Tuza [33] proved that the conjecture is valid for all resolvable Steiner systems of type $S(2,k,n)$. For several other hypergraphs the same result is obtained from known combinatorial theorems (see [10, 15]).

It is important to mention that the first conjecture is narrowly related to a famous conjecture that Chvátal posed in 1972 during the hypergraph seminar [23]:

**Chvátal's conjecture:** Every hypergraph $H$ satisfies $\Delta_0(H^C)=\Delta(H^C)$.

This conjecture also remains open (see e.g. [16, 56]). If the first conjecture is valid. then every linear hypergraph $H$ satisfies $\Delta(H^C)\leqslant\Delta_0(H^C)\leqslant q(H^C)=\Delta(H^C)$, so the

Chvátal conjecture is true for linear hypergraphs. This weaker statement has been proved first by Sterboul [54] if $H$ is linear and uniform (every edge has the same cardinality), then by Stein [53] if $H$ is linear and not uniform.

## 4. The second conjecture to extend Vizing's theorem

At the third international conference of the New York Academy of Science (1985), I also presented as an open problem the following conjecture:

**The second Vizing-type conjecture:** Every linear hypergraph $H$ with no repeated loop (edge of cardinality 1) satisfies

$$q(H) \leqslant \Delta(H^{(2)}),$$

where $H^{(2)}$ denotes the 2-section of $H$, i.e. the collection of all the nonempty subsets of an edge of $H$ which have a cardinality $\leqslant 2$.

(See [14].) If $H$ reduces to a simple graph $G$, then $\Delta(G^{(2)}) = \Delta(G) + 1$, so the above inequality becomes equivalent to the Vizing inequality: $q(G) \leqslant \Delta(G) + 1$. If $H$ is the hypergraph dual of a Steiner triple system, the same bound for $q(H)$ was also conjectured by J. Colbourn and M. Colbourn [24]. Independently, Meyniel [46] conjectured that the inequality holds for every linear hypergraph $H$. Also, on the way to a RUTCOR combinatorial conference in 1986, I had the opportunity to mention this problem to Lovász, and we were surprised to see that Füredi presented the same conjecture as an open problem during the conference (see [28]). Thus, one can say that this conjecture had in fact four fathers!

Very few results have been obtained so far, but if the 'big' edges are pairwise disjoint, we know that the conjecture is valid [20].

The most interesting fact is that the validity for every $H$ implies the validity of a famous conjecture that Erdős, Faber and Lovász proposed during the hypergraph seminar in Colombus (Ohio):

**The Erdős–Faber–Lovász conjecture.** If a graph $G$ is the union of $m$ cliques of size $m$, no two of them having more than one vertex in common, then the chromatic number of $G$ is equal to $m$.

Clearly, the clique-hypergraph of $G$ is linear, uniform of rank $m$ and with only $m$ edges; its dual is a linear hypergraph, regular of degree $m$, with only $m$ vertices. Consequently, an equivalent formulation of the Erdős–Faber–Lovász conjecture is: *Let $H$ be an $m$-regular linear hypergraph with only $m$ vertices, then $q(H)$ is equal to $\Delta(H^{(2)}) = m$.*

The Erdős–Faber–Lovász conjecture was proved up to $m = 10$ by computer search [36], and asymptotic results have been obtained in [38].

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