Optimal Deterministic Ring Exploration with Oblivious Asynchronous Robots

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Abstract. We consider the problem of exploring an anonymous unoriented ring of size n by k identical, oblivious, asynchronous mobile robots, that are unable to communicate, yet have the ability to sense their environment and take decisions based on their local view. Previous works in this weak scenario prove that k must not divide n for a deterministic solution to exist. Also, it is known that the minimum number of robots (either deterministic or probabilistic) to explore a ring of size n is 4. An upper bound of 17 robots holds in the deterministic case while 4 probabilistic robots are sufficient. In this paper, we close the complexity gap in the deterministic setting, by proving that no deterministic exploration is feasible with less than five robots, and that five robots are sufficient for any n that is coprime with five. Our protocol completes exploration in $O(n)$ robot moves, which is also optimal.

Keywords: Robots, Anonymity, Obliviousness, Exploration, Asynchronous system, Ring.

1 Introduction

Recent research focused on systems of autonomous mobile entities (that are hereafter referred to as *robots*) that have to collaborate in order to accomplish collective tasks. Two universes have been studied: the continuous euclidean space [8,14,4] where the robots entities can freely move on a plane, and the discrete universe in which space is partitioned into a finite number of locations, conventionally represented by a graph, where the nodes represent the possible locations that a robot can take and the edges the possibility for a robot to move from one location to the other [7,11,2,1,10,9,5,6,3]. In this paper we pursue research in the discrete universe and focus on the exploration problem when the network is an anonymous unoriented ring, using a team of autonomous mobile robots. The robots we consider are unable to communicate, however they can sense their environment and take decisions according to their local view. We assume anonymous and uniform robots (*i.e* they execute the same protocol and there is no way to distinguish between them using their appearance). In addition they are oblivious, *i.e* they do not remember their past actions. In this context, robots asynchronously operate in cycles of three phases: look, compute and move phases. In the first phase, robots observe their environment in order to get the position of all the other robots in the ring. In the second phase, they perform a

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local computation using the previously obtained view and decide on their target destination to which they will move in the last phase.

Related Work. In the discrete model, two main problems are investigated assuming very weak asynchronous, identical, and oblivious robots: the gathering and the exploration problem. In the gathering problem, robots have to gather in one location not known in advance *i.e* there exists an instant $t > 0$ where all robots share the same location (one node of the ring). In the exploration problem, robots have to explore a given graph, every node of the graph must be visited by at least one robot and the protocol eventually terminates (that is, all robots are idle).

For the problem of gathering in the discrete robot model, the aforementioned weak assumptions have been introduced in [10]. The authors proved that the gathering problem is not feasible in some symmetric configurations and proposed a protocol based on breaking the symmetry of the system. By contrast in [9], the authors proposed a gathering protocol that exploits this symmetry for a large number of robots ($k > 18$) closing the open problem of characterizing symmetric situations on the ring which admit a gathering.

For the exploration problem, the fact that the robots have to stop after the exploration process implies that the robots somehow have to remember which part of the graph has been explored. Nevertheless, in this weak scenario, robots have no memory and thus are unable to remember the various steps taken before. In addition, they are unable to communicate explicitly, therefore the positions of the other robots remain the only way to distinguish different stages of the exploration process. The main complexity measure here is the minimal number of robots necessary in order to explore a given graph. It is clear that a single robot is not sufficient for the exploration in the case where it is not allowed to use labels. In [6], it has been shown that $\Omega(n)$ robots are necessary in order to explore trees of size n, however, when the maximum degree of the tree is equal to three then the exploration can be done with a sub-linear robot complexity. In the case where the graph is a ring, it has been shown in [5] that k (the number of robots) must not divide n (the size of the ring) to enable a deterministic solution. This implies that for a general n, $\log(n)$ robots are necessary. The authors also present in [5] a deterministic protocol using 17 robots for every n that is coprime with 17. By contrast, [3] presents a probabilistic exploration algorithm for a ring topology of size $n > 8$. Four probabilistic robots are proved optimal since the same paper shows that no protocol (probabilistic or deterministic) can explore a ring with three robots.

Contribution. In this paper, we close the complexity gap in the deterministic setting. In more details, we prove that there exists no deterministic protocol that can explore an even sized ring with $k \leq 4$ robots. This impossibility result is written for the ATOM model [14] where robots execute their look, compute and move phases in an atomic manner, and thus extend naturally in the non-atomic CORDA model. We complement the result with a deterministic protocol using five robots and performing in the fully asynchronous non-atomic CORDA model