Multi-agent Logic with Distances Based on Linear Temporal Frames*

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Abstract. The paper investigates a new temporal logic $\mathcal{TL}^M_{Dist}$, which combines temporal operations with the operations of localised agent's knowledge and operations responsible for measuring distances. The main goal is to construct a logical framework for modelling logical laws, which describe interactions between such operations. We consider issues of satisfiability and decidability for $\mathcal{TL}^M_{Dist}$. Our principal result is the algorithm which recognizes theorems of $\mathcal{TL}^M_{Dist}$, which implies that $\mathcal{TL}^M_{Dist}$ is decidable, and the satisfiability problem for $\mathcal{TL}^M_{Dist}$ is solvable.

Keywords: multi-agent systems, multi-modal logics, decision algorithms, satisfiability, Kripke semantics, distance measuring.

1 Introduction

This paper attempts to simulate evolutions of a distributed system of agents in a logical framework, based on linear temporal frames with distances. Temporal logics were first suggested to specify properties of programs in the late 1970's (cf. Pnueli [1]). The most used temporal framework is linear discreet-time frames, which have been extensively studied from the point of view of various prospects of applications (cf. e.g. Manna and Pnueli [2], Barringer, Fisher, Gabbay and Gough [3]). Model checking for such frames formed a strong direction in Logic in Computer Science, which uses, in particular, automata theory (cf. Vardi [4]). Temporal logics themselves can be considered as a special cases of hybrid logics, e.g. as bimodal logics with some laws imposed on the interaction of modalities to imitate the flow of time.

The mathematical theory devoted to the study of axiomatizations of temporal logics and development of their semantic theory based on Kripke/Hintikka-like models and temporal Boolean algebras formed a highly technical branch in the area of Non-classical logic (cf. van Benthem [5,6], Gabbay and Hodkinson [7], Hodkinson [8]). Axiomatizations of various (uni)-temporal linear logics are summarized in de Jongh et al. [9].

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The introduction of agents into the picture, brings additional challenges for the researcher looking to simulate such systems. One of the problems is that introducing agents might lead to undecidability (cf. Kacprzak [10]). Another problem is that, non withstanding some previous successful attempts to apply multi-modal propositional logics toward the task of describing evolutions of multi-agent systems (cf. Fagin et al. [11]), there are still multiple technical problems remain, some of mathematical and some of conceptual nature. For instance, picking out only discreet, already accomplished states of affairs, prevents from looking into the structure of the transition process, which would be the main interest from the simulation point of view.

To bridge this gap, at this paper we introduce a new temporal logic $\mathcal{TL}^{M}_{Dist}$, which describes frames where transition periods are filled with intermediate states. This construction is necessarily based on some tools of general fusion theory. The issues of satisfiability and decidability for $\mathcal{TL}^{M}_{Dist}$ is our main interest: we find an algorithm which recognizes theorems of logic $\mathcal{TL}^{M}_{Dist}$ (which implies that it is decidable, and the satisfiability problem for it is decidable). The algorithm works as follows: an arbitrary formula in the language of $\mathcal{TL}^{M}_{Dist}$ is, first, transformed into a rule in a special normal reduced form, which, then, is checked for validity on special models (of size efficiently bounded in the size of the rule) with respect to special kind of valuations.

The general methodology of this paper is borrowed from [12] and [13].

2 Language and Semantics of $\mathcal{TL}^{M}_{Dist}$

The main goal of this paper is to understand how one can "fill in" the intervals in seemingly discreet temporal moments. In other words, to answer the question: "Can we simulate by finitary means how we arrive at the (next) discreet moment-state of affairs?"

The way we chose to investigate, is to suppose that there is a finite chain (may be more than one) of intermediate events, reached by interaction of agents, that starts late "Today" and ends up as "Tomorrow". This somewhat reminds the software development cycles, where stable releases are "interspersed" with alpha or beta builds (each of them can be "promoted" to the stable status or on the contrary outright abandoned).

We start from the formal definition of the logical syntax.

Language of $\mathcal{TL}^{M}_{Dist}$ To build logical language, we add to usual temporal operations Next (next) and Until (until), new unary logical operations K_i for agents' knowledge, a special operator Today, together with a countable set of operations for measuring temporal distances $\{\Diamond^{+}_{k}\}_{k\in\mathbb{N}}$.

Altogether, propositional language $\mathcal{L}$ includes (logical connectives are given with their arities as upper-right indices):

$$\mathcal{L} := \langle \vee^2, \wedge^2, \rightarrow^2, \neg^1, \mathrm{N}^1, \{K_i^1\}_{i=1}^m, \{\Diamond_k^{+1}\}_{k\in\mathbb{N}}, \mathrm{Today}^1, \top^0, \bot^0 \rangle.$$