A Linguistic Truth-Valued Uncertainty Reasoning Model Based on Lattice-Valued Logic

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Abstract. The subject of this work is to establish a mathematical framework that provide the basis and tool for uncertainty reasoning based on linguistic information. This paper focuses on a flexible and realistic approach, i.e., the use of linguistic terms, specially, the symbolic approach acts by direct computation on linguistic terms. An algebra model with linguistic terms, which is based on a logical algebraic structure, i.e., lattice implication algebra, is applied to represent imprecise information and deals with both comparable and incomparable linguistic terms (i.e., non-ordered linguistic terms). Within this framework, some inferential rules are analyzed and extended to deal with these kinds of lattice-valued linguistic information.

1 Introduction

One of the fundamental goals of artificial intelligence (AI) is to build artificially computer- based systems which make computer simulate, extend and expand human's intelligence and empower computers to perform tasks which are routinely performed by human beings. Due to the fact that human intelligence actions are always involved with uncertainty information processing, one important task of AI is to study how to make the computer simulate human being to deal with uncertainty information. Among major ways in which human being deal with uncertainty information, the uncertainty reasoning becomes an essential mechanism in AI.

In real uncertainty reasoning problem, most information, which are always propositions with truth-values, can be very qualitative in nature, i.e., described in natural language. Usually, in a quantitative setting the information is expressed by means of numerical values. However, when we work in a qualitative setting, that is, with vague or imprecise knowledge, this cannot be estimated with an exact numerical value. Then, it may be more realistic to use linguistic truth-values instead of numerical values.

Since 1990, there have been some important conclusions on inference with linguistic terms. In 1990, Ho [3] constructed a distributive lattice-Hedge algebra, which can be used to deal with linguistic terms. He [4] gave a measure function between two linguistic terms, and obtained the fuzzy inference theory and method, which based on linguistic term Hedge algebra. In 1996, Zadeh [17]

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discussed the formalization of some words and proposition of natural language, and given the standard form of language propositional and production ruler, and he discussed the linguistic terms fuzzy inference based on the fuzzy sets theory and fuzzy inference method. In 1998 and 1999, Turksen [9,10] studied the formalization and inference of descriptive words, substantive words and declarative sentence. In 2003, Xu [5], and in 2004 Pei [8] proposed a kind of simple lattice implication algebra with linguistic truth-values.

Based on the symbolic approaches, a linguistic truth-valued algebra model, which is based on a logical algebraic structure, i.e., lattice implication algebra, is applied to represent imprecise information and deals with both comparable and incomparable linguistic terms (i.e., non-ordered linguistic values). Within this framework, some inferential rules are analyzed and extended to deal with these kinds of lattice-value linguistic information.

The paper is organized as follows: Section 2 analyzes the structure of lattice implication algebras with linguistic terms. Based on it, a linguistic truth-valued uncertainty reasoning model based on lattice-valued logic is proposed in Section 3 with an illustration. Section 4 comes to the conclusion.

2 Lattice Implication Algebra with Linguistic Terms

2.1 Lattice Structure and Lattice Implication Algebra

Lattice structures [1] have been successfully applied to many fields, such as reliability theory, rough theory, knowledge representation and inference etc. Among them, the introduction of L-fuzzy sets by Goguen [2] provides a general framework for Zadeh's fuzzy set theory.

Two important cases of L are of interest and often being used: when L is a finite simple ordered set; and when L is the unit interval [0, 1]. More general, L should be a lattice with suitable operations like $\wedge$, $\vee$, $\rightarrow$, $'$. The question of the appropriate operation and lattice structure has generated much literature. Goguen [2] established L-fuzzy logic of which truth value set is a complete lattice-ordered monoid, also called a complete residuated lattice in Pavelka and Novak's L-fuzzy logic [7,6]. Since this algebraic structure is quite general, we specify the algebraic structure to lattice implication algebras introduced by Xu [11], which was established by combining lattice and implication algebra with the attempt to model and deal with the comparable and incomparable information. There have been much work about lattice implication algebra, as well as the corresponding lattice valued logic system, lattice-valued reasoning theory and methods [11,12,13,16]. The lattice implication algebra is defined axiomatically as:

Definition 1. [11] *Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order- reversing involution $'$, I and O the greatest and the smallest element of L respectively, and $\rightarrow$: $L \times L \rightarrow L$ be a mapping. $(L, \vee, \wedge, O, I)$ is called a lattice implication algebra (LIA) if the following conditions hold for any x, y, $z \in L$:*