

# Relational Databases and Homogeneity in Logics with Counting

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## Abstract

We define a new hierarchy in the class of computable queries to relational databases, in terms of the preservation of equality of theories in fragments of first order logic with bounded number of variables with the addition of counting quantifiers ( $C^k$ ). We prove that the hierarchy is strict, and it turns out that it is orthogonal to the TIME-SPACE hierarchy defined with respect to the Turing machine complexity. We introduce a model of computation of queries to characterize the different layers of our hierarchy which is based on the reflective relational machine of S. Abiteboul, C. Papadimitriou, and V. Vianu. In our model the databases are represented by their  $C^k$  theories. Then we define and study several properties of databases related to homogeneity in  $C^k$  getting various results on the change in the computation power of the introduced machine, when working on classes of databases with such properties. We study the relation between our hierarchy and a similar one which we defined in a previous work, in terms of the preservation of equality of theories in fragments of first order logic with bounded number of variables, but *without* counting quantifiers ( $FO^k$ ). Finally, we give a characterization of the layers of the two hierarchies in terms of the infinitary logics  $C_{\infty\omega}^k$  and  $\mathcal{L}_{\infty\omega}^k$ , respectively.

**Keywords:** query languages, database machines, query computability, completeness of models, counting

## 1 Introduction

Given a relational database schema, it is natural to think about the whole class of queries which might be computed over databases of that schema. That is, if we do not restrict ourselves to a given implementation of certain query language on some computer, in the same way as the notion of computable function over the natural numbers was raised in computability theory. In [CH80], A. Chandra and D. Harel devised a formalization for that notion. They defined a *computable query*

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