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ADLER-BELL-JACKIW ANOMALY AND FERMION-NUMBER BREAKING IN THE PRESENCE OF A MAGNETIC MONOPOLE

V.A. RUBAKOV

Institute for Nuclear Research of the Academy of Sciences of the USSR, 60th October Anniversary prospect 7 a, Moscow 117312, USSR

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In (V − A) theories, fermion number is broken in the presence of the 't Hooft-Polyakov magnetic monopole through the Adler-Bell-Jackiw anomaly. An exactly solvable zeroth-order approximation for evaluating Green functions of zero-angular-momentum fermions in the presence of a monopole is developed in the case of an SU(2) model with massless left-handed fermions. Within this approximation the density of the fermion-number breaking condensate is calculated. This density is found to be O(1), i.e. to be independent of the coupling constant and of the vacuum expectation value of the Higgs field. The corrections to the approximation are estimated. It is argued that the above effect can give rise to the strong baryon-number breaking in monopole-fermion interactions in SU(5) grand unified theory.

1. Introduction

The existence of the 't Hooft-Polyakov magnetic monopoles [1] is one of the most interesting features of spontaneously broken gauge theories. The monopoles are inherent in all models with compact $U(1)_{EM}$ group [2], including [3,4] grand unified theories [5]. Experimental observation of relic superheavy magnetic monopoles would provide a strong argument in favour of grand unification [6], so further investigation of the monopole properties is important from both theoretical and experimental points of view.

Most of the known characteristics of the 't Hooft-Polyakov magnetic monopoles (mass, magnetic charge etc.) manifest themselves already at the classical level, the quantum effects giving rise to $O(e^2)$ corrections (for a review see, e.g., [7]). The only known exception is the deep relationship [8-10] between the magnetic charge and the winding number [11] of the gauge field. In theories without massless fermions this results in the Witten value of the charge of the quantum dyon [8], $Q_D = -e\theta/2\pi$, where θ is the *CP* non-conservation angle. In the present paper we consider theories with massless left-handed fermions [(V − A) theories]. Our main purpose is to show that in these theories the above relationship leads to strong fermion-number non-conservation in monopole-fermion interactions. We also develop a suitable approximation for calculating some fermion number breaking matrix elements in the presence of a monopole.

It is well known that in (V − A) theories the divergence of the (euclidean) fermionic current is anomalous [12],

$$\partial_\mu J_\mu^{\mathrm{F}} = \mathrm{const}\ \mathrm{Sp}\, F_{\mu\nu}\tilde{F}_{\mu\nu} = \mathrm{const}\ \boldsymbol{E}^a\boldsymbol{H}^a ,$$

so that the fermion number

$$N_{\mathrm{F}} = \int J_0^{\mathrm{F}}\, \mathrm{d}^3x$$

is not conserved in external fields with a non-zero winding number q, where

$$q = -\frac{1}{32\pi^2}\varepsilon_{\mu\nu\lambda\rho}\int \mathrm{Sp}\, F_{\mu\nu}F_{\lambda\rho}\, \mathrm{d}^4x . \tag{1.1}$$

In the vacuum sector this effect is associated with instantons [11, 13–15] and the fermion-number breaking amplitudes are suppressed by the factor $\exp(-\mathrm{const}/e^2)$ as well as by negative powers of the vacuum expectation value of the Higgs field [13, 14]. The first suppression results from the large values of action for the configurations with $q \neq 0$, while the second one is due to the small value of the instanton size, which is cut off at the Compton length of the massive vector boson.

Since in the presence of a monopole there exists a non-zero classical magnetic field, $\boldsymbol{H}^{\mathrm{cl}} \neq 0$, the fluctuation of the electric field can give rise to non-zero q, thus leading to non-zero change in the fermion number. This means that in the monopole sector the anomalous fermion number breaking can be associated with purely electromagnetic configurations which are abelian and massless. So, one expects no suppression factors in the fermion-number breaking amplitudes* (these arguments are further developed in sect. 2). In other words, one expects the anomalous fermion number breaking in the presence of a magnetic monopole to be strong, presumably O(1). This effect can have far reaching consequences, the most interesting one being the strong baryon-number non-conservation in fermion-monopole interactions in grand unified theories** [17, 18].

From the above arguments it is clear that the actual calculation of Green functions with fermion number breaking in the presence of a monopole will be

* Note that these observations are close to those of Marciano and Pagels [16] who argued that the non-abelian dyons ($\boldsymbol{H}^{\mathrm{cl}} \neq 0$, $\boldsymbol{E}^{\mathrm{cl}} \neq 0$) could give rise to the strong chirality breaking in quantum chromodynamics (see also [40]).

** Note that this effect has nothing to do with leptoquark exchange or with the leptoquark kern of the SU(5) monopole. The latter was considered in ref. [3] and shown to lead to $\mathrm{O}(M_{\mathrm{X}}^{-2})$ baryon-number breaking cross sections.

rather non-standard. The effect is neither perturbative (the $\bar{\Psi}\Psi A$ vertex conserves the fermion number) nor quasiclassical (since the factor $\exp(\text{const}/e^2)$ does not appear). This difficulty is also inherent in Schwinger model [19] where an exact solution (either operator [20,21] or functional [19]) is needed to investigate the chirality and fermion number breaking [21–25]. Since we are unable to obtain an exact solution of the spontaneously broken four-dimensional gauge theory, we are faced with the problem of developing a suitable zeroth-order approximation. At present we cannot solve this problem in general; however, the natural approximation does exist if we restrict ourselves to the dynamics of spherically symmetric fermions*. Within this approximation one assumes the relevant gauge field configurations to be spherically symmetric and neglects the contribution of fermions with non-zero angular momentum to the fermionic determinant. Under these assumptions the problem becomes effectively two-dimensional and one can find an exact solution which is quite similar to the solution of the Schwinger model. The main part of the present paper is devoted to the description and solution of this approximation and to estimating corrections. Within this approximation it becomes possible to confirm the heuristic arguments of sect. 2 and find the e^2 dependence of fermion number breaking matrix elements in the presence of monopole.

The paper is organized in the following way. In sect. 2 we present heuristic arguments showing that fermion-number breaking amplitudes in the presence of a monopole are not suppressed by $\exp(-\text{const}/e^2)$ or negative powers of the vacuum expectation value of the Higgs field. These arguments are also useful in the investigations of the effects of anomalous non-conservation in more complicated cases (e.g. in the case of a chromomagnetically neutral SU(5) monopole [27]). Before proceeding further, in sect. 3 we summarize the relevant properties of massless left-handed fermions in the field of a magnetic monopole. In sect. 4 the zeroth-order approximation is described and solved. As an example, in sect. 5 we calculate the density of fermion number breaking condensate of zero-angular-momentum fermions in the presence of a monopole and discuss the relevant gauge field configurations. In sect. 6 we study the corrections to our approximation and show that they are finite and small in the limit of vanishing monopole size. Sect. 7 contains some concluding remarks, including a discussion of the relation between the anomalous fermion-number breaking in the presence of a monopole and the θ vacuum structure [28] of gauge theories as well as a preliminary consideration of the baryon-number breaking in monopole-fermion interactions in SU(5). Some properties of special functions used in this paper are listed in appendix A. Appendices B and C contain some technical details needed to estimate corrections to the zeroth-order approximation.

* Note that in the presence of a monopole the total fermionic angular momentum can be integer valued (see, e.g. [26] and references therein). Throughout this paper we consider the model in which it is indeed integer valued.

2. The heuristic arguments

Throughout this paper we consider an SU(2) gauge theory with a Higgs triplet φ^a and two left-handed fermionic doublets $\Psi^{(s)}$ ($s=1,2$ is the "flavour" index). We always use the euclidean formulation of the field theory, so the action functional is

$$S=S_{A,\varphi}+S_{\Psi}, \tag{2.1}$$

where

$$S_{A,\varphi}=\int \mathrm{d}t\left\{\int \mathrm{d}^3x\left[-\frac{1}{2e^2}\,\mathrm{Sp}\,F_{\mu\nu}^2+\tfrac{1}{4}\mathrm{Sp}(D_\mu\varphi)^2+\lambda(\mathrm{Sp}\,\varphi^2-2c^2)^2\right]-M_{\mathrm{MON}}\right\} \tag{2.2}$$

is the bosonic part and

$$S_{A,\Psi}=-i\int \mathrm{d}^3x\,\mathrm{d}t\sum_{s=1,2}\overline{\Psi}^{(s)}\gamma_{\mathrm{L}}^{\mu}(\partial_\mu+A_\mu)\Psi^{(s)} \tag{2.3}$$

is the fermionic part of the action. Since we are interested in the monopole sector, it is convenient to normalize the zero-point energy so that the monopole energy is equal to zero,

$$E_{\mathrm{MON}}=0. \tag{2.4}$$

According to this prescription, the last term (the monopole mass) on the r.h.s. of (2.2) is added to the standard bosonic part of the action of the Georgi-Glashow SU(2) model. The "left-handed γ-matrices" are defined by the following relations,

$$\gamma^\mu\frac{1-\gamma^5}{2}=\begin{pmatrix}0 & \gamma_{\mathrm{L}}^{\mu}\\ 0 & 0\end{pmatrix},$$

or, explicitly,

$$\gamma_{\mathrm{L}}^{0}=1,\qquad \gamma_{\mathrm{L}}^{i}=i\sigma_i,$$

σ_i being Pauli matrices. The matrix notation for A_μ and φ,

$$A_\mu=\frac{e}{2i}A_\mu^a\tau^a,\qquad \varphi=\varphi^a\tau^a,$$

is used in (2.2), (2.3).

The Higgs field φ develops a non-zero vacuum expectation value, so that in the unitary gauge

$$\langle\varphi\rangle_{\mathrm{vac}}=c\tau_3$$

and only the third component A^3_μ (photon) remains massless. In this gauge the fermionic sector consists of four massless left-handed fermions $\Psi^{(s)}_- \equiv \Psi^{(s)}_1$ and $\Psi^{(s)}_+ \equiv \Psi^{(s)}_2$ carrying the electromagnetic charge $(-\frac{1}{2}e)$ and $(+\frac{1}{2}e)$, respectively (the lower index 1,2 is the SU(2) group one). The gauge-invariant current of sth fermion,

$$J^{(s)}_\mu = \overline{\Psi}^{(s)}\gamma^\mu_{\mathrm{L}}\Psi^{(s)},$$

has the anomalous divergence

$$\partial_\mu J^{(s)}_\mu = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\lambda\rho}\mathrm{Sp}\, F_{\mu\nu}F_{\lambda\rho}. \tag{2.5}$$

The 't Hooft-Polyakov magnetic monopole solution is

$$A^{\mathrm{cl}}_0 = 0,$$

$$A^{\mathrm{cl}}_i = \varepsilon_{aij}\tau_a n_j \frac{1-F(r)}{2ir},$$

$$\varphi^{\mathrm{cl}} = c\tau^a n^a(1-H(r)), \tag{2.6}$$

where $r=\sqrt{x^2}$, $n = x/r$; $F(r)$ and $H(r)$ obey the following boundary conditions:

$$F(0) = H(0) = 1, \qquad F(\infty) = H(\infty) = 0, \tag{2.7}$$

$F(r)$ and $H(r)$ are exponentially small at $r \gg c^{-1}/e$, $r \gg c^{-1}/\lambda$. Throughout this paper we are primarily interested in the dynamical properties of fermions far from the monopole center, i.e. we assume the limit

$$c \to \infty, \qquad F \to 0 \tag{2.8a}$$

to be taken whenever possible (otherwise the function $F(r)$ will be explicitly indicated). Note that in this limit the monopole size vanishes

$$r_{\mathrm{M}} \to 0. \tag{2.8b}$$

Throughout this paper we treat the configuration (2.6) as a classical background one, though we do not assume the perturbations to be small. Thus, the generating functional for the fermionic Green functions in the presence of a monopole,

$$Z^{\mathrm{MON}}[\bar{\zeta}, \zeta] = \left\langle \exp\left[\int (\bar{\zeta}\Psi + \overline{\Psi}\zeta)\,\mathrm{d}^4x\right]\right\rangle^{\mathrm{MON}}$$

(we omit the flavour index s whenever possible as well as the summation over s) is represented by the functional integral*

$$Z^{\mathrm{MON}}[\bar{\zeta}, \zeta] = \int \mathrm{d}A_\mu \mathrm{d}\varphi \exp\left[-S_{A,\varphi} + \text{gauge-fixing terms} + \text{ghost terms}\right]$$

$$\times \int \prod_{s=1}^{2} \mathrm{d}\Psi^{(s)} \mathrm{d}\bar{\Psi}^{(s)} \exp\left[-S_{A,\Psi} + \int (\bar{\zeta}\Psi + \bar{\Psi}\zeta)\, \mathrm{d}^4x\right], \qquad (2.9)$$

with the following boundary conditions:

$$A_\mu(\boldsymbol{x}, t) \to A_\mu^{\mathrm{cl}}(\boldsymbol{x}),$$

$$\varphi(\boldsymbol{x}, t) \to \varphi^{\mathrm{cl}}(\boldsymbol{x}), \qquad t \to \pm\infty. \qquad (2.10)$$

Eq. (2.5) implies [13] that the change of the sth flavour is equal up to a sign to the winding number of the gauge field (1.1),

$$\Delta N^{(s)} = -q.$$

We first show that there exist configurations of the bosonic fields, obeying the boundary conditions (2.10) and having $q = -1$ and that, as opposed to the vacuum sector, in the monopole sector the action $S_{A,\varphi}$ for these configurations can be arbitrarily close to zero. This means that the suppression factor $\exp(-\mathrm{const}/e^2)$ does not appear in fermion number non-conserving matrix elements. Consider the configuration

$$A_0 = \tau^a n^a a_0(r, t)/i,$$

$$A_i = \tau^a n^a n_i a_1(r, t)/i + A_i^{\mathrm{cl}},$$

$$\varphi = \varphi^{\mathrm{cl}}, \qquad (2.11)$$

where $a_0(r, t)$ and $a_1(r, t)$ obey

$$a_0(r, \pm\infty) = a_1(r, \pm\infty) = 0. \qquad (2.12)$$

The action functional for this configuration reads

$$S_{A,\varphi} = \frac{4\pi}{e^2}\int_0^\infty \mathrm{d}r \int_{-\infty}^{\infty} \mathrm{d}t\left[(\partial_t a_1 - \partial_r a_0)^2 r^2 + 2F^2(a_0^2 + a_1^2)\right], \qquad (2.13)$$

* Hereafter the standard denominator needed to ensure $Z^{\mathrm{MON}}[0,0] = 1$ [see (2.4)] is not explicitly indicated for simplicity of notation.

and the winding number (1.1) reads

$$q=\frac{1}{\pi}\int_0^{\infty}\mathrm{d}r\int_{-\infty}^{+\infty}\mathrm{d}t\left\{\partial_r\left[a_0(1-F)^2\right]-\partial_t\left[a_1(1-F)^2\right]\right\}, \tag{2.14}$$

i.e., in virtue of (2.7) and (2.12),

$$q=\frac{1}{\pi}\lim_{r\to\infty}\int_{-\infty}^{+\infty}\mathrm{d}t\,a_0(r,t). \tag{2.15}$$

Note that the last expression can be obtained from (2.14) in the limit (2.8) only if $a_0(r,t)$ satisfies

$$a_0(r=0,t)=0. \tag{2.16}$$

An explicit example of a configuration obeying (2.12), (2.16) and having $q=-1$ is

$$a_0(\rho/r,t)=-\partial_r\rho(r,t),$$

$$a_1(\rho/r,t)=\partial_t\rho(r,t), \tag{2.17}$$

with

$$\rho(r,t)=\tfrac{1}{2}\log\left[\mu_1^2(r^2+t^2)+1\right]+\frac{1}{2\varepsilon}\left[\mu_1^2(r^2+t^2)+1\right]^{-\varepsilon}, \tag{2.18}$$

where μ_1 is some mass scale and ε is a positive number. The value of action for the field (2.11), (2.17) is

$$S_{A,\varphi}=\frac{5\pi^2}{3e^2}\varepsilon\left(1+\mathrm{O}(\varepsilon)+\mathrm{O}(\mu_1/c)\right)$$

and can be arbitrarily small for small ε and μ_1, Q.E.D.

In the vacuum sector, fermion number breaking matrix elements are also suppressed by negative powers of c [14]. This suppression occurs because the zero fermion modes far from the instanton are proportional to $\lambda_{\mathrm{inst}}^{-3/2}$ and the instanton size λ_{inst} is bounded from above by c^{-1}. Thus, it is instructive to investigate the zero fermion modes in the external field (2.11) in order to find their c dependence. For the sake of convenience we consider the fields a_0, a_1 of the form (2.17). Since the external field is spherically symmetric, it is natural to choose a spherically symmetric ansatz for the zero modes. A most general spherically symmetric fermionic field has the following form:

$$\Psi_{\alpha l}^{(0)}(x,t)=\left(\sqrt{8\pi}\,r\right)^{-1}\exp\left(\int_{\infty}^{r}F(r')\frac{\mathrm{d}r'}{r'}\right)\chi_{\alpha l}(x,t), \tag{2.19}$$

where $\alpha = 1, 2$ and $l = 1, 2$ are Lorentz and gauge group indices respectively, and

$$\chi_{\alpha l}(x, t) = \varepsilon_{\alpha l}\chi_1(r, t) - i\tau^a_{\alpha\beta}\varepsilon_{\beta l}n_a\chi_2(r, t). \tag{2.20}$$

Introducing the compact notation

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix},$$

we obtain the following equation for the zero mode $\chi^{(0)}$:

$$\left[(\partial_t + i\tau_2\,\partial_r\rho) + i\tau_2(\partial_r - i\tau_2\,\partial_t\rho) + \frac{F}{r}(\tau_1 + i\tau_2)\right]\chi^{(0)} = 0. \tag{2.21}$$

In order that the solution be non-singular at $r = 0$, one should impose the boundary condition

$$(\tau_1 + i\tau_2)\chi^{(0)}(r = 0) = 0 \tag{2.22}$$

(this point will be clarified in sect. 3). The solution of eqs. (2.21), (2.22) is

$$\chi^{(0)}(r, t) = N\exp[-\rho(r, t)]\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{2.23}$$

where N is a normalization factor. Since the zero mode (2.19), (2.23) is square integrable near $x = 0$ in the limit (2.8), the factor N is independent of c and the zero mode is independent of c far from the monopole center. This implies that the fermion-number breaking matrix elements in the presence of a monopole are not suppressed by negative powers of c. Note that, as is seen from (2.18), the zero mode has the following asymptotic behaviour as $r^2 + t^2 \to \infty$,

$$\Psi \sim r^{-1}(r^2 + t^2)^{-1/2},$$

so that its norm $\int \Psi^+\Psi\,d^4x$ is logarithmically divergent in the infrared region. As will be shown in sects. 4, 5, this fact is inessential, in complete analogy to the Schwinger model [24, 25].

The nature of the configuration (2.11) is most transparent in the unitary gauge of ref. [29]. Performing the transformation to this gauge, we obtain in the limit (2.8)

$$A_0^{(u)} = \frac{1}{i}\tau_3 a_0,$$

$$A_i^{(u)} = \frac{1}{i}\tau_3 a_1 + A_i^{D}\frac{\tau_3 e}{2i},$$

$$\varphi^{(u)} = c\tau_3, \tag{2.24}$$

where A_i^{D} is the Dirac expression for the vector-potential of the magnetic monopole carrying two Diraç units of magnetic charge. From (2.24) it is clear that the configuration (2.11) is purely electromagnetic. Moreover, the magnetic field of this configuration is just the monopole one, while the electric field $E_i = (i/e)\mathrm{Sp}\, F_{0i}\tau_3$ is

$$E_i = \frac{4n_i}{e}(\partial_0 a_1 - \partial_1 a_0)$$

and is directed along the magnetic field, so that $\boldsymbol{HE} \neq 0$ (cf. sect. 1).

Thus, the heuristic arguments of this section imply strong fermion-number breaking in the presence of a monopole. Guided by the above observations, in sect. 4 we shall develop a suitable approximation for evaluation of spherically symmetric matrix elements in the presence of a monopole. Before doing this, we consider the massless left-handed fermions in the external field (2.11).

3. Massless fermions in the field of a magnetic monopole

In this section we consider the left-handed massless fermions in the external field (2.11) in the limit (2.8). It is convenient to introduce the operator of total angular momentum [26],

$$M_i = -i\varepsilon_{ijk}x_j\partial_k + \tfrac{1}{2}\sigma_i + \tfrac{1}{2}\tau_i .$$

The operator M_i commutes with the Dirac operator $\not{D}$,

$$\not{D} \equiv \gamma^\mu_{\mathrm{L}}(\partial_\mu + A_\mu),$$

as well as with the operators $\boldsymbol{\tau n}$ and $\boldsymbol{\sigma n}$. The angular part of the Dirac operator,

$$\not{D}_\Omega \equiv ir\sigma_k(\delta_{kl} - n_k n_l)(\partial_l + A_l) - i\sigma_k n_k ,$$

commutes with $\boldsymbol{\tau n}$ and anticommutes with $\boldsymbol{\sigma n}$,

$$[\sigma_i n_i, \not{D}_\Omega]_+ = 0. \tag{3.1}$$

It is a matter of straightforward calculation to verify the following identity:

$$\not{D}_\Omega^2 = M^2 .$$

There exist two eigenfunctions of $\boldsymbol{M}$ with zero eigenvalue, namely $\varepsilon_{\alpha l}$ and $\tau^a_{\alpha\beta}\varepsilon_{\beta l}n_a$ (α and l are the Lorentz and gauge group indices respectively), and $4(2J+1)$ eigenfunctions $\Psi_{JM\delta\nu}$ of $\boldsymbol{M}^2$ which can be chosen to satisfy

$$\boldsymbol{M}^2\Psi_{JM\delta\nu} = J(J+1)\Psi_{JM\delta\nu}, \qquad J = 1,2\dots,$$

$$M_3\Psi_{JM\delta\nu} = M\Psi_{JM\delta\nu}, \qquad M = 0, \pm 1, \dots, \pm J,$$

$$\boldsymbol{\tau n}\Psi_{JM\delta\nu} = \delta\Psi_{JM\delta\nu}, \qquad \delta = \pm 1,$$

$$\boldsymbol{\sigma n}\Psi_{JM\delta\nu} = \nu\Psi_{JM\delta\nu}, \qquad \nu = \pm 1.$$

The functions $\Psi_{JM\delta\nu}$ ($J \neq 0$) form a set of functions, which is complete in the subspace with $J \neq 0$ and orthonormal on a sphere. Thus, the fermion field Ψ can be decomposed in the following way:

$$\Psi(\boldsymbol{x}, t) = \Psi^{(0)}(\boldsymbol{x}, t) + \frac{1}{r}\sum_{JM\delta}\sum_{\nu} u_\nu^{JM\delta}(r, t)\Psi_{JM\delta\nu}(\Theta, \Phi), \tag{3.2}$$

where $\Psi^{(0)}(\boldsymbol{x}, t)$ is given by (2.19), (2.20) (but the field χ need not satisfy the Dirac equation). It is convenient to introduce the compact notation

$$u^{JM\delta} = \frac{1 - i\tau_1}{\sqrt{2}}\begin{pmatrix} u_{+1}^{JM\delta} \\ u_{-1}^{JM\delta} \end{pmatrix}$$

and to rewrite the fermionic part of the action, (2.3), in the following form,

$$S_{A,\Psi} = S_{J=0} + \sum_{J\neq 0}\sum_{M,\delta} S_{JM\delta}, \tag{3.3}$$

where

$$S_{J=0} = -i\int \mathrm{d}r\,\mathrm{d}t\,\bar{\chi}D_{J=0}\chi, \tag{3.4}$$

$$S_{JM\delta} = -i\int \mathrm{d}r\,\mathrm{d}t\,\bar{u}^{JM\delta}D_{J\delta}u^{JM\delta}, \tag{3.5}$$

$$D_{J=0} = \partial_t - i\tau_2 a_0 + i\tau_2(\partial_r - i\tau_2 a_1), \tag{3.6}$$

$$D_{J,\delta} = \partial_t - i\delta a_0 - i\tau_2(\partial_r - i\delta a_1) + \frac{\sqrt{J(J+1)}}{r}\tau_1 \tag{3.7}$$

[the limit (2.8) is assumed].

The form of the vector potential (2.11), the decomposition (3.2) and the actions $S_{J=0}$ and $S_{JM\delta}$ are invariant under the following transformation:

$$\chi \to \mathrm{e}^{i\tau_2\beta}\chi,$$

$$u^{JM\delta} \to \mathrm{e}^{i\delta\beta}u^{JM\delta},$$

$$a_0 \to a_0 + \partial_t\beta, \qquad a_1 \to a_1 + \partial_r\beta, \tag{3.8}$$

where $\beta(r, t)$ is some real function. The transformation (3.8) is a special case of gauge transformation, the gauge function

$$g(x,t) = \exp[i\tau^a n^a \beta(r, t)] \tag{3.9}$$

being spherically symmetric. For this gauge function to be non-singular at $r = 0$, the function β should vanish at the origin,

$$\beta(r=0, t) = 0. \tag{3.10}$$

According to the decomposition (3.2), the functional measure in (2.9) can be rewritten in the following form:

$$\prod_{s=1}^{2} \prod_{x,t} \mathrm{d}\Psi^{(s)} \mathrm{d}\overline{\Psi}^{(s)} = \prod_{r,t} \prod_{s=1}^{2} \left[\mathrm{d}\chi^{(s)} \mathrm{d}\overline{\chi}^{(s)} \prod_{JM\delta} \mathrm{d}u^{(s)JM\delta} \mathrm{d}\overline{u}^{(s)JM\delta} \right]. \tag{3.11}$$

Thus, the functional integral over fermions in the external field (2.11) reduces to an infinite product of functional integrals over the two-dimensional fermionic fields $\chi(r, t)$ and $u^{JM\delta}(r, t)$ (defined on a half-plane), the relevant action functionals being given by (3.4), (3.5).

We begin the discussion of the above action functionals by deriving the Green function of zero-angular-momentum fermions. Since the Dirac operator (3.6) in the limit of a pointlike monopole is ill defined at $r = 0$ [30], we consider the full operator for the field χ [cf. (2.21)],

$$D_{J=0}^{\mathrm{full}} = \partial_t - i\tau_2 a_0 + i\tau_2(\partial_r - i\tau_2 a_1) + \frac{F}{r}(\tau_1 + i\tau_2). \tag{3.12}$$

The Green function $G(rt; r't')$ obeys the following equation:

$$D_{J=0}^{\mathrm{full}} G(rt; r't') = \delta(r - r')\,\delta(t - t'). \tag{3.13}$$

To derive the boundary condition for G, we assume for simplicity that the function F is a step function,

$$F(r) = \theta(r - r_{\mathrm{M}})$$

where r_{M} is the monopole radius. At the end of our derivation we shall take the limit (2.8b). We also assume that the functions a_0 and a_1 are finite and smooth at $r = 0$. The standard arguments of the theory of differential equations lead to the following behaviour of $G(rt; r't')$ near the origin $r = 0$:

$$G_1(rt; r't') = \mathrm{O}(1),$$

$$G_2(rt; r't') = \mathrm{O}(r), \tag{3.14}$$

where

$$G_{1,2}=\frac{1\pm\tau_3}{2}G.$$

From (3.12), (3.13) it follows that $G(rt;\, r't')$ is continuous at $r=r_{\mathrm{M}}$ and from (3.14) in the limit (2.8b) we obtain the following boundary condition:

$$(1-\tau_3)G(0t;\, r't')=0. \tag{3.15}$$

Note that in terms of the field $\chi(r,t)$ this boundary condition corresponds to (2.22). Thus, in the limit (2.8) the Green function of the field χ obeys the equation

$$D_{J=0}G(rt;\, r't')=\delta(r-r')\,\delta(t-t')$$

and the boundary condition (3.15). Generalization of the above arguments to the general case of an arbitrary function $F(r)$ is straightforward, provided $F(r)$ rapidly tends to zero at $r\gg r_{\mathrm{M}}$.

It is convenient to proceed further in the temporal gauge

$$A_0=0 \tag{3.16a}$$

or

$$a_0=0 \tag{3.16b}$$

and consider the function $a_1(r,t)$ obeying the boundary condition (2.12). In this case $G(rt;\, r't')$ can be obtained in the closed form, namely,

$$G(rt;\, r't')=\exp\left[-\sigma(r,t)+\sigma(r',t')+i\tau_2\gamma(r,t)\right]G_0(rt;\, r't')\exp\left[i\tau_2\gamma(r',t')\right], \tag{3.17}$$

where

$$\sigma(r,t)=\int_0^{\infty}\mathrm{d}r''\int_{-\infty}^{\infty}\mathrm{d}t''\left[\mathcal{D}(r-r'',t-t'')+\mathcal{D}(r+r'',t-t'')\right]$$
$$\times\partial_{t''}a_1(r'',t''), \tag{3.18}$$

$$\gamma(r,t)=\int_{-\infty}^{t}\partial_r\sigma(r,t'')\,\mathrm{d}t''. \tag{3.19}$$

Here $\mathcal{D}(r,t)$ is the propagator of the two-dimensional massless scalar field (the inverse two-dimensional laplacian),

$$\mathcal{D}(r,t)=\frac{1}{4\pi}\log\mu_2^2(r^2+t^2) \tag{3.20}$$

(μ_2 is an arbitrary mass scale [31]), and G_0 is the solution of the "free" equation

$$(\partial_t+i\tau_2\partial_r)G_0(rt;r't')=\delta(r-r')\,\delta(t-t'),$$

obeying the boundary condition

$$(1-\tau_3)G_0(0t;r't')=0.$$

Explicitly

$$\begin{aligned}G_0(rt;r't')&=(\partial_t-i\tau_2\partial_r)[\mathcal{D}(r-r',t-t')+\mathcal{D}(r+r',t-t')\tau_3]\\&=\frac{1}{2\pi}\left[\frac{(t-t')-i\tau_2(r-r')}{(r-r')^2+(t-t')^2}+\frac{(t-t')-i\tau_2(r+r')}{(r+r')^2+(t-t')^2}\tau_3\right].\end{aligned} \tag{3.21}$$

Note that the definitions (3.18), (3.19) imply

$$\partial_r\sigma(0,t)=0,\qquad \gamma(0,t)=0. \tag{3.22}$$

Now we turn to the discussion of the action (3.5). In this case we cannot find the exact Green function of the operator $D_{J,\delta}$ so we develop perturbation theory around $a_0=a_1=0$. The free propagator G^J corresponding to the action (3.5), obeys the equation

$$\left(\partial_t-i\tau_2\partial_r+\frac{b(J)}{r}\right)G^J(rt;r't')=\delta(r-r')\,\delta(t-t'), \tag{3.23}$$

where

$$b(J)=\sqrt{J(J+1)}\,.$$

It is straightforward to prove that the solution of (3.23) has the following form

$$G^J=\begin{pmatrix}\partial_t\mathcal{R}_{b^2+b} & \left(\partial_r-\frac{b}{r}\right)\mathcal{R}_{b^2-b}\\ \left(-\partial_r-\frac{b}{r}\right)\mathcal{R}_{b^2+b} & \partial_t\mathcal{R}_{b^2-b}\end{pmatrix}, \tag{3.24}$$

where the function $\mathcal{R}_\kappa(rt;r't')$ obeys the equation

$$\left(\partial_t^2+\partial_r^2+\frac{\kappa}{r^2}\right)\mathcal{R}_\kappa(rt;r't')=\delta(r-r')\,\delta(t-t'). \tag{3.25}$$

Using the properties of the Legendre function $Q_m(z)$ listed in appendix A, one can verify that the solution of (3.25) is

$$\mathcal{R}_\kappa(rt; r't') = -\frac{1}{2\pi} Q_{d(\kappa)}\left[1 + \frac{(r-r')^2 + (t-t')^2}{2rr'}\right], \tag{3.26}$$

where

$$d(\kappa) = \sqrt{\kappa + \tfrac{1}{4}} - \tfrac{1}{2}. \tag{3.27}$$

The free propagator (3.24) vanishes as $(r-r')^2 + (t-t')^2$ tends to infinity as well as at $r = 0$ (see appendix A).

To conclude this section we summarize the analogous properties of the left-handed fermions in an external gauge field of the form

$$\tilde{A}_0 = \frac{1}{i}\tau_3 a_0(r,t),$$

$$\tilde{A}_i = \frac{1}{i}\tau_3 n_i a_1(r,t),$$

$$\tilde{\varphi} = c\tau_3. \tag{3.28}$$

This field is purely electromagnetic and differs from the unitary gauge configuration (2.24) by the Dirac vector potential A_i^{D}. In this case the angular momentum operator is the standard one,

$$\tilde{M}_i = -i\varepsilon_{ijk}x_j\partial_k + \tfrac{1}{2}\sigma_i,$$

and the decomposition analogous to (3.2) reads

$$\Psi(x,t) = \frac{1}{r}\sum_{n,k,\delta}\sum_{\nu} v_\nu^{n,k,\delta}(r,t)\tilde{\Psi}_{nk\delta\nu}(\Theta,\Phi),$$

where $\tilde{\Psi}_{nk\delta\nu}$ are the eigenfunctions of $\tilde{\boldsymbol{M}}^2$, $\tilde{M}_3$, τ_3 and $\boldsymbol{\sigma n}$ with the eigenvalues $n - \frac{1}{2}$ ($n = 1,2,\ldots$), k ($k = \pm\frac{1}{2},\ldots,\pm(n-\frac{1}{2})$), δ ($\delta = \pm 1$) and ν ($\nu = \pm 1$), respectively. The fermionic action in the external field (3.28) can be rewritten as [cf. (3.3)]

$$S_{A\Psi} = \sum_{nk\delta}\tilde{S}_{nk\delta},$$

where

$$\tilde{S}_{nk\delta} = -i\int \mathrm{d}r\,\mathrm{d}t\,\bar{v}^{nk\delta}(r,t)\tilde{D}_{n,\delta}v^{nk\delta}(r,t), \tag{3.29}$$

and

$$\tilde{D}_{n,\delta} = \partial_t - i\delta a_0 - i\tau_2(\partial_r - i\delta a_1) + \frac{n}{r}\tau_1 .$$

The free propagator $\tilde{G}^n$ corresponding to the action (3.29) can be found in the same way as G^J,

$$\tilde{G}^n = \begin{pmatrix} \partial_t \mathfrak{R}_{n^2+n} & \left(\partial_r - \frac{n}{r}\right)\mathfrak{R}_{n^2-n} \\ \left(-\partial_r - \frac{n}{r}\right)\mathfrak{R}_{n^2+n} & \partial_t \mathfrak{R}_{n^2-n} \end{pmatrix} . \tag{3.30}$$

4. The zeroth-order approximation

In this section we describe an approximation for evaluating matrix elements of zero-angular-momentum fermionic fields in the presence of a monopole, i.e. the matrix elements of the following form:

$$W(r_1t_1,\dots,r_N't_N') = \langle \chi(r_1t_1)\dots\chi(r_Nt_N)\bar{\chi}(r_1't_1')\dots\bar{\chi}(r_N't_N')\rangle^{\mathrm{MON}} . \tag{4.1}$$

Using the representations [which are inverse to (3.2), (2.19)]

$$\chi_1^{(s)} = (8\pi)^{-1/2} r \int \varepsilon_{\alpha l}\Psi_{\alpha l}^{(s)}(x,t)\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi ,$$

$$\chi_2^{(s)} = i(8\pi)^{-1/2} r \int \varepsilon_{l\beta}\tau_{\beta\alpha}^a n^a \Psi_{\alpha l}^{(s)}(x,t)\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi , \tag{4.2}$$

one can relate the matrix elements (4.1) to the matrix elements of the initial fields $\Psi^{(s)}$ in the presence of a monopole.

The functional integral representation (2.9) for the matrix elements (4.1) can be rewritten in the following way*:

$$W(r_1t_1,\dots,r_N't_N') = \int \mathrm{d}A_\mu \,\mathrm{d}\varphi \exp\Big[-S_{A,\varphi} - \hat{S}[A,\varphi; r_1t_1,\dots,r_N't_N']$$

$$+ \text{gauge-fixing terms} + \text{ghost terms}\Big] ,$$

where the fields A_μ, φ obey the boundary conditions (2.10) and*

$$\mathrm{e}^{-\hat{S}} = \int \prod_{s=1}^{2} \mathrm{d}\bar{\Psi}^{(s)}\mathrm{d}\Psi^{(s)}\mathrm{e}^{-S_{A,\Psi}}\chi(r_1t_1)\dots\bar{\chi}(r_N't_N') .$$

* We still omit the standard denominator in the r.h.s. of these equations.

We search for the minimum of the effective action $S_{A,\varphi}+\hat{S}$ and assume that, to the lowest order in e^2 and c^{-1}, the matrix element (4.1) is

$$W(r_1t_1,\dots,r'_Nt'_N)=\exp\left[-\left(S_{A,\varphi}+\hat{S}\right)_{\min}\right].$$

We also assume that the fields A,φ realising this minimum take the form (2.11), where the field $a_1(r,t)$ obeys the boundary condition (2.12) [we are still proceeding in the temporal gauge (3.16)]. Under the above assumptions the fermionic contribution to the effective action, $\hat{S}$, takes a particularly simple form

$$\begin{aligned}\hat{S}=&-2\sum_{J\neq 0}\sum_{\delta}(2J+1)\log\mathrm{Det}\left[iD_{J\delta}(a_1)\right]\\&-2\log\mathrm{Det}\left[iD_{J=0}(a_1)\right]+\sum_{p=1}^{N}\left[\sigma(r_p,t_p)-\sigma(r'_p,t'_p)\right]\\&-\log\left\{\exp\left[\sum_{p=1}^{N}i\tau_2^{(p)}\gamma(r_p,t_p)+\sum_{p'=1}^{N}i\tau_2^{(p')}\gamma(r'_p,t'_p)\right]\right.\\&\left.\times W^{(0)}(r_1t_1,\dots,r'_Nt'_N)\right\},\end{aligned}\tag{4.3}$$

where the operators $D_{J=0}$ and $D_{J,\delta}$ are defined by (3.6) and (3.7), σ and γ are defined by (3.18) and (3.19), and $W^{(0)}(r_1t_1,\dots,r'_Nt'_N)$ is the "free" (no interaction with a_1) matrix element (4.1), i.e. the Wick expansion of (4.1) with the pairing (3.21). Eq. (4.3) is a direct consequence of (3.2)–(3.4) and (3.17), the factor 2 in the first two terms on the r.h.s. of (4.3) comes from the summation over the flavour s, while the factor $(2J+1)$ in the first term of the r.h.s. of (4.3) comes from the summation over the third component of angular momentum.

Now we make another assumption which will be justified in sect. 6. We assume that the first term on the r.h.s. of (4.3) is negligible. Since the a_1 dependences of the third and fourth terms in (4.3) are explicit, we only have to evaluate the second term. This can be done in the same way as in the Schwinger model [19], so we only sketch the derivation. It is convenient to adopt the following unified notation. By ξ_ι ($\iota=0,1$) we denote the coordinates in the (t,r) half-plane:

$$\xi_0=t,\qquad \xi_1=r,\tag{4.4a}$$

so that

$$\xi^2\equiv\xi_\iota\xi_\iota=r^2+t^2,\qquad \mathrm{d}^2\xi=\mathrm{d}r\,\mathrm{d}t.\tag{4.4b}$$

The variation of the second term on the r.h.s. of (4.3) with respect to the variation of a_1 is

$$\delta(-2\log \mathrm{Det}\, iD_{J=0}) = -2\int \mathrm{d}^2\xi\, \mathrm{Sp}\, G(\xi,\xi)\, \delta a_1(\xi). \tag{4.5}$$

From the explicit expressions (3.17), (3.21) it follows that the contribution of the second (non-singular) term on the r.h.s. of (3.21) vanishes. Using the point-splitting regularization,

$$G(\xi,\xi) = \tfrac{1}{2} \lim_{\varepsilon_t \to 0,\, \varepsilon^2 \neq 0} \left[G(\xi|\varepsilon) + G(\xi|-\varepsilon) \right],$$

$$G(\xi|\varepsilon) = \exp\left(i\int_{\xi}^{\xi+\varepsilon} a_t(\xi')\, \mathrm{d}\xi'_t \right) G(\xi, \xi+\varepsilon),$$

which is invariant under the gauge transformation (3.8), we obtain

$$\mathrm{Sp}\, G(\xi,\xi) = -\frac{1}{\pi}\partial_t \sigma(r,t),$$

where σ is defined by (3.18). From (4.5) we get

$$\delta(-2\log \mathrm{Det}\, iD_{J=0}) = \frac{2}{\pi}\int \mathrm{d}r\,\mathrm{d}t\, \partial_t\sigma \cdot \delta a_1$$

$$= -\frac{2}{\pi}\int \sigma\delta\left[(\partial_r^2 + \partial_t^2)\sigma \right] \mathrm{d}r\,\mathrm{d}t. \tag{4.6}$$

The last expression has been obtained by integration by parts with the use of (3.22) [this is another way to understand the necessity of the boundary condition (3.22)]. Finally, from (4.6) we find

$$-2\log \mathrm{Det}\, iD_{J=0} = -\frac{1}{\pi}\int \sigma(\partial_r^2 + \partial_t^2)\sigma\, \mathrm{d}r\,\mathrm{d}t. \tag{4.7}$$

In terms of the variable σ, the action $S_{A,\varphi}$ can be rewritten as [see (2.13); we still take the limit (2.8)]

$$S_{A,\varphi} = \frac{4\pi}{e^2}\int \mathrm{d}r\,\mathrm{d}t \left[(\partial_r^2 + \partial_t^2)\sigma \right]^2 \cdot r^2 \tag{4.8}$$

so the effective action $S_{A,\varphi} + \hat{S}$, within our approximation, is at most quadratic in σ [the last term in (4.3) is, in fact, linear in γ and hence in σ] and the quadratic part is the sum of (4.7) and (4.8),

$$S_2(\sigma) = S_{A,\varphi} - 2\log\operatorname{Det} iD_{J=0}$$

$$= \tfrac{1}{2}\int \sigma(r,t) L_{r,t}\sigma(r,t)\,\mathrm{d}r\,\mathrm{d}t, \tag{4.9}$$

where

$$L_{r,t} = -\frac{2}{\pi}(\partial_r^2 + \partial_t^2) + \frac{8\pi}{e^2}(\partial_r^2+\partial_t^2) r^2 (\partial_r^2 + \partial_t^2). \tag{4.10}$$

We conclude that, within our approximation, the matrix elements (4.1) are equal to

$$W(r_1 t_1, \ldots, r'_N t'_N) = \exp\left\{ -\left(S_2 + \int \sigma j\,\mathrm{d}r\,\mathrm{d}t \right)_{\min} \right\} W^0(r_1 t_1, \ldots, r'_N t'_N), \tag{4.11}$$

where

$$\int \mathrm{d}r\,\mathrm{d}t\,\sigma j = \sum_{p=1}^{N} \left[\sigma(r_p, t_p) - \sigma(r'_p, t'_p) \right]$$

$$+ \sum_{p=1}^{N} i\tau_2^{(p)}\gamma(r_p, t_p) + \sum_{p'=1}^{N} i\tau_2^{(p')}\gamma(r'_p, t'_p) \tag{4.12}$$

is the linear term in (4.3). To find the explicit expression for the exponential in (4.11), it is sufficient to determine the Green function $\mathcal{P}(rt; r't')$ of the operator (4.10). This function obeys the following equation

$$L_{rt}\mathcal{P}(rt; r't') = \delta(r-r')\,\delta(t-t'). \tag{4.13}$$

Since the function

$$\sigma(r,t|j) = -\int \mathcal{P}(rt; r't') j(r', t')\,\mathrm{d}r'\,\mathrm{d}t',$$

realizing the minimum of $S_2 + \int j\sigma$, should obey the boundary condition (3.22), the defining equation (4.13) should be supplemented by the following boundary condition:

$$\partial_r \mathcal{P}(0t; r't') = 0. \tag{4.14}$$

As is clear from (3.25) and (A.15), the solution of (4.13), (4.14) is

$$\mathcal{P}(rt; r't') = \tfrac{1}{2}\pi\left[\mathcal{R}_{e^2/4\pi^2}(rt; r't') - \mathcal{D}(r-r', t-t') - \mathcal{D}(r+r', t-t') \right], \tag{4.15}$$

where the function $\mathcal{D}$ is defined by (3.20). Eqs. (4.11), (4.12), (4.15) are sufficient to evaluate the matrix elements (4.1) within our approximation. Rather than present explicit expressions which are somewhat complicated, we prefer to describe the functional integral fit for these matrix elements. From (4.11), (4.12) and (4.15) we find

$$W(r_1t_1,\ldots,r'_Nt'_N) = \int \prod_{r,t,s} \mathrm{d}\chi_0^{(s)} \mathrm{d}\bar{\chi}_0^{(s)} \mathrm{d}\Sigma\, \mathrm{d}\eta$$

$$\times \exp(-S_\Sigma - S_\eta - S_{\chi_0}) \chi(r_1t_1)\ldots\bar{\chi}(r'_Nt'_N), \qquad (4.16)$$

where the fields Σ, η and χ_0 are defined on a half-plane $\{r \in [0,\infty), t \in (-\infty,+\infty)\}$ and obey the boundary conditions

$$\partial_r\Sigma(0,t) = \partial_r\eta(0,t) = (1-\tau_3)\chi_0(0,t) = 0.$$

The effective actions are

$$S_\Sigma = -\tfrac{1}{2}\int \mathrm{d}r\,\mathrm{d}t\, \Sigma\left(\partial_r^2 + \partial_t^2 - \frac{e^2}{4\pi r^2}\right)\Sigma,$$

$$S_\eta = +\tfrac{1}{2}\int \mathrm{d}r\,\mathrm{d}t\, \eta(\partial_r^2+\partial_t^2)\eta,$$

$$S_{\chi_0} = \int \mathrm{d}r\,\mathrm{d}t\, \bar{\chi}_0 D_{J=0}(a=0)\chi_0,$$

with $D_{J=0}$ defined by (3.6), and

$$\chi^{(s)}(r,t) = \exp\left[-\tilde{\sigma}(r,t) + i\tau_2\int_{-\infty}^{t} \partial_r\tilde{\sigma}(r,t')\,\mathrm{d}t'\right]\chi_0^{(s)},$$

with

$$\tilde{\sigma}(r,t) = \sqrt{\tfrac{1}{2}\pi}\,[\Sigma(r,t)+\eta(r,t)].$$

Note that the integrals (4.16) are gaussian and the propagators of the fields Σ and η are

$$\mathcal{R}_{e^2/4\pi^2}(rt;r't'),\ [-\mathcal{D}(r-r',t-t') - \mathcal{D}(r+r',t-t')],$$

respectively, while the propagator of χ_0 is given by (3.21). Note also, that the fit

(4.16) is analogous to the (euclidean) functional integral counterpart of the VLS-like operator solution [20, 21] of the γ^5 analogue [32] of the Schwinger model, transformed to the temporal gauge. In the next section we further exploit this analogy to discuss the fermion-number breaking in the presence of monopole.

5. Density of the condensate of zero-angular-momentum fermions: the zeroth-order approximation

This section is devoted to discussing the fermion number violating matrix element

$$\langle f(r_1, t_1)\rangle^{\mathrm{MON}} = \langle f\rangle^{\mathrm{MON}}, \tag{5.1}$$

of the operator

$$f(r, t) = \chi_1^{(1)}(r, t)\chi_1^{(2)}(r, t) + \chi_2^{(1)}(r, t)\chi_2^{(2)}(r, t) \tag{5.2}$$

in the presence of a monopole*. The operator (5.2) carries one unit of each flavour; it is invariant under the gauge transformation (3.8).

Guided by the analogy with the Schwinger model, we begin the evaluation of (5.1) with the calculation of the two-point function (cf. [23], [32])

$$\langle f(r_1, t_1) f^{\dagger}(r_2, t_2)\rangle^{\mathrm{MON}} \equiv \mathcal{F}(r_1 t_1; r_2 t_2), \tag{5.3}$$

within the approximation of sect. 4. The general formula (4.11) applied to the function (5.3) yields

$$\mathcal{F}(r_1 t_1; r_2 t_2) = \exp\{[-S_2(\sigma) - 2\sigma(r_1, t_1) + 2\sigma(r_2, t_2)]_{\min}\} \times \mathrm{Sp}[G_0(r_1 t_1; r_2 t_2) G_0^{\mathrm{T}}(r_1 t_1; r_2 t_2)]. \tag{5.4}$$

From (4.9) and (4.13) we find that the exponential in (5.4) is minimized by the following function

$$\sigma_c(r, t) = \sigma^-(r, t | r_1, t_1) + \sigma^+(r, t | r_2, t_2), \tag{5.5}$$

where

$$\sigma^{\pm}(r, t | r_k, t_k) = \mp 2\mathcal{P}(rt; r_k t_k), \qquad k = 1, 2. \tag{5.6}$$

The function (5.5) corresponds to the following temporal gauge saddle-point field a_1^{t},

$$a_1^{\mathrm{t}} = a_1^{\mathrm{t},-} + a_1^{\mathrm{t},+}, \tag{5.7}$$

* Note that $\langle \bar{\Psi}^{C(1)}\Psi^{(2)}\rangle^{\mathrm{MON}} = (4\pi r^2)^{-1}\langle f\rangle^{\mathrm{MON}}$ + contributions from higher angular momenta. That is why we call (5.1) the "density of the condensate of zero-angular-momentum fermions".

where

$$a_1^{t,\mp}(r,t|r_k,t_k)=\int_{-\infty}^{t}(\partial_r^2+\partial_{t'}^2)\sigma^{\mp}(r,t'|r_k,t_k)\,\mathrm{d}t' \tag{5.8}$$

[in fact, (5.8) is the inverse of (3.18)]. From (5.4)–(5.6) within our approximation we obtain

$$\begin{aligned}\mathcal{F}(r_1t_1;r_2t_2)&=\exp[-4\mathcal{P}(r_1t_1;r_2t_2)+2\mathcal{P}(r_1t_1;r_1t_1)+2\mathcal{P}(r_2t_2;r_2t_2)]\\&\quad\times\mathrm{Sp}[G_0(r_1t_1;r_2t_2)G_0^{\mathrm{T}}(r_1t_1;r_2t_2)]\,.\end{aligned} \tag{5.9}$$

Note that the function $\mathcal{P}$ is finite at coinciding arguments [see (4.15) and (A.14)] so the whole expression (5.9) is finite. Note also that (5.9) is independent of the infrared mass scale μ_2. We are interested in the asymptotic behaviour of $\mathcal{F}$ as $|t_1-t_2|\to\infty$. From (3.21), (4.15), (A.16) and (A.14) we find in this limit

$$\mathcal{F}(r_1t_1;r_2t_2)=\frac{1}{16\pi^2r_1r_2}\exp\left\{-2\pi\psi\left[d\left(\frac{e^2}{4\pi^2}\right)+1\right]+2\pi\psi(1)\right\}(1+\mathrm{o}(1)),$$

where the function $d(\kappa)$ is defined by (3.27). Thus,

$$\lim_{|t_1-t_2|\to\infty}\mathcal{F}(r_1t_1;r_2t_2)=\frac{1}{16\pi^2r_1r_2}(1+\mathrm{O}(e^2))\,. \tag{5.10}$$

To evaluate (5.1) we use the cluster property (cf. [23, 32]),

$$\lim_{|t_1-t_2|\to\infty}\mathcal{F}(r_1t_1;r_2t_2)=\langle f(r_1t_1)\rangle^{\mathrm{MON}}\langle f^{\dagger}(r_2t_2)\rangle^{\mathrm{MON}}, \tag{5.11}$$

which is valid since the monopole state is the lowest energy state with non-zero magnetic charge and the operator f carries zero magnetic charge [note also the normalization condition (2.4)]. From (5.10) and (5.11) we find

$$\langle f(r_1,t_1)\rangle^{\mathrm{MON}}=\frac{\mathrm{e}^{i\theta}}{4\pi r_1}(1+\mathrm{O}(e^2)), \tag{5.12}$$

where θ is an unknown real parameter. Thus, the fermion number is indeed broken and density of the condensate of zero-angular-momentum fermions is e^2 and c independent.

We wish to discuss the saddle-point field (5.7) in some detail. Before doing this it is convenient to perform the gauge transformation of the type of eq. (3.8) with the gauge function

$$\beta(r,t) = -\gamma_c(r,t) \equiv -\int_{-\infty}^{t} \partial_r \sigma_c(r,t')\,dt'. \tag{5.13}$$

After this transformation the saddle-point field a_ι^c becomes

$$a_\iota^c(r,t) = a_\iota^-(r,t|r_1,t_1) + a_\iota^+(r,t|r_2,t_2), \qquad \iota = 0,1, \tag{5.14}$$

where

$$\begin{aligned} a_0^{\mp}(r,t|r_k,t_k) &= -\partial_r \sigma^{\mp}(r,t|r_k,t_k), \\ a_1^{\mp}(r,t|r_k,t_k) &= \partial_t \sigma^{\mp}(r,t|r_k,t_k). \end{aligned} \tag{5.15}$$

In a perfect analogy to the Schwinger model [23,32] the decomposition (5.14) implies that it is the field $a_\iota^-(r,t|r_1,t_1)$ that is responsible for the non-zero value of (5.1), while the field $a_\iota^+(r,t|r_2 t_2)$ is responsible for $\langle f^\dagger(r_2,t_2)\rangle^{\rm MON} \neq 0$. In other words, the neighbourhood of the configuration a_ι^- gives the largest contribution to the functional integral for $\langle f(r_1,t_1)\rangle^{\rm MON}$. Using either (2.14) or (2.15) as well as the explicit expression for σ^-, eq. (5.6), and asymptotics of $\mathfrak{R}$, eq. (A.16), one can calculate the winding number of the saddle-point field corresponding to the functions a_ι^-. One finds

$$q(a_\iota^-) = -1,$$

in agreement with sect. 2.

There is another argument of sect. 2 which is also justified within our approximation. Using the methods developed in this context for the Schwinger model [24,25], one can obtain (5.12) by direct evaluation of the functional integral (under the assumptions of sect. 4). One finds that the largest contribution comes indeed from the neighbourhood of the saddle-point field characterized by the functions a_ι^-, and the zero fermion mode giving rise to non-zero value of (5.1) (cf. [24,25]) coincides with (2.23) with σ^- substituted for ρ. Since this direct evaluation is quite similar to the Schwinger model one and leads to no new results, we do not reproduce it here.

6. Density of the condensate of zero-angular-momentum fermions: an estimate of corrections

In this section we argue that the corrections to (5.12) are $O(e^2)$ and/or $O(c^{-1})$. The sources of these corrections are: (i) the first term on the r.h.s. of (4.3), (ii) the contribution of the bosonic determinant to the effective action. In both cases we have to estimate the functional determinants in an external field of the form (2.11) with $a_\iota = a_\iota^-(r,t|r_1,t_1)$ (the gauge transformation (5.13) is assumed).

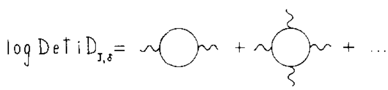

Fig. 1.

We begin with the first of these corrections,

$$\log \mathrm{Det}_{J\neq 0} iD(a^-) \equiv -2 \sum_{J\neq 0,\,\delta} (2J+1) \log \mathrm{Det}\left[iD_{J,\delta}(a^-)\right]. \tag{6.1}$$

Using the results of sect. 3 we depict the summand of (6.1) in fig. 1 where the wavy line, the solid line and the vertex correspond to $a_\iota^-(\xi)$, $G^J(\xi,\xi')$ and $-i\int \mathrm{d}^2\xi\,\zeta_\iota$, respectively [we use the compact notations of sect. 4, see eqs. (4.4)]. Here

$$\zeta_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \zeta_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{6.2}$$

From (3.24), (A.15), (A.16) it follows that the function $G^J(\xi,\xi')$ $(J\neq 0)$ vanishes at $r=0$ as well as when $(r^2+t^2)\to\infty$, so the series of fig. 1 is invariant under the gauge transformation (3.8) with the gauge function $\beta(\xi)$ which need not obey the boundary condition (3.10). Indeed, one can perform integration by parts in

$$\int \mathrm{d}^2\xi\, G^J(\xi',\xi)\zeta_\iota G^J(\xi,\xi'')\,\partial_\iota\beta(\xi)$$

and, in virtue of (3.23) and (6.2), one finds that this integral is equal to zero. We use this gauge freedom to perform the gauge transformation with the gauge function

$$\beta(r,t) = \tfrac{1}{2}\arctan\frac{t-t_1}{r+r_1}. \tag{6.3}$$

After this gauge transformation, (6.1) becomes the sum of the graphs shown in fig. 1 but with the wavy line corresponding to

$$\hat{a}_\iota(\xi) \equiv \hat{a}_\iota(r,t\,|\,r_1,t_1),$$

where

$$\hat{a}_0 = -\partial_r\hat{\sigma}, \qquad \hat{a}_1 = \partial_t\hat{\sigma}, \tag{6.4a}$$

$$\hat{\sigma}(r,t\,|\,r_1,t_1) = -\pi\left[\mathfrak{R}_{e^2/4\pi^2}(rt;r_1t_1) - \mathfrak{R}_0(rt;r_1t_1)\right], \tag{6.4b}$$

and

$$\mathfrak{R}_0(rt;r_1t_1) = \lim_{\kappa\to 0}\mathfrak{R}_\kappa(rt;r_1t_1) = \frac{1}{4\pi}\log\frac{(r-r_1)^2+(t-t_1)^2}{(r+r_1)^2+(t-t_1)^2}. \tag{6.5}$$

Note that the boundary condition (3.15) [or, equivalently, (2.22)] is not invariant under the gauge transformation (6.3), so the arguments of the present section are not applicable to the zero-angular-momentum contribution [i.e. to the second term on the r.h.s. of (4.3)].

From (6.4) it follows that the field $\hat{a}_\iota$ is formally $O(e^2)$, namely

$$\hat{a}_\iota = -\frac{e^2}{4\pi} a_\iota^{(1)} + O(e^4), \tag{6.6}$$

where

$$a_0^{(1)} = -\partial_r \sigma^{(1)}, \qquad a_1^{(1)} = \partial_t \sigma^{(1)}, \tag{6.7}$$

$$\sigma^{(1)}(r, t | r_1, t_1) = \left.\frac{\partial \mathcal{R}_\kappa(rt; r_1 t_1)}{\partial \kappa}\right|_{\kappa=0}. \tag{6.8}$$

Thus, (6.1) is formally $O(e^4)$. To make this argument precise, one has to show that each term in the expansion

$$\log \mathrm{Det}_{J \neq 0} iD(\hat{a}) = \left(\frac{e^2}{4\pi}\right)^2 V^{(1)} + \left(\frac{e^2}{4\pi}\right)^3 V^{(2)} + \cdots \tag{6.9}$$

is finite. In the present paper we study only the $O(e^4)$ contribution to (6.1), i.e. the first term on the r.h.s. of (6.9). As follows from (6.6), this contribution comes from the sum over J of the graphs with two external legs (see fig. 1). Its explicit expression is

$$V^{(1)} = -4 \int d^2\xi\, d^2\xi' \sum_{\iota, \iota'=1}^{2} \Pi_{\iota\iota'}(\xi, \xi') a_\iota^{(1)}(\xi) a_{\iota'}^{(1)}(\xi'), \tag{6.10}$$

where

$$\Pi_{\iota\iota'}(\xi, \xi') = \sum_{J=1}^{\infty} (2J+1) \mathrm{Sp}\left[G^J(\xi, \xi') \zeta_{\iota'} G^J(\xi', \xi) \zeta_\iota\right]. \tag{6.11}$$

Naively, we have to show that (6.10) is finite. However, since (6.9) is the fermionic determinant in an external field of the form (2.11) (with $a^{(1)}$ substituted for a), $V^{(1)}$ is expected to be infinite because of standard ultraviolet divergences. Thus, we have first to eliminate these divergences and then show that the renormalized $V^{(1)}$ is finite.

To realize this program, we decompose the function $\Pi_{\iota\iota'}$ in the following way,

$$\Pi_{\iota\iota'} = \Pi_{\iota\iota'}^{\mathrm{reg}} + \tilde{\Pi}_{\iota\iota'} - \tfrac{1}{2}\tilde{\Pi}_{\iota\iota'}^{(1/2)}, \tag{6.12}$$

where

$$\Pi^{\text{reg}}_{\iota\iota'}(\xi,\xi') = \sum_{J=1}^{\infty} (2J+1)\{\text{Sp}[G^J(\xi,\xi')\zeta_{\iota'}G^J(\xi',\xi)\zeta_\iota - \tfrac{1}{2}\tilde{G}^J(\xi,\xi')\zeta_{\iota'}\tilde{G}^J(\xi',\xi)\zeta_\iota$$

$$-\tfrac{1}{2}\tilde{G}^{J+1}(\xi,\xi')\zeta_{\iota'}G^{J+1}(\xi',\xi)\zeta_\iota]\}, \tag{6.13}$$

$$\tilde{\Pi}_{\iota\iota'}(\xi,\xi') = \sum_{n=1}^{\infty} 2n\,\text{Sp}[\tilde{G}^n(\xi,\xi')\zeta_{\iota'}\tilde{G}^n(\xi',\xi)\zeta_\iota], \tag{6.14}$$

$$\tilde{\Pi}^{(1/2)}_{\iota\iota'}(\xi,\xi') = \text{Sp}[\tilde{G}^1(\xi,\xi')\zeta_{\iota'}\tilde{G}^1(\xi',\xi)\zeta_\iota], \tag{6.15}$$

and the Green functions $\tilde{G}^n$ are given by (3.30). Note that the three terms on the r.h.s. of (6.12) are separately invariant under the gauge transformation (6.3). According to the decomposition (6.12), the integral (6.10) is decomposed as

$$V^{(1)} = V^{(1)\text{reg}} + \tilde{V}^{(1)} - \tfrac{1}{2}\tilde{V}^{(1)(1/2)}, \tag{6.16}$$

where

$$V^{(1)\text{reg}} = -4\int d^2\xi\, d^2\xi' \sum_{\iota,\iota'} \Pi^{\text{reg}}_{\iota\iota'}(\xi,\xi') a^{(1)}_\iota(\xi) a^{(1)}_{\iota'}(\xi'), \tag{6.17}$$

etc. The second term on the r.h.s. of (6.16) is just the $O(a^{(1)2})$ contribution to the fermionic determinant in an external field (3.28) with $a^{(1)}$ substituted for a. Thus, the standard ultraviolet divergences are contained in this term and its renormalized value is

$$\tilde{V}^{(1)} = -\frac{1}{3\pi^2}\int F^{(1)}_{0i}(\boldsymbol{x},t) F^{(1)}_{0i}(\boldsymbol{x}',t') \tilde{\Pi}(\boldsymbol{x}-\boldsymbol{x}', t-t')\, d^3\boldsymbol{x}\, d^3\boldsymbol{x}'\, dt\, dt', \tag{6.18}$$

where

$$F^{(1)}_{0i} = n_i(\partial_t a^{(1)}_1 - \partial_r a^{(1)}_0) = -\frac{n_i}{r^2}\mathfrak{R}_0(rt; r_1 t_1) \tag{6.19}$$

is the electric field for the configuration (3.28) (the magnetic field vanishes) and the Fourier transform of $\tilde{\Pi}$ is

$$\tilde{\Pi}(\boldsymbol{p}, p_0) = \log(\boldsymbol{p}^2 + p_0^2)/\mu_0^2, \tag{6.20}$$

μ_0 being the normalization point. Using (6.18)–(6.20) one can prove that $\tilde{V}^{(1)}$ is finite; this is done in appendix B.

Now we estimate the first term on the r.h.s. of (6.16). First, we have to show that (6.13) is integrable near $\xi=\xi'$ [the natural measure is $d^2\xi$, see (6.17)]. From (3.24), (3.30) and (A.14) it follows that $G^J(\xi,\xi')$ and $\tilde{G}^J(\xi,\xi')$ behave near $\xi=\xi'$ as

$$G^J(\xi,\xi')=\frac{1}{2\pi}\frac{\zeta_\iota^+(\xi-\xi')_\iota}{(\xi-\xi')^2}+\mathrm{O}\left[\log(\xi-\xi')^2\right],$$

$$\tilde{G}^J(\xi,\xi')=G^J(\xi,\xi')+\mathrm{O}\left[\log(\xi-\xi')^2\right]. \tag{6.21}$$

From (6.21) we find that the summand in (6.13) is integrable near $\xi=\xi'$. From (3.26) and (A.5) it follows that the series (6.13) is convergent at fixed $\xi\neq\xi'$. The proof of integrability of the whole $\Pi^{\mathrm{reg}}_{\iota\iota'}$ near $\xi=\xi'$ is more involved; it is outlined in appendix C. To proceed further, we need to specify the behaviour of $\Pi^{\mathrm{reg}}_{\iota\iota'}(\xi,\xi')\equiv\Pi^{\mathrm{reg}}_{\iota\iota'}(rt;r't')$ at $r=0$ and at $(\xi-\xi')^2\to\infty$. From (3.24), (3.30) and (A.15) it follows that

$$\Pi^{\mathrm{reg}}_{\iota\iota'}(rt;r't')=\mathrm{O}(r^2), \tag{6.22}$$

at small r, while from (A.16) we get

$$\Pi^{\mathrm{reg}}_{\iota\iota'}(\xi,\xi')=\mathrm{O}\left[1/(\xi-\xi')^2\right] \tag{6.23}$$

at large $(\xi-\xi')^2$. From (6.7), (6.8) and (A.15) we find that $a_\iota^{(1)}(r)=\mathrm{O}(\log r)$ at small r, and, in view of (6.22), the integral (6.17) is convergent near $r=0$ and/or $r'=0$. Eq. (A.16) leads to the following estimate of $a_\iota^{(1)}$ in the infrared region,

$$a_\iota^{(1)}(\xi)=\mathrm{O}(\xi^{-2}),\qquad \xi^2\to\infty.$$

Using (6.23), it is straightforward to prove that the integral (6.17) is convergent at large ξ^2 and/or ξ'^2. Since at finite ξ and ξ' the integrand of (6.17) is an integrable function, the above statements complete the proof of finiteness of $V^{(1)\mathrm{reg}}$.

We are left with the third term on the r.h.s. of (6.16). Naively, it is logarithmically divergent because of the $(\xi-\xi')^{-2}$ singularity of $\tilde{\Pi}^{(1/2)}_{\iota\iota'}$. However, the structure of this singularity coincides with that of the Schwinger model [19]. This is clear from the identity $\tilde{G}^1(\xi,\xi')\zeta_{\iota'}\equiv\tilde{G}^1(\xi,\xi')\tau_3\tau_3\zeta_{\iota'}$ since the singular part of $\tilde{G}^1\tau_3$ is just the two-dimensional massless fermionic propagator (cf. (6.21) and [30]), if the two-dimensional γ-matrices are equal to $\gamma_\iota=\tau_3\zeta_\iota\equiv(\tau_3,-\tau_1)$. It is well known that the polarization operator of the Schwinger model is integrable, so $\tilde{\Pi}^{(1/2)}_{\iota\iota'}$ is also integrable. This can be demonstrated explicitly with the use of the point-splitting regularization technique analogous to that used in sect. 4. Repeating the arguments which have led to the finiteness of $V^{(1)\mathrm{reg}}$, we find that $\tilde{V}^{(1)(1/2)}$ is finite. Thus, the $\mathrm{O}(e^4)$ term in (6.1) is finite, which is the desired result.

Another source of corrections to (5.12) is the bosonic determinant in the external field (2.11) with $a^{(1)}$ substituted for a. Far from the monopole center this configuration is purely electromagnetic (see the discussion at the end of sect. 2), so the contributions of the Higgs field and the charged vector fields to the effective action are $O(m_{\text{Higgs}}^{-2})$ and $O(m_{\text{vector}}^{-2})$, respectively, while the electromagnetic field gives no contribution because of linearity. Near the monopole center the purely electromagnetic nature of the saddle-point field is lost, but the relevant spatial volume is $O(r_M^3)$, so the corrections from the bosonic fields vanish in the limit (2.8).

Thus, the above arguments show that the corrections to the zeroth-order approximation are $O(e^2)$ or $O(c^{-1})$, as has been claimed in sect. 1. We conclude that this approximation is reasonable at least for the evaluation of Green functions of fermions with zero total angular momentum, including fermion-number breaking Green functions.

7. Discussion

We would like to make some comments concerning different aspects of the above effect.

7.1. RELATION TO THE θ-VACUUM STRUCTURE

Since the vacuum structure of the gauge theories is most apparent in the temporal gauge (3.16a) [28], it is convenient to proceed in this gauge. The temporal gauge saddle-point field $a_1^{t,-}$ giving rise to a non-zero value of (5.1) is defined by (5.6), (5.8). From (4.15) we find that $a_1^{t,-}$ can be represented as

$$a_1^{t,-}(r,t;r_1,t_1)=\pi\theta(t-t_1)\,\delta(r-r_1)$$
$$-\partial_r^2\int_{-\infty}^{t}\mathcal{R}_{e^2/4\pi^2}(rt';r_1t_1)\,\mathrm{d}t'-\partial_t\mathcal{R}_{e^2/4\pi^2}(rt;r_1t_1)\,. \quad (7.1)$$

From (A.16) it follows that the last term vanishes as $t\to\mp\infty$ so the field $a_1^{t,-}$ interpolates between the following two configurations:

$$a_1^{t,-}(r,t=-\infty\,|\,r_1)=0\,, \quad (7.2a)$$

$$a_1^{t,-}(r,t=+\infty\,|\,r_1)=\partial_r\Omega(r\,|\,r_1)\,, \quad (7.2b)$$

where

$$\Omega(r\,|\,r_1)=\pi\theta(r-r_1)-\partial_r\int_{-\infty}^{+\infty}\mathcal{R}_{e^2/4\pi^2}(r,t';0,r_1)\,\mathrm{d}t'\,.$$

The field (7.2b) is a pure gauge [see (3.8)]; from (A.15), (A.16) we find the following asymptotics of the gauge function Ω:

$$\Omega(0\,|\,r_1)=0\,,\qquad \Omega(\infty\,|\,r_1)=\pi\,. \quad (7.3)$$

Now we recall the fact that the gauge transformation (3.8) in terms of the initial fields A_μ, φ, Ψ is just the usual gauge transformation with the gauge function (3.9). Thus, the saddle-point configuration (i.e. the configuration (2.11) with $a^{t,-}$ substituted for a) interpolates between the fields

$$A_i(t=-\infty)=A_i^{\mathrm{cl}}, \qquad \varphi(-\infty)=\varphi^{\mathrm{cl}},$$

$$A_i(t=+\infty)=g_\Omega A_i^{\mathrm{cl}} g_\Omega^{-1}+g_\Omega \partial_i g_\Omega^{-1},$$

$$\varphi(t=+\infty)=g_\Omega \varphi^{\mathrm{cl}} g_\Omega^{-1}=\varphi^{\mathrm{cl}},$$

where

$$g_\Omega=\exp(i\tau^a n^a \Omega). \tag{7.4}$$

From (7.3) we conclude that the gauge function (7.4) has just the same form as that considered in ref. [28] and its topological number is equal to -1. This could be anticipated since, as has been shown in sect. 5, the saddle-point field has the winding number -1 and in the temporal gauge the winding number of any configuration obeying $\{A_i(t=+\infty)=\text{gauge transform of } A_i(t=-\infty)\}$, coincides with the topological number of the gauge transformation [9]. The same arguments as those of refs. [28, 33] show that the vector $U[g_\Omega]|M,0\rangle$ ($U[g_\Omega]$ being the operator of the gauge transformation with the gauge function (7.4)), which is the gauge transform of the perturbation theory monopole state $|M,0\rangle$, carries one unit of each flavour. This could also be anticipated, since the operator $U[g_\Omega]$ carries one unit of each flavour, as follows from the considerations of refs. [28, 33]. The gauge-invariant monopole state is a linear superposition of the form*

$$|M,\theta\rangle=\sum_{n=-\infty}^{+\infty} \mathrm{e}^{in\theta}(U[g_\Omega])^n|M,0\rangle\,; \tag{7.5}$$

this is another way to understand the fermion-number breaking in the presence of a monopole. In fact, the heuristic arguments of sect. 2 are simplified in the temporal gauge; indeed, the unboundedness from below (by any positive number) of the action (2.13) can be established by the Derrick-like [34] time rescaling (for details see [35]).

7.2. THE UNITARY GAUGE

The particle content of the theory with the action (2.1) is most apparent in the unitary gauge. In this gauge it makes sense to consider the matrix element

* In the theories without massless fermions, the operator $U[g_\Omega]$ carries no superselection charge. Nevertheless, the representation (7.5) is valid and leads to Witten's value of the charge of quantum dyon [8, 10].

$\langle \varepsilon_{\alpha\beta} \Psi^{(1)}_{+\alpha} \Psi^{(2)}_{-\beta} \rangle^{\mathrm{MON}}$ ($\alpha, \beta = 1, 2$ are Lorentz indices, the fields $\Psi^{(s)}_{\pm}$ are defined in sect. 2). Since the operator $\Psi^{(1)}_{+}\Psi^{(2)}_{-}$ carries one unit of each flavour, the non-zero contributions to this matrix element come from the unitary gauge configurations with the winding number equal to -1, in particular, from the field (2.24), (2.17) [or (2.24), (5.15)]. The latter contribution is proportional to the zero fermion modes in the external field (2.24), (2.17), namely, it is proportional to

$$\varepsilon_{\alpha\beta} \Psi^{\mathrm{u}}_{+\alpha} \Psi^{\mathrm{u}}_{-\beta}, \tag{7.6}$$

where the zero mode Ψ^{u} is just the zero mode (2.19), (2.23) transformed to the unitary gauge. Performing this gauge transformation (the corresponding gauge function is described, e.g., in ref. [29]) far from the monopole center, we obtain

$$\begin{aligned} \Psi^{\mathrm{u}}_{+} &= B(r,t) \begin{pmatrix} \sin\frac{1}{2}\Theta\, \mathrm{e}^{-i\Phi} \\ -\cos\frac{1}{2}\Theta \end{pmatrix}, \\ \Psi^{\mathrm{u}}_{-} &= B(r,t) \begin{pmatrix} \cos\frac{1}{2}\Theta \\ \sin\frac{1}{2}\Theta\, \mathrm{e}^{i\Phi} \end{pmatrix}, \end{aligned} \tag{7.7}$$

where Θ, Φ are polar angles and

$$B(r,t) = \frac{N}{\sqrt{8\pi}} \frac{\mathrm{e}^{-\rho(r,t)}}{r}.$$

Note that Ψ^{u}_{-} is the CP conjugate of Ψ^{u}_{+}. From (7.6), (7.7) we conclude that $\langle \varepsilon_{\alpha\beta} \Psi^{(1)}_{+\alpha} \Psi^{(2)}_{-\beta} \rangle^{\mathrm{MON}} \neq 0$, i.e. the Adler-Bell-Jackiw anomaly gives rise to flavour-non-conserving and fermion-number-non-conserving transitions with charge conservation.

7.3. BARYON-NUMBER BREAKING IN THE PRESENCE OF THE FUNDAMENTAL SU(5) MONOPOLE

The fundamental monopole [3, 4] of the SU(5) grand unified theory [5] coincides asymptotically with the 't Hooft-Polyakov one for the SU(2) group imbedded into SU(5) in the following way,

$$\boldsymbol{T} = \tfrac{1}{2}\,\mathrm{diag}(0, 0, \boldsymbol{\tau}, 0). \tag{7.8}$$

This monopole is fundamental in the sense that it is characterized by minimal magnetic charge. With respect to SU(2) specified by (7.8), the first generation fermions form the following left-handed doublets (in the unitary gauge),

$$\begin{pmatrix} -\bar{\mathrm{u}}^2 \\ \mathrm{u}^1 \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \bar{\mathrm{u}}^1 \\ \mathrm{u}^2 \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \mathrm{d}^3 \\ \mathrm{e}^+ \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \mathrm{e}^- \\ -\bar{\mathrm{d}}^3 \end{pmatrix}_{\mathrm{L}}, \tag{7.9}$$

others being singlets. In (7.9) the superscripts 1, 2, 3 are colour indices.

If u and d quarks and electrons were massless, the above arguments would be directly applicable to this case, so the matrix element

$$\langle u^1 u^2 d^3 e^- \rangle^{MON} \tag{7.10}$$

would be non-zero, and coupling-constant and unification scale independent. This conclusion remains unchanged if other (massive) generations are taken into account [17]. The matrix element (7.10) corresponds to the process

$$p + \text{monopole} \to e^+ + \text{monopole} + \text{everything}, \tag{7.11}$$

and the arguments of the present paper imply that the cross section of this process is independent of the coupling constant and the unification scale*, i.e. it is roughly $O(1\ \text{GeV}^{-2})$. Unfortunately, the above discussion is not quite decisive. First, electrons and quarks are massive. Naively, this seems to be inessential at distances small compared to the Compton wavelengths of electron and light quarks. However, in the massive case the higher order corrections could destroy the boundary conditions (2.22), (3.15) thus invalidating the above analysis. For example, the boundary conditions for fermions with extra magnetic moment [30] differ from those given by (2.22). Second, in the above considerations we completely ignored gluon self-interaction. So, further investigations are required to establish the existence of processes like (7.11) and to estimate the cross sections of these processes.

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Appendix A

In this appendix we summarize some relevant properties of the special functions.

A.1. LEGENDRE FUNCTION

The Legendre function $Q_m(x)$ obeys the following equation [36, 37, 38]:

$$(1-x^2)\frac{d^2Q_m}{dx^2} - 2x\frac{dQ_m}{dx} + m(m+1)Q_m = 0. \tag{A.1}$$

* For the discussion of some possible experimental consequences of this effect see [18].

Its explicit expression for $m=0$ is

$$Q_0(x) = -\tfrac{1}{2}\log\frac{x-1}{x+1}\,. \tag{A.2}$$

It has the following asymptotic behaviour as $x\to 1$ [38]:

$$Q_m(x) = -\tfrac{1}{2}\log\frac{x-1}{2} - \psi(m+1) + \psi(1) + \mathrm{O}[(x-1)\log(x-1)]\,. \tag{A.3}$$

From the representation [36–38]

$$Q_m(x) = 2^{-m-1}\pi^{1/2}\frac{\Gamma(m+1)}{\Gamma(m+\frac{3}{2})}x^{-m-1}F\left(1+\frac{m}{2},\frac{1+m}{2};m+\tfrac{3}{2};\frac{1}{x^2}\right),$$

where $F(\alpha,\beta;\gamma;x)$ is the hypergeometric function, it follows that

$$Q_m(x) = 2^{-m-1}\pi^{1/2}\frac{\Gamma(m+1)}{\Gamma(m+\frac{3}{2})}x^{-m-1}(1+\mathrm{O}(x^{-2})) \tag{A.4}$$

at large x. Q_m can be also expressed as [36, 38]

$$Q_m(x) = (\tfrac{1}{2}\pi)^{1/2}(x^2-1)^{-1/4}\frac{\Gamma(m+1)}{\Gamma(m+\frac{3}{2})}\left[x-(x^2-1)^{1/2}\right]^{m+1/2}$$

$$\times F\left(\tfrac{1}{2},\tfrac{1}{2};m+\tfrac{1}{2};-\frac{x-(x^2-1)^{1/2}}{2(x^2-1)^{1/2}}\right).$$

Using the Stirling formula,

$$\Gamma(m) = \mathrm{e}^{-m+m\log m}m^{-1/2}(2\pi)^{1/2}(1+\mathrm{O}(m^{-1}))\,,$$

as well as the definition of the hypergeometric series, we find at large m and x fixed

$$Q_m(x) = (\tfrac{1}{2}\pi)^{1/2}m^{-1/2}(x^2-1)^{-1/4}\left(x-(x^2-1)^{1/2}\right)^{m+1/2}. \tag{A.5}$$

Now we derive the asymptotic expansion of $Q_m(x)$ as $m\to\infty$ which is uniformly valid at $1<x<\infty$. We use the method described by Thorne [39] and consider the function $y(\tau)$ defined by

$$y(\tau) = \left(\frac{\sinh\tau}{\tau}\right)^{1/2}Q_m(\cosh\tau)\,.$$

From (A.1) we obtain the following equation for $y(\tau)$:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + \frac{1}{\tau}\frac{\mathrm{d}y}{\mathrm{d}\tau} - \Lambda^2 y + w(\tau) y = 0, \tag{A.6}$$

where

$$\Lambda = m + \tfrac{1}{2}, \qquad w(\tau) = \tfrac{1}{4}(\sinh^{-2}\tau - \tau^{-2}). \tag{A.7}$$

We search for the solution of (A.6) in the form of asymptotic series

$$y(\tau) = K_0(\Lambda\tau) \sum_{n=0}^{\infty} \frac{T_n(\tau)}{\Lambda^{2n}} - \frac{K_1(\Lambda\tau)}{\Lambda} \sum_{n=0}^{\infty} \frac{R_n(\tau)}{\Lambda^{2n}}, \tag{A.8}$$

where K_m are modified Bessel functions. Inserting (A.8) into (A.6), we obtain the following recurrent relations:

$$R_n(\tau) = -\tfrac{1}{2}\int_0^{\tau}\left[T_n''(\tau') + \frac{T_n'(\tau')}{\tau'} + w(\tau')T_n(\tau')\right]\mathrm{d}\tau',$$

$$T_{n+1}(\tau) = -\tfrac{1}{2}\int_0^{\tau}\left[R_n''(\tau') - \frac{R_n'(\tau')}{\tau'} + \frac{R_n(\tau')}{\tau'^2} + w(\tau')R_n(\tau')\right]\mathrm{d}\tau'. \tag{A.9}$$

By comparing the behaviours of $y(\tau)$ and $K_0(\Lambda\tau)$ at small τ, namely (see (A.3) and [36, 37])

$$y(\tau) = -\log\tau + \mathrm{O}(1),$$

$$K_0(\Lambda\tau) = -\log\tau + \mathrm{O}(1),$$

we find that

$$T_0 = 1. \tag{A.10}$$

Eqs. (A.9), (A.10) are sufficient to determine the unknown functions T_n and R_n. Note that at small τ

$$R_n = \mathrm{O}(\tau), \qquad n \geqslant 0,$$

$$T_n = \mathrm{O}(\tau^2), \qquad n > 0.$$

Thus, the desired expansion is

$$Q_m(\cosh\tau) = \left(\frac{\tau}{\sinh\tau}\right)^{1/2}\left\{K_0\left[(m+\tfrac{1}{2})\tau\right]\sum_{n=0}^{\infty}\frac{T_n(\tau)}{(m+\frac{1}{2})^{2n}} - \frac{K_1\left[(m+\frac{1}{2})\tau\right]}{m+\frac{1}{2}}\sum_{n=0}^{\infty}\frac{R_n(\tau)}{(m+\frac{1}{2})^{2n}}\right\}. \tag{A.11}$$

Note that the asymptotic expansion (A.11) is a particular case of Thorne's [39] and is uniformly valid in the region $0<\tau<\infty$. Performing the change of variables, $\tau=z/(m+\frac{1}{2})$, we find another asymptotic expansion,

$$Q_m\left(\cosh\frac{z}{m+\frac{1}{2}}\right)=K_0(z)\left\{1+\frac{z^2}{\left(m+\frac{1}{2}\right)^2}\sum_{n=0}^{\infty}\frac{\tilde{T}_n(z)}{\left(m+\frac{1}{2}\right)^{2n}}\right\}$$

$$-\frac{K_1(z)z}{\left(m+\frac{1}{2}\right)^2}\sum_{n=0}^{\infty}\frac{\tilde{R}_n(z)}{\left(m+\frac{1}{2}\right)^{2n}}, \tag{A.12}$$

where

$$\tilde{R}_n=T_n=\mathrm{O}(1), \qquad z\to 0. \tag{A.13}$$

A.2. THE FUNCTION $\mathfrak{R}_\kappa(rt;r't')$

$\mathfrak{R}_\kappa(rt;r't')$ is defined by (3.26), (3.27). From (A.1) and (A.3) it follows that this function obeys (3.25). From (A.3) we find

$$\mathfrak{R}_\kappa(rt;r't')=\frac{1}{4\pi}\log\frac{(r-r')^2+(t-t')^2}{4r^2}+\frac{1}{2\pi}\{\psi[d(\kappa+1)]-\psi(1)\}$$

$$+\mathrm{O}\left\{\left[(r-r')^2+(t-t')^2\right]\log\left[(r-r')^2+(t-t')^2\right]\right\} \tag{A.14}$$

at small $(r-r')^2+(t-t')^2$. Eq. (A.4) yields

$$\mathfrak{R}_\kappa=\alpha(\kappa)\left[\frac{rr'}{r'^2+(t-t')^2}\right]^{1/d(\kappa)}, \qquad \text{as } r\to 0, \tag{A.15}$$

as well as

$$\mathfrak{R}_\kappa=\alpha(\kappa)\left[\frac{(r-r')^2+(t-t')^2}{rr'}\right]^{-1-d(\kappa)}, \qquad r^2+t^2\to\infty,$$

$$\alpha(\kappa)=-\frac{1}{2\sqrt{\pi}}\frac{\Gamma(1+d(\kappa))}{\Gamma(\frac{3}{2}+d(\kappa))}. \tag{A.16}$$

A.3. THE FUNCTION $E_1(z)$

$E_1(z)$ is defined by the following relation [36]:

$$E_1(z)=\int_z^{\infty}\frac{\mathrm{e}^{-z'}}{z'}\,\mathrm{d}z'. \tag{A.17}$$

This function is analytical in the complex z plane cut along the real negative semiaxis. The function $E_1(z)+\log z$ is analytical [36] in the z plane. The function $E_1(z)$ has the following behaviour near $z=0$ [36]:

$$E_1(z) = -\log z + \psi(1) + \mathrm{O}(z) . \tag{A.18}$$

From the analyticity of $E_1(z)+\log z$ we find

$$\lim_{\delta\to+0} \operatorname{Im} E_1(x+i\delta) = \lim_{\delta\to-0} \operatorname{Im} E_1(-x+i\delta) + \pi = 0 \tag{A.19}$$

for real $x>0$. The asymptotic expansion for $E_1 + \log z$ at large $|z|$ reads [36]

$$E_1(z) + \log z = \frac{\mathrm{e}^{-z}}{z}\left[1 - \frac{1}{z} + \mathrm{O}\left(\frac{1}{z^2}\right)\right] + \log z . \tag{A.20}$$

Appendix B

In this appendix we show that the integral (6.18) is finite. It is convenient to perform the Fourier transformation of (6.19); from (6.5) we find

$$F_{0i}^{(1)}(\boldsymbol{p}, p_0; r_1) = i\pi \frac{p_i}{|\boldsymbol{p}|} I(|\boldsymbol{p}|, p_0; r_1)$$

(without loss of generality we have chosen $t_1=0$), where

$$I(|\boldsymbol{p}|, p_0; r_1) = \int_0^\infty \mathrm{d}r\, \varepsilon(r-r_1) \frac{\sin|\boldsymbol{p}|r}{r}\, \mathrm{e}^{-|p_0||r-r_1|} - \int_0^\infty \mathrm{d}r \frac{\sin|\boldsymbol{p}|r}{r}\, \mathrm{e}^{-|p_0|(r+r_1)} ,$$

or [36]

$$I(|\boldsymbol{p}|, p_0; r_1) = -\mathrm{e}^{-|p_0|r}\{\pi + \operatorname{Im} E_1[-r_1(|p_0| - i|\boldsymbol{p}|]\} - \mathrm{e}^{|p_0|r} \operatorname{Im} E_1[r_1(|p_0| + i|\boldsymbol{p}|)] .$$

The function E_1 is defined by (A.17). In terms of the function I, the integral (6.18) reads

$$\tilde{V}^{(1)} = \tfrac{4}{3}\pi \int I(|\boldsymbol{p}|, p_0; r_1) I(|\boldsymbol{p}|, -p_0; r_1) \times \log\left[\left(|\boldsymbol{p}|^2 + p_0^2\right)/\mu_0^2\right] \mathrm{d}|\boldsymbol{p}|\, \mathrm{d}p_0/|\boldsymbol{p}|^2 . \tag{B.1}$$

Since the integrand is finite at $|\boldsymbol{p}| \neq 0$, $p_0^2 + \boldsymbol{p}^2 \neq 0$, there are three potentially dangerous regions, (i) $\boldsymbol{p}^2 + p_0^2 \to 0$, (ii) $|\boldsymbol{p}| \to 0$, (iii) $\boldsymbol{p}^2 + p_0^2 \to \infty$. From (A.18) we find

$$I(|\boldsymbol{p}|, p_0; r_1) = 2 p_0 r_1 \arctan|\boldsymbol{p}/p_0| + \mathrm{O}(\boldsymbol{p}^2 + p_0^2),$$

so the integral (B.1) is convergent in region (i). As follows from (A.19), the integral (B.1) is also convergent in region (ii). Finally, from (A.20) we get in region (iii)

$$I = -\frac{1}{(\boldsymbol{p}^2 + p_0^2)^{1/2}} \{\sin(|\boldsymbol{p}| r_1 - \arctan|\boldsymbol{p}/p_0|) - \sin(|\boldsymbol{p}| r_1 + \arctan|\boldsymbol{p}/p_0|)\},$$

so the integral (B.1) converges in this region.

Appendix C

In this appendix we outline the proof of integrability of the function $\Pi^{\text{reg}}_{\iota\iota'}(\xi, \xi')$ defined by (6.13). This function is a weighted sum over $u, v, u', v' = 1,2$ of

$$\Pi^{\text{reg}}_{uv;\,u'v'}(\xi, \xi') = \sum_{J=1}^{\infty} (2J+1)\Big[G^J_{uv}(\xi, \xi') G^J_{u'v'}(\xi', \xi) - \tfrac{1}{2}(G^J \to \tilde{G}^J) - \tfrac{1}{2}(G^J \to \tilde{G}^{J+1})\Big].$$

For the sake of definiteness we consider only $\Pi^{\text{reg}}_{11;\,11}$; other cases are treated in the same way. Instead of proving integrability of $\Pi^{\text{reg}}_{11;\,11}$ directly, we solve an equivalent problem of proving the finiteness of its momenta, i.e. the finiteness of the integrals of the form

$$\int \Pi^{\text{reg}}_{11;\,11}(\xi, \xi')(\xi - \xi')_{\iota_1} \ldots (\xi - \xi')_{\iota_k} \, \mathrm{d}^2\xi' \tag{C.1}$$

over some small but finite region [say $(\xi - \xi')^2 < a^2$]. From (3.24) it follows that the integrand in (C.1) has no angular singularities, so from (6.13) we conclude that it is sufficient to prove that the series

$$\sum_{J=1}^{\infty} (2J+1) \mathfrak{M}_k(a|J), \qquad k = 0, 1, \ldots \tag{C.2}$$

is convergent. Here

$$\mathfrak{M}_k(a|J) = \int_0^a \mathfrak{T}(\tau|J) \tau^{k+1} \, \mathrm{d}\tau,$$

$$\mathfrak{T}(\tau|J) = \left[\frac{\partial}{\partial\tau} Q_{\sqrt{J(J+1)}}(\cosh\tau)\right]^2 - \tfrac{1}{2}\left[\frac{\partial}{\partial\tau} Q_J(\cosh\tau)\right]^2 - \tfrac{1}{2}\left[\frac{\partial}{\partial\tau} Q_{J+1}(\cosh\tau)\right]^2, \tag{C.3}$$

and we have used the explicit expressions for G^J, $\tilde{G}^J$ and $\mathfrak{R}_\kappa$, eqs. (3.24), (3.30) and (3.26) and performed the change of variables

$$\cosh\tau = 1 + \frac{(r-r')^2 + (t-t')^2}{2rr'} .$$

We consider the case $k=0$ (other cases are treated in the same way) and rewrite (C.3) as

$$\mathfrak{M}_0(a|J) = \lim_{\delta\to 0}\left[\mathfrak{M}(J|\delta) - \tfrac{1}{2}\tilde{\mathfrak{M}}(J|\delta) - \tfrac{1}{2}\tilde{\mathfrak{M}}(J+1|\delta)\right], \qquad \text{(C.4)}$$

where

$$\mathfrak{M}(J|\delta) = \int_\delta^a \left[\frac{\partial}{\partial\tau} Q_{\sqrt{J(J+1)}}(\cosh\tau)\right]^2 \tau\, \mathrm{d}\tau, \qquad \text{(C.5)}$$

$$\tilde{\mathfrak{M}}(J|\delta) = \int_\delta^a \left[\frac{\partial}{\partial\tau} Q_J(\cosh\tau)\right]^2 \tau\, \mathrm{d}\tau. \qquad \text{(C.6)}$$

We decompose the integral (C.5) in the following way:

$$\int_\delta^a = \int_\delta^{\delta(\sqrt{1+1/J}+1/2J)} + \int_{\delta(\sqrt{1+1/J}+1/2J)}^{a(\sqrt{1+1/J}+1/2J)} + \int_{a(\sqrt{1+1/J}+1/2J)}^{a} .$$

The first integral is evaluated using (A.3); it is equal to $-\log\left(\sqrt{1+1/J}+1/2J\right) + \mathrm{O}(\delta)$. From (A.5) one finds that the third integral yields a summable (with the weight $(2J+1)$) contribution to $\mathfrak{M}_0$. To estimate the second integral, we perform the change of variables,

$$\tau = z/\left(\sqrt{J(J+1)} + \tfrac{1}{2}\right).$$

Applying the analogous procedure to the second and third terms on the r.h.s. of (C.4), we get

$$\mathfrak{M}_0(a|J) = \tfrac{1}{2}\log\frac{(J+\frac{1}{2})(J+\frac{3}{2})}{\left(\sqrt{J(J+1)}+\frac{1}{2}\right)^2}$$

$$+ \lim_{\delta\to 0}\int_{\delta J}^{aJ} z\,\mathrm{d}z\left\{\left[\frac{\partial}{\partial z} Q_{\sqrt{J(J+1)}}\left(\cosh\frac{z}{\sqrt{J(J+1)}+\frac{1}{2}}\right)\right]^2\right.$$

$$\left. -\tfrac{1}{2}\left(\sqrt{J(J+1)}\to J\right) - \tfrac{1}{2}\left(\sqrt{J(J+1)}\to J+1\right)\right\}, \qquad \text{(C.7)}$$

up to summable (with the weight $2J+1$) terms. The first term on the r.h.s. of (C.7) is easily estimated to be $O(1/J^3)$. From (A.12) we find that the second term is also $O(1/J^3)$, so the series (C.2) is convergent for $k=0$, which is the desired result.

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