Code-Fed Omnidirectional Arrays

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Abstract—The synthesis of linear and planar arrays of desired omnidirectional intensity patterns is considered. A new approach that utilizes the relation between the array intensity pattern and the correlation function of the used feeding sequence, or code, is adopted. The basic theory of such code-fed arrays is reviewed and it is shown that almost omnidirectional patterns result when codes with sharp autocorrelation functions are used as the feeding sequences. Examples of omnidirectional linear and planar arrays fed with Barker codes, Kuttruff–Quadt trial and error two-dimensional binary codes, and nonbinary Huffman-type codes are presented. The results of the paper have direct applications in underwater communication systems, public address systems, and in acoustical imaging systems. They can also be easily adapted to antenna arrays.

I. INTRODUCTION

RADIATION from arrays has many important applications. The use of arrays instead of single radiators makes it possible to tailor the radiation or sound intensity pattern to almost any desired shape, to increase the gain and the power-handling capability, to use electronic scanning, and to automatically adapt the pattern to match the changing environment. In this paper we consider the design of omnidirectional arrays. Starting with omnidirectional radiators, the source level of which falls short of satisfying a certain engineering purpose, we describe a method by which a number of these radiators can be used in an array configuration to increase the required source level without spoiling the omnidirectional nature of the pattern.

The concept of code-fed arrays as an approach for the synthesis of omnidirectional arrays has been recently proposed by Kuttruff and Quadt [1], [2] and El-Khamy et al. [3]–[5]. This approach is based on relating the intensity pattern of an array to the correlation properties of the sequence used to feed the array elements, henceforth referred to as the feeding code. This relation can be shown to be of the form of a discrete Fourier transform. Thus, feeding codes with sharp autocorrelation functions are expected to result in almost omnidirectional patterns. The parameter considered for evaluating the

resultant design should not be the radiation efficiency in the classical sense used with directive systems because it is irrelevant to our present purposes. Our measure of a good design is a small intensity pattern variance rather than a high radiation efficiency.

The dependence of the array intensity pattern on the autocorrelation functions of the feeding codes for linear and planar arrays are considered in Section II. Exact and approximate expressions for the average values and variances of the intensity patterns are also presented in the two cases of linear and planar arrays.

Barker codes [6]–[8] are the optimum binary sequences for feeding linear omnidirectional arrays [4] since they are characterized by the minimum possible correlation peak-to-sidelobe ratios among all binary sequences of the same length. Their performance is discussed in Section III. They are also used to feed planar arrays with one Barker code feeding the rows and another Barker code feeding the columns of the array [5]. In both cases of linear and planar arrays fed by Barker codes, the resulting intensity patterns are shown to be characterized by some sharp peaks or deep nulls superimposed on omnidirectional patterns.

As two-dimensional optimum binary codes of the Barker type do not exist, the search for two-dimensional binary codes that result in the minimum possible pattern variances has been of interest. Kuttruff and Quadt [2] made a numerical search of such codes using a trial and error procedure. Their search was limited to array sizes not greater than 5×5 elements, which are described in Section IV. Although the performance of these trial and error codes compares favorably with that of multiplied Barker codes, they still suffer from some isolated peaks, nulls, and uneven intensity distributions.

The search for code-fed linear arrays with improved performance compared with that of Barker-code-fed arrays has led to the introduction of some nonbinary sequences that are generated by special combinations of Barker codes with different lengths [3]. These codes are characterized by having almost zero sidelobes in their correlation functions and correlation properties which are similar to those of Huffman codes [9]. Thus they are referred to as "Huffman-type codes." A detailed description of the performance of Huffman-type code-fed linear and planar arrays is presented in Section V. It is shown that they are characterized by very good omnidirectional patterns.

The paper is concluded by a performance comparison of the various considered types of linear and planar arrays as well as a discussion of the different applications, important results, and possible extensions of this work.

II. BASIC THEORY OF CODE-FED ARRAYS

A. Linear Code-Fed Arrays

1) Field-Code Relations for Linear Arrays: Consider a linear array consisting of M isotropic elements which are uniformly spaced with a spacing d between any two successive elements. The far-field pressure pattern $P(u)$ due to that array can be written in terms of the sequence of volume velocity feedings Q_r of the elements; namely [10],

$$P(u) = \sum_{r=0}^{M-1} Q_r \exp(-jkrdu) \tag{1}$$

with $u = \sin \theta$, where θ is the angle from the array broadside direction, and $k = 2\pi/\lambda$, where λ is the used wavelength. The total sound intensity $I(u)$ in the far field is given by the squared amplitude of the sound pressure; i.e.,

$$I(u) = |P(u)|^2 \tag{2}$$

and hence it can be shown to be equal to the discrete Fourier transform of the aperiodic discrete autocorrelation function $C_Q(m)$ of the feeding volume velocity sequence; i.e.,

$$I(u) = \sum_{m=1-M}^{M-1} C_Q(m) \exp(jkmdu) \tag{3}$$

where $C_Q(m)$ is given by

$$C_Q(m) = \begin{cases} \sum_{r=0}^{M-1-|m|} Q_r Q_{r+|m|}, & |m| \leqslant M-1 \\ 0, & |m| > M-1. \end{cases} \tag{4}$$

It is clear from (4) that

$$C_Q(-m) = C_Q(m). \tag{5}$$

Hence, $I(u)$ can be recast into the following form:

$$I(u) = C_Q(0) + 2 \sum_{m=1}^{M-1} C_Q(m) \cos(kmdu). \tag{6}$$

In the case of a binary feeding $Q_r = q_r Q$, where $q_r = -1$ or $+1$, (6) simplifies to

$$I(u) = Q^2 \left[M + 2 \sum_{m=1}^{M-1} C_q(m) \cos(kmdu) \right]. \tag{7}$$

2) The Variance Formula: A good measure of the isotropy or uniformity of the two-dimensional pattern $I(u)$ of sound intensity among the different directions is the normalized variance V defined by [1].

$$V = \langle (I - \langle I \rangle)^2 \rangle / \langle I \rangle^2 = (\langle I^2 \rangle - \langle I \rangle^2) / \langle I \rangle^2 \tag{8}$$

where the operator $\langle \cdot \rangle$ means averaging with respect to the angle θ. The lower the value of V is, the more omnidirectional

is the pattern. In the high frequency limit $kd \gg 1$, V can be shown to be given by [1].

$$V = \left[\sum_{\substack{m=1-M \\ m \neq 0}}^{M-1} C_Q^2(m) \right] \Big/ C_Q^2(0)$$

$$= 2 \left[\sum_{m=1}^{M-1} C_Q^2(m) \right] \Big/ C_Q^2(0). \tag{9}$$

For binary feeding, (9) takes the form

$$V = \frac{2}{M^2} \sum_{m=1}^{M-1} C_q^2(m). \tag{10}$$

From (9) or (10), it is seen that V equals the ratio of the sum of the squares of the sidelobe values of $C_Q(m)$ to the square of its main-lobe value.

The upper bound of the variance for binary codes occurs for uniform feeding that is, when $Q_r = 1$ for $r = 0, 1, \cdots M - 1$, so that (4) yields

$$C_Q(m) = C_q(m) = M - |m| \tag{11}$$

and hence (10) reduces to the following expression for V (henceforth called V_0):

$$V_0 = \frac{2}{M^2} \sum_{m=1}^{M-1} (M - |m|)^2 = \frac{2}{M^2} \frac{M}{6} (2M^2 - 3M + 1)$$

$$= (2M^2 + 1)/3M - 1. \tag{12}$$

The lower bound for the variance of binary codes for $M = 2, 3, 4, 5, 7, 11,$ and 13 occurs for Barker code feeding, and this will be discussed in Section III. The lower bound for the variance for other values of M, up to $M = 40$, occurs for the binary codes described by Lindner [11].

B. Planar Coded Arrays

1) Field-Feeding Code Relation for Planar Arrays: Consider a rectangular planar array of $M \times N$ isotropic elements uniformly spaced with spacings d_1 and d_2 in the x and y directions, respectively, as shown in Fig. 1. The far-field pressure pattern $P(u, v)$ due to that array is known to be related to the two-dimensional sequence of volume velocity feedings Q_{rs} of the elements [12] as

$$P(u, v) = \sum_{r=0}^{M-1} \sum_{s=0}^{N-1} Q_{rs} \exp[-jk(rd_1 u + sd_2 v)] \tag{13}$$

where $u = \sin \theta \cos \Phi$ and $v = \sin \theta \sin \Phi$ are the directional cosines of the propagation direction relative to the x and y directions, and θ and Φ are the elevation and azimuthal angles of the propagation direction in the three-dimensional cartesian coordinate system of Fig. 1. The total sound intensity

$$I(u, v) = |P(u, v)|^2 \tag{14}$$

can be shown [2], [13] to be equal to the two-dimensional

discrete Fourier transform of the two-dimensional aperiodic discrete autocorrelation function $C_Q(m, n)$ of the feeding volume velocity matrix; i.e.,

$$I(u, v) = \sum_{m=1-M}^{M-1} \sum_{n=1-N}^{N-1} C_Q(m, n)$$

$$\cdot \exp\left[jk(md_1u + nd_2v)\right] \quad (15)$$

where $C_Q(m, n)$ is given for $1 - M \le m \le M - 1$ and $1 - N \le n \le N - 1$ by

$$C_Q(m, n) = \sum_{r=0}^{M-1-|m|} \sum_{s=0}^{N-1-|n|} Q_{rs} Q_{r+|m|, s+|n|} \quad (16)$$

when m and n have the same sign, and by

$$C_Q(m, n) = \sum_{r=0}^{M-1-|m|} \sum_{s=0}^{N-1-|n|} Q_{r+|m|, s} Q_{r, s+|n|} \quad (17)$$

when m and n have different signs. Whenever $|m| \ge M$ or $|n| \ge N$, the aperiodic autocorrelation $C_Q(m, n)$ is identically zero.

It is clear from (16) and (17) that $C_Q(m, n)$ satisfies the following identities:

$$C_Q(-m, -n) = C_Q(m, n) \quad (18a)$$

when m and n have the same sign, and

$$C_Q(-m, n) = C_Q(m, -n) \quad (18b)$$

when m and n have opposite signs. Hence, $I(u, v)$ can be recast into the following form:

$$I(u, v) = C_Q(0, 0)$$

$$+ 2 \sum_{m=1}^{M-1} C_Q(m, 0) \cos(kmd_1u)$$

$$+ 2 \sum_{n=1}^{N-1} C_Q(0, n) \cos(knd_2v)$$

$$+ 4 \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} C_Q(m, n) \cos(kmd_1u) \cos(knd_2v).$$

$$(19)$$

If the feeding Q_{rs} of the array elements is the product of two independent linear codes; i.e., if

$$Q_{rs} = Q_{r0} Q_{0s} = a_r b_s \quad (20)$$

for $r = 0, 1, \cdots M - 1$ and $s = 0, 1, \cdots N - 1$, then it immediately results that $C_Q(m, n)$ is the product of the linear aperiodic discrete autocorrelations $C_a(m)$ and $C_b(n)$; i.e.,

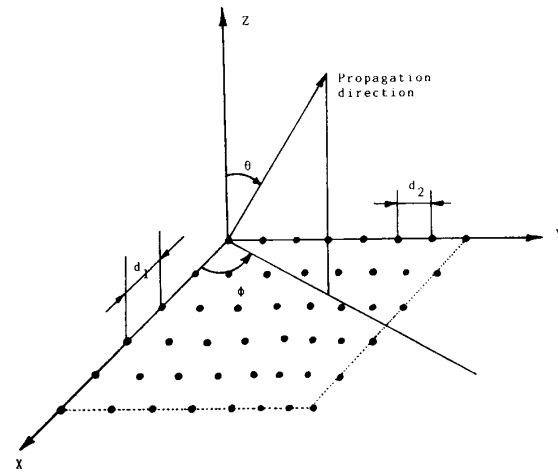$$C_Q(m, n) = C_a(m) C_b(n) \quad (21)$$

Fig. 1. Planar array configuration with $M = 6$ and $N = 8$.

where

$$C_a(m) = \sum_{r=0}^{M-1-|m|} a_r a_{r+|m|} \quad (22)$$

and

$$C_b(n) = \sum_{s=0}^{N-1-|n|} b_s b_{s+|n|}. \quad (23)$$

As a consequence, the "multiplication of patterns" formula results; namely,

$$I(u, v) = I_a(u) I_b(v) \quad (24)$$

where

$$I_a(u) = \sum_{m=1-M}^{M-1} C_a(m) \exp(jkmd_1u) \quad (25)$$

$$I_b(v) = \sum_{n=1-N}^{N-1} C_b(n) \exp(jknd_2v). \quad (26)$$

2) The Variance Formula: A good measure of the uniformity of the three-dimensional pattern $I(u, v)$ of sound intensity among the different directions (θ, Φ) is the normalized variance V defined by (8), wherein the average operator $\langle \cdot \rangle$ is now interpreted to mean averaging with respect to the two variables θ and Φ rather than to a single variable only. It can be proved that [2], [13]

$$V = \left[\sum_{\substack{m=1-M \\ (m,n) \ne (0,0)}}^{M-1} \sum_{n=1-N}^{N-1} C_Q^2(m, n) \right] \bigg/ C_Q^2(0, 0). \quad (27)$$

This formula means that V can be visualized as the total sidelobe energy of the two-dimensional discrete autocorrelation function $C_Q(m, n)$ divided by its main-lobe energy. In

view of (18), the expression (27) for V can be rewritten as

$$V = 2 \left[\sum_{m=1}^{M-1} C_Q^2(m, 0) + \sum_{n=1}^{N-1} C_Q^2(0, n) \right.$$

$$\left. + \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} (C_Q^2(m, n) + C_Q^2(m, -n)) \right] \Bigg/ C_Q^2(0, 0). \quad (28)$$

If the factorization relation (20) holds, then (28) simplifies to

$$V = 2 \left[C_b^2(0) \sum_{m=1}^{M-1} C_a^2(m) + C_a^2(0) \sum_{n=1}^{N-1} C_b^2(n) \right.$$

$$\left. + 2 \left(\sum_{m=1}^{M-1} C_a^2(m) \right) \left(\sum_{n=1}^{N-1} C_b^2(n) \right) \right] \Bigg/ C_a^2(0) C_b^2(0). \quad (29)$$

3) Upper and Lower Bounds for the Variance: The upper bound for the variance of binary codes occurs for uniform feeding; i.e., when

$$Q_{rs} = 1, \quad \{r = 0, 1, \cdots M - 1 \quad \text{and}$$

$$s = 0, 1, \cdots N - 1\}. \quad (30)$$

Equation (30) is a special case of (20) with $a_r = 1$ and $b_s = 1$. Hence (22) and (23) yield expressions for $C_a(m)$ and $C_b(n)$ on the form of (11) and the following summations can be proved:

$$\sum_{m=1}^{M-1} C_a^2(m) = \sum_{m=1}^{M-1} (M - |m|)^2 = (M/6)(2M^2 - 3M + 1)$$

$$(31a)$$

$$\sum_{n=1}^{N-1} C_b^2(n) = \sum_{n=1}^{N-1} (N - |n|)^2 = (N/6)(2N^2 - 3N + 1). \quad (31b)$$

By substitution in (29), the upper bound for V (henceforth denoted by V_0) is obtained as

$$V_0 = (2M^2 + 1)(2N^2 + 1)/9MN - 1. \quad (32)$$

If a two-dimensional Barker code was to exist, its variance (henceforth denoted by V_B) would have been the minimum possible. Therefore, V_B sets a lower bound for V. It can be calculated by noting that a two-dimensional Barker code would have had the autocorrelation function [2] (see Fig. 2)

$$C_Q(0, 0) = MN \quad (33a)$$

$$C_Q(m, n) = \begin{cases} 0 & \text{for } (m + n) \text{ odd} \\ 1 & \text{for } (m + n) \text{ even} \\ & \text{and } (m, n) \neq (0, 0). \end{cases} \quad (33b)$$

The corresponding value of V_B can be shown to be given by
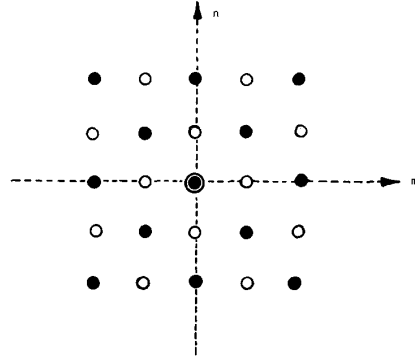
$$V_B = (2MN - M - N)/M^2N^2 \quad (34a)$$

Fig. 2. Distribution of $C_Q(m, n)$ for a fictitious two-dimensional Barker code with $M = N = 3$. The central double circle represents MN, while a white circle stands for 0 and a black circle represents $+1$.

if M and N are both even or both odd, and

$$V_B = (2MN - M - N - 1)/M^2N^2 \quad (34b)$$

if one of M and N is even while the other is odd.

III. BARKER CODE-FED ARRAYS

A. Definition and Correlation Properties of Barker Codes

As discussed in the previous section, and in particular according to (6) or (9), isotropic intensity patterns can be obtained by using feeding codes that possess the sharpest autocorrelation functions; i.e., have autocorrelation functions with the lowest possible sidelobe levels with respect to the peek value. Among the finite-length binary codes, Barker codes constitute an optimum set of codes in the sense that a Barker code has a minimum variance (9) for a specific code length (array length) M. That is because the sidelobe level of a Barker code is either 0 or $1/M$ of its peak value [6], [7]. There are eight known Barker codes, the longest among them has $M = 13$, and no Barker code of $M > 13$ is expected to exist. Table I lists the known Barker codes $B_M(r) = \{q_r\}, r = 0, \ldots M - 1$, together with their discrete autocorrelation functions.

B. Performance in Code-Fed Linear Arrays

Closed forms for the intensity patterns of linear arrays fed by Barker codes can be obtained by substituting the values of their autocorrelation functions in (3). The resulting expressions for the normalized intensity patterns [4] are given in Table II for $M = 5, 7, 11$, and 13.

The variances V_B of the intensity patterns of Barker code-fed arrays are listed in Table III for all known Barker codes. The comparison of the values of V_B with those of the upper bounds V_0 shows that Barker code-fed arrays are characterized by intensity patterns of relatively small variances.

The intensity pattern of a typical Barker code-fed array (i.e., that for B_{13}) is plotted in Fig. 3. The intensity patterns for B_5 and B_{13} are characterized by some large peaks in certain directions which are superimposed on otherwise omnidirectional patterns, while those for B_7 and B_{11} are characterized by

TABLE I
BARKER CODES AND THEIR APERIODIC AUTOCORRELATION FUNCTIONS

$B_M(r) = q_r$

M \ r	0	1	2	3	4	5	6	7	8	9	10	11	12
2	+1	-1											
3	+1	+1	-1										
4a	+1	+1	-1	+1									
4b	+1	+1	+1	-1									
5	+1	+1	+1	-1	+1								
7	+1	+1	+1	-1	-1	+1	-1						
11	+1	+1	+1	-1	-1	-1	+1	-1	-1	+1	-1		
13	+1	+1	+1	+1	+1	-1	-1	+1	+1	-1	+1	-1	+1

Nonzero $C_q(m)$ for m=0,1,2,........,12 , $C_q(-m)=C_q(m)$

M \ m	0	1	2	3	4	5	6	7	8	9	10	11	12
2	2	-1											
3	3	0	-1										
4a	4	-1	0	+1									
4b	4	+1	0	-1									
5	5	0	+1	0	+1								
7	7	0	-1	0	-1	0	-1						
11	11	0	-1	0	-1	0	-1	0	-1	0	-1		
13	13	0	+1	0	+1	0	+1	0	+1	0	+1	0	+1

TABLE II
NORMALIZED INTENSITY PATTERNS FOR BARKER CODE-FED ARRAYS

M	$I(u) / I_{max}$
5	$(1/9) [4 + \sin(5kdu)/\sin(kdu)]$
7	$(1/9.6) [8 - \sin(7kdu)/\sin(kdu)]$
11	$(1/14.4) [12 - \sin(11kdu)/\sin(kdu)]$
13	$(1/25) [12 + \sin(13kdu)/\sin(kdu)]$

TABLE III
VARIANCES OF INTENSITY PATTERNS FOR BARKER CODE-FED ARRAYS
COMPARED WITH THE CORRESPONDING UPPER BOUNDS

M	V_B	V_0	V_B/V_0
2	0.5	0.5	1.0
3	0.2222	1.111	0.2
4	0.25	1.75	0.1428
5	0.16	2.4	6.667×10^{-2}
7	0.1225	3.714	3.298×10^{-2}
11	0.0824	6.364	1.294×10^{-2}
13	0.0707	7.692	9.191×10^{-3}

some deep nulls in certain directions which are superimposed on otherwise omnidirectional patterns. This can be explained by noting that the autocorrelation function $C_Q(m)$ of B_5 and B_{13} are characterized by positive sidelobes, while those of B_7 and B_{11} are characterized by negative sidelobes, as seen from Table I.

C. Multiplied Barker Code-Fed Planar Arrays

An omnidirectional planar array can be excited in rows and columns by a linear Barker code. However, the autocorrela-tion function $C_Q(m, n)$ may have large sidelobe values on the two main axes, resulting in a relatively high value for the intensity variance. Therefore the three-dimensional pattern $I(u, v)$ suffers from sharp peaks or deep nulls [14]. Table IV shows two different combinations of Barker codes used for obtaining two-dimensional binary codes for feeding planar arrays. The corresponding two-dimensional autocorrelation functions and the pattern variances are also given. Table V compares the pattern variances V_{MB} of several multiplied Barker code-fed arrays with the corresponding minimum and upper bounds V_B and V_0. This table shows that in all cases, the planar arrays obtained are characterized by pattern variances that are much less than the upper bound but are still relatively high with respect to the minimum values (V_B) which can be approached only theoretically.

The three-dimensional patterns of planar arrays fed by multiplied Barker codes have been computer generated. As a first example, the θ- and Φ-patterns for a planar array fed by the $B_5 \times B_5$ code are shown in Fig. 4 (a)-(d) for $\theta = 90°$, $\Phi = 0°$, $\Phi = 45°$, and $\Phi = 90°$, respectively. The expected isolated peaks and the omnidirectional background are quite clear from these patterns. The corresponding three-dimen-sional pattern is given in Fig. 5. The vertical axis (z axis) gives the value of intensity, the x axis represents $\Phi = 0°$, and the y axis represents $\Phi = 90°$. Any vertical plane gives the intensity as a function of θ for a given value of Φ. Fig. 6 shows another three-dimensional pattern (that for the $B_5 \times B_7$ feeding) which exhibits some isolated nulls superimposed on an omnidirectional background.

IV. KUTTRUFF–QUADT TRIAL AND ERROR CODE-FED PLANAR ARRAYS

A. Description of Kuttruff-Quadt Codes

The selection of binary codes for feeding planar arrays so that they result in the best omnidirectional intensity patterns can be achieved by exhaustive search. For an $M \times N$ array of binary elements, the number of trials can be as much as 2^{MN}. Kuttruff and Quadt [2] have considered this task for small values of M and N that are no greater than 5. The largest array size considered by them (5×5) requires $2^{25} \approx 3.5 \times 10^7$ trials, and hence requires a considerable amount of computer time. Instead of exhaustive searching, Kuttruff and Quadt [2] adopted a trial and error procedure to select the two-dimensional binary codes. From the various codes attempted, the one with a minimum variance for specific values of M and N is selected as the optimal code. Table VI gives the correlation properties and pattern variances for the 4×3 and 5×5 trial and error codes [2].

B. Pattern Variances

Table VII lists the pattern variances of planar arrays fed by seven Kuttruff–Quadt trial and error codes and compares them with the upper bound V_0 and the lower bound V_B for a two-dimensional code. For all table entries other than $M \times N = 2 \times 2$, the variance V is strictly greater than V_B. We conjecture that no two-dimensional Barker code exists for all values of M and N such that M and/or $N > 2$.
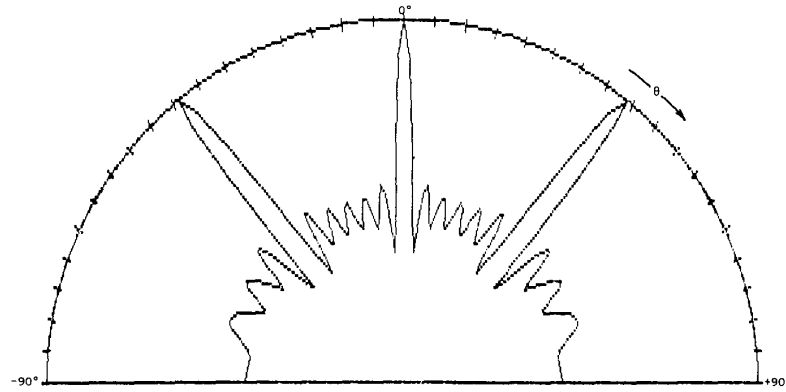
Fig. 3. Intensity pattern of a Barker code-fed linear array, $M = 13$, $kd = 5$. Scale: Linear.

TABLE IV
CORRELATION PROPERTIES AND PATTERN VARIANCES FOR TYPICAL MULTIPLIED BARKER CODES FEEDING PLANAR ARRAYS

------------------BARKER CODE [B4xB3]----------------

THE FEEDING CODE:

```
+1  +1  -1  +1
+1  +1  -1  +1
-1  -1  +1  -1
```

THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

```
-1   0  +1  -4  +1   0  -1
 0   0   0   0   0   0   0
+3   0  -3 +12  -3   0  +3
 0   0   0   0   0   0   0
-1   0  +1  -4  +1   0  -1
```

VARIANCE = 0.52777782

------------------BARKER CODE [B5XB5]----------------

THE FEEDING CODE:

```
+1  +1  +1  -1  +1
+1  +1  +1  -1  +1
+1  +1  +1  -1  +1
-1  -1  -1  +1  -1
+1  +1  +1  -1  +1
```

THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

```
+1   0  +1   0  +5   0  +1   0  +1
 0   0   0   0   0   0   0   0   0
+1   0  +1   0  +5   0  +1   0  +1
 0   0   0   0   0   0   0   0   0
+5   0  +5   0 +25   0  +5   0  +5
 0   0   0   0   0   0   0   0   0
+1   0  +1   0  +5   0  +1   0  +1
 0   0   0   0   0   0   0   0   0
+1   0  +1   0  +5   0  +1   0  +1
```

VARIANCE = 0.3456

C. Intensity Patterns

The θ- and Φ-patterns and the three-dimensional intensity pattern of a 5 × 5 Kuttruff–Quadt code-fed planar array are given in Figs. 7 (a)–(d) and 8. These patterns show that such arrays give a better performance as omnidirectional arrays than the multiplied Barker code-fed arrays. However, the patterns are still characterized by some isolated peaks and nulls as well as by uneven intensity distributions.

TABLE V
COMPARISON OF THE VARIANCES OF MULTIPLIED BARKER CODE-FED TWO-DIMENSIONAL ARRAYS (V_{MB}) WITH THE LOWER (V_B) AND UPPER (V_0) BOUNDS

MXN	V_B	V_{MB}	V_0
2X2	0.25	1.25	1.25
3X2	0.1667	0.8333	2.167
3X3	0.1481	0.4938	3.457
4X3	0.1111	0.5278	4.806
4X4	0.0937	0.5625	6.563
5X4	0.075	0.45	8.35
5X5	0.064	0.3456	10.56
7X5	0.0474	0.3020	15.028
7X7	0.035	0.2599	21.224

V. NONBINARY HUFFMAN-TYPE CODE-FED ARRAYS

A. Generation of Huffman-Type Codes From Barker Codes

The appearance of large peaks or deep nulls in the otherwise almost isotropic intensity patterns obtained by Barker codes feeding has led to the search for integer nonbinary codes that are characterized by sharper aperiodic autocorrelation functions. Such codes have been obtained by combining Barker codes of different lengths [3] as described below.

Combined Barker codes can be chosen to have very sharp autocorrelation functions with zero sidelobes everywhere except at points $r = \pm(M - 1)$ far from the main lobe. Such codes are of the Huffman-code-type in the sense that their autocorrelation functions resemble those of Huffman codes [9]. Lists of some combined Barker or Huffman-type codes along with their aperiodic autocorrelation functions and variances of the intensity patterns of linear arrays fed by these codes are given in Table VIII.

If the Huffman-type code (S_7) is compared with the Barker code of the same length (B_7), it turns out that the pattern variance for this code is $V_{S7} = 2/(18)^2 = 6.173 \times 10^{-3}$, which is much less than $V_{B7} = 0.1225$. As a bonus, (S_7)
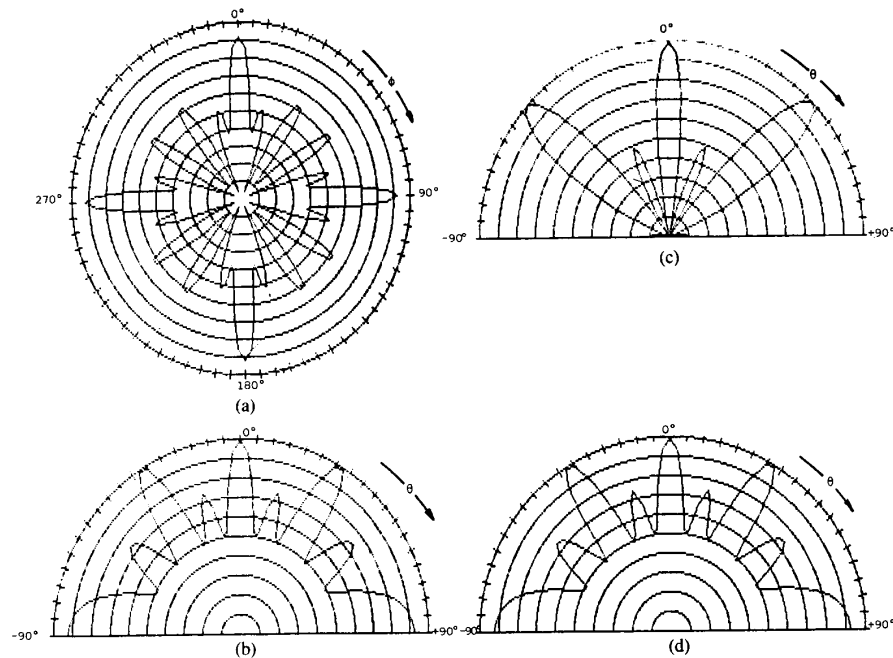
Fig. 4. Intensity patterns of a multiplied Barker 5 × 5 code-fed planar array $k_1d = k_2d = 6$. (a) $\theta = 90°$. (b) $\phi = 0°$. (c) $\phi = 45°$. (d) $\phi = 90°$. Scale: 1 dB/circle.
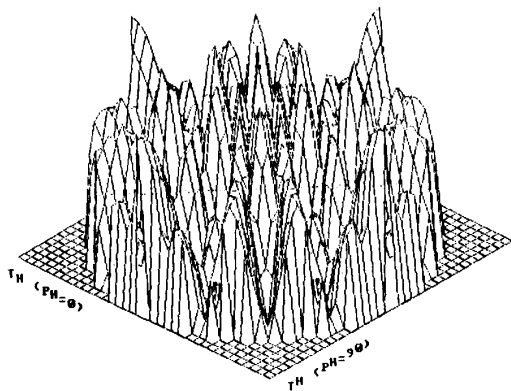

Fig. 5. The three-dimensional intensity pattern of a multiplied Barker 5 × 5 code-fed planar array $k_1d = k_2d = 6$ (+10 dB).
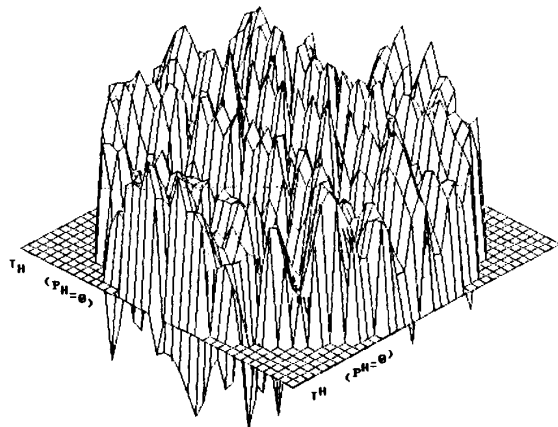

Fig. 6. The three-dimensional intensity pattern of a multiplied Barker 5 × 7 code-fed planar array $k_1d = k_2d = 6$ (+10 dB).

TABLE VI
CORRELATION PROPERTIES AND PATTERN VARIANCES FOR TYPICAL
TRIAL AND ERROR CODES FEEDING PLANAR ARRAYS

```
--------------TRIAL AND ERROR CODE [TE4X3]--------------

THE FEEDING CODE:

                    +1  +1  -1  -1
                    +1  +1  +1  +1
                    +1  -1  +1  -1

THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

              -1   0  +1   0  +1   0  -1
               0  +2   0   0   0  -2   0
              -1  +2  +1 +12  +1  +2  -1
               0  -2   0   0   0  +2   0
              -1   0  +1   0  +1   0  -1

        VARIANCE = 0.25
```

```
-------------TRIAL AND ERROR CODE [TE5X5]-------------

THE FEEDING CODE:

                  +1  +1  +1  +1  +1
                  -1  -1  -1  +1  +1
                  +1  -1  -1  +1  +1
                  -1  +1  +1  -1  +1
                  +1  -1  +1  -1  +1

THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

        +1   0  +1   0  +1   0  +1   0  +1
         0   0   0  +4   0   0   0   0   0
        +1   0  +1   0  +1   0  +1   0  +1
         0   0   0  +4   0  -4   0  -4   0
        +1   0  +1   0 +25   0  +1   0  +1
         0  -4   0  -4   0  +4   0   0   0
        +1   0  +1   0  +1   0  +1   0  +1
         0   0   0   0   0  +4   0   0   0
        +1   0  +1   0  +1   0  +1   0  +1

        VARIANCE = 0.2432
```

TABLE VII
COMPARISON OF THE VARIANCES OF THE TRIAL AND ERROR CODES
WITH THE CORRESPONDING LOWER (V_B) AND UPPER (V_0) BOUNDS

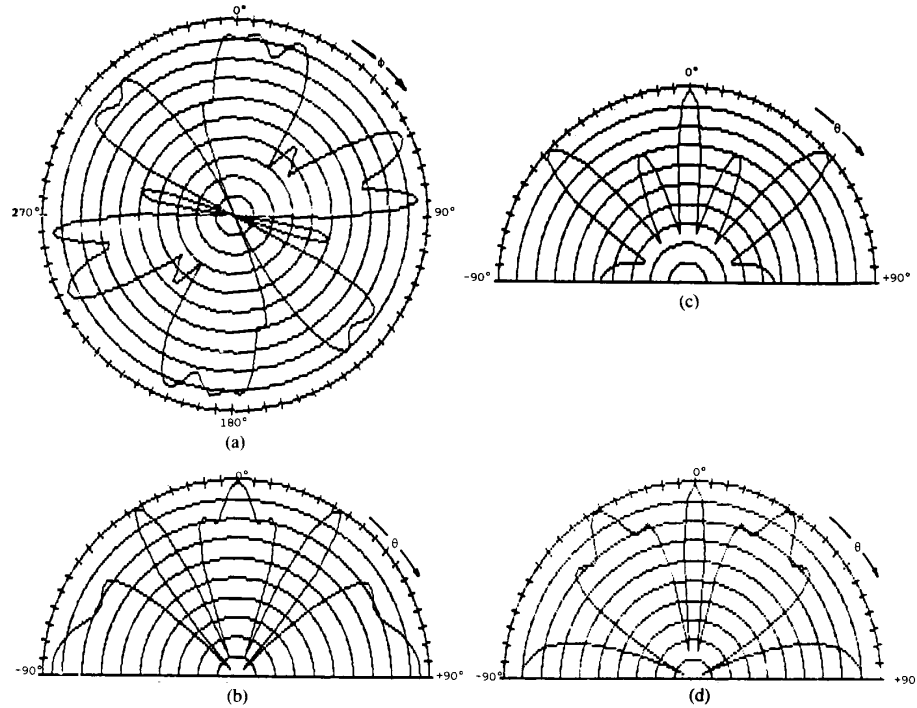| M x N	V_B	V	V_0
2 x 2	.25	.25	1.25
3 x 2	.1667	.3889	2.167
4 x 2	.1563	.3750	3.125
3 x 3	.1481	.3950	3.457
4 x 3	.1111	.25	4.806
4 x 4	.09375	.1875	6.563
5 x 5	.064	.2432	10.56

Fig. 7. Intensity patterns of a trial and error 5×5 code-fed planar array $k_1 d = k_2 d = 6$. (a) $\theta = 90°$. (b) $\phi = 0°$. (c) $\phi = 45°$. (d) $\phi = 90°$. Scale: 1 dB/circle.
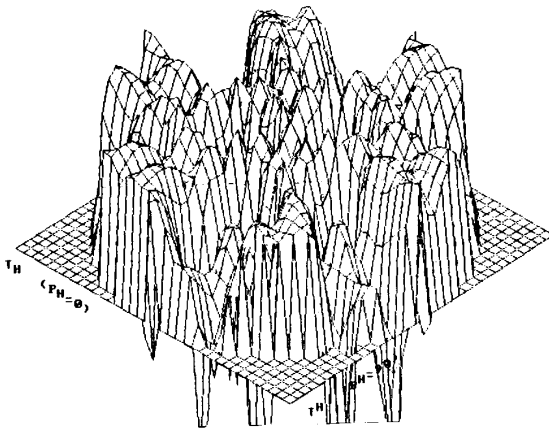

Fig. 8. The three-dimensional intensity pattern of a trial and error 5×5 code-fed planar array $k_1 d = k_2 d = 6$ (+10 dB).

requires six rather than seven array elements since, as seen from Table VIII, the central element does not need to be excited and hence does not need to exist at all. In that sense, (S_7) may be thought of as a nonuniform array feeding code consisting of only six elements. The code (S_7^*) leads to even more saving in the number of needed elements since it consists of three elements only. Its variance of $2/(6)^2 = 5.555 \times 10^{-2}$ is greater than V_{S7} but is much better than $V_{B3} = 0.2222$.

B. Huffman-Type-Code-Fed Linear Arrays

Since the pattern variances of Huffman-type codes are much less than those obtained by using the optimum-binary Barker codes, linear arrays with much improved omnidirectional intensity patterns are obtained by using Huffman-type codes feeding. These codes are nonbinary and hence are more difficult to realize than binary codes. However, the maximum number of excitation levels required by them is only four.

TABLE VIII
CORRELATION PROPERTIES AND PATTERN VARIANCES FOR TYPICAL
HUFFMAN-TYPE CODES FEEDING LINEAR ARRAYS

```
--------------HUFFMAN-TYPE CODE [S5]---------------

          {S_5(r) = B_5(r) +B_3(r-1),  0 ≤ r ≤ 4}

THE FEEDING CODE:

              +1  +2  +2  -2  +1

THE APERIODIC CORRELATION FUNCTION:

          +1   0   0   0  +14   0   0   0  +1

VARIANCE = 0.0010204
```

```
--------------HUFFMAN-TYPE CODE [S7]---------------

        {S_7(r) = B_7(r) + B_5(r-1) ,   0 ≤ r ≤ 6}

THE FEEDING CODE:

            +1  +2  +2   0  -2  +2  -1

THE APERIODIC CORRELATION FUNCTION:

   -1   0   0   0   0   0  +18   0   0   0   0   0  -1

VARIANCE = 0.00061728
```

```
--------------HUFFMAN-TYPE CODE [S7*]---------------

        {S_7*(r) = B_7(r) - B_5(r-1) ,  0 ≤ r ≤ 6 }

THE FEEDING CODE:

            +1   0   0  -2   0   0  -1

THE APERIODIC CORRELATION FUNCTION:

   -1   0   0   0   0   0  +6   0   0   0   0   0  -1

VARIANCE = 0.00555555
```
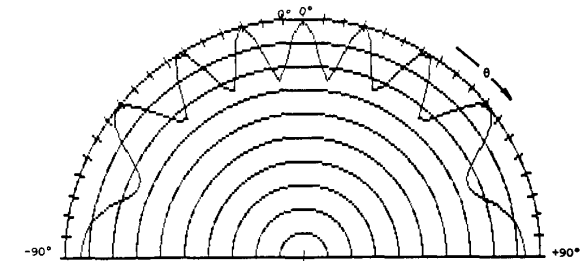--

Fig. 9.   Intensity pattern of a Huffman-type code-fed array with $M = 5$ and $kd = 6$. Scale: 0.5 dB/circle.

namely ± 1 and ± 2 (zero indicates that no element is placed in the corresponding position).

The simple form of the aperiodic correlation functions of Huffman-type codes makes it easy to find closed-form expressions for their intensity patterns through the use of (6). For example, the intensity patterns for linear arrays fed by the codes (S_5) and (S_7) are given by

$$I_{S5}(u) = 14 + 2 \cos (4kdu) \qquad (35)$$

and

$$I_{S7}(u) = 18 - 2 \cos (6kdu). \qquad (36)$$

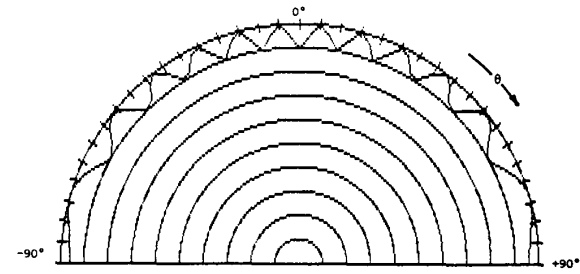These intensity patterns as plotted in Figs. 9 and 10 are

Fig. 10.   Intensity pattern of a Huffman-type code-fed array with $M = 7$ and $kd = 6$. Scale: 1 dB/circle.

TABLE IX
CORRELATION PROPERTIES AND PATTERN VARIANCES FOR TYPICAL
MULTIPLIED HUFFMAN-TYPE CODES FEEDING PLANAR ARRAYS

```
--------------HUFFMAN-TYPE CODE [S5xS5]-----------

THE FEEDING CODE:
              +1  +2  +2  -2  +1
              +2  +4  +4  -4  +2
              +2  +4  +4  -4  +2
              -2  -4  -4  +4  -2
              +1  +2  +2  -2  +1
THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

   +1   0   0   0 +14   0   0   0  +1
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
  +14   0   0   0 +196   0   0   0 +14
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
   +1   0   0   0 +14   0   0   0  +1

VARIANCE = 0.0205123
```

```
--------------HUFFMAN-TYPE CODE [S5xS7*]-----------

THE FEEDING CODE:
              +1  +2  +2  -2  +1
               0   0   0   0   0
               0   0   0   0   0
              -2  -4  -4  +4  -2
               0   0   0   0   0
               0   0   0   0   0
              -1  -2  -2  +2  -1
THE TWO-DIMENSIONAL APERIODIC CORRELATION FUNCTION:

   -1   0   0   0 -14   0   0   0  -1
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
   +6   0   0   0 +84   0   0   0  +6
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0
   -1   0   0   0 -14   0   0   0  -1

VARIANCE = 0.0663265
```
----

almost omnidirectional and devoid of any deep nulls or sharp peaks such as those which characterize Barker codes patterns.

C. Multiplied Huffman-Type-Code-Fed Planar Arrays

Three-dimensional intensity patterns $I(u, v)$ of good omnidirectional properties can be obtained by feeding arrays in their rows and columns by nonbinary combined Barker codes of the Huffman type. Examples of such two-dimensional codes are listed in Table IX for $M \times N = 5 \times 5$ and 5×7. The two-dimensional autocorrelation functions of these codes, as well as the intensity pattern variances, are also listed in Table IX.
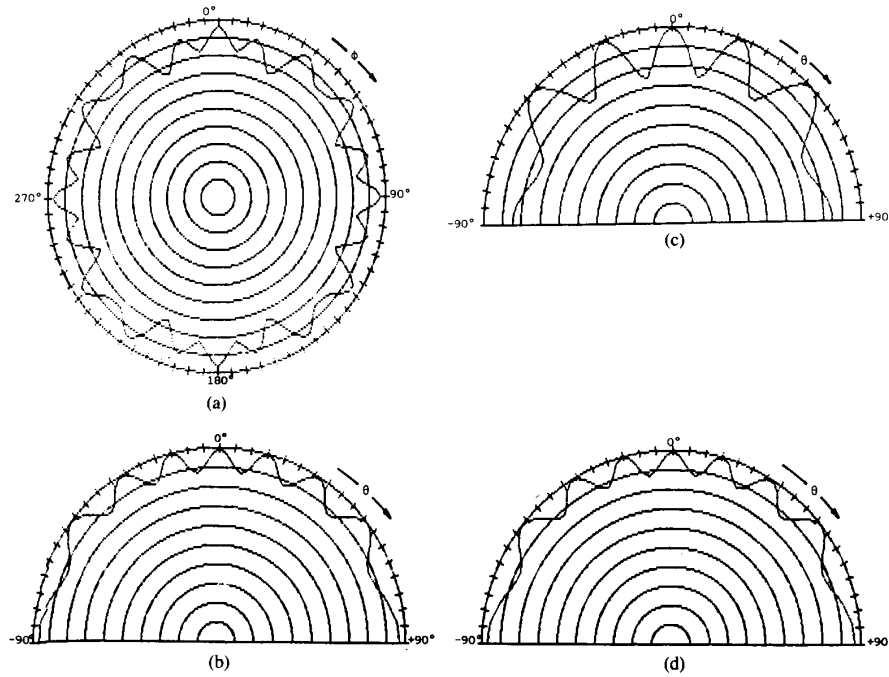
Fig. 11. Intensity patterns of a Huffman-type 5 × 5 code-fed planar array $k_1d = k_2d = 6$. (a) $\theta = 90°$. (b) $\phi = 0°$. (c) $\phi = 45°$. (d) $\phi = 90°$. Scale: 1 dB/circle.

As an illustration, consider the 5 × 5 array fed in rows and columns by the code (S_5). The variance of this code is given by 0.0205, which is much less than the corresponding value for a 5 × 5 multiplied Barker code of $V_{MB} = 0.3456$ and that of a Kuttruff-Quadt binary trial and error 5 × 5 code of $V_{KQ} = 0.2432$, and is even less than the lower bound $V_B = 0.064$ that is theoretically predicted for a 5 × 5 binary code. The exact form of the resulting intensity pattern for the multiplied Huffman-type 5 × 5 code-fed planar array can be directly obtained from (35) and the pattern multiplication formula (24) as

$$I_{S5,S5}(u, v) = [14 + 2 \cos (4kd_1u)]$$

$$\cdot [14 + 2 \cos (4kd_2v)]. \quad (37)$$

The Φ-pattern (with $\theta = 90°$) and the θ-patterns (with $\Phi = 0°, 45°$, and $90°$) for the $S_5 \times S_5$ feeding code are plotted in Fig. 11 (a)-(d), while its three-dimensional pattern is plotted in Fig. 12. These patterns exhibit the expected highly omnidirectional characteristics of the Huffman-type code-fed arrays.

VI. DISCUSSION AND CONCLUSIONS

An interesting problem in array synthesis has been investigated. It deals with the use of codes with good correlation properties which are familiar in the design of modern pulse compression radar and spread spectrum communication systems for feeding arrays to result in omnidirectional patterns. The designed arrays are very useful in many applications, such as underwater acoustical communication systems, loudspeaker arrays for public address systems (e.g., lecture halls, theaters,
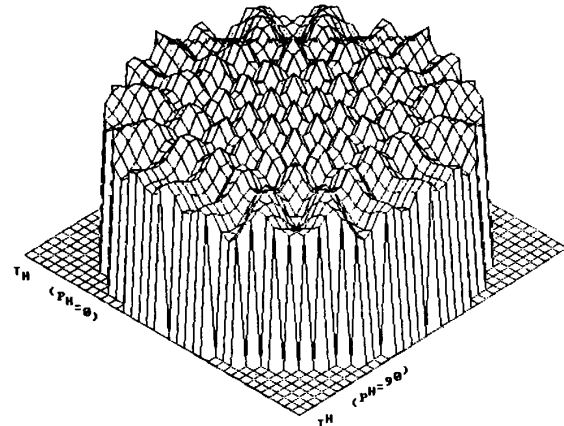

Fig. 12. The three-dimensional intensity pattern of a multiplied Huffman-type 5 × 5 code-fed planar array $k_1d = k_2d = 6$ (+10 dB).

etc.), and biomedical acoustical-imaging systems. The same procedure can also be applied to the synthesis of omnidirectional antenna arrays.

Table X presents a comprehensive comparison of all the considered types of linear code-fed arrays. In this table, the arrays are arranged according to the number of used elements, size, type, number of code discrete levels (= 2 for binary codes and > 2 for nonbinary codes), and finally the resultant pattern variance. The table indicates that for a given number of used elements, Huffman-type codes are characterized by the least variance and that Barker codes are the optimum binary feeding codes for linear arrays with sizes of 2, 3, 4, 5, 7, 11,

TABLE X

COMPARISON OF THE PERFORMANCE OF THE DIFFERENT CONSIDERED CODE-FED LINEAR ARRAYS

#	No. of Elements	Size	Type		No. of Levels	Variance
1	2	2	Barker Code	[B2]	2	0.5
2	3	3	Barker Code	[B3]	2	0.2222
3	3	3	Huffman-Type	[S3]	3	0.0555
4	3	7	Huffman-Type	[S7*]	3	0.0555
5	4	4	Barker Code	[B4]	2	0.25
6	5	5	Barker Code	[B5]	2	0.16
7	5	5	Huffman-Type	[S5]	3	0.00102
8	6	7	Huffman-Type	[S7]	4	0.000617
9	7	7	Barker Code	[B7]	2	0.1235
10	11	11	Barker Code	[B11]	2	0.0824
11	13	13	Barker Code	[B13]	2	0.0707

TABLE XI

COMPARISON OF THE PERFORMANCE OF THE DIFFERENT CONSIDERED CODE-FED PLANAR ARRAYS

#	No. of Elements	Size	Type		No. of Levels	Variance
1	4	2x2	Mult. Barker Codes	[B2xB2]	2	1.25
2	4	2x2	Trial & Error Code	[TE2x2]	2	0.25
3	6	3x2	Mult. Barker Codes	[B3xB2]	2	0.8333
4	6	3x2	Trial & Error Code	[TE3x2]	2	0.3889
5	8	4x2	Trial & Error Code	[TE4x2]	2	0.375
6	9	3x3	Mult. Barker Codes	[B3xB3]	2	0.4938
7	9	3x3	Trial & Error Code	[TE3x3]	2	0.395
8	12	4x3	Mult. Barker Codes	[B4xB3]	2	0.5278
9	12	4x3	Trial & Error Code	[TE4x3]	2	0.25
10	15	5x3	Mult. Huffman-Type	[S5xS3]	6	0.066326
11	15	5x7	MULT. Huffman-Type	[S5xS7*]	6	0.066326
12	16	4x4	Mult. Barker Codes	[B4xB4]	2	0.5625
13	16	4X4	Trial & Error Code	[TE4x4]	2	0.1875
14	18	7x3	Mult. Huffman-Type	[S7xS3]	6	0.06207
15	20	5x4	Mult. Barker Codes	[B5xB4]	2	0.45
16	25	5x5	Mult. Barker Codes	[B5xB5]	2	0.3456
17	25	5x5	Trial & Error Code	[TE5x5]	2	0.2432
18	25	5x5	Mult. Huffman-Type	[S5xS5]	5	0.0205
19	30	5x7	Mult. Huffman-Type	[S5xS7]	6	0.016439
20	35	7x5	Mult. Barker Codes	[B7xB5]	2	0.3020
21	49	7x7	Mult. Barker Codes	[B7xB7]	2	0.2599

and 13. Table XI gives a comparison of all of the considered types of planar arrays. This table indicates that although Barker codes are the optimum binary feeding codes for linear arrays, multiplied Barker codes do not perform similarly well. For array sizes up to 5 × 5, Kuttruff–Quadt trial and error codes are the best-known binary feeding codes. The table also indicates that multiplied nonbinary Huffman-type codes are the best codes (among those considered) for feeding omnidirectional planar arrays with arbitrary sizes.

For further investigation, the authors have considered binary codes with good periodic correlation properties such as m-sequences, Hadamard matrices, and the two-dimensional PN codes [13], [14] as well as the whole class of nonbinary Huffman codes [9]. The results of the last point are very interesting and will be published soon [16]. Another natural extension of the present work that we are presently working on is to study the sensitivity of the omnidirectional properties of the intensity patterns to weighting errors and element positioning errors. We hope to report our findings soon in a forthcoming publication.

ACKNOWLEDGMENT

REFERENCES

[1] H. Kuttruff and H. P. Quadt, "Lautsprecherzeilen mit ungebündelter Schallabstrahlung (Loudspeaker feeding with omnidirectional sound radiation)," presented at the *DAGA 78*, VDE Verlag, 1978.

[2] H. Kuttruff and H. P. Quadt, "Ebene Schallstrahlergruppen mit Ungebündelter Abstrahlung (Plane sound source arrays with omnidirectional radiation)," *Acoustica*, vol. 50, pp. 273–279, 1982.

[3] S. E. El-Khamy and O. A. Abdel-Alim, "Omnidirectional coded loudspeaker arrays," in *Proc. Int. Conf. Acoust., Speech, Signal Processing* (Boston, MA), Apr. 1983.

[4] O. A. Abdel-Alim, S. E. El-Khamy, and I. A. Mandour, "Directivity improvement of loudspeaker arrays using Barker codes feeding," *Z. Elect. Inform. u. Energietechnik*, vol. 13, pp. 1–7, 1983.

[5] S. E. El-Khamy and O. A. Abdel-Alim, "Omnidirectional coded planar loudspeaker arrays," in *Proc. Conf. Acoust. Speech, Signal Processing* (Tampa, FL), Mar. 1985.

[6] R. H. Barker, "Group synchronization of binary digital systems" in *Communication Theory*, W. Jackson, Ed. New York: Academic, 1953, pp. 273–287.

[7] S. E. El-Khamy, "On generation, correlation and spectrum measurement of Barker codes," *Int. J. Electron.*, vol. 50, pp. 353–361, 1981.

[8] S. E. El-Khamy, "On Generation and measurement of combined Barker codes," *Int. J. Electron.*, vol. 50, pp. 429–436, 1981.

[9] D. A. Huffman, "The generation of impulse-equivalent pulse trains," *IRE Trans. Inform. Theory*, vol. IT-8, pp. 510–516, 1962.

[10] B. D. Steinberg, *Principles of Aperture and Array System Design*. New York: Wiley, 1976.

[11] J. Lindner, "Binary sequences up to length 40 with best possible autocorrelation function," *Electron. Let.*, vol. 11, p. 507, 1975.

[12] M. T. Ma, *Theory and Application of Antenna Arrays*. New York: Wiley, 1974.

[13] S. E. El-Khamy, A. M. Rushdi, and A. H. Banah, "Code-fed arrays," Dir. Sci. Res., College of Engineering, King Abdulaziz Univ., Jeddah, Saudi Arabia, Final. Rep., Proj. 06-509, 1987.

[14] F. A. MacWilliams and N. J. A. Sloane, "Pseudo-random sequences," *Proc. IEEE*, vol. 64, pp. 1715–1729, 1976.

[15] D. V. Sarwate and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," *Proc. IEEE*, vol. 68, pp. 593–619, 1980.

[16] S. E. El-Khamy and A. H. Banah, "Huffman codes-fed omnidirectional acoustical arrays," *IEEE Trans. Acoust., Speech, Signal Processing*, to be published.

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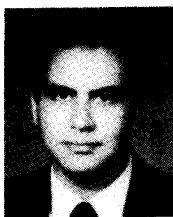
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