Cost of Business Cycles with Indivisibilities and Liquidity Constraints

Ayşe İmrohoroğlu

University of Southern California

It is almost universally agreed that individuals face incomplete insur-
ance markets and cannot perfectly insure against the idiosyncratic
risk. In this paper simple general equilibrium models with incom-
plete insurance markets are examined in order to assess the impact
of imperfect insurance on the magnitude of the welfare costs of
business cycles. Two versions of incomplete insurance markets are
considered, and certain statistical properties of the equilibrium sto-
chastic processes in these environments are compared with those of a
perfect insurance economy.

I. Introduction

In an interesting study, Lucas (1987) estimates the magnitude of the
costs of business cycles to be remarkably small, 0.1 percent of total
U.S. consumption. His approach assumes perfect insurance of the
idiosyncratic risk. The purpose of this study is to examine whether
the magnitude of the costs of business cycles in economies with incom-
plete insurance markets differs significantly from the cost estimates
found in an environment with perfect insurance. However, it is not
obvious how one should depart from the assumption of perfect insur-
ance. One way would be to limit insurance arrangements endoge-
nously by using moral hazard or incomplete information models as

1364

pursued in Green (1987), Atkeson (1988), or Townsend (1988). That is not the approach taken in this research. Here, perfect insurance is precluded exogenously, in line with Scheinkman and Weiss (1986), and the effects of different versions of incomplete insurance markets on the issue of costs of business cycles are studied. Restrictions on the set of asset holdings of the consumer are utilized to generate the incomplete insurance markets. These restrictions can be described as different specifications of liquidity constraints.

There is strong evidence for liquidity constraints at the micro level.[1] Furthermore, liquidity constraints have proved to be important for some issues. For example, Tobin and Dolde (1971) examined the implications of liquidity constraints in a deterministic framework and showed that capital accumulation in the economy analyzed increases by a factor of two because of liquidity constraints. The incorporation of liquidity constraints into general equilibrium models, however, has been quite limited. A notable exception is by Scheinkman and Weiss (1986). They introduce borrowing constraints in a two-type agent, equilibrium model and emphasize the role of uninsured risk in affecting aggregate outcomes. Their economy generates more asset price variability relative to dividend variability and aggregate consumption variability than if there were perfect insurance markets. My approach, however, is closer to the permanent income hypothesis than Scheinkman and Weiss's. In a life cycle framework, Bewley (1980) shows that, in an economy with borrowing constraints, if the subjective time discount rate is sufficiently close to zero, then the allocation would approach that of a perfect insurance economy since, when the rate of interest equals the rate of time preference of the consumer, self-insurance would be costless. But we do not have a quantitative feel for how close the time discount rate must be to zero for the costs of self-insurance through the holdings of non-interest-bearing assets to be negligible.

The purpose of this study is to develop tools for computing the equilibria for economies with two different forms of incomplete insurance markets and to apply these tools to estimate the magnitude of the costs of business cycles. The labor supply decision is not endogenized in the economies studied. Agents will work whenever the stochastic work option is available. On the other hand, if the option is not available, the worker will be unemployed and receive a much lower compensation through home production. If there were a full set of Arrow-Debreu contingent claims markets, agents could attain a

---

[1] For example, Zeldes (1989) tests the behavior of consumption in the presence of liquidity constraints using data from the Panel Study of Income Dynamics. His results suggest that borrowing constraints exist and affect consumption significantly. For a detailed survey of the empirical literature on liquidity constraints, see Hayashi (1985).

consumption stream that would be affected only by aggregate uncertainty and not by their stochastic employment opportunities. But I assume that such markets do not exist. Indeed, in the first environment studied, the only technology that the consumer has access to is a storage technology; no borrowing is allowed. In this environment agents will self-insure through holdings of precautionary assets. In the second environment, in addition to the storage technology, there is an intermediation technology that allows consumers to borrow, but at a rate that exceeds the lending rate. The findings from these environments are compared with the cost estimates from an environment with perfect insurance.

Within each environment, two economies are considered. In the first, the economy displays business cycles. There are good and bad times, and the probability of finding employment differs across these times. These probabilities are selected so that the model economy mimics some key observations in the U.S. aggregate data. In particular, the differences in the rate and duration of unemployment between peaks and troughs are used to calibrate the probability of employment in good and bad times. The average utility of the agent in the economy with business cycles is compared with that of an agent in the second economy, where no business cycle fluctuations are observed. The average rate of unemployment and the average level of consumption are the same for the two model economies. This enables one to isolate the effect of a fluctuating consumption stream on the welfare of the consumer. In particular, the magnitude of the increase in average consumption that is necessary to compensate the individual for the costs of fluctuations is computed. The key issue addressed by this paper is whether the increase in average consumption needed in an economy with liquidity constraints differs significantly from the amount of increase found in an economy with perfect insurance of the idiosyncratic risk. If it turns out that the findings are similar, then abstracting from liquidity constraints in estimating the cost of fluctuations is justifiable.

Tools for computing equilibria for economies with liquidity constraints are limited. The approach taken in this research is to discretize the economy and use numerical methods to compute the equilibrium for the approximate economies. The number of discrete levels of the state space and the control space is sufficiently large that adding intermittent levels changes the results hardly at all. The statistical properties of the equilibrium stochastic process are determined and examined for these approximate economies.

The paper is organized as follows: Section II presents the economies studied. The calibration of the model is described in Section III, and the computation techniques used to determine the value function

and the decision rules are described in Section IV. In Section V the results are discussed and the statistical properties of the equilibrium are examined. Section VI presents concluding remarks.

II. Structure of the Economies

The economy consists of many infinitely lived individuals who are different at a point in time only in their asset holdings and employment opportunities. They maximize

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where $0 < \beta < 1$ is their subjective time discount factor and c_t is their consumption in period t. The utility function is twice continuously differentiable, increasing, and concave in c_t and has the following form:

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0. \tag{2}$$

Agents are endowed with one indivisible unit of time in each period and each face an individual-specific stochastic employment opportunity that has two states, $i = e$ or $i = u$. If the employed state occurs ($i = e$), an agent produces y units of the consumption good using the time allocation. In the unemployed state ($i = u$), the agent produces θy units of consumption good through household production, where $0 < \theta < 1$.

Let a_{t+1} be an agent's asset holdings at the beginning of period $t + 1$ and r be the rate of return on stored assets. Then an individual's asset holdings evolve through time according to

$$a_{t+1} = \begin{cases} (1 + r)(a_t - c_t + y) & \text{if } i = e, \\ (1 + r)(a_t - c_t + \theta y) & \text{if } i = u. \end{cases} \tag{3}$$

In order to assess the cost of business cycles, two economies are considered. In the first, the economy experiences business cycles, whereas in the second there is no aggregate uncertainty. The average rates of unemployment for these two economies are the same.

Economy 1.—The economy evolves through good and bad times, which are modeled as variations in the process of employment prospects faced by the individuals. The state of the national economy, n, is assumed to follow a first-order Markov chain. The economy experiences good times if $n = g$ and bad times if $n = b$. The transition matrix of n is a 2×2 matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \tag{4}$$

where $\Pr\{n_{t+1} = g|n_t = g\} = p_{11}$ and $\Pr\{n_{t+1} = b|n_t = b\} = p_{22}$.

Variable i denotes the individual-specific employment state and is assumed to follow a first-order Markov chain. There are only two possible states, e and u, which stand for employed and unemployed, respectively. The transition matrix for i is $\mathbf{P}^g$ in good times and $\mathbf{P}^b$ in bad times. Let

$$\mathbf{P}^g = \begin{bmatrix} p_{u/u}^g & p_{e/u}^g \\ p_{u/e}^g & p_{e/e}^g \end{bmatrix}, \ \mathbf{P}^b = \begin{bmatrix} p_{u/u}^b & p_{e/u}^b \\ p_{u/e}^b & p_{e/e}^b \end{bmatrix}, \tag{5}$$

where, for example, $\Pr\{i_{t+1} = u^g|i_t = e\} = p_{u/e}^g$ is the probability that an agent will be unemployed in good times at period $t + 1$ given that the agent was employed at period t.

The following structure on the transition probabilities of $\mathbf{P}^g$ and $\mathbf{P}^b$ summarizes the differences in employment prospects between good and bad times:

$$\begin{align} &\text{(i)} \quad p_{e/e}^g > p_{e/e}^b \qquad \text{(iii)} \quad p_{u/u}^g < p_{u/u}^b \notag \\ &\text{(ii)} \quad p_{e/u}^g > p_{e/u}^b \qquad \text{(iv)} \quad p_{u/e}^g < p_{u/e}^b. \end{align} \tag{6}$$

The overall employment prospects state, s, faced by each individual is a combination of the aggregate and individual states, that is, $s = \{i, n\}$. It has four possible values, s_1, s_2, s_3, and s_4, which stand for employed in good times, unemployed in good times, employed in bad times, and unemployed in bad times, respectively. The process governing s is a first-order Markov chain with the transition matrix $\mathbf{\Pi} = [\pi_{ij}]$, where $\Pr\{s_{t+1} = s_j|s_t = s_i\} = \pi_{ij}$. The transition probabilities of this matrix are determined from the $\mathbf{P}$, $\mathbf{P}^g$, and $\mathbf{P}^b$ matrices. For example, if $s_t = s_1$, then the probability of s_{t+1} being equal to s_2 is given by $\pi_{21} = p_{11}P_{u/e}^g$.

Economy 2.—In this economy, there is no aggregate uncertainty. The state of employment, i, is assumed to follow a Markov chain with two possible states, u and e. The transition function for this process is given by the matrix $\mathbf{\chi} = [\chi_{ij}]$, $i, j = e, u$.

A. Environment with Storage Technology

In this environment borrowing is not allowed; a_{t+1} is required to be nonnegative. Since event-contingent insurance is not permitted, individuals can insure only through holdings of liquid assets. In equilibrium they will accumulate assets during the periods when they work to provide for consumption during the periods when they are unemployed.

The equilibrium processes for the economies with and without business cycles are computed by using numerical methods. These will be explained in Section IV.

B.   Environment with Intermediation Technology

In this environment there is a perfectly competitive intermediation sector. Because of this new technology, agents can now borrow as well as save; a_{t+1} is permitted to be negative. However, agents are allowed to borrow from the intermediary at a borrowing rate that exceeds the lending rate. The difference between these rates reflects the costs of intermediation.

Two economies are studied: one with aggregate fluctuations and the other without fluctuations. The equilibrium processes for the two economies in this environment are also computed by using numerical methods, and certain statistical properties of these economies are compared.

C.   Environment with Perfect Insurance

In this environment there is perfect insurance of the idiosyncratic shock. An event-contingent insurance scheme is assumed to exist that eliminates all but aggregate uncertainty.

At a given point in time a certain fraction of the population is employed, producing y units of the consumption good. On the other hand, those who are unemployed produce θy units of the consumption good, where $0 < \theta < 1$. However, regardless of the individual-specific employment state, each agent receives the per capita income. At each period, the amount of income produced in the economy depends on the aggregate shock and the fraction of the people employed.

Let κ be the fraction of people employed in the current period and $\bar{y}^n$ be the per capita income in the current period, where $n = g, b$. Then

$$\bar{y}^n = \kappa y + (1 - \kappa)\theta y. \tag{7}$$

The fraction of the people employed next period, κ', is

$$\kappa' = \kappa \pi_{e/e}^n + (1 - \kappa)\pi_{e/u}^n. \tag{8}$$

Two economies are studied, one with aggregate fluctuations and the other without fluctuations. Statistical properties of this environment are examined by using Monte Carlo methods.

III.   Calibration

For the economies to be fully specified, it is necessary to choose the invariant transition probabilities for $\mathbf{P}^g$, $\mathbf{P}^b$, and $\mathbf{P}$ and specific parameter values for β, σ, r, and θ. I follow Kydland and Prescott (1982) and

choose these so that certain key statistics for the model economies match those for the U.S. economy. The net real return on stored assets, r, is assumed to be zero. Since the assets in these economies are liquid assets, the assumption of zero real interest is justified by the findings of Ibbotson and Sinquefield (1979). They report that for the 1926–78 period, average real returns on highly liquid short-term debt were near zero. The micro evidence on the spread between the borrowing and lending rates facilitates restrictions on the economies with an intermediation technology. Large spreads between these rates exist even for collateralized loans.[2] For the model economies studied, the rate on borrowing is chosen to be 8 percent while the rate on storage is kept at 0 percent. The income of an unemployed individual, θ, is assumed to be one-fourth of that of an employed individual.[3] This is selected to be somewhat less than the level of unemployment insurance payments for the U.S. economy. This choice was motivated by the fact that 61 percent of the unemployed receive no benefits (see Clark and Summers 1977). The time period is selected to be 6 weeks. The subjective time discount factor, β, is assumed to be .995, which implies an annual subjective time discount rate of 4 percent. This is a typical value used in applied general equilibrium studies. The coefficient of risk aversion, σ, has been estimated in a variety of ways using a variety of data,[4] and the estimates vary widely. In order to compare the results here with the findings in Lucas (1987), the costs of business cycles are computed for $\sigma = 6.2$. The implications of taking σ to be 1.5 are also analyzed since most studies estimate it to be between one and two. The value of the risk aversion parameter utilized for the economies with an intermediation technology is 1.5.

The time-invariant transition probabilities for $\mathbf{P}^g$, $\mathbf{P}^b$, and $\mathbf{P}$, for all the environments studied, are selected so that the variation in per capita employment between good and bad times is 8 percent. This implies a variation in unemployment of 8 percent, namely from 4 percent in good times to 12 percent in bad times. This is much larger than the variation in measured unemployment. But the unemployment rate in the model economy does not correspond directly to what the Bureau of Labor Statistics measures. For the purposes of the

---

[2] According to the *Federal Reserve Bulletin*, the yields on 2-year U.S. Treasury notes in 1982, 1983, and 1984 were 12.80, 10.21, and 11.65 percent, respectively, while the average interest rates on 24-month personal loans over the same period were 18.65, 16.50, and 16.47 percent, respectively.

[3] The utility function specified for these economies is homogeneous of degree $1 - \sigma$. The homogeneity property is useful, for only the value function for $y = 1$ need be computed (as is done here). Subsequently y, the income of an employed individual, is assumed to be equal to one, unless otherwise stated.

[4] See, e.g., Mehra and Prescott (1985) for a survey of the literature on the risk aversion parameter.

model, there is no distinction between in and out of the labor force. Thus the important variable to match is the variation in employment. Given that the rows of the transition matrices sum to one, there really are only five parameters for an economy with business cycles and two parameters for an economy without business cycles that need to be selected.

For the economy with business cycles, the transition probabilities are selected so that the average duration of unemployment and the rate of employment in good times (D_u^g, N^g) and in bad times (D_u^b, N^b) are

$$D_u^g = 10 \text{ weeks} \quad (1.66 \text{ model periods}), \tag{9}$$

$$D_u^b = 14 \text{ weeks} \quad (2.33 \text{ model periods}), \tag{10}$$

$$N^g = .96, \tag{11}$$

$$N^b = .88. \tag{12}$$

This requires the value of $P_{u/u}^g$ to be the one that satisfies[5]

$$D_u^g = \frac{1}{1 - P_{u/u}^g} \tag{13}$$

since the average duration of any state s is $(1 - P_{s/s})^{-1}$. Given $P_{u/u}^g$, $P_{e/e}^g$ is selected such that the fraction employed at the peak, N^g, is .96. Similarly, $P_{u/u}^b$ and $P_{e/e}^b$ are determined using D^b and N^b, respectively. One final parameter remains to be selected: the probability that good or bad times will continue for another period. The value selected is .9375, which implies an average duration of both good and bad times of 24 months or an average business cycle duration of 4 years (see DeLong and Summers 1986). As the model's time period is 6 weeks, the average duration of good or bad times for the model economies is 16 periods.

With these parameter values, the transition probabilities matrix Π is

$$\Pi = \begin{bmatrix} .9141 & .0234 & .0587 & .0038 \\ .5625 & .3750 & .0269 & .0356 \\ .0608 & .0016 & .8813 & .0563 \\ .0375 & .0250 & .4031 & .5344 \end{bmatrix} \tag{14}$$

The next step is to select the transition probabilities for the state of employment in an economy that does not display any business cycles. Requiring the average rate of unemployment and the average dura-

<hr/>

[5] Let D_u be the average duration of unemployment, $m_{i,t+1}$ be the number of people unemployed for i periods at $t + 1$, and ϕ be the proportion of people who stay unemployed. Then $m_{i,t+1} = \phi m_{i,t}$ and $D_u = \Sigma_i m_i/\Sigma m_i = 1/(1 - \phi)$. Hence, $p_{s/s}$ is given by $1/(1 - p_{s/s})$.

tion of unemployment to be the same across the economies with and without business cycles determines these transition probabilities. Consequently the required transition probabilities matrix for an economy with no business cycles is

$$\mathbf{P} = \begin{bmatrix} .9565 & .0435 \\ .5000 & .5000 \end{bmatrix}. \tag{15}$$

When comparisons are made, the values for the risk aversion parameter and the time discount rate are always the same for economies with and without business cycles.

IV. Computation of the Equilibrium

A. Economies with Imperfect Insurance

The maximization problem faced by an individual in an economy with business cycles can be represented as a dynamic programming problem in which a_t and s_t are the state variables and a_{t+1} is the decision variable. The optimality equation is

$$V(a, s) = \max\left\{ U(c) + \beta \sum_{s'} \mathbf{\Pi}(s, s') V(a', s') \right\}, \tag{16}$$

where maximization is over a' and is subject to constraint (3). Using (3) to substitute for c_t in the utility function, we obtain an indirect utility function $U(a, s, a')$. Then

$$V_{k+1}(a, s) = \max\left\{ U(a, s, a') + \beta \sum_{s'} \mathbf{\Pi}(s, s') V_k(a', s') \right\}, \tag{17}$$

where $V(a, s)$ is the value function that will be computed by successive approximations and $V_k(a, s)$ is the kth approximation.

Notice that the dynamic programming problem for the economy with no business cycles is the same as that of the economy with business cycles except for the fact that overall employment prospects state, s, in this economy is the same as the individual-specific employment state, i. In fact, in this economy transition matrices $\mathbf{P}^g$, $\mathbf{P}^b$, and χ are identical.

Tools for computing the equilibria for the economies above are limited. The linear quadratic local approach, which has proved to be useful for many applications, is not suitable for computing the equilibrium stochastic process for this economy. This approach involves constructing a quadratic approximation of the objective function around the steady state after all the random shocks are set equal to their unconditional means. However, in the certainty version of the economy in which individuals would receive the average income each

period, the steady-state asset holdings will be zero. Apparently, for these economies, in which the nonnegativity constraint on the asset holdings is essential, the approximations around this steady state are not feasible. Another approach to the analysis of models with nonconvexities or models with inequality constraints is to study their properties directly by using numerical methods. This is the approach taken in this paper.

The first step is to discretize the state space and the control space. The maximum level of liquid assets that an agent is permitted to hold is assumed to be eight, which is a little more than average annual per capita income if the employed state continues for a year. It turns out that in equilibrium this constraint is never binding. For the economy with only the storage technology, a grid of 301 points with increments of 0.027 in asset holdings is utilized. The sensitivity of the results to the limit on maximum asset holdings and to the tightness of the grid is examined by increasing the limit to 10 and keeping the grid at 301 and also by widening the grid to 251 while keeping the limit on savings at eight. These changes had a negligible effect on the results. For the economies with the intermediation technology, the maximum amount of borrowing permitted is set at eight. The precise level of this limit is not important, provided it is sufficiently large that in equilibrium it is never binding. If this is so, increasing the borrowing limit does not change any of the results. But a borrowing limit is essential even though it is never reached. Otherwise agents could finance any consumption stream by borrowing increasing amounts. The limit on borrowing rules out these Ponzi games. Given this limit, a grid of 601 points with increments of 0.027 in asset holdings was utilized. The overall state of employment prospects takes one of four possible values, s_1, s_2, s_3, and s_4. The total number of possible states for the individual is then 1,204, the number of employment states times the number of asset states. At each point in time the number of possible outcomes is finite, never exceeding 301. Consequently the problem is a finite state, discounted dynamic program.

The optimal value function and the decision rule for asset holdings are obtained by the method of successive approximations. The basic approach is to start with the initial approximation, $V_0(a, s)$, compute the next approximation of the value function, and continue this process until the sequence of value functions converges. We have found that the sequence of decision rules converged after 120 iterations.[6]

[6] Several measures are taken to reduce the cost of computations. For example, to speed up convergence, the initial approximation of the value function, $V_0(a, s)$, is set equal to the steady-state value function for the deterministic economy obtained by setting all random variables equal to their means. Another measure that reduced the cost of computations is the use of tabulations. The value of the utility function is

After the decision rules are found, the $1{,}204 \times 1{,}204$ equilibrium state transition probability matrix is checked for ergodicity. (The ergodicity of the transition matrix is established in the Appendix.)

Given the ergodicity of the Markov chain, there exists a unique invariant distribution with probabilities $\lambda^*(x)$, where $x = (a, s)$, for the equilibrium Markov process governing an individual's state. Moreover, the law of large numbers holds; that is, for any function $f(x)$, the sample average of $f(x)$ converges to the expected value of f with respect to the invariant measure.

For ergodic processes, there are two different methods for computing the statistical properties of the equilibrium Markov process. The first is to create long time series for each model economy by using Monte Carlo methods. In this study, individual time series that consist of 500,000 periods are generated and average utility, consumption, and asset holdings for the two economies are found. For all practical purposes, these averages are independent of the initial wealth and employment conditions.

The second approach for examining the properties of the equilibrium process involves computing the invariant distribution, $\lambda^*(x)$, governing an individual's state of asset holdings and employment prospects for each economy. This distribution can be used to compute the probability limit of the average value of any function of the state; for example, average consumption converges to the expected value of consumption with respect to the invariant distribution.

The interpretation of the invariant distribution is different for the economies with and without business cycles. For both economies this invariant distribution specifies the limits of the fractions of the time a particular individual is in these various asset-employment states as the sample period goes to infinity. Moreover, for the economy without aggregate fluctuations, this distribution is also the same as the distribution of people's states at a given point in time, given the independence of the processes over individuals. For the economy with business cycles, however, it is *not* the distribution of people indexed by their asset holdings and employment status at a given point in time—a distribution that is not constant over time. The invariant distribution is the limit of the predictive probability distribution of an individual n periods in the future as n goes to infinity for both economies.

The invariant distribution $\lambda^*(x)$ for an economy with business cycles is obtained in the following way. Let $\lambda_t(x)$ be the fraction of the time an individual attains a particular state (a, s). The probability that state x'

<hr>

tabulated for each state variable before the value iterations are executed. Therefore, during the successive approximations, the value of the utility function is just "read" from these tables, which are stored in the core memory.

$= (a', s')$ occurs, given the last period's state $x = (a, s)$ and the decision rule $a_{t+1} = f(x)$, is

$$\lambda_{t+1}(x') = \sum_{\substack{\{x' \\ a' = f(x)\}}} \sum_{s'} \Pi(s, s') \lambda_t(x). \tag{18}$$
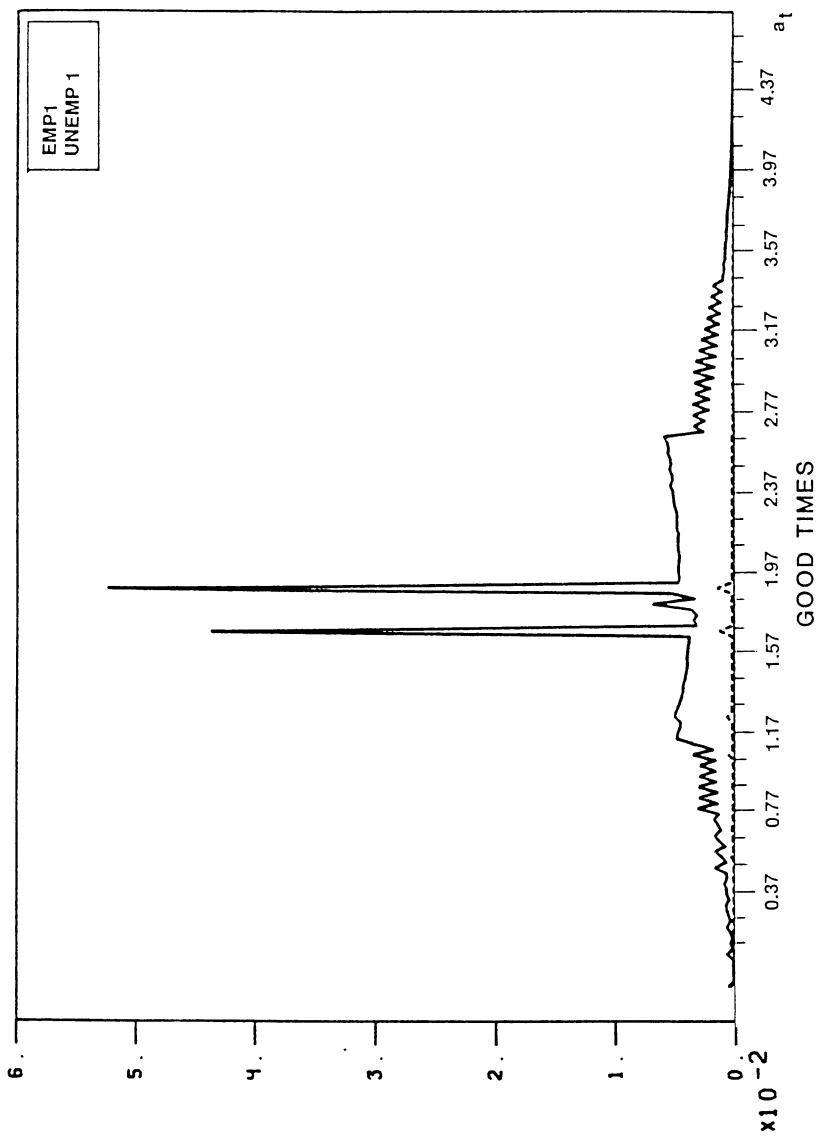
The ergodicity of the Markov process and the absence of cyclically moving subsets guarantee that this sequence of recursively defined distributions converges to a unique invariant distribution $\lambda^*(a, s)$ from any initial distribution.

The invariant distribution of assets conditional on the state of employment prospects for the business cycle economy with only the storage technology is shown in figure 1. For example, emp1 is the distribution of asset holdings conditional on $s = (e, g)$. The two spikes in this curve occur at asset levels 1.79 and 1.95. The probability of reaching asset level 1.79 is particularly large since this is the level that assets will reach and remain at if current assets are less than or equal to 1.79 and the employment state is and remains for a sufficiently long time at $s = (e, g)$. A similar statement holds for $a = 1.95$ if current asset levels are greater than or equal to 1.95. These spikes are artifacts of the fact that there is so little household heterogeneity and limited individual variability over time.

B. Economies with Perfect Insurance

The equilibrium process for the economies with perfect insurance is given by the equality between per capita consumption and per capita income each period because storage of the aggregate output is not allowed in this study. An interesting economy to analyze would have been the perfect insurance economy with storage. However, the dynamic programming problem that would have to be solved in such an economy is computationally more complicated than the ones solved for the economies with imperfect insurance. The reason for that is the additional state variable that would have to be considered if storage is allowed, namely the fraction of the people employed at a given time. Once storage is precluded, the statistical properties of this economy are easily computed. However, the cost figures obtained from this environment should be considered as an upper bound.

In order to compute the steady-state average utility in the economy with business cycles, it is necessary to generate time series by using Monte Carlo methods. Average income fluctuates each period and is a function of the fraction of the people employed. For this economy, individual time series that consist of 500,000 periods are generated and average utility is found. For the economy with no business cycles,
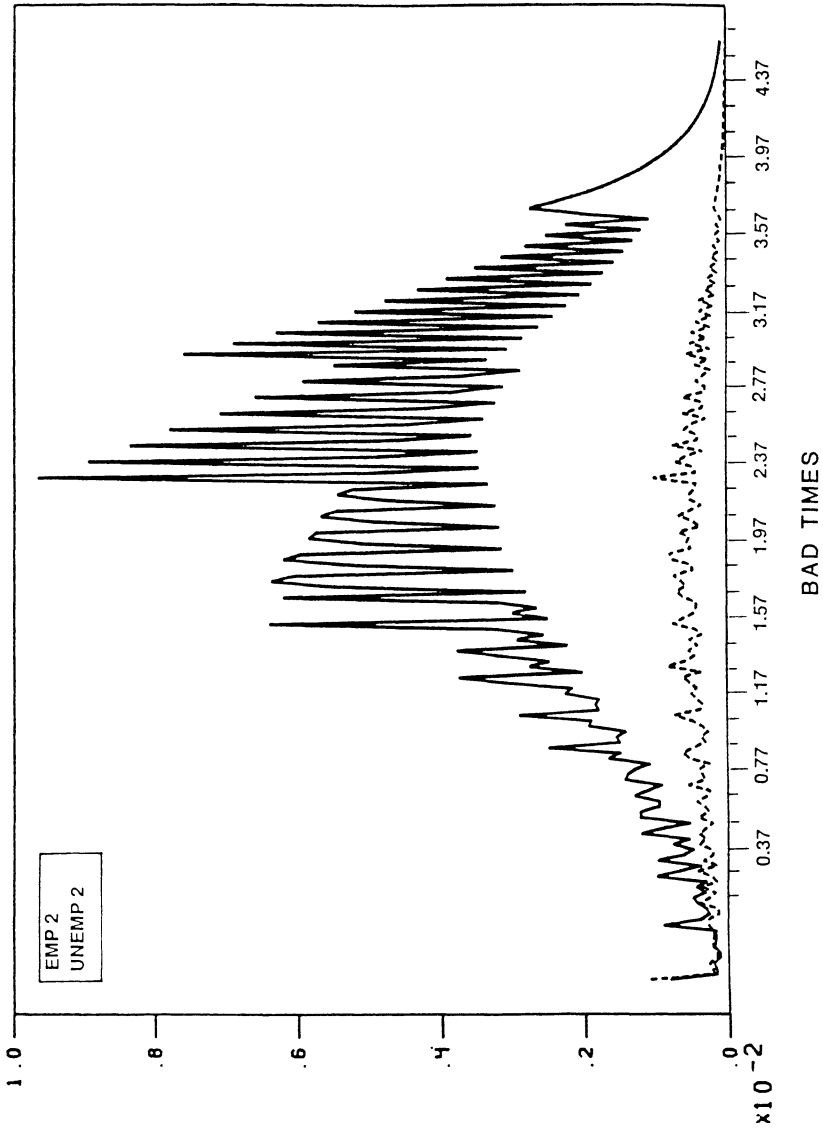
EMP1
UNEMP 1

GOOD TIMES

a_t

4.37 3.97 3.57 3.17 2.77 2.37 1.97 1.57 1.17 0.77 0.37

6. 5. 4. 3. 2. 1. 0. $^{-2}$

$\times 10$

1376

FIG. 1.—Distribution of assets in an economy with a storage technology

1377

average income each period is constant. Thus average utility is computed simply by setting average consumption equal to average income.

V.   Findings

A.   *Economies with a Storage Technology*

In this section, the increase in average consumption that is necessary to compensate the individual for the loss of utility due to business cycles in the environment with only a storage technology is reported. Table 1 summarizes the cost estimates found for this environment and for the perfect insurance environment.

For the case $\sigma = 6.2$, in the economy with perfect insurance, eliminating business cycle fluctuations is equivalent in utility terms to an increase in average consumption of 0.3 percent. If total U.S. consumption is taken to be \$2 trillion (1983 figure), this implies a cost of \$6 billion for the U.S. economy per year, which is \$25.50 per person per year.[7] But with $\sigma = 6.2$ and just the storage technology, an increase in average consumption of 1.5 percent is needed to compensate the individual for the cost of business cycles. This implies a cost of \$128 per person or \$30 billion for the economy per year. This cost estimate is five times larger than the one in an economy with perfect insurance.

For $\sigma = 1.5$, the cost estimate based on the economy with liquidity constraints is four times larger than the one with perfect insurance. For $\sigma = 1.5$, however, the cost estimate is only 0.3 percent of total consumption.

An economy in which workers are half as productive in the household sector as in the market sector is also studied; that is, $\theta = \frac{1}{2}$ instead of $\frac{1}{4}$. In this case, eliminating business cycles is equivalent in utility terms to a 0.5 percent increase in average consumption. The

TABLE 1

COST OF BUSINESS CYCLES AS A PERCENTAGE OF CONSUMPTION

Risk Aversion Parameter	For Economies with Perfect Insurance	For Economies with Only a Storage Technology
$\sigma = 1.5$	.080	.300
$\sigma = 6.2$	.300	1.500

[7] The cost estimates found for the perfect insurance economy in this study are different from those found in Lucas (1987), mainly because of the differences in the volatility of individual consumption used in these studies.

behavior of precautionary asset holdings at the steady state for the economies with and without business cycles for different risk aversion parameters was also examined. Increasing the risk aversion coefficient from 1.5 to 6.2 increased the average asset holdings by a factor of 2.6. This confirms the fact that as individuals get more risk averse, their precautionary asset holdings increase. The time average of asset holdings in the economy with business cycles is 2.20 for $\sigma = 1.5$ and 5.67 for $\sigma = 6.2$. The corresponding averages for the economy without business cycles are 2.26 and 5.76, respectively. It is important to note that the steady-state comparisons of average utilities would not have been sensible if the average asset holdings for the economies with and without business cycles, for a given environment and a given σ, were significantly different. In all the comparisons made, this criterion was met.

Also, given the risk aversion parameter of 6.2, increasing the unemployment compensation from one-fourth to one-half resulted in a reduction in the average asset holdings by a factor of two. This result suggests that as the average unemployment compensation increases, holdings of liquid assets will decrease.

The sensitivity of these results to the specification of the subjective time discount factor β is also examined. For $\sigma = 6.2$, the costs of business cycles with $\beta = .9925$ and $\beta = .9975$, which imply steady-state annual real interest rates of 6 and 2 percent, respectively, are computed. The corresponding cost estimates are 1.6 percent and 1.3 percent of consumption. Compared to the 1.5 percent cost estimate found for $\beta = .995$, these changes in β hardly affect our results at all.

B. Economies with an Intermediation Technology

In the economies in which borrowing is allowed and $\sigma = 1.5$, only a 0.05 percent increase in average consumption is needed to compensate the individual for the loss of utility due to business cycles. That is, the cost estimate is reduced by a factor of six when borrowing is permitted, even though the borrowing rate exceeds the lending rate by 8 percent.[8] Apparently, the findings seem to suggest that the ability to store along with an intermediation technology significantly reduces the magnitude of the cost of fluctuations. This was an unanticipated result.

The time averages of income, consumption, and assets borrowed, stored, and saved in the economies with business cycles and with and

---

[8] Notice that the cost estimates found for the economies with an intermediation technology are lower than those found for the economies with perfect insurance. The reason is that storage against the aggregate uncertainty is allowed in the environment with intermediation but not in the environment with perfect insurance.

TABLE 2

PROPERTIES OF THE EQUILIBRIUM

Time Average of	Economies with an Intermediation Technology	Economies with Only a Storage Technology
Assets borrowed	.480	.000
Assets stored	.220	2.400
Assets saved	.700	2.400
Income	.940	.940
Consumption	.935	.940

without borrowing are summarized in table 2 for the risk aversion parameter $\sigma = 1.5$.

Table 2 shows that that average level of assets saved in the economy with borrowing is equal to 0.70; 60 percent of this is in the form of lending. For the economies computed, the amount held in storage was always positive, no matter what the state of the overall employment prospects. These results would not constitute an equilibrium if that was not the case. Owing to intermediation, average income in the economy with borrowing is no longer equal to average consumption. The difference between them, 0.048, reflects the costs of intermediation: per period spread between the borrowing and lending rates times the average borrowing. Overall, the findings suggest that, even though the equilibrium amount of borrowing is not very high, the magnitude of the costs of business cycles is much lower, once borrowing is allowed.

VI. Concluding Remarks

In this paper simple general equilibrium models with liquidity constraints were examined in order to assess the magnitude of the costs of business cycles. Two versions of incomplete insurance markets were considered. In the first, the consumer had access to only a storage technology, and in the second there was an intermediation technology that allowed the consumer to borrow as well as to save, but at a rate that exceeded the lending rate. Certain statistical properties of the equilibrium stochastic process in these environments were compared with those in a perfect insurance economy. In order to compute the costs of business cycles, two economies in each environment were examined. The agents in these economies faced an uncertain income because of the variability of work option, and they held liquid assets during the periods in which they worked to provide for consumption during the periods in which they were unable to work. The

first economy displayed business cycle fluctuations. There were good and bad times, and the probability of finding employment differed across these times. These probabilities were selected so that the model economy would mimic some key observations in the U.S. aggregate data. The average utility of the agent in the economy with aggregate fluctuations was compared with that of an agent in the second economy, in which no business cycle fluctuations were observed. In particular, the magnitude of the increase in average consumption that was necessary to compensate the individual for the loss of utility due to business cycles was computed. The cost estimates found for different environments were compared with the estimates for an environment with perfect insurance. The results indicate that liquidity constraints in the form of unavailability of borrowing alter the magnitude of the costs of business cycles by a factor of four to five. However, results obtained from the economies with borrowing were surprisingly lower. The cost estimates were reduced by a factor of six when borrowing was permitted, even though the borrowing rate exceeded the lending rate by 8 percent. This was an unexpected result.

Appendix

Ergodicity of the transition matrix for the economy with business cycles in which borrowing is not allowed is established in this Appendix.[9]

The equilibrium law of motion for assets is denoted by $f_s(a)$ for any s, that is, $a' = f_s(a)$. Optimal asset holdings for $s = (e, g)$, $s = (e, b)$, $s = (u, g)$, and $s = (u, b)$ are denoted by emp1, emp2, unemp1 and unemp2, respectively. As can be seen in figure A1, $f_s(\cdot)$ is an increasing function of asset holdings, a, for each of the four employment states s. (Here 1 indicates good times and 2 indicates bad times.)

Result 1.—The state $x_0 = (a_0 = 0, s_0 = (u, b))$ is recurrent and therefore the equilibrium Markov chain is ergodic.

Proof.—The curve $f_s(a)$ for $s = (u, b)$ lies uniformly below the 45° line. Consequently there is a positive probability of reaching $a_0 = 0$ in a finite number of periods if the sequence of unemployment is sufficiently long. Given that all $\Pi(s, s')$ are positive, the state x_0 must be recurrent. Q.E.D.

Result 2.—The ergodic set for the economy with business cycles is $E = \{x \in X: a < 3.8\}$.

Proof.—The largest function is the one for $s = (e, b)$. As it crosses the 45° line from above at $a = 3.8$, starting from any s there is a positive probability of reaching $a = 3.8$ in a finite number of periods, given that all $\Pi(s, s')$ are positive. Consequently, given any x, there is a positive probability of reaching an asset position with $a < 3.8$ in some finite number of periods. Further, if $a_0 > 3.8$, no point with $a > 3.8$ can be reached with positive probability. Therefore, states with $a > 3.8$ are transient. In other words, $P^n(x_0, x) = 0$ for all n if $a > 3.8$ and $P^n(x_0, x) > 0$ for some n and for all $a < 3.8$. This result follows from having $\Pi(s, s') > 0$ for all s and $0 < \Delta f_s(a)/\Delta a < 1$ for each s. Q.E.D.

---

[9] The proofs for the ergodicity of the transition matrices for the other economies can be obtained from the author.

a_{t+1}

8.50

7.40

6.37

5.30

4.24

3.17

2.11

1.04

0.00

45 degree	—
EMP1	---
UNEMP 1	--

0.69   1.42   2.13   2.85   3.57   4.29   5.01   5.73   6.45   7.17   7.89

GOOD TIMES a_t

a_{t+1}

8.50

7.40

6.37

5.30

4.24

3.17

2.11

1.04

0.00

45 degree	—
EMP 2	---
UNEMP 2	--

0.69   1.42   2.13   2.85   3.57   4.29   5.01   5.73   6.45   7.17   7.89
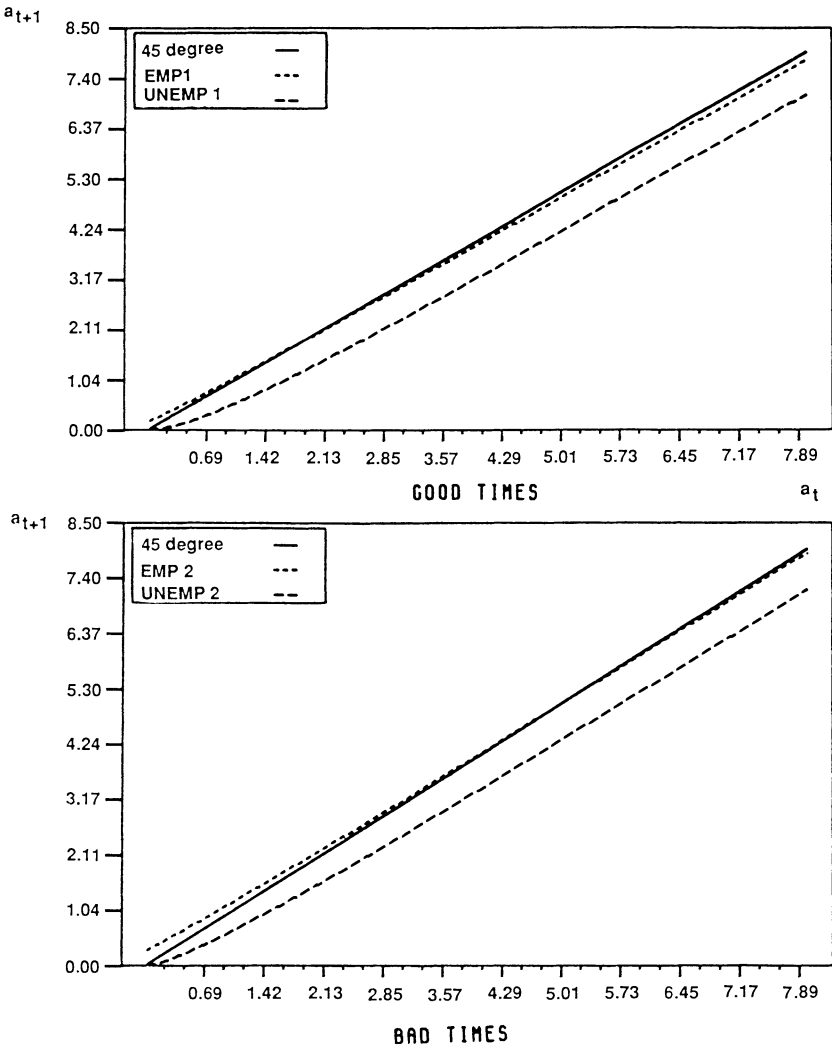
BAD TIMES

FIG. A1.—Decision rules in an economy with a storage technology

References

Atkeson, Andrew. "International Lending with Moral Hazard and Risk of Repudiation." Manuscript. Chicago: Univ. Chicago, 1988.

Bewley, Truman. "The Optimum Quantity of Money." In *Models of Monetary Economies,* edited by John Kareken and Neil Wallace. Minneapolis: Fed. Reserve Bank Minneapolis, 1980.

Clark, S. Kim, and Summers, Lawrence H. "Unemployment Insurance and Labor Market Transitions." Reprint no. 520. Cambridge, Mass.: NBER, 1977.

DeLong, J. Bradford, and Summers, Lawrence H. "Are Business Cycles Symmetrical?" In *The American Business Cycle: Continuity and Change,* edited by Robert J. Gordon. Chicago: Univ. Chicago Press (for NBER), 1986.

Green, Edward. "Lending and the Smoothing of Uninsurable Income." In *Contractual Arrangements for Intertemporal Trade,* edited by Edward C. Prescott and Neil Wallace. Minnesota Studies in Macroeconomics, vol. 1. Minneapolis: Univ. Minnesota Press, 1987.

Hayashi, Fumio. "Tests for Liquidity Constraints: A Critical Survey." Working Paper no. 1720. Cambridge, Mass.: NBER, October 1985.

Ibbotson, Roger G., and Sinquefield, Rex A. *Stocks, Bonds, Bills, and Inflation: Historical Returns (1926–1978).* 2d ed. Charlottesville, Va.: Financial Analysts Res. Found., 1979.

Kydland, Finn E., and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (November 1982): 1345–70.

Lucas, Robert E., Jr. *Models of Business Cycles.* New York: Blackwell, 1987.

Mehra, Rajnish, and Prescott, Edward C. "The Equity Premium: A Puzzle." *J. Monetary Econ.* 15 (March 1985): 145–61.

Scheinkman, José A., and Weiss, Laurence. "Borrowing Constraints and Aggregate Economic Activity." *Econometrica* 54 (January 1986): 23–45.

Tobin, James, and Dolde, Walter. "Wealth, Liquidity and Consumption." In *Consumer Spending and Monetary Policy: The Linkages.* Monetary Conference Series, no. 5. Boston: Fed. Reserve Bank Boston, 1971.

Townsend, Robert M. "Information-constrained Insurance: The Revelation Principle Extended." *J. Monetary Econ.* 21 (March/May 1988): 411–50.

Zeldes, Stephen P. "Consumption and Liquidity Constraints: An Empirical Investigation." *J.P.E.* 97 (April 1989): 305–46.