R68-16 A Mathematical Model for the Investigation of Three-Dimensional Fields with Asymmetric Boundaries—E. S. Ip (*Proc. Internat'l Assoc. for Analog Computation*, vol. 9, no. 3, pp. 122–130, July 1967).

The purpose of the investigation reported in this paper was to test the feasibility of solving Laplace's equation in a three-dimensional region having asymmetric boundaries by means of an electrolytic tank and also by a three-dimensional network of resistors. The region considered was one octant of an ellipsoid.

The author first presents an analytical solution for the potential distribution in the interior of an ellipsoid in terms of ellipsoidal coordinates. The boundary conditions consist of the voltages being specified on plane electrodes with elliptic or hyperbolic boundaries. These electrodes are located on the plane sides of the octant under consideration.

The analytic results were evaluated numerically with the aid of a digital computer and were compared with results measured directly from an electrolytic tank having walls of ellipsoidal shape. A system of fixed and movable probes was used to determine the location of the equipotential surfaces.

Finally, a three-dimensional resistor network was used to solve the same problem and the results of the three methods were compared. The resistor network was, of course, a lumped model. Taking 15 cells along the semimajor axis, several thousand resistors of 2-percent accuracy were used in the network. It turned out that the electrolytic tank gave somewhat better accuracy than the network, although the accuracy of either was within approximately 2 percent of the applied potential difference. Estimates were made of the errors due to resistor tolerances and due to truncation effects.

The number of hours of work required to perform the research reported in this paper must have been very great indeed, since each of the experimental portions was quite a project in itself. The principal result obtained was the demonstration that an effective three-dimensional probe can be used to obtain good accuracy for electrolytic tank problems having asymmetric boundaries.

The three-dimensional resistor network used well-known lumping techniques and arrived at results of the accuracy that one might expect. Consequently, this portion of the paper may not be of as much general interest as the report on the electrolytic tank.

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E. DIGITAL-TO-ANALOG CONVERSION

R68-17 Synthesis of Resistive Digital-to-Analog Conversion Ladders for Arbitrary Codes with Fixed Positive Weights—M. R. Aaron and S. K. Mitra (*IEEE Trans. Electronic Computers*, vol. EC-16, pp. 277–281, June 1967).

The purpose of this paper is to develop design equations for switched positive-resistance digital-to-analog converters. The authors state that no synthesis method heretofore has existed for the design of this type of ladder network. The problem to which the paper addresses itself is the following.

1) Given a set of fixed positive weights (A_j) and a set of positive source conductances (g_j), find the necessary and sufficient conditions that these quantities and the input admittance must satisfy in order for the resulting ladder decoder to be realizable with positive conductance (G_j) and ideal switches.

2) Derive explicit equations relating the (G_j) to the (A_j), (g_j) and input admittance.

The source conductances are, of course, a part of the ladder. The constraint placed upon them is found to be

$$\frac{g_j A_{j+1}}{g_{j+1} A_j} \leq 1$$

where A_{j+1} signifies a lesser weight of the desired code than A_j.

Thus, it is seen that the source conductances need not be equal, the way they are usually designed, but must be in ratios constrained by the weight of the code selected for synthesis.

The input admittance Y_1 is constrained by the following relationship:

$$Y_1 \geq g_1 \sum_{j=2}^{n} \prod_{r=1}^{j} \frac{A_r}{A_{r-1}} .$$

These two inequalities answer part 1) of the problem statement. The second part, the explicit equations relating the shunt conductances (G_j) to the weights (A_j) and to the series of source conductances (g_j) and input admittance, are:

$$G_j = \frac{Y_j}{1 - b_{j+1,j}}$$

$$Y_{j+1} = \frac{Y_j}{b_{j+1,j}} - g_{j-1}$$

where

$$A_{mp} = \frac{A_m}{A_p}, \qquad m > p$$

$$b_{mj} = \frac{g_p}{g_m} .$$

In the special cases where the (g_j) are ordered such that

$$g_{j+1} \geq g_j, \qquad \text{all } j \leq n-1,$$

or where g_j are equal to each other for all $j \leq n-1$, the realizability condition becomes

$$\frac{A_{j+1}}{A_j} \leq 1 \qquad \text{all } j \leq n-1;$$

that is, the weights of the selected code form a nonincreasing sequence.

After arriving at the general solution for the design problem, the special cases of repetitive codes and the straight binary system are treated, and simplified design equations for these cases are developed. These sections and those following assume equal (g_j) normalized to one ohm.

The paper concludes with a treatment of binary-coded decimal systems which are repetitive structures. General-design equations are again developed and an example is given utilizing the equations to develop an 8421 weighted code.

The authors have written a letter which appeared in the Correspondence section of the October, 1967, issue of this TRANSACTIONS, entitled "A Note on the Design of D to A Converters." The substance of this letter is to show how the generalized ladder reduces to the weighted resistor grouping—sometimes called a star, or a ladder grouping—by placing constraints on the ladder parameters and then applying the equations which have been developed. Since these special codes are extremely useful, the letter should be read along with the paper by anyone interested in the subject.

The paper concludes with a tabulation of normalized conductances of the repetitive DAC ladder for 17 different BCD codes.

The authors suggest that their paper is written to have engineering appeal. If this means that it can easily be used by engineers, I would take some issue with them. Except for the tabulation at the end of the paper, which allows the ladder resistors to be chosen simply by selecting from the table and multiplying by a constant, the paper is not written so that it is especially easy to read. Some of the notation is not defined prior to its use, but must be inferred from the text, which compels one to reread sections several times or to work out examples to make sure one has not gone astray. Also, the mathematical notation was not as clear as it could have been if some additional explanations had been given. Much of the algebraic manipulation of the derivation is left to be worked out by the reader. Finally, the paper gives no hint as to why it was undertaken, except to say that no synthesis previously existed. If, by way of background, a brief description of the reason for synthesizing all possible DAC ladders were included in the paper, it would have made it more interesting and less academic.

However, these are minor points. The paper is instructive and of interest to those involved in the design of digital-to-analog converters or analog computers.

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F. STOCHASTIC COMPUTATION

R68-18 Random Pulse Machines—S. T. Ribeiro (*IEEE Trans. Electronic Computers*, vol. EC-16, pp. 261–276, June 1967).

In conventional digital computation, continuous quantities are quantized and represented by binary words in which the number of bits determines the precision of representation. In stochastic computation, as described in this paper, continuous quantities are represented as a sequence of one-bit binary words (i.e., as a pulse stream or sequence of logic levels) in which the probability of an ON logic level is a measure of the quantity. Since probability is a continuous variable in the range $0 \leq p \leq 1$, this removes the effects of quantization. However, a probability cannot be measured precisely, only estimated subject to random variance, and hence there is an effective random noise in the output of the computer.

Alternatively, the stochastic computer may be compared with other forms of computer in terms of efficiency. Representation of quantities as random variables enables very simple elements to be used to perform arithmetic operations. For example, a single EXCLUSIVE-OR gate acts as a multiplier for two signed numbers; far more complex and expensive hardware is required for four-quadrant multiplication in either conventional analog or digital computers. Similarly, simple digital elements may be used in the stochastic computer to perform all the various operations—addition, subtraction, integration, and so on—commonly associated with the analog computer.

The stochastic computer is thus very efficient in its utilization of hardware. On the other hand, the noise inherent in stochastic computation makes N^2 pulses necessary to define a quantity with a precision of 1 part in N. This compares unfavorably with other pulse-counting computers, such as the DDA, which need N pulses for the same precision, and even more unfavorably with the general-purpose digital computer, which requires $\log_2 N$ bits to obtain this precision. The stochastic computer is thus inefficient in its use of the potential information content of a data stream, and hence its operations are slow compared with those of other machines. It scores when large amounts of low-bandwidth (10 Hz) data have to be processed in parallel in a fairly complex fashion, for example, in pattern-recognition or multivariable adaptive control.

It is difficult to trace the origins of stochastic computing back very far, although one has the suspicion that the extremely simple concept of representing analog variables by the probability of occurrence of discrete events may well have a fairly ancient lineage. However, although the early Chinese had a grasp of number theory which gives them some right to claim prior disclosure on patents for residue-number machines, the concept of a quantitative representation of random events is itself sufficiently recent to preclude this for stochastic machines. On the other hand, the human brain seems to use stochastic computing as its mainstay (perhaps), and might be considered prior art!

Certainly the possible genesis of stochastic computing may be found in the use of high-frequency sawtooth "dither" to round off the corners in the output of diode function generators, and, more recently, to linearize polarity coincidence correlators, but none of this work has ventured beyond the immediate application and generated a complete computing system based on the representation of continuous variables as probabilities.

Suddenly this has been done, with papers presented at both the Spring and Fall Joint Computer Conferences this year, together with the excellent introduction to stochastic computing principles which is the subject of the present review.

The author first introduces the concept that a logical interaction between two independent binary sequences in a two-input, single-output gate is equivalent to an arithmetic operation on their generating probabilities, and tabulates this operation for the 16 possible Boolean functions of two variables. He then presents the problem of coding any analog variable as a probability in such a way that this effect may be used to perform arithmetic. Finally, the author considers time-dependent operations, such as integration, gives a brief analysis of the errors inherent in stochastic computation, and suggests some possible applications.

In its treatment of memoryless computing elements and stochastic representations of quantity, this paper is particularly comprehensive and much to be welcomed. Time-dependent operations are not covered in as great a depth. In particular, the stochastic integrator with feedback, which is usually the outward interface of the computer and fundamental to the solution of differential equations on the stochastic computer, is hardly mentioned. Because of this, the wide range of potential applications in control systems, both for identification and optimization, is not described.

However, these are minor criticisms of the first paper to appear in this TRANSACTIONS on a new type of computer which presents exciting problems and a fruitful area for research. The main obstacle to the practical application of the stochastic computer is, at present, the generation of the random variables required in a reliable and economical manner. It may well be that we should look to truly random physical processes, such as photon–photon interactions, to provide the hardware foundation for stochastic computing systems.

This is a relatively unexplored region, however, where much further effort by mathematicians, engineers, and physicists will be required before we can fully define the potential impact of stochastic techniques on future computing systems. Fortunately, there is sufficient commercial promise and intellectual novelty in the concept of stochastic computing to attract ever-increasing research effort in a wide range of locations.

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