Chapter 9
Introduction to Triadic Graphical Calculus

9.1 INTRODUCTION

This chapter introduces the binary and triadic notations which appear in the referenced papers on the parallel Boolean processor and mosaic functions.

Included is the triadic map algorithm for finding the complete sum of the function y = F(x), which is part of the set of applications for the parallel Boolean processor. This algorithm is programmable and is based upon the theorems of Chapter 11. The chapter concludes with the triadic and binary logical instrument card decks which have been successfully used to demonstrate the algorithm.

9.2 BINARY SPACE NOTATION

A system X of n Boolean variables where

$$x_i \in \{0,1\} \text{ for } 0 \le i \le n-1$$

may be represented in terms of the ALGEBRA OF SETS by a LOGICAL SPACE which is the universe of the system.

The logical space consists of all points x where each point represents a particular validity configuration of the variables of the system. The total number of points in the logical space is 2^n. The Marquand map will be used to represent the logical space.

The point x in the logical space is identified by:

$$\begin{aligned} x &= (x_{n-1}\ x_{n-2}\ \dots\ x_1\ x_0)_2 \\ &= x_{n-1}\cdot 2^{n-1} + x_{n-2}\cdot 2^{n-2} + \dots + x_1\cdot 2^1 + x_0\cdot 2^0 \end{aligned}$$

The integer x is termed a POINT IDENTIFIER.

A MINTERM in Boolean algebra is a function defined by:

$$m_a = \dot{x}_{n-1}\ \dot{x}_{n-2}\ \dots\ \dot{x}_1\ \dot{x}_0$$

where

$$\dot{x}_i \in \{x_i, \bar{x}_i\} \quad \text{for } 0 \le i \le n-1$$

The function m_a is equal to 0 everywhere except for a single configuration of values for the variables. When this function is charted in the logical space, the space is filled with zeros everywhere except at a single point where it is 1.

It has been an accepted policy to give the index of m_a the value of the point identifier for the point where $m_a = 1$. Under that convention:

$$m_a = 0 \text{ for } x \ne a$$

$$m_a = 1 \text{ for } x = a$$

The index a is termed the MINTERM IDENTIFIER and has the form of a binary number

$$a = a_{n-1} \cdot 2^{n-1} + a_{n-2} \cdot 2^{n-2} + \dots + a_1 \cdot 2^1 + a_0 \cdot 2^0$$

where

$$a_j \in \{0,1\}$$

The following is the correspondence between a and the minterm identified:

$\dot{x}_j$	x_j	$\bar{x}_j$
a_j	1	0

Or:

$$(\dot{x}_j = x_j) <=> (a_j = 1)$$

$$(\dot{x}_j = \bar{x}_j) <=> (a_j = 0)$$

Figure 9.1 demonstrates the Marquand map for $m_a = m_{10}$.

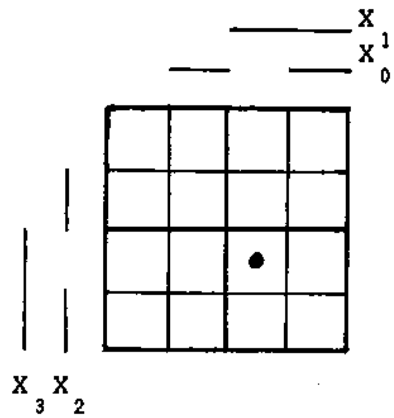


Figure 9.1. Marquand map for the function $m_a = x_3\overline{x_2}x_1\overline{x_0}$ ($m_a = m_{10}$).

9.3 TRIADIC SPACE NOTATION

A TERM is a Boolean function defined by:

$$t_h = t_h(x) = \ddot{x}_{n-1}\ \ddot{x}_{n-2}\ \dots\ \ddot{x}_1\ \ddot{x}_0$$

where

$$\ddot{x}_i \in \{1,\ x_i,\ \overline{x_i}\}$$

For a logical system X with n Boolean variables, Svoboda created a TRIADIC SPACE of 3^n points in order to represent all possible terms belonging to the logical system. A triadic map is used to represent the triadic space.

The TERM IDENTIFIER h is defined as a triadic number:

$$h = h_{n-1}\cdot 3^{n-1} + h_{n-2}\cdot 3^{n-2} + \dots + h_1\cdot 3^1 + h_0\cdot 3^0$$

where

$$h_i \in \{0,1,2\}$$

The following is the correspondence between h_j and the term identified:

$\ddot{x}_j$	1	x_j	$\overline{x_j}$
h_j	0	1	2

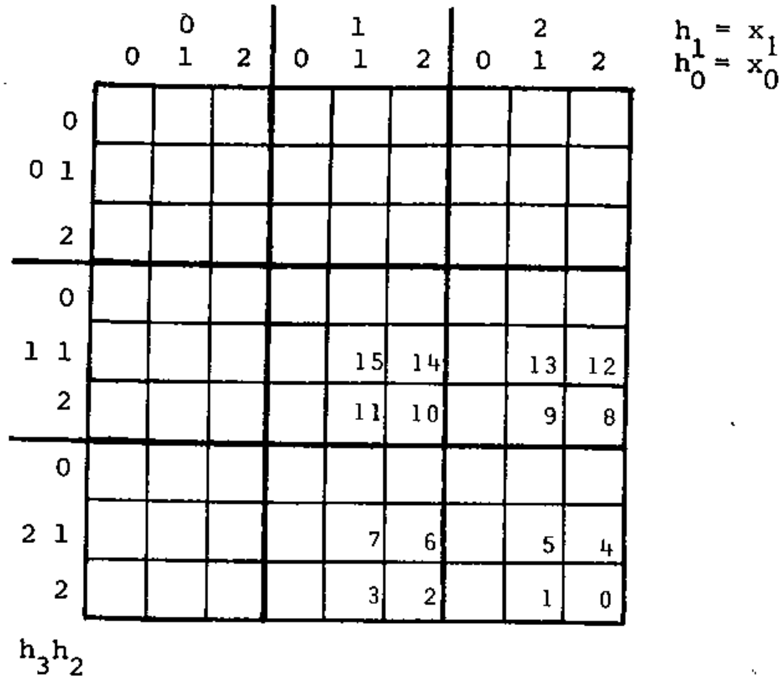


Figure 9.2. Triadic map with minterm space labeled with point identifiers.

Or:

$$(\ddot{x}_j = 1) \Leftrightarrow (h_j = 0)$$

$$(\ddot{x}_j = x_j) \Leftrightarrow (h_j = 1)$$

$$(\ddot{x}_j = \overline{x}_j) \Leftrightarrow (h_j = 2)$$

The triadic map was devloped by Svoboda as a graphical medium for the representation of Boolean forms. A triadic map for four-variable space is shown in Figure 9.2. Note that the space of minterms (or minterm cube) is a subspace of the triadic space. The minterm cube on the triadic map includes only those terms t_h which have only nonzero digits as the coordinates of their identifiers.

Or,

$$\text{Space of minterms} = \{t_h \mid h_j \in \{1,2\}\}$$

9.4 CORRELATIONS BETWEEN BINARY AND TRIADIC SPACE

The Karnaugh map, Marquand map, and triadic map for a four-variable system are shown in Figure 9.3. Each is labeled with point identifiers. Figure 9.3c demonstrates the relationship between triadic terms 53 and 80 and minterms 8 and 0 respectively.

Figure 9.4 demonstrates the relationships between the terms of the triadic space and the configurations of the binary space for four variables using the Marquand map. Note that the larger structures have the lower-numbered point identifier. This is an important feature of the triadic map and relates to the "ordering of implicants" referred to previously.

9.5 TRIADS

A TRIAD is a set of three points:

$$\{t_h \mid h \in \{a,b,c\}\}$$

from the triadic space whose identifier coordinates h_i agree for all variables except one, x_j. For the variable x_j, the coordinates of the points of a triad are bound through the relationships:

h	a	b	c
h_j	0	1	2

The terms of the points of any triad are logically related through:

$$t_c = \bar{x}_j t_a$$
$$t_b = x_j t_a$$
$$t_a = t_b + t_c$$

X_1, X_0

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

$X_3 X_2$ a. Karnaugh map.

X_1, X_0

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

$X_3 X_2$ b. Marquand Map.

h_3h_2		0			1			2		$h_1 = x_1$
		0	1	2	0	1	2	0	1	2 (h_0)
0	0	0	1	2	3	4	5	6	7	8
0	1	9	10	11	12	13	14	15	16	17
0	2	18	19	20	21	22	23	24	25	26
1	0	27	28	29	30	31	32	33	34	35
1	1	36	37	38	39	40	41	42	43	44
1	2	45	46	47	48	49	50	51	52	53
2	0	54	55	56	57	58	59	60	61	62
2	1	63	64	65	66	67	68	69	70	71
2	2	72	73	74	75	76	77	78	79	80

$t_{53} = m_8$

$t_{80} = m_0$

c. Triadic map.

Figure 9.3. Maps and their identifiers.

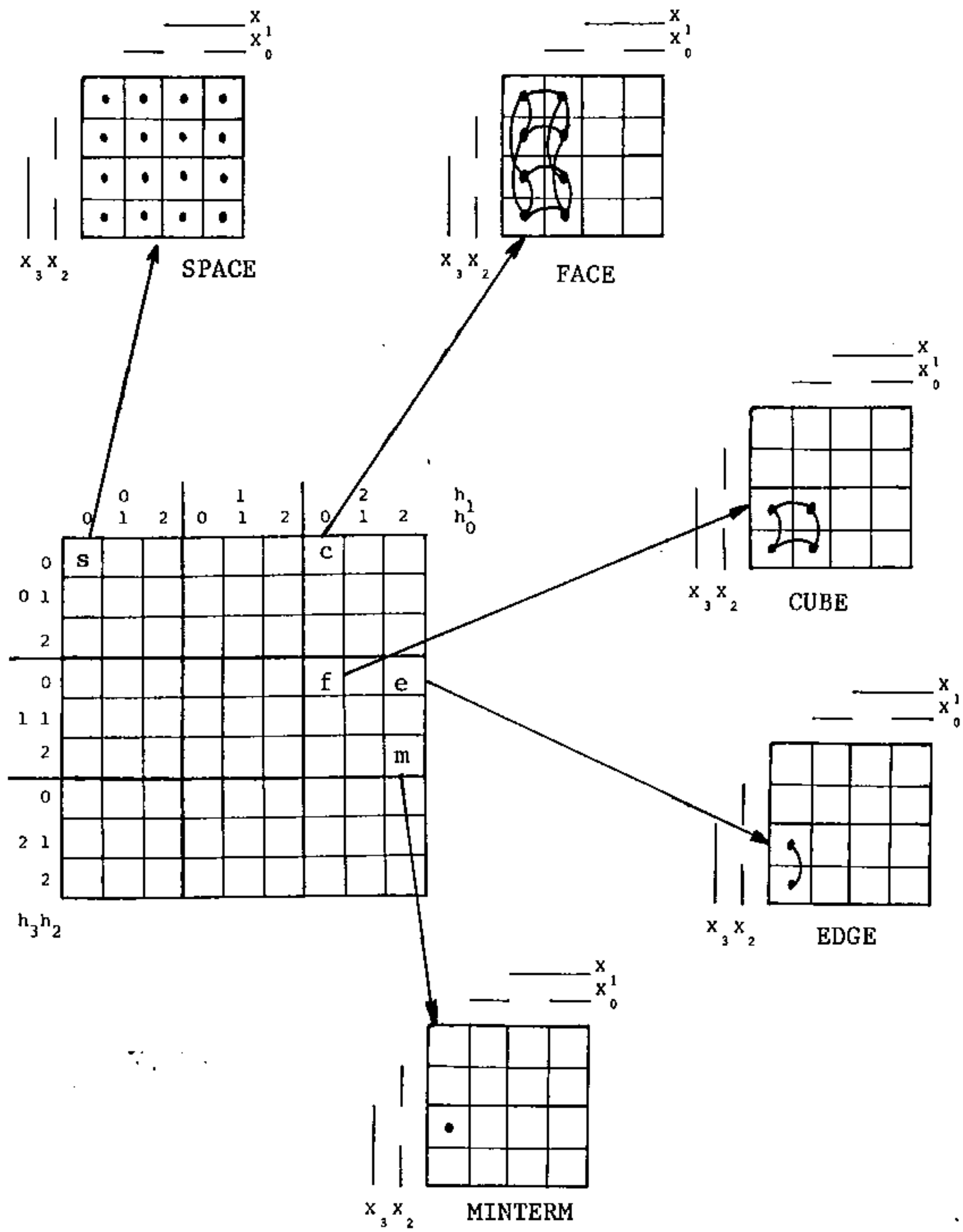


Figure 9.4. Structures in binary and triadic space.

And also, note that

$$c - b = b - a = 3^j$$

Three points which are a triad form a triadic line in the triadic space, as shown in Figure 9.5. Several triads are shown in Figure 9.6. This corresponds to the edge formed in binary space by two points whose logical distance is 1. If t_b and t_c are minterms, then t_a is the term representing the edge in binary space. This and other relationships are detailed in Figure 9.7 for a four-variable space. (The four-variable space was chosen for convenience and no limitation on problem size is to be inferred.)

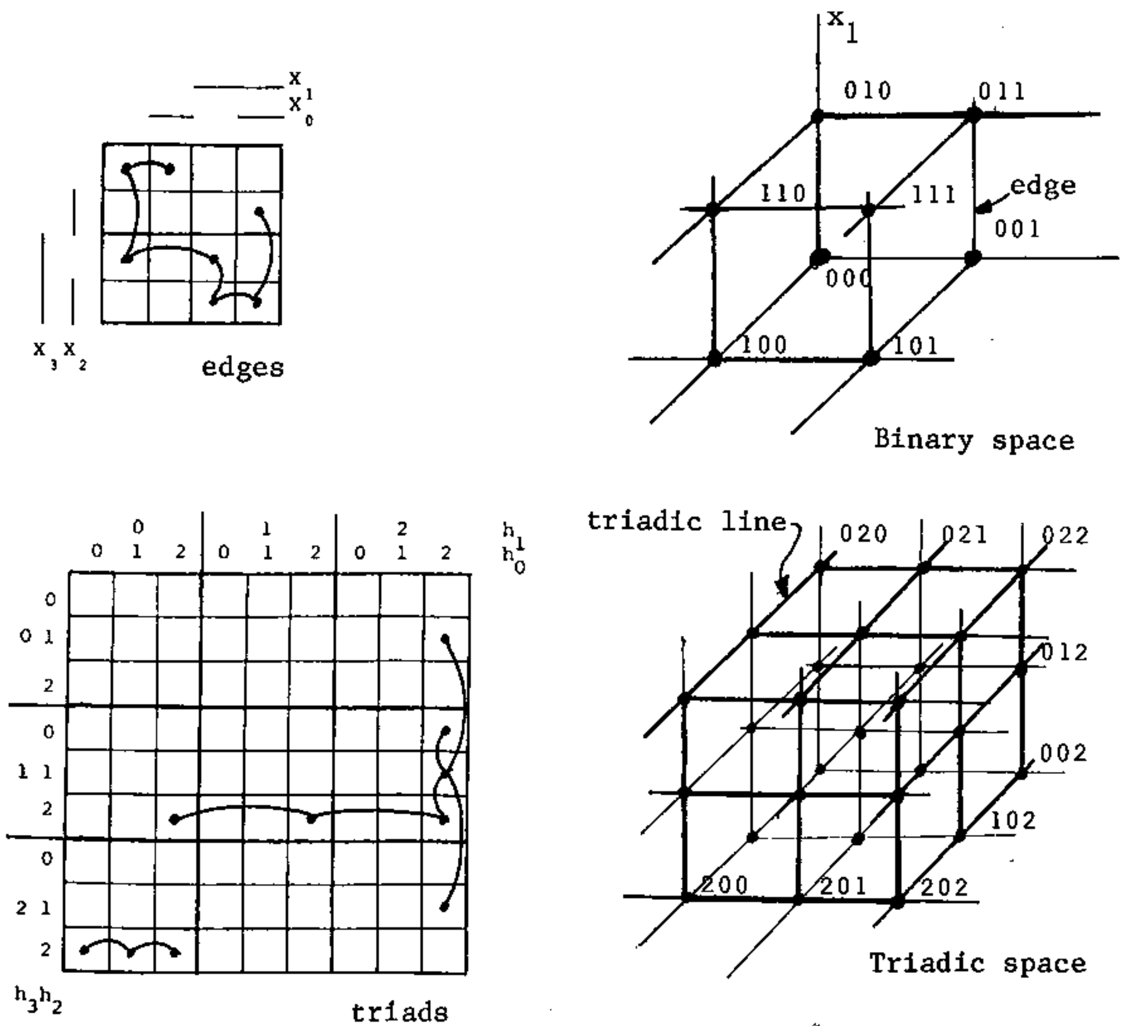
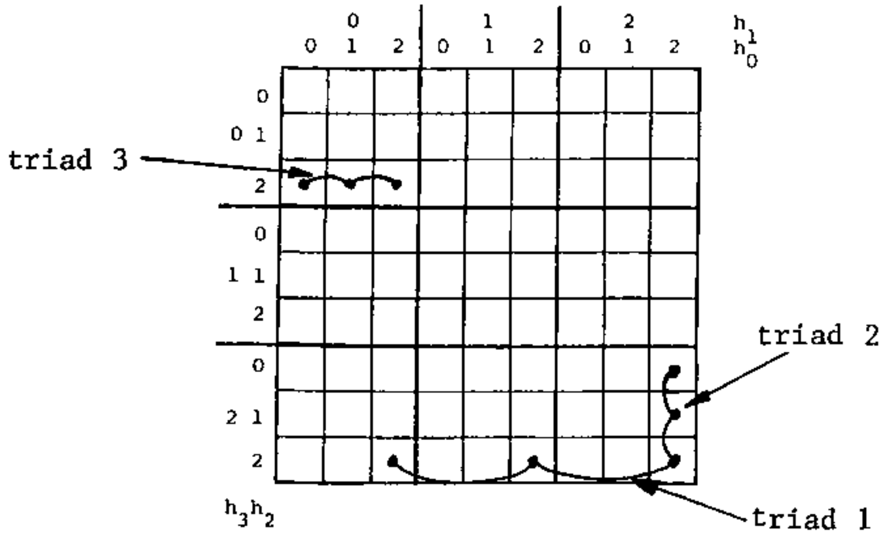


Figure 9.5. Spatial relationships.



		$h_3h_2h_1h_0$
Triad 1:	$t_a = \bar{x}_3\bar{x}_2\ \ \bar{x}_0$	$a = 74 = (\ 2\ 2\ 0\ 2\ )_3$
	$t_b = \bar{x}_3\bar{x}_2x_1\bar{x}_0$	$b = 77 = (\ 2\ 2\ 1\ 2\ )_3$
	$t_c = \bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0$	$c = 80 = (\ 2\ 2\ 2\ 2\ )_3$
Triad 2:	$t_a = x_3\ \ x_1x_0$	$a = 62 = (\ 2\ 0\ 2\ 2\ )_3$
	$t_b = x_3x_2x_1x_0$	$b = 71 = (\ 2\ 1\ 2\ 2\ )_3$
	$t_c = x_3\bar{x}_2x_1x_0$	$c = 80 = (\ 2\ 2\ 2\ 2\ )_3$
Triad 3:	$t_a = x_2$	$a = 18 = (\ 0\ 2\ 0\ 0\ )_3$
	$t_b = x_2\ \ x_0$	$b = 19 = (\ 0\ 2\ 0\ 1\ )_3$
	$t_c = x_2\ \ \bar{x}_0$	$c = 20 = (\ 0\ 2\ 0\ 2\ )_3$

Figure 9.6. Example triads.

9.6 INFORMATIONAL CONTENT OF MAPS

The amount of information that can be presented in each of the two graphic mediums, binary maps (Marquand, Karnaugh, Veitch) and ternary or triadic maps, highlights their differences. There are three symbols which are used for binary maps, as shown in Table 9.1; there are eight symbols for the triadic map (Table 9.2). The position of a square within a triadic map also imparts data (this is due to the ordering of implicants).

Also, note that reduction figures (edges, faces, cubes, etc.) must be drawn or visualized for the binary maps, while for the triadic map such structures are represented by a single term.

Referring to Figure 9.7, given the t_b and t_c terms of a triad, the notation for the t_a term is found using Figure 9.8. For example, if two minterms of a function form an edge, the t_a term of their triad is noted as I, an implicant of the function. If only one of the two minterms belongs to y and the other belongs to $Y = \bar{y}$, then the edge as a structure does not exist as noted by the symbol $\emptyset$.

9.7 BOOLEAN NOTATIONAL FORMS

There are several notational forms which can occur for Boolean functions. For the function $y = F(x)$, these are (1) the canonical $\Sigma\Pi$ form, (2) the $\Sigma\Pi$ form, (3) the maximal $\Sigma\Pi$ form, and (4) the complete sum. Equivalent notational forms exist for the complement function $Y = \bar{y}$.

Symbol	Definition	Name
•	$m_a \Rightarrow y$	Minterm of y
0	$m_a \Rightarrow Y$	Minterm of Y
#	$(m_a \Rightarrow y) \vee (m_a \Rightarrow Y)$	Don't care

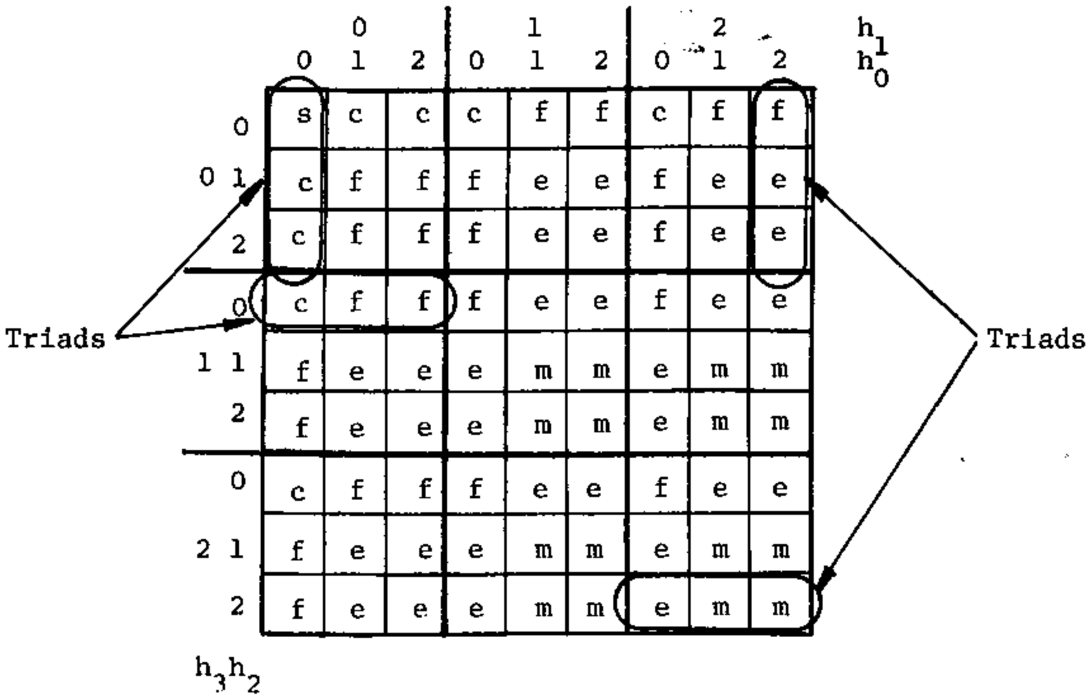
Table 9.1. Binary map symbols.

Symbol	Definition	Name
#	$(t_h = m_a) \forall (t_h = m_a)$ $\wedge (t_h \rightarrow y) \forall (t_h \rightarrow Y)$	Don't care
•	$(t_h = m_a) \wedge (t_h \rightarrow y)$; $\sim (t_h \rightarrow Y)$	Minterm of y
0	$(t_h = m_a) \wedge (t_h \rightarrow Y)$; $\sim (t_h \rightarrow y)$	Minterm of Y
I	$(t_h \rightarrow y); \sim (t_h \rightarrow Y)$	Implicant of y
N	$(t_h \rightarrow Y); \sim (t_h \rightarrow y)$	Implicant of Y
/	$\sim (t_h \rightarrow Y) \wedge (t_h \rightarrow y)$ $\forall \sim (t_h \rightarrow y)$	Nonimplicant of y
o	$\sim (t_h \rightarrow y) \wedge (t_h \rightarrow Y)$ $\forall \sim (t_h \rightarrow Y)$	Nonimplicant of Y
∅	$\sim (t_h \rightarrow y) \wedge (t_h \rightarrow Y)$	Nonimplicant of y and nonimplicant of Y

Also used:

II	Given term of ΣΠ-form of y
N	Given term of ΣΠ-form of Y

Table 9.2. Triadic map symbols.



Key:

m: minterms

e: edge: edges are t_a terms of triads whose t_b and t_c terms are minterms.

f: face: faces are t_a terms of triads whose t_b and t_c terms are edges.

c: cube: cubes are t_a terms of triads whose t_b and t_c terms are faces. Cubes are extended to n dimentions.

s: space:(universe)the t_a term of triads whose t_b and t_c terms are cubes where $n' = n - 1$.

Figure 9.7. Detail of term relationships for a four-variable map.

t_c →

			II		N				
	t_a	#	.	I	0	N	/	o	∅
t_b ↓	#	#	/	/	o	o	/	o	∅
II	.	/	I	I	∅	∅	/	∅	∅
II	I	/	I	I	∅	∅	/	∅	∅
N	0	o	∅	∅	N	N	∅	o	∅
N	N	o	∅	∅	N	N	∅	o	∅
	/	/	/	/	∅	∅	/	∅	∅
	o	o	∅	∅	o	o	∅	o	∅
	∅	∅	∅	∅	∅	∅	∅	∅	∅

Figure 9.8. Triadic map symbol intersection chart.
(Note that some intersections are impossible but are defined.)

9.7.1. Canonical ΣΠ-form.

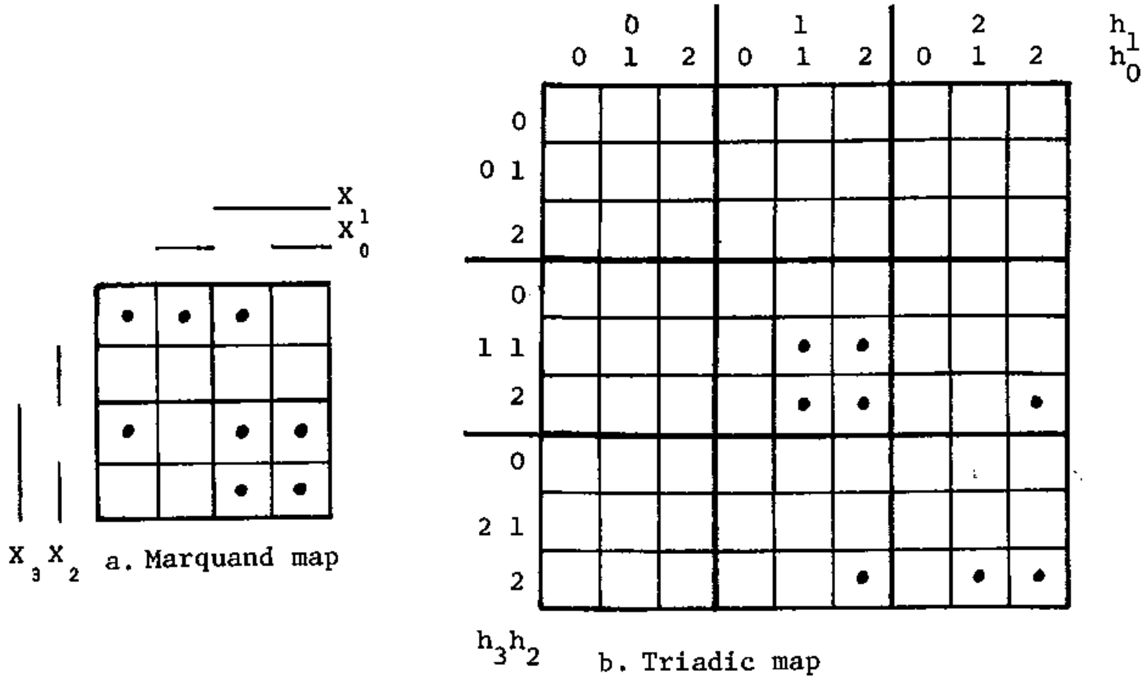
The canonical ΣΠ-form of a Boolean function y is the unique sum of all of the minterms for which for function y = F(x) is true.

$$y = \sum_a m_a \cdot f_a \quad \text{for all } a \in \{\, a \mid f_a = 1 \,\} = \{a\}_y$$

The Marquand and triadic maps of an example function are shown in Figure 9.9.

The canonical ΣΠ-form of the complement Boolean function Y where $Y = \bar{y}$ is the unique sum of all of the minterms for which the function y = F(x) is false.

$$Y = \sum_{a'} m_{a'} \cdot f_{a'} \quad \text{for all } a' \in \{\, a \mid f_{a'} = 0 \,\} = \{a'\}_Y$$



$$y = m_0 + m_1 + m_2 + m_8 + m_{10} + m_{11} + m_{14} + m_{15}$$

Figure 9.9. Canonical ΣΠ-form of an example function.

9.7.2 ΣΠ-form.

The ΣΠ-form of a Boolean function y is a not-necessarily-unique sum of terms for which the function is true; i.e., there may be more than one ΣΠ-form for any given function. The sum of terms, each of which is an implicant of the function, includes sufficient terms to cover all of the minterms for which the function is true.

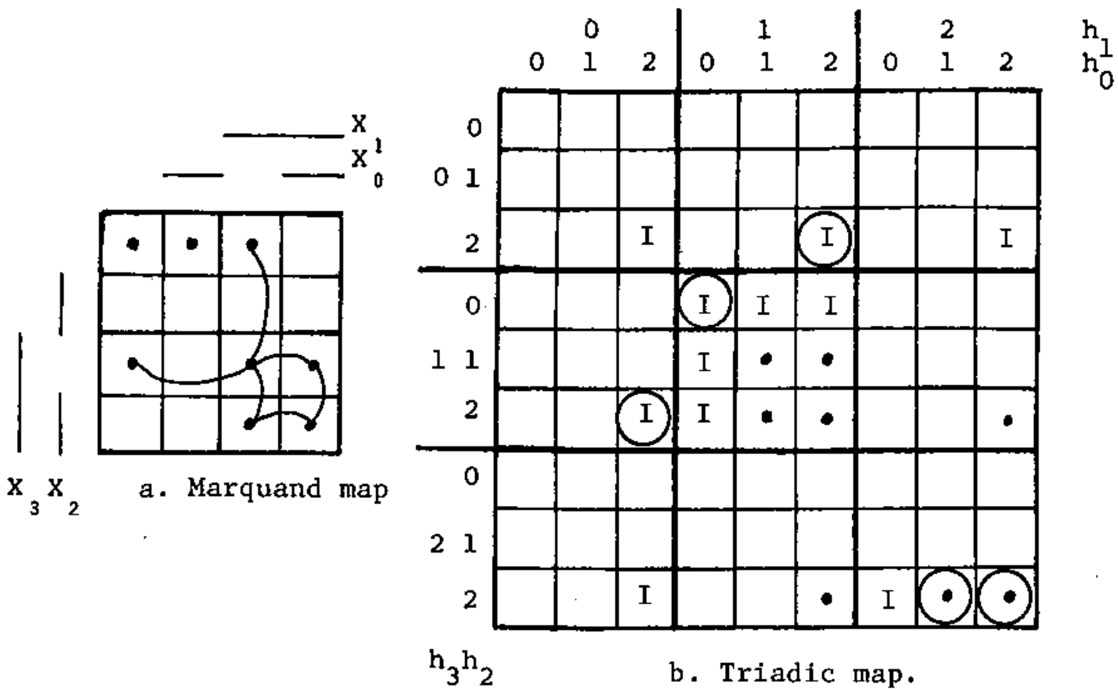
For y:

$$y = \sum_h t_h \text{ where every } h \in \{ h \mid t_h \rightarrow y \} \equiv \{h\}_y$$

for Y:

$$Y = \sum_{h'} t_{h'} \text{ where every } h' \in \{ h \mid t_h \rightarrow Y \} \equiv \{h\}_Y$$

The Marquand and triadic maps of the ΣΠ-form of the previous example function are shown in Figure 9.10.



$$y = t_{23} + t_{30} + t_{47} + t_{79} + t_{80}$$

Figure 9.10. Map of a ΣΠ-form of y.

9.7.3. Maximal ΣΠ-form.

The maximal ΣΠ-form of a Boolean function y is the unique sum of all of the terms t_h for which the function y is true.

A ΣΠ-form for y is a subset of the maximal ΣΠ-form of y.

For y:

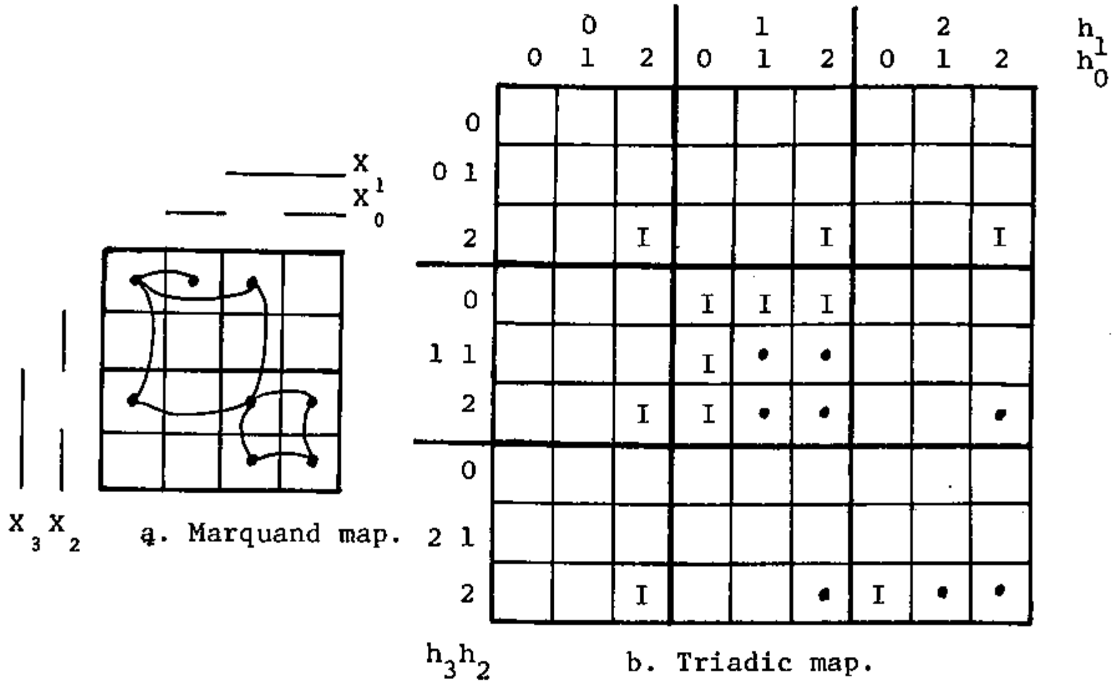
$$y = \sum_h t_h \text{ for all } h \in \{ h \mid t_h \rightarrow y \}$$

$$y = \{ t_h \}_{max}$$

For Y:

$$Y = \sum_{h'} t_{h'} \text{ for all } h' \in \{ h \mid t_h \rightarrow Y \}$$

The Marquand and triadic maps of the maximal ΣΠ-form for the previous example are shown in Figure 9.11.



$$y = t_{20} + t_{23} + t_{26} + t_{30} + t_{31} + t_{32} + t_{39} + t_{40} + t_{41} + t_{47}$$

$$+ t_{48} + t_{49} + t_{50} + t_{53} + t_{74} + t_{77} + t_{78} + t_{79} + t_{80}$$

Figure 9.11. Map of the maximal ΣΠ-form of y.

9.7.4. Complete sum.

The complete sum of a Boolean function y is the sum of all of the prime implicants of y:

$$y = \sum_h t_h \text{ where all } h \in \{ h \mid t_h \rightarrow y \} \equiv \{ h \}_y$$

$$\text{and } \sim(t_p \rightarrow t_h) \text{ for all } p \neq h$$

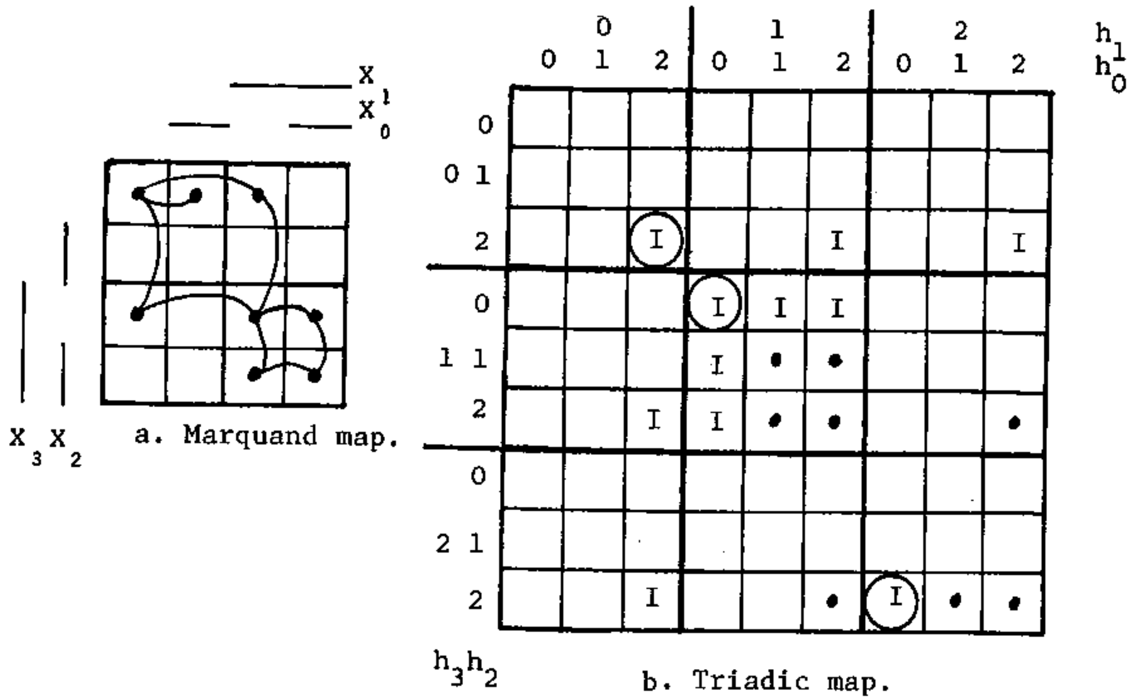
$$\text{where } p \in \{ h \}_y$$

$$y = \{ t_h \}_{h*}$$

The Marquand and triadic maps of the complete sum of the previous example function are shown in figure 9.12.

A prime implicant of a Boolean function y is an implicant of y which is not implied by any other implicant of the maximal ΣΠ-form of y except itself.

There is a corresponding sum for Y.



$y = t_{20} + t_{30} + t_{78}$

Figure 9.12. Map of the complete sum of y.

9.8 THE TRIADIC MAP ALGORITHM FOR FINDING THE COMPLETE SUM OF y

To solve (manually or via a computer program) a function for its complete sum, proceed as follows:

1. Map the function on the triadic map.
2. Use the triadic intersection chart (Figure 9.8) to complete the triads. It is not necessary to complete terms for Y as well as y, since the maps would be dual. The complete map for an example function is shown in Figure 9.13.

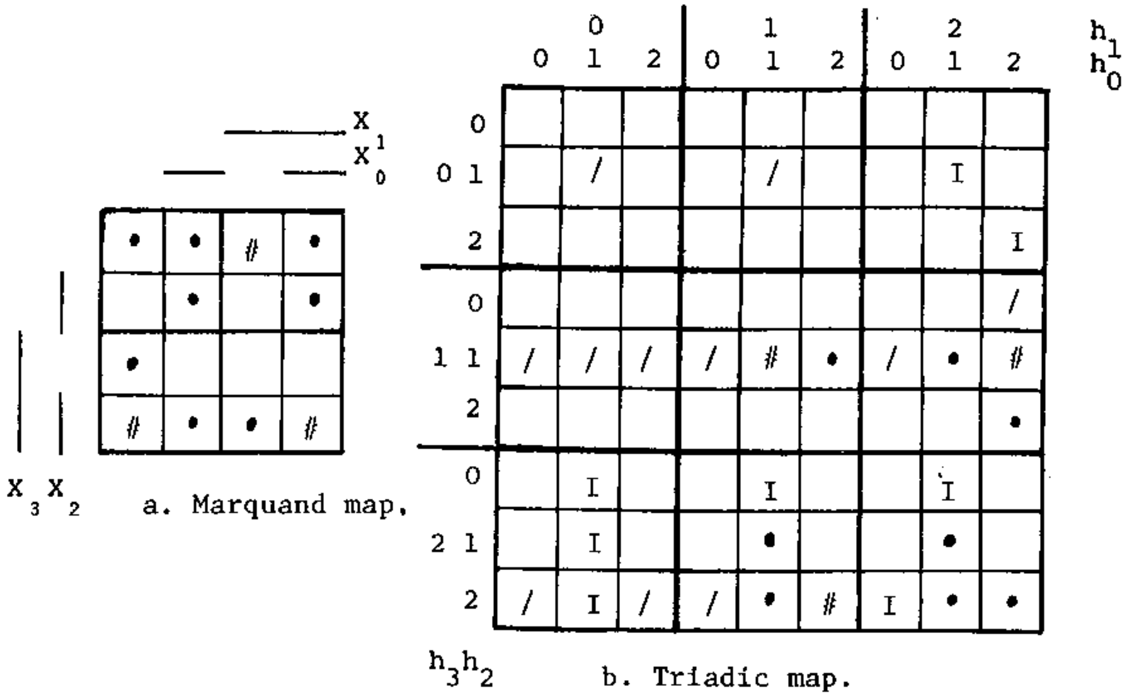


Figure 9.13. Finding the complete sum.

3. Find the lowest-ordered term which does not imply Y but at least <u>may</u> imply y (denoted by / or I). This is a prime implicant. Record it and cancel it on the map. For the example function, the first term found would be $t_{10} = x_2 x_0$ (see Figure 9.14a).

4. Cancel all t_b and t_c terms for which this term is t_a (which form a triad with the chosen term). Also, cancel terms which form a triad with any of these terms. For t_{10}, terms t_{13}, t_{16}, t_{37}, t_{41}, t_{43}, t_{64}, t_{67}, and t_{70} would be cancelled.

5. Repeat, finding the next-lowest-indexed, uncancelled term each time. The resulting set of terms form the complete sum of the function y. (See Figure 9.14b and 9.14c. The complete sum for the example is:

$$y = t_{10} + t_{26} + t_{35} + t_{36} + t_{55} + t_{72}$$

9.9 THE PETRICK FUNCTION SOLUTION FOR THE MINIMAL ΣΠ-FORM OF y

Once the complete sum of y has been found, by whatever algorithm has been chosen, the minimal ΣΠ-form may be found via a reduction of the prime implicant table.

The prime implicant table for the example function of Figures 9.13 and 9.14 is shown in Figure 9.15.

The prime implicant table is a cross-reference of the prime implicants and the minterms ($m_i \rightarrow y$) of the function y.

From the columns of the table, one per minterm, form the Petrick function as follows:

1. Each column contains a ✓ for each prime implicant which covers the minterm of that column. In the example, minterm m_0 is shown to be covered by prime implicants p_1 and p_5.

   Form a sum term for each minterm using the symbol p_i for each of the prime implicants for that minterm. In the example, the sum term for m_0 is $(p_1 + p_5)$; for m_3, it is $(p_4 + p_5)$. The complete expression is shown in Figure 9.15.

2. Solve the completed equation in ΠΣ-form by expanding it to ΣΠ-form and selecting the term with the minimum number of literals. This product term has as its literals the symbols of the prime implicants of the minimal ΣΠ-form of y.

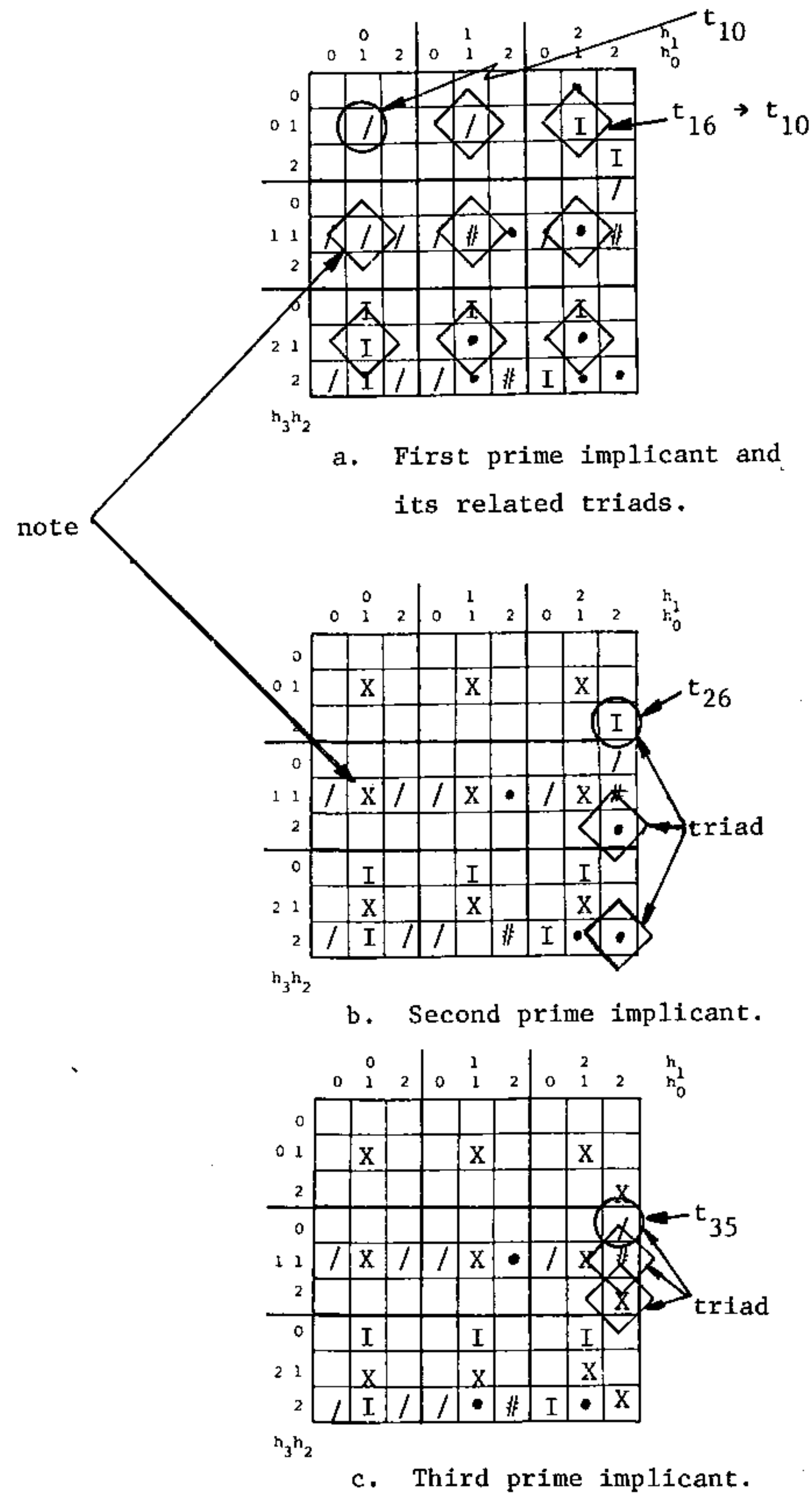


Figure 9.14. Finding prime implicants via the triadic map.

term	literals	symbol	m_0	m_1	m_3	m_5	m_7	m_8	m_{13}	m_{15}
t_{10}	x_2x_0	p_0				✓	✓		✓	
t_{26}	$\bar{x}_2\bar{x}_1\bar{x}_0$	p_1	✓					✓		
t_{35}	$x_3x_1x_0$	p_2						✓		
t_{36}	x_3x_2	p_3							✓	✓
t_{55}	$\bar{x}_3x_0$	p_4		✓	✓	✓	✓			
t_{72}	$\bar{x}_3\bar{x}_2$	p_5	✓	✓	✓					

Notes:

1. Ordering of terms, and therefore of subscripts of p_i, is arbitrary and has no effect on the solution.
2. To "cover" m_0 there is a choice of p_1 or p_5. To cover m_{15} there is no choice, therefore prime implicant x_3x_2 (p_3) is present in every solution.

$$S = \underset{m_0}{(p_1 + p_5)}\underset{m_1}{(p_4 + p_5)}\underset{m_3}{(p_4 + p_5)}\underset{m_5}{(p_0 + p_4)}\underset{m_7}{(p_0 + p_4)}\underset{m_8}{(p_1 + p_2)}\underset{m_{13}}{(p_0 + p_3)}\underset{m_{15}}{(p_3)}$$

Expand S into:

$$S = p_1p_3p_4 + p_1p_2p_3p_4 + \ldots$$

where $p_1p_3p_4$ is (in this case) the minimal literal product term representing the minimal ΣΠ-form of y.

Figure 9.15. Prime implicant table.

For the example, $p_1p_3p_4$ has the least number of literals. Therefore, the minimal ΣΠ-form of y is

$$y = \bar{x}_2\bar{x}_1\bar{x}_0 + x_3x_2 + \bar{x}_3x_0 .$$

The Boolean equation expansion is also available on the parallel Boolean processor.

9.10 LOGICAL INSTRUMENTS -- PRIME IMPLICANT GENERATION

9.10.1 Introduction

The triadic deck set is a pair of 80-column card decks which implement the triadic approach to minimization presented in Section 9.8. The first deck is for the representation of the function by its minterms, and is referred to as the binary or minterm deck. The second deck, known as the triadic or term deck, is for the representation of the function in triadic form.

The use of 80-column cards limits the function to a maximum of four variables where the term $t_0 = 1$ is not represented.

9.10.2 The Minterm Deck

The minterm deck consists of 16 cards labeled 0 through 15, one for each point identifier on a four-variable map (one for each point in a four-variable binary space).

Each card has a punch in every column in either row 0 or row 7. A 7-punch is in any column whose numeric label (1 through 80) is the same as the term identifier for a term implied by the minterm.

$$(\text{7-punch in column a of card i}) \equiv (m_i \rightarrow t_a)$$

A 0-punch is used in every column which represents a term <u>not</u> implied by the minterm.

$$(\text{0-punch in column a of card i}) \equiv \sim(m_i \rightarrow t_a)$$

A 7-punch is referred to a non-punch with reference to row 0.

A triadic map is used to explain the cards throughout this section. Figure 9.16 is a map representing the punch-nonpunch configuration for card number 4 (which represents minterm m_4).

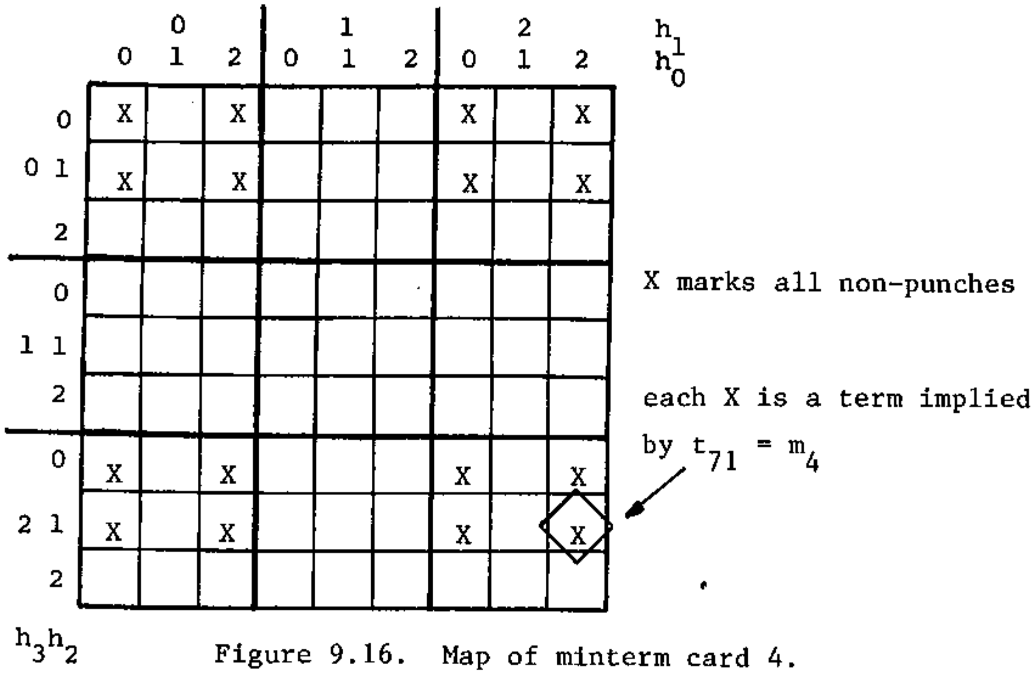


Figure 9.16. Map of minterm card 4.

The Marquand map of an example function is shown in Figure 9.17. To use the deck, proceed as follows:

1. Rather than choose all cards from the minterm deck where $p_i = 1$ or $p_i = \#$, choose all cards where $p_i = 0$. The minterms chosen are the minterms of Y where $Y = \bar{y}$. For the example function, cards 3, 4, 5, 6, 7, 9, 12, and 13 would be chosen.

2. Form a deck with these cards. Hold the deck to a light with the cards face-up and with the 9-edge down.

3. Observe the punches in the 0-row. All columns which have no punches visible are terms which <u>do not</u> belong to the maximal ΣΠ-form of y. All "windows" in the 0-row represent terms which <u>do</u> belong to the maximal ΣΠ-form of y. (The triadic map in Figure 9.18 is labeled with the term identifiers of these terms for the example function.)

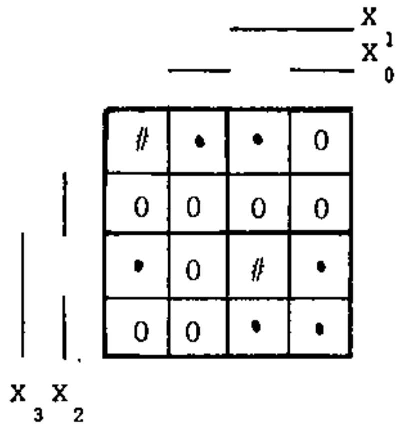


Figure 9.17. Example function.

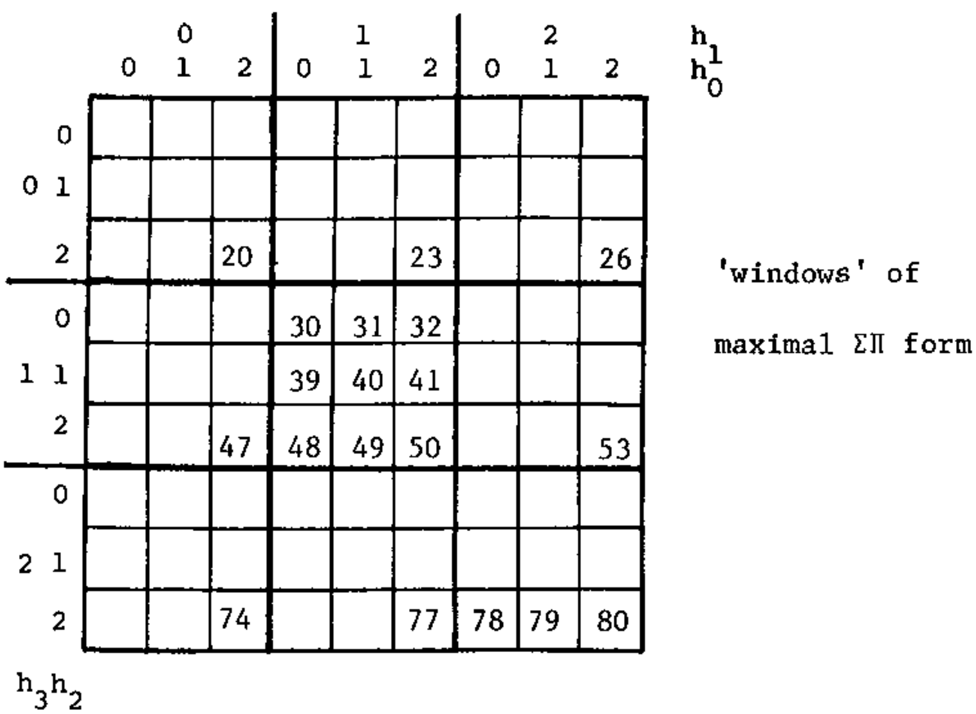


Figure 9.18. The maximal ΣΠ-form of y.

9.10.3 The Term Deck

The term deck contains 80 cards labeled 1 through 80 for each of the term identifiers for terms in a four-variable triadic space. (Term t_0 is excluded as trivial as it represents y = 1.)

Each card contains punches in the column corresponding to the term identifier of the term represented by the card. These are in rows 3 (index punch) and 0 (prime implicant punch).

Each card has been punched in row zero in each column that represents a non-implicant of the term which the card represents.

Each card is non-punched in row 0 in each column that represents an implicant of the term which the card represents. The exception is the prime implicant punch referenced above.

Figure 9.19 is a triadic map representing the punch/non-punch configuration for card number 10 (which represents term t_{10}).

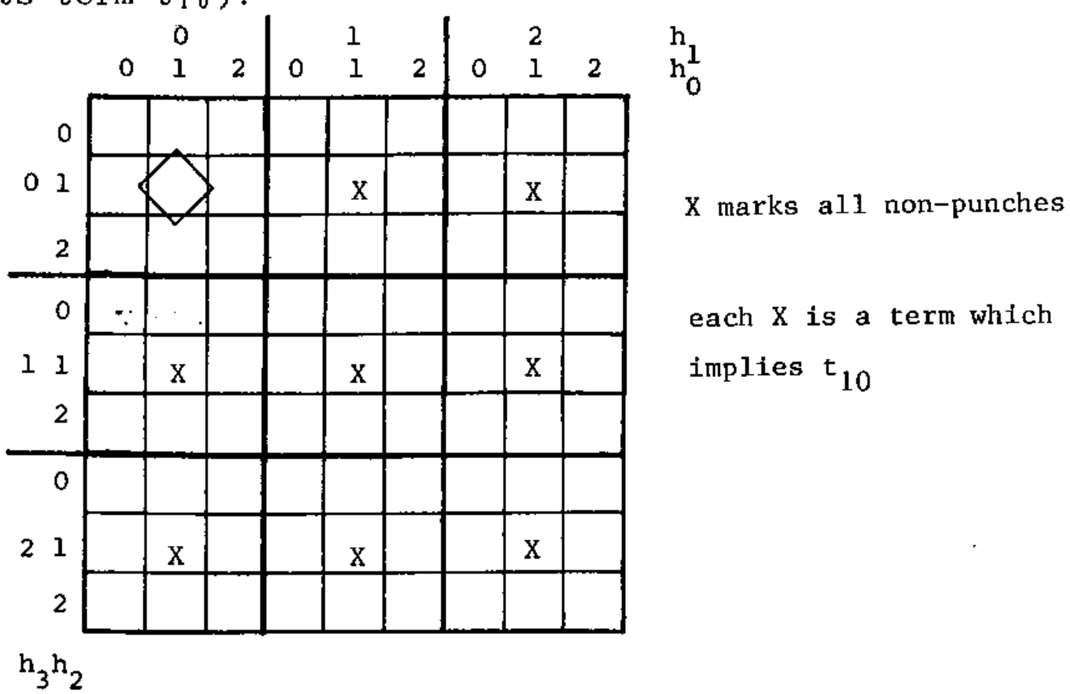
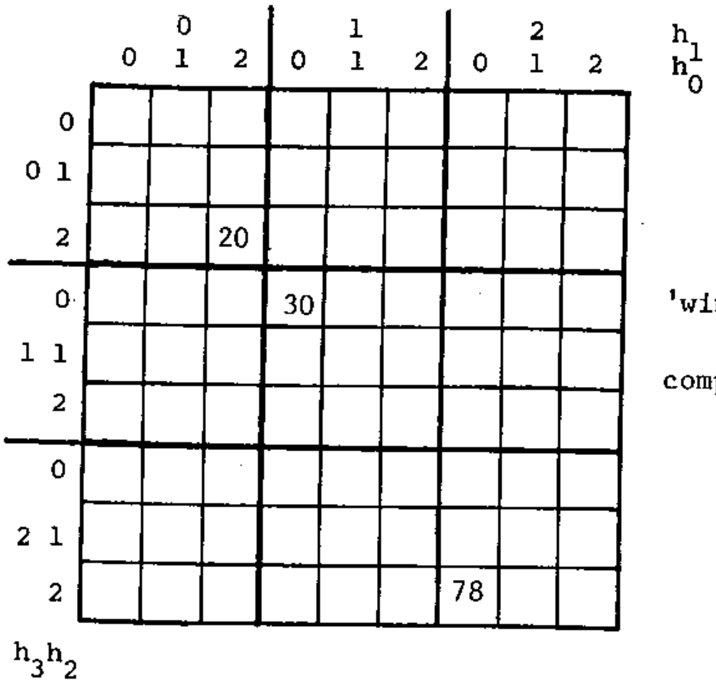


Figure 9.19. Map of triadic card 10.



'windows' of complete sum

Figure 9.20. The complete sum.

To continue the example of Section 9.10.2:

4. Choose one triadic term card for each "window" or term in the set of implicants of y (one card for each term in the maximal ΣΠ-form of y).

5. Add this set of cards to the set formed from the minterm deck and, keeping them face-forward and 9-edge down, hold them to a light so that the holes may be examined.

6. This time, all columns which have visible "windows" represent terms which are prime implicants of y. This is the complete sum of y. (The complete sum is not necessarily the minimal ΣΠ-form.) The triadic map of the complete sum for the example function is shown in Figure 9.20.

7. To find the minimal ΣΠ-form of y, the cards are used to generate the prime implicant table. Take from the unused cards of the minterm deck those cards which represent only the "1"s of the function, i.e., the minterms of the function y.

8. One at a time, place these cards upside down behind the combined decks (i.e., 9-edge up, face down).

9. The remaining visible windows represent the prime implicants which cover the minterm represented by the inverted card. (The inversion uses the punches that are or are not present in row 7.)

10. Record this information in the prime implicant table, remove the inverted card, and proceed to the next until the table is completed.

11. Find the minimal ΣΠ-form using the Petrick function.