DEXTERITY THROUGH ROLLING: TOWARDS MANIPULATION OF UNKNOWN OBJECTS

ANTONIO BICCHI, YACINE CHITOUR,
ALESSIA MARIGO, DOMENICO PRATTICHIZZO

Centro “E. Piaggio”, Università di Pisa, via Diotisalvi, 2 – 56100 Pisa – Italia
Fax:+39–50–565333, Tel.:+39–50–565328
e–mail: {bicchi, chitour, alessia, domenico}@piaggio.ccii.unipi.it

Abstract. The nonholonomy exhibited by kinematic systems consisting of bodies rolling on top of each other can be used to the purpose of building dexterous mechanisms with a minimum hardware complication. Such desirable an engineering feature can be fully exploited, however, only if the capability of planning and controlling rolling motions of arbitrary objects is achieved. In this paper we present some results on the description of the set of reachable positions and orientations of manipulated objects of different shapes, along with some advances in realizing a robot hand system for manipulation of objects whose shape is not known a priori, but is reconstructed as manipulation proceeds.

Key Words. Dextrous robot hands, Nonholonomic systems, differential geometric control.

1 Introduction

Few recent works in mechanism design and robotics reported on the possibility of exploiting nonholonomic mechanical phenomena in order to design devices that achieve complex tasks with a reduced number of actuators (Brockett [1989], Ostrowski *et al.* [1994], Sordalen and Nakamura [1994], Bicchi and Sorrentino [1995], [De Luca *et al.*, 1996]). Although this seems to be a promising new approach to reducing the complexity, cost, weight, and unreliability of the hardware used in such devices, it is true in general that planning and controlling nonholonomic systems is more difficult than holonomic ones. Indeed, notwithstanding the large efforts spent by applied mathematicians, control engineers, and roboticists on the subject, many open problems remain unsolved at the theoretical level, and even more at the computational and implementation level.
In this paper we report on some results that have been obtained in the study of the operation of rolling objects, in view of the realization of a robot gripper that exploits rolling to achieve dexterity, i.e., the ability to arbitrarily change the location and orientation of the manipulated objects. A first prototype of such device was presented in [Bicchi and Sorrentino, 1995], along with some preliminary experiments in planning and controlling motions of a sphere manipulated by rolling. In order for the proposed gripper to be a viable solution in practice, it is necessary that more general objects are dealt with effectively. Furthermore, the case that the object shape is not known a priori should also be faced in real–world applications. This paper discusses a method for building an approximation of the surface of an unknown object from data gathered by exploring the object by rolling. Further, we report on the implementation of a planning method of Sussmann and Chitour [1993] that can be applied to the case of rolling of a general (regular) convex surface.

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2 Background

For the reader’s convenience, we report here some preliminaries that help in fixing the notation and resume the background. For more details, see e.g. [Murray, Li and Sastry, 1994] [Bicchi and Sorrentino, 1995] [Bicchi, Prattichizzo, and Sastry 1995], and [Chitour *et al.*, 1996].

2.1 Regular surfaces

The kinematic equations of motion of the contact points between two bodies with regular surface (i.e., with no edges or cusps) rolling on top of each other describe the evolution of the (local) coordinates of the contact point on the finger surface, $\alpha_f \in \mathbb{R}^2$, and on the object surface, $\alpha_o \in \mathbb{R}^2$, along with the holonomy angle ψ between the x–axes of two gauss frames fixed on the surfaces at the contact points, as they change according to the rigid relative motion of the finger and the object described by the relative velocity $\mathbf{v}$ and angular velocity ω. According to the derivation of Montana [1988], in the presence of friction one has

$$\begin{aligned} \dot{\alpha}_f &= \mathbf{M}_f^{-1}\mathbf{K}_r^{-1}\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix}; \\ \dot{\alpha}_o &= \mathbf{M}_o^{-1}\mathbf{R}_\psi\mathbf{K}_r^{-1}\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix}; \\ \dot{\psi} &= \mathbf{T}_f\mathbf{M}_f\dot{\alpha}_f + \mathbf{T}_o\mathbf{M}_o\dot{\alpha}_o; \end{aligned} \tag{1}$$

where $\mathbf{K}_r = \mathbf{K}_f + \mathbf{R}_\psi\mathbf{K}_o\mathbf{R}_\psi$ is the relative curvature form, and

$$\mathbf{R}_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}.$$

The rolling kinematics (1) can readily be written, upon specialization of the object surfaces, in the standard control form

$$\dot{\xi} = \mathrm{g}_1(\xi)v_1 + \mathrm{g}_2(\xi)v_2, \tag{2}$$

where the state vector $\xi \in \mathbb{R}^5$ represents a local parametrization of the configuration manifold, and the system inputs are taken as the relative angular velocities $v_1 = \omega_x$ and $v_2 = \omega_y$. Applying known results from nonlinear system theory, some interesting properties of rolling pairs have been shown. The first two concern controllability of the system:

Theorem 1 **[Li and Canny, 1990]** *A kinematic system comprised of a sphere rolling on a plane is completely controllable. The same holds for a sphere rolling on another sphere, provided that the radii are different and neither is zero.*

Theorem 2 **[Bicchi, Prattichizzo, and Sastry 1995]** *A kinematic system comprised of any smooth convex surface of revolution rolling on a plane is completely controllable.*

Remark 1. Motivated by the above results, it seems reasonable to conjecture that a kinematic system comprised of *almost any* pair of surfaces is controllable. Such fact is indeed important in order to guarantee the possibility of building a dextrous hand manipulating arbitrary (up to practical constraints) objects. The following propositions concern the possibility of finding coordinate transforms and static state feedback laws which put the plate–ball system in special forms, which are of interest for designing planning and control algorithms:

Proposition 1 *The plate–ball system can not be put in* chained form *[Murray, 1994]; it is not* differentially flat *[Rouchon* et al.*, 1993]; it is not* nilpotent *[Guyot and Petitot, 1995].*

The above results prevent the few powerful planning and control algorithms known in the literature to be applied to kinematic rolling systems (of which the plate–ball system is a prototype). The following positive result is of some use in planning:

Theorem 3 **[Bicchi, Prattichizzo, and Sastry, 1994].** *Assuming that either surface in contact is (locally) a plane, there exist a state diffeomorphism and a regular static state feedback law such that the kinematic equations of contact (1) assume a strictly triangular structure.*

2.2 Polyhedral Objects

The simple experiment of rolling a die onto a plane without slipping, and bringing it back after any sufficiently rich path, shows that its orientation has changed in general, and hints to the fact that manipulation of parts with non–smooth (e.g., polyhedral) surface can be advantageously performed by rolling. However, while for analysing rolling of regular surfaces the powerful tools of differential geometry and nonlinear control theory are readily available, the surface regularity assumption is rarely verified with industrial parts, which often have edges and vertices.

Although some aspects of graspless manipulation of polyhedral objects by rolling have been already considered in the robotics literature, a complete study on the analysis, planning, and control of rolling manipulation for polyhedral parts is far from being available, and indeed it comprehends many aspects, some of which appear to be non–trivial. In particular, the lack of a differentiable structure on the configuration space of a rolling polyhedron deprives us of most techniques used with regular surfaces. Moreover, peculiar phenomena may happen with polyhedra, which have no direct counterpart with regular objects. For instance, in the examples reported in figures 2.2 and 2.2, it is shown that two apparently similar objects can reach configurations belonging to a very fine and to a coarse grid, respectively. In the first case, the mesh of the grid can actually be made arbitarily small by manipulating the object long enough; in such case, the reachable set is called "dense".

The question whether a sequence of rolling operations exists that can bring a given polyhedron arbitrarily close to any arbitrary configuration, can be answered completely in terms of the curvature of the vertices of the polyhedron:

Theorem 4 **[Chitour, Marigo, Prattichizzo, and Bicchi, 1996]** *The set of reachable configurations of a polyhedron is globally dense in its configuration manifold if and only if there exists a vertex on the polyhedron whose curvature is irrational with respect to π.*

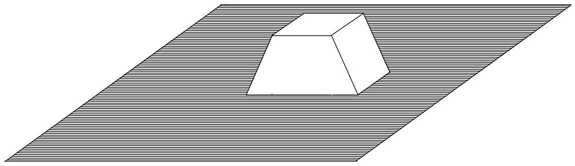

Figure 1: A polyhedron whose reachable set is everywhere dense

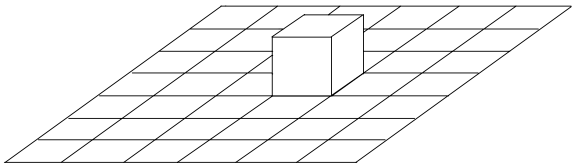

Figure 2: A polyhedron whose reachable set is nowhere dense

For the case of polyhedra with vertices whose curvature is rational w.r.t. π (and for the practically most relevant case of parts whose physical dimensions are measured within a tolerance), it is important to have a description of the lattice structure and mesh size of the reachable set. While results concerning cubic objects are reported by Chitour *et al.*, [1996], general solutions are not available at present in the literature.

3 Exploration of Unknown Objects

As already mentioned, parts to be manipulated are sometimes not known a priori to the robot, and information on their shape need to be gathered before manipulation can be planned and executed. In this section we describe the means by which it is possible to elicit shape information from rolling, with particular reference to the case of regular surfaces.
The dextrous gripper used in our experiments consists of two parallel plates, whose motions are actuated by three electrical motors (see figure 3) The procedure used to reconstruct the surface of unknown objects is as follows:

i) The hand (with fingers open) is put around the object to be explored, and then closed in guarded mode with a contact force threshold;

ii) While the actuator commanding the distance between the fingers regulates a suitable grasping force to avoid slippage of the object on the fingers, the actuators that command translations of one finger in its plane follow pseudo–random trajectories causing the object to roll between the fingers;

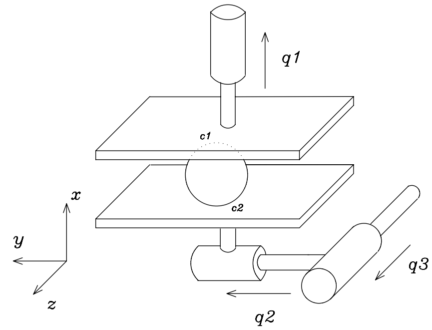

Figure 3: A schematic of the "dextrous gripper" of the University of Pisa

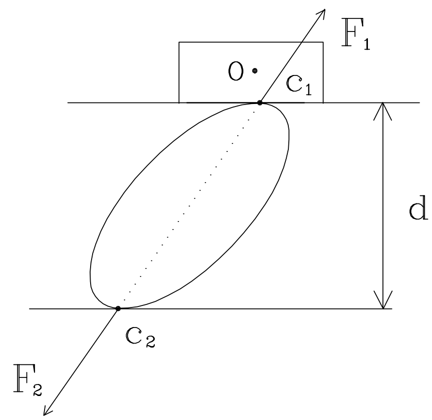

Figure 4: Contact points and contact forces in the manipulation process.

iii) the position of the contact point on the surface of the upper and lower fingers, as well as the position of the hand actuators, are measured during exploration; this information is used to calculate the position of the contact point on the object surface.

In order to control the grasping force and to detect the location of contact points on the fingers, a six–axis force/torque sensor is used on the upper finger. The calculation of the location $\mathbf{c_1}$ of the contact point on the surface of the upper finger from the measured resultant force $\mathbf{F_1}$ and torque $\mathbf{M_1}$ (see figure 4) is obtained by simply intersecting the finger plane with the wrench axis of the contact force. This tactile sensing technique is very fast and accurate with respect to other possibilities, such as employing arrays of pressure sensitive sensors on the fingers. The method can be used also for fingers with a general convex surface, by using the "intrinsic" tactile sensing algorithms described in [Bicchi, Salisbury, and Brock, 1993]). Another advantage of using such force/torque based tactile sensing method is that it allows to detect also the contact point $\mathbf{c_2}$ on the lower finger, provided that any other force acting on the object (includ-

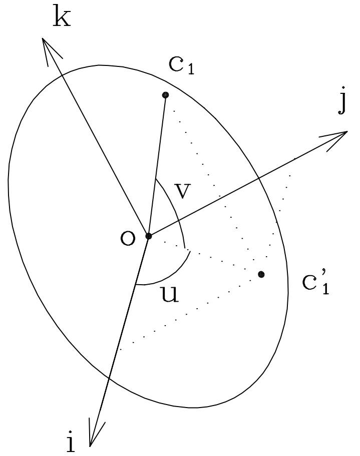

Figure 5: Spherical coordinates on the manipulated object.

ing weight) are negligible or known. In this case in fact, for balance reasons, $\mathbf{c}_2$ is at the intersection of the wrench axis of the first contact force with the lower finger plane (see figure 4).

To reconstruct an approximation of the surface of the object, it is necessary to evaluate the instantaneous position of the contact points with respect to a frame fixed with the object. Let the origin of this frame be denoted by $\mathbf{o}$, and let three unit vectors parallel to the x, y, and z axes of the body frame be denoted by $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, respectively (see figure 5). We choose to describe the object surface in spherical coordinates, i.e., the position of a generic point (except the north and south poles) of the surface in the body–fixed frame is given in terms of azimuth $u \in [-\pi, \pi)$ and elevation $v \in (-\pi/2, \pi/2)$ angles as

$$\begin{cases} x = \rho(u,v)\cos v \cos u \\ y = \rho(u,v)\cos v \sin u \\ z = \rho(u,v)\sin v \end{cases} \tag{3}$$

where $\rho(u, v)$ is a continuous function of the azimuth and elevation u, v. Notice that spherical coordinates are convenient for several reasons, among which is the fact that they provide an orthogonal parametrization of all surfaces of revolution (i.e., surfaces with an axis of symmetry), except at their poles. Notice that for surfaces of revolution, $\frac{\partial \rho(u,v)}{\partial u} = 0$. The velocity of the points of the upper and lower finger in base frame are easily expressed in terms of joint velocities as

$$\mathbf{c}_1 = \begin{pmatrix} \dot{q}_1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{c}_2 = \begin{pmatrix} 0 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}$$

Let $\boldsymbol{\omega}$ denote the angular velocity of the object w.r.t. the base frame. Under the assumption that the object is rigid and that there is no slippage at the contacts, it holds

$$\dot{\mathbf{c}}_1 = \dot{\mathbf{c}}_2 + \boldsymbol{\omega} \times (\mathbf{c}_1 - \mathbf{c}_2).$$

Furthermore, notice from figure 4 that, in the absence of forces on the object other than contact forces, there are no torques acting about the axis joining the contact points. Since manipulation is assumed to proceed slowly, and dynamics are negligible, the angular velocity of the object can be safely assumed to have no component along the axis through contacts, i.e.

$$\boldsymbol{\omega}^T(\mathbf{c}_1 - \mathbf{c}_2) = 0.$$

Hence, the angular velocity of the body is evaluated as

$$\boldsymbol{\omega} = \frac{(\mathbf{c}_1 - \mathbf{c}_2) \times (\dot{\mathbf{c}}_1 - \dot{\mathbf{c}}_2)}{\|\mathbf{c}_1 - \mathbf{c}_2\|}.$$

Letting $\mathbf{R} = [\mathbf{ijk}]$ denote the orientation matrix of the frame fixed to the body, the evolution of $\mathbf{R}$ is described by the following differential equations:

$$\begin{aligned} \dot{\mathbf{o}} &= \dot{\mathbf{c}}_1 + \boldsymbol{\omega} \times (\mathbf{o} - \mathbf{c}_1) \\ \dot{\mathbf{R}} &= \boldsymbol{\omega} \times \mathbf{R} \end{aligned}$$

Integrating these equations during the exploration time, the instantaneous position and orientation of the body can be obtained. From geometric considerations (see figure 5) we obtain the desired information on the spherical coordinates of the contact points from sensor measurements as

$$\begin{aligned} \rho &= \|\mathbf{c}_1 - \mathbf{o}\|; \\ v &= \arcsin \frac{(\mathbf{c}_1 - \mathbf{o})^T \mathbf{k}}{\rho} \\ u &= \operatorname{atan2}\left((\mathbf{c}_1' - \mathbf{o})^T \mathbf{j}, \quad (\mathbf{c}_1' - o)^T \mathbf{i} \right), \end{aligned}$$

where

$$\mathbf{c}_1' = \mathbf{c}_1 - \rho \sin v \mathbf{k}.$$

Similar relations hold for the coordinates of the contact point on the lower finger.

4 Reconstructing surfaces

The problem of reconstructing a surface from knowledge of a number of points laying on it is an important issue common to several fields of science and engineering. In robotics, the problem has been studied extensively in relation with processing data from cameras, range finders, and/or tactile sensors. Part of the literature is concerned with the "object recognition", or model matching problem (see e.g. [Faugeras *et al.*, 1984], [Luo *et al.*, 1984], [Grimson and Lozano–Pérez, 1984], [Ellis, 1992]). Works concerned with shape reconstruction deal with fitting experimental data with general models of surfaces (see e.g. [Brady, Ponce, and Yuille, 1984], [Grimson, 1987]). Various methods are distinguished by the information used and the surface model adopted to fit data. Allen [1986] used bicubic (Coons') patches to fit data from vision and touch sensors, while Allen and Roberts [1989] used superquadrics. Berkemeyer and Fearing [1989] approximated objects by surfaces of revolution, and were able to determine their axis of symmetry by using tactile measurement of contact points, contact normals, and curvatures at the contact points. Caselli, Magnanini, and Zanichelli [1995] considered haptic recognition of objects based on polyhedral shape approximations.

With respect to the existing literature, where surface reconstruction is mostly intended for object recognition, the problem we consider is to gather the surface information necessary to obtain sufficiently accurate formulae for the control vector fields appearing in the rolling equation (2). As these vector fields are computed through differential operations from the surface description, it is necessary not only that the reconstruction is given in terms of analytic functions which are defined on as large a domain as possible, but also are sufficiently smooth to avoid noise amplification through differentiation.

In order to master completely the accuracy/smoothness tradeoff in reconstruction, we found tools made available from regularization theory to be most effective (see e.g. Tikhonov and Arsenin [1977], and Wahba [1990]).

In that framework, the problem of finding the "best" function approximating a multivariate function $y(\mathbf{x})$, whose values y_i at k points $\mathbf{x}_i$ are known (albeit with errors), is formulated as the minimization of the variational expression

$$H(f) = \sum_{i=0}^{k} (y_i - f(\mathbf{x}_i))^2 + \lambda \|Pf\|^2 \qquad (4)$$

where P is a differential operator used to weigh the "bumpiness" of the approximating function, and λ is a regularization parameter, that controls the compromise between the degree of smoothness of the solution, and its closeness to data ([Poggio and Girosi, 1990]). Such standard regularization technique provides solutions that are equivalent to generalized splines: for example, for single variable functions, it can be shown that with the differential operator

$$\|Pf\|^2 = \int_R \left[\ \frac{\partial^2 f(x)}{\partial x^2} \ \right]^2 \partial x$$

the solution of the regularization problem is given by cubic splines. In general, solution of (4) leads to the associated Euler-Lagrange equation

$$\hat{P} P f(x) = \frac{1}{\lambda} \sum_{i=0}^{k} (y_i - f(\mathbf{x})) \delta(\mathbf{x} - \mathbf{x}_i) \qquad (5)$$

where $\hat{P}$ is the adjoint operator of P and δ is the Dirac delta function. The solution of (5) can be written as

$$f(\mathbf{x}) = \frac{1}{\lambda} \sum_{i=0}^{k} (y_i - f(\mathbf{x}_i)) G(\mathbf{x}; \mathbf{x}_i) \qquad (6)$$

where $G(\mathbf{x}; \mathbf{x}_i)$ are the Green functions of the differential operator $\hat{P}P$. Green functions are actually *radial* functions of their arguments $G(\mathbf{x}; \mathbf{y}) = G(\|\mathbf{x} - \mathbf{y}\|)$ when P is rotationally and translationally invariant. In such case, the solution of the regularization problem is a sum of *radial basis functions*:

$$f(\mathbf{x}) = \sum_{i=0}^{k} c_i G(\|\mathbf{x} - \mathbf{x}_i\|), \qquad (7)$$

where the weights c_i can be evaluated by simple linear algebraic operations. Some commonly encountered radial basis functions used in regularization theory and in the closely allied field of neural networks are (see [Poggio and Girosi, 1990])

$$\mathbf{G}(r) = \begin{cases} r & \text{(linear interploation)} \\ r^3 & \text{(cubic interpolation)} \\ \sqrt{r^2 + c^2} & \text{(multiquadric)} \\ \frac{1}{\sqrt{r^2 + c^2}} & \text{(invers multiquadric)} \\ e^{-\frac{r^2}{\sigma^2}} & \text{(gaussian)} \end{cases}$$

where $r_i = \|\mathbf{x} - \mathbf{x}_i\|$.

The problem of reconstructing a surface described in spherical coordinates (3) amounts to approximating a smooth function $\rho : S^2 \to \mathbb{R}$, $\rho = \rho(u, v)$ of the azimuth and elevation angles u, v, for which a set of points $\rho(u_i, v_i) = \rho_i$ are given from exploration data. With respect to the theory above resumed, the fact that the domain manifold S^2 is not globally equivalent to $\mathbb{R}^2$ imposes some modifications in the choice of basis functions. Following Wahba [1990], we choose

$$\rho = \sum_{l=0}^{n} \sum_{s=-l}^{l} f_{ls} Y_{ls} \qquad (8)$$

where f_{ls} are coefficients, and Y_{ls} are the eigenfunctions of the (surface) Laplacian on the sphere, i.e. the *spherical harmonics*, whose expression in coordinates is

$$\begin{aligned} Y_{ls}(u, v) &= U_{ls} \cos(us) P_l^s(\sin v) & 0 < s \le l \\ &= U_{ls} \sin(us) P_l^{|s|}(\sin v) & -l \le s < 0 \\ &= U_{l0} P_l(\sin v) & s = 0 \end{aligned}$$

for $l = 0, 1, \ldots$. Here,

$$\begin{aligned} U_{ls} &= \sqrt{2} \sqrt{\tfrac{2l+1}{4\pi} \tfrac{(l-|s|)!}{(l+|s|)!}} & s \neq 0 \\ &= \sqrt{\tfrac{2l+1}{4\pi}} & s = 0 \end{aligned}$$

$P_l, l = 0, 1, \ldots$, are the Legendre polynomials, and P_l^s are the Legendre functions

$$P_l^s(z) = (1 - z^2)^{\frac{s}{2}} \frac{\partial^s}{\partial z^s} P_l(z)$$

The unknown coefficients are obtained by minimizing

$$\frac{1}{n} \sum_{i=1}^{k} \left(\ \rho_i - \sum_{l=0}^{n} \sum_{s=-l}^{l} f_{ls} Y_{ls}(u_i, v_i) \ \right)^2 \qquad (9)$$

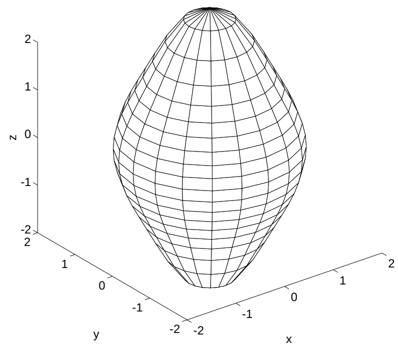

Figure 6: Exact description of the manipulated object.

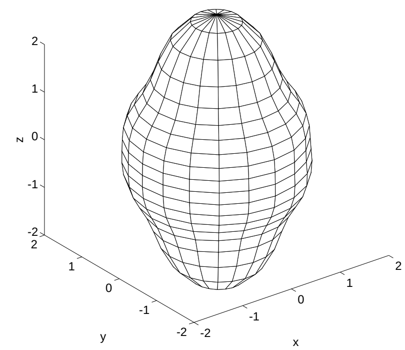

Figure 7: Approximation with $\Lambda = 0,002$ and $N = 7$.

$$+\lambda \sum_{l=0}^{n} \sum_{s=-l}^{l} [l(l+1)]^m f_{ls}^2$$

Arranging the index set $\{(l,s)\}$ in a convenient order, and letting $\mathbf{f}$ be the vector of f_{ls} and $\mathbf{X}$ be the matrix with $(i, ls)_{th}$ entry $Y_{ls}(u_i, v_i)$, (10) becomes

$$\frac{1}{n}\|\mathbf{y} - \mathbf{X}\mathbf{f}\|^2 + \lambda \mathbf{f}^T \mathbf{D} \mathbf{f}$$

where $\mathbf{D}$ is the diagonal matrix with $(ls, ls)_{th}$ entry $[l(l+1)]^m$. The minimizing vector $\mathbf{f}_\lambda$ is then given by

$$\mathbf{f}_\lambda = (\mathbf{X}^T\mathbf{X} + \lambda \mathbf{D})^{-1}\mathbf{X}^T\mathbf{y}$$

4.1 Experimental

The experimental results obtained by exploring an unknown object and reconstructing it by the above described techniques, are reported in figures 6, 7, 8, and 9. Figures show apparently how using few spherical harmonics (low N) and/or low regularization weights λ provides "bumpy" reconstructions, while heavy regularization tends to round up the object shape excessively.

5 Planning for general surfaces

Although methods for planning rolling motions of objects that possess a known axis of revolution are available ([Bicchi, Prattichizzo, and Sastry 1995]), for the general case of a regular surface without such symmetry, no specialized technique is available. A very general method for planning motions of nonholonomic systems was presented by Sussmann and Chitour [1993], which is suitable for application to our case.

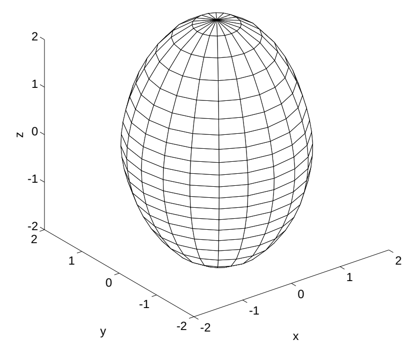

Figure 8: Approximation with $\Lambda = 0,05$ and $N = 9$

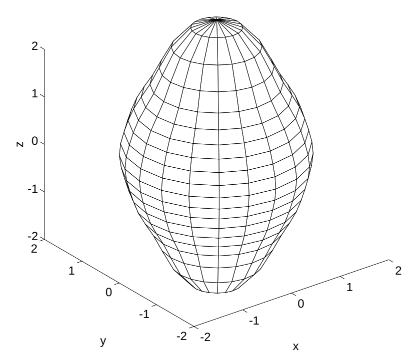

Figure 9: Approximation with $\Lambda = 0,002$ and $N = 9$

Consider a general driftless nonholonomic system in the form

$$\dot{\mathbf{x}} = \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{x}) v_i, \tag{10}$$

with the state evolving on some manifold $\mathbf{x} \in M$, and controls defined as functions of time in a finite interval, e.g. $v_i : [0,1] \rightarrow \mathbb{R}$, and belonging to a sufficiently rich class of functions V (say e.g. piecewise constant, or integrable functions over $[0,1]$). Under some well known conditions on the vectorfields $\mathbf{g}_i$, to any initial configuration $\mathbf{x}(0) = \mathbf{x}_o \in M$ and any control $\mathbf{v}(t) \in V^m$, the differential equation (10) associates an unique end point $\hat{\mathbf{x}} = \mathbf{x}(1)$, thus defining a so–called "endpoint map" $\mathcal{E}_p : M \times V^m \rightarrow \mathbf{M}$.
The basic idea of continuation methods consists in guessing first an input $\mathbf{v}_o(t)$, and compute $\hat{\mathbf{x}}_o = \mathcal{E}_p(\mathbf{x}_o, \mathbf{v}_o)$. Such generally incorrect guess is joined to the desired final configuration $\mathbf{x}_d$ through any path γ with curvilinear abscissa s, such that $\gamma(s = 0) = \hat{\mathbf{x}}_o$, and $\gamma(s = 1) = \mathbf{x}_d$. The path γ is then "lifted" to a path $\Gamma(s)$ in the space of control functions by solving the "path lifting equation" (PLE), given by

$$\dot{\Gamma}(s) = d\hat{\mathcal{E}}_p (d\mathcal{E}_p d\hat{\mathcal{E}}_p)^{-1} \dot{\gamma}(s)$$

where $d\mathcal{E}_p$ is the differential of the endpoint map, and $d\hat{\mathcal{E}}_p$ its adjoint operator. In order for the PLE equation to make sense, it is assumed that the endpoint map differential is onto. This condition is always verified except at most within a subset of the state manifold of zero measure (the subset is composed of so–called *abnormal extremals* for the system). Solving the PLE, along trajectories that avoid abnormal extremals, allows one to continuously "deform" the initial guess into a new control, satisfying the specified final position.
In our practical implementation of the continuation method for the case of rolling surfaces, the space of control functions was restricted to the linear combinations of a finite number of elements of a Fourier basis, so that a finite–dimensional PLE was obtained, suitable for numerical solution. Similar techniques have been reported by other authors (see e.g. Divelbiss and Wen [1993], and Fernandes, Gurvitz, and Li, [1994]).
Abnormal extremals can occur in rolling problems. Although a study of the abnormal extremals for the case of a sphere rolling on a plane was performed, not much could be said for a general surface. However, a simple modification of the algorithm consisting in applying the pseudo–inverse of the endpoint map differential to a randomly perturbed tangent vector to the path to be lifted, effectively solved any abnormal extremal–related problem in practice. An intuition why such simple modification works can be gained by considering that, since the system is controllable, there cannot exist any closed submanifold in M encircling $\mathbf{x}_d$ and whose tangent space everywhere contains the range of the endpoint map differential.
In our experimental implementation, the continuation method proved considerably slower than other methods available for particular surfaces. However, besides being, as already mentioned, one of the very few available approaches to planning general systems, the continuation method can be readily adapted to take into account further constraints, such as those generated by the limits of the actuator workspace in our prototype hand.

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