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Manipulation of Polyhedral Parts by Rolling*

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Abstract

The nonholonomy exhibited by kinematic systems consisting of bodies rolling on top of each other can be used to the purpose of building dexterous mechanism with a minimum hardware complication. Previous work concentrated on manipulation of objects possessing a regular surface. On the other hand, industrial parts are most often irregular, possessing vertices and edges. In this paper we present some results on the description of the set of positions and orientations that polyhedral objects can reach when manipulated by rolling without slipping. An algorithm for planning the manipulation of a polyhedral part from a given configuration to another reachable one, is also presented.

1 Introduction

Manipulation of industrial parts has been one of the core problems of robotics since its beginnings, and it still attracts large attention. Solutions have been proposed that vary in philosophy according to the different application domain. Thus multifingered hands apply where flexibility is at a premium, while more factory–oriented solutions privilege simplicity of the manipulator and use regrasping ([15], [8]) and/or pushing and tilting actions ([13], [7]) in conjunction with such simple end–effectors as parallel–jaw grippers. In this paper, we focus on tasks requiring much flexibility, but where the hardware complexity of the end–effector is to be minimized, in the interest of weight, unreliability, and cost reduction.

The nonholonomic behaviour of some systems have been exploited for achieving dexterous manipulation by means of simple mechanical design. Montana[11], and Li and Canny[9] used tools from differential geometry and nonlinear control to model manipulation by rolling and discuss its geometry. Bicchi and Sorrentino [5] designed and implemented a dexterous hand exploiting rolling, which used only three motors. Such hand is able to arbitrarily change the position and orientation of the manipulated object, provided that its surface complies with some assumptions (see [6]), including regularity and convexity. In order to approach genuine industrial problems, in this paper we consider a similar style of manipulation as applied to polyhedral parts.

The rolling of a polyhedron on a plane is itself a nonholonomic phenomenon, although a wider defini-

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Figure 1: Slightly different polyhedra may reach dramatically different sets of configurations by rolling

tion of nonholonmy is to be accepted than the one most engineers may be familiar with. Consider the example of rolling a die on a table about its edges, tumbling it first rightwards, then upwards, hence leftwards, and finally downwards. While the final position of the center of the die is unchanged, its orientation is, and a different face of the die is now on the table. The correspondence to a cyclic path of some ***base*** variables (in this case, the position in the plane) to a net change in other ***fiber*** variables (the orientation angles), is syntomatic of nonholonomy. This suggests that manipulation of parts with non-smooth surface can be advantageously performed by rolling.

Previous work on graspless manipulation of polyhedral parts by rolling in the robotics literature include that of Inoue and Aiyama [4], Sawasaki, Inoue and Inaba [14]), and Erdmann and Mason [7]. As of today, however, study of this subject is far from complete, and some peculiar phenomena may occur in its consideration, that call for an oculate analysis. One such phenomenon consists in the dramatically different structure of the set of configurations that slightly different polyhedra can be brought to reach by rolling about their edges. For instance, it is intuitive that a cube will only reach positions that lie on a square mesh, with orientations that are multiples of $\pi/2$. However, by inducing the results of simple computer simulations, it can be seen that rolling for long enough a truncated pyramid, however slightly different it may be from the cube, any arbitrary configuration can be reached as close as one wishes (see fig. 1). It is also possible to have a polyhedron whose reachable set of positions is dense, while it is discrete for orientations (e.g., an equilateral pyramid). In this paper, we explain these phenomena and characterize the set of positions and orientations reachable by a general polyhedron. The constructive method used in order to study the reachability problem also allow us to propose an algorithm for planning manipulation of polyhedral parts by rolling.

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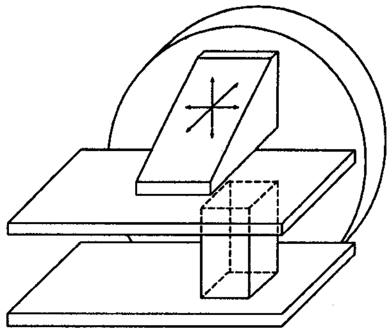

Figure 2: A parallel–jaw gripper can manipulate polyhedral parts

2 Problem Formulation

Consider a simple manipulator as the one in fig. 2, consisting of two plates one of which is fixed, while the other can translate remaining parallel to the first. A part is put between the plates, which are covered by compliant high–friction pads. By coordinated motion of the jaws, the object can be made to roll from a face to another adjacent one through the connecting edge. The goal of manipulation is to bring the part from a given initial configuration (a point in $SE(3)$) to another desired one. Without loss of generality, we only consider here different configurations modulo a rigid vertical translation of the whole mechanism (i.e., we restrict to $\mathbb{R}^2 \times SO(3)$).

Manipulated parts are considered that have a piecewise flat, closed surface, comprised of a finite number of faces, edges, and vertices. Observe that actual parts need not be convex, in general. However, the finger plates being assumed to be large w.r.t. the diameter of parts, we only need to be concerned with the convex hull of parts themselves.

Several kinds of motions for a polyhedron on a plane are possible, as e.g. by sliding on a face, pivoting about a vertex or tumbling about an edge. In this paper, however, we rule out the former two possibilities, and only consider sequences of rotations about one of the edges in contact, by the amount that exactly brings another face to ground. This action on the parts, which will be referred to as an elementary tumble (ET for short), appears to be more reliable than slipping or pivoting, as it will be discussed later on.

Let $\mathcal{P}$ be a convex polyhedron rolling on a plane P, and let $\mathcal{V} = \{V_1, \ldots, V_m\}$ be the set of vertices, $\mathcal{E} = \{E_1, \ldots, E_k\}$ the set of edges, and $\mathcal{F} = \{F_1, \ldots, F_l\}$ the set of faces of $\mathcal{P}$. The configuration space $\widetilde{M}$ of the system is given by the set of points of type (p, θ, i) where i is the index of the face in contact with the plane P, p is the projection onto P of some point c fixed on the part (e.g., its center of gravity), and θ is the angle between two reference systems fixed respectively on face F_i and on P. Briefly, $\widetilde{M}$ is the union of l copies of $SE(2)$, i.e.

$$\widetilde{M} = \mathbb{R}^2 \times S^1 \times \widetilde{F}. \tag{1}$$

In this terms, the problem of deciding whether the polyhedron $\mathcal{P}$ can be dextrously manipulated is solved by studying the subset of reachable configurations $\mathcal{R}_1 \subset \widetilde{M}$, given by all configurations (p_f, θ_f, i_f) such that there exists some sequence of ET's bringing $\mathcal{P}$ from a given initial configuration $(p_0, \theta_0, 1)$ to (p_f, θ_f, i_f). Such sequence of ET's will be referred to as a "walk", and will be described by the sequence of faces brought successively in contact with P, $\{F_{S_n}\}$, where $\{S_n\}_{n\in I}$, $I \subset N$, $S_n \in \{1, ..., l\}$ is a sequence of face indices. Thus $\{F_{S_n}\}$ represents a walk if F_{S_k} and $F_{S_{k+1}}$, $\forall k \in I$, are adjacent faces. Let then S be the set of all the sequences $\{S_n\}$ such that $\{F_{S_n}\}$ represent a "walk" of $\mathcal{P}$ on P. For a walk $\{F_{S_n}\}$ steering configuration $(p_0, \theta_0, i_0) \in \widetilde{M}$ in configuration $(p_f, \theta_f, i_f) \in \widetilde{M}$ we will use the notation

$$\{F_{S_n}\} : (p_0, \theta_0, i_0) \mapsto (p_f, \theta_f, i_f) \tag{2}$$

or briefly, when we are not interested in position and orientation, as $\{F_{S_n}\} : F_{i_0} \mapsto F_{i_f}$. While in analysing the rolling motions of regular surfaces the central role was played by the surface metric, curvature, and torsion forms ([11]), to study the structure of the reachable set of a polyhedron it is instrumental to refer to the geometrical quantities introduced below:

Definition 1 *Let D_{lk} denote the lenght of the edge incident to vertices V_l and V_k. Also, denote α_{ij} the angles of face F_j at vertex V_i. The defect angle β_i at vertex V_i is defined as the complement to 2π of the sum of angles α_{ij} for all j such that face F_j is adjacent to V_i.*

Being convex polyhedral parts topological spheres, their total curvature is 4π. Because faces and edges of a polyhedron have zero Gauss curvature, all curvature is concentrated at vertices. In fact, the defect angle represents how much of the curvature of the object is concentrated at V_i, and clearly we have $\sum_{i=1}^{m} \beta_i = 4\pi$. The fact that all the curvature of a polyhedral part (and hence, sensitivity to rolling) is concentrated at its vertices, along with the fact that such vertices are never perfectly sharp in real–world parts, suggests that pivoting about vertices may be much less robust a means of manipulation for polyhedral parts, than that of tumbling about edges.

The main theoretical results reported in this paper concern the structure of the set of configurations reachable by rolling a given polyhedron. We will say that such set is ***dense*** w.r.t. positions if for any desired position p_f in the plane, and any given tolerance δ_p, there exists a walk such that the polyhedron reaches a position closer to p_f than δ_p. Analogously, the reachable set is dense w.r.t. orientations if, for any θ_f and δ_θ, there exists a walk leading to an orientation closer to θ_f than δ_θ. The reachable set will be called dense in $\widetilde{M}$, or dense ***tout-court***, if the polyhedron can be brought arbitrarily close to any desired position with an orientation arbitrarily close to any desired orientation. The term ***discrete*** will be used for the negation of ***dense***. Our results are as follows:

Theorem 1 *The set of configurations reachable by a polyhedron is dense in $\widetilde{M}$ if and only if there exists a vertex V_i whose defect angle is irrational with π, i.e., iff $\exists \beta_i : \frac{\beta_i}{\pi} \notin Q$.*

Theorem 2 *The reachable set is discrete in both positions and orientations if and only if either of these conditions hold:*

i) *all angles of all faces (hence all defect angles) are $\pi/2$, and all lenghts of the edges are rational w.r.t. each other (i.e., $D_{ij}/D_{kl} \in Q, \forall i,j,k,l \in \{1,...,m\}$;*

ii) *all angles of all faces (hence all defect angles) are integer multiples of $\pi/3$, and all lenghts of the edges are rational w.r.t. each other (in other words, $\alpha_{ij} \in \{\frac{\pi}{3}, \frac{2\pi}{3}\}$, $\forall i \in \{1,...,m\}$ and $\forall j \in \{1,...,l\}$, and $D_{ij}/D_{kl} \in Q, \forall i,j,k,l \in \{1,...,m\}$;*

Theorem 3 *The reachable set is dense in positions and discrete in orientations if and only if the defect angles are all rational w.r.t. π, and neither conditions i) or ii) of theorem 2 apply.*

Remark 1. Polyhedra satisfying condition i) of theorem 2 are rectangular parallelepipeds, as e.g. a cube or a sum of cubes which is convex. Polyhedra as in condition ii) are those whose surface can be covered by a tessellation of equilateral triangles, as e.g. any Platonic solid except the dodecahedron.

Before giving a sketch of the proof of these results in the next section, it is perhaps the case to underline that they concern theoretical idealizations of real–world polyhedral parts. No part can be measured with accuracy fine enough to say whether the characteristic ratios above are rational or irrational. However, from the very machinery developed for the proof, an intuition of what goes on in the real case and an useful planning algorithm will result.

3 Mathematical development

We first reduce our problem of studying the set $\mathcal{R}_1$ reachable from a configuration with face F_1 in contact with P to the study of one of its subsets. In fact, the density of the subset of reachable configurations with face F_1 in contact with P is the same for every face: the problem does not depend on the initial configuration. Thus if through some sequence $\{F_{S_n}\} = t_i$, that will be referred to as "transit walk", we bring face F_i in contact with P then F_i can be brought in any position and orientation if and only if this is true for F_1. Therefore, what is actually to be studied is the subset of reachable configurations with face F_1 in contact with P. By doing this, it is clear that such subset is the orbit of the initial configuration under the action of the subset of all walks bringing face F_1 back in contact with $P : L_1 = \{F_{S_n} : F_1 \mapsto F_1\}$. Such set of walks L_1, together with the operation of composition of walks by concatenation, is clearly a group, and possesses a finite set of generators which can be described, using some tools of the theory of graphs and from topology, as follows.

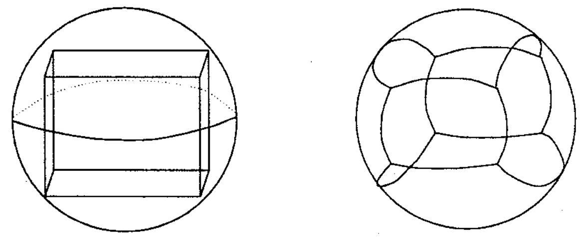

Figure 3: A convex polyhedron projected onto a sphere

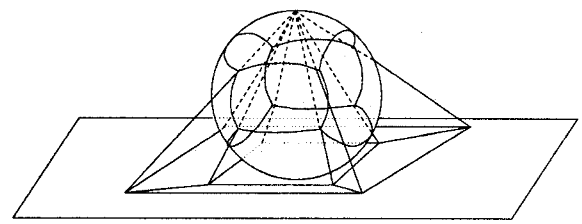

Figure 4: The stereographic projection

3.1 A set of canonical movements for the motion of the polyhedron

By the assumption of convexity, parts are topological spheres, i.e. they can be continously deformed onto spheres. As an example of such process, consider "blowing up" a polyhedron $\mathcal{P}$ onto a sphere $\mathcal{S}$ large enough to encompass all of it (see fig. 3) by projecting the surface of $\mathcal{P}$ on the sphere from a point inside the polyhedron. Consider the tiling X induced on $\mathcal{S}$ by the image of the edges and the vertices of $\mathcal{P}$, i.e. the covering of its surface by a number of connected components (cells) which are the image on the sphere of the faces of the polyhedron. The Schlegel map ([1]) of $\mathcal{P}$ associates to the polyhedron a graph on the plane built by the stereographic projection (fig.4)

$$\pi_N : \mathcal{S} \setminus \{N\} \longrightarrow \Pi$$

from a point $N \in \mathcal{S}$ (the "north" pole) onto a plane Π, the latter being tangent to $\mathcal{S}$ through a point S (the "south" pole) provided that both poles do not belong to the projection of any edge or vertex of the polyhedron on the sphere.

The stereographic projection $\pi_N|_X$ produces a tiling X' on the plane P with one infinite component (corresponding to the cell of $\mathcal{S}$ containing N) and $l-1$ bounded connected components. The graph naturally associated to such tiling X' is the so–called Schlegel map of the polyhedron. The Schlegel map has the same number k of edges and m of vertices (nodes) as the original polyhedron. Furthermore, since we consider convex polyhedra for which the Euler relation $l+m-k=2$ holds, the map is line–crossing free ([1]).

The dual of the Schlegel map can now be built by taking the following steps (see fig.5):

1. take new vertices as interior points of the cells of

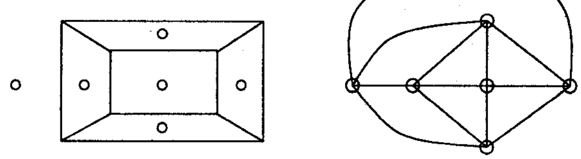

Figure 5: The Schlegel map and its dual

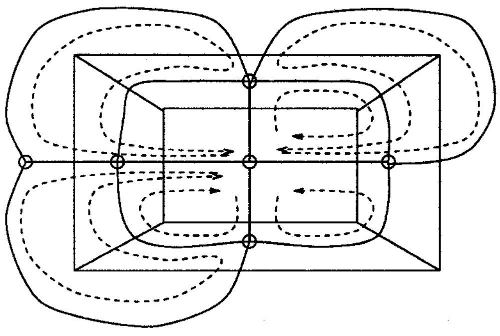

Figure 6: The generators of the fundamental group are the segment paths on the dual graph which appears overlapping the Schlegel map.

X', one per cell;

2. draw new edges joining pairs of new vertices, each new edge crossing one and only one of the edges of the original map. Observe that k such new edges are drawn, and that two new vertices are connected by a new edge if and only if the two corresponding cells of the Schlegel map are adjacent, i.e. iff the corresponding faces of the polyhedron share an edge.

In the resulting dual graph, described by $X'' = (\mathcal{V}'', \mathcal{E}'')$ with $\mathcal{V}''$ the set of l new vertices and $\mathcal{E}''$ the set of k new arcs, vertices correspond one–to–one to faces of the polyhedron, and two vertices on the graph are adjacent if and only if the corresponding faces of the polyhedron are adjacent. Thus we can uniquely associate to any sequence of elementary tumbles one path on the dual of the Schlegel map, and to L_1 (the group of walks eventually bringing face F_1 in contact with P), the fundamental group on the dual graph based at point F_1. Using a well–known theorem from the theory of groups stating that the foundamental group of a graph is a free group generated by a set of $e-v+1$ generators, where e is the number of edges and v is the number of vertices of the graph, along with the Euler relation, we have that L_1 can be generated by a set of $m-1$ generators.

In order to describe such generators, and hence the group L_1 and the reachable set, we proceed as follows. Consider that in the dual of the Schlegel map there are $m-1$ bounded cells. For the i–th of such bounded cells, consider a node F_j belonging to the cell and a transit path t_j (possibly void) joining the base node F_1 to F_j (see fig.6). Concatenate with t_j a loop ℓ_i that touches all and only the nodes on the i–th cell boundary. Finally return to the base point F_1 by following t_j in the inverse sense. By repeating the procedure for each bounded cell, we obtain $m-1$ independent paths on the graph of type $R_i = t_j \circ \ell_i \circ t_j^{-1}$, which form a set of generators of all possible paths on the graph. Recalling that nodes and cells of the dual Schlegel map correspond to faces and vertices of the polyhedron, respectively, we can view each generator as an operator acting on $\widetilde{M}$:

$$R_i : (p_0, \theta_0, 1) \mapsto (p_f, \theta_f, 1),$$

Such map can be explicitly calculated based on the geometry of the polyhedron. Consider a plane development of the polyhedron based at the position of face 1 (i.e., glue face 1 on the plane, then "spread" the polyhedron cutting along its edges when necessary so as to bring all faces on the plane while leaving the surface connected, see fig.7). Let $\hat{F}_j$ be the image on the plane of face F_j, and let $\hat{V}_i$ be the image of vertex V_i on the boundary of $\hat{F}_j$. Recalling the geometrical meaning of the defect angle β_i, therefore, the action of the generators is described as

$$R_i(p_0, \theta_0, 1) = (p_0 + (\hat{V}_i - p_0)e^{\imath\beta_i}, \beta_i + \theta_0, 1).$$

3.2 Proof of theorems (sketch)

Observe first that the action on S^1 of the generators R_i is transitive. Thus the structure of the projection of the reachable set on S^1, i.e., the orientation part of the three theorems, is proved at once: the set of reachable orientations is in fact given by all θ such that the Diophantine equation

$$\sum_i \alpha_i \beta_i = \theta + 2k\pi, \tag{3}$$

has a solution with $\alpha_i, i = 1, \ldots, m-1$, and k integers. If all β_i are rational w.r.t. π, the set of such solutions is discrete (actually, finite modulo 2π), and easily characterized as the integer multiples of the greatest common divisor of the β_i's, denoted by $\tilde{\beta}$.

Concerning the structure of the projection of the reachable set in $\mathbb{R}^2$, i.e. displacements of the polyhedra, for the case of existence of a defect angle irrational w.r.t. π, we recall the proof of theorem 1 given in [3].

If otherwise all β_i's are rational w.r.t. π, it is possible to focus on the subgroup T of the translations of L_1, i.e., the set of all walks of type $(p_0, \theta_0, 1) \mapsto (p_f, \theta_0, 1)$. In fact, the density of T implies and is implied by the density of the whole set of reachable positions in the plane. Again, being T a transitive subgroup of the group generated by the $R_i's$, the description of a complete set of generators is sufficient to fully characterize its action. To describe such a set of generators, a "virtual rotation" R_V is introduced which is comprised of a composition of rotations R_i such that the total rotation is $\tilde{\beta}$, the G.C.D. of the defect angles. Any solution set α_i, $i = 1, \ldots, m-1$ of (3) with $\theta = \tilde{\beta}$ is taken as such a virtual rotation. Denote by $\tilde{C}$ the point in the plane about which R_V occurs (this is computed easily given the α_i's and the polyhedron geometric parameters). The generators of T can be written as translations in the direction of the vectors

$$\mathbf{g}_{hj} = (\hat{V}_j - \tilde{C})(e^{\imath\beta_j} - 1)e^{\imath h\tilde{\beta}},$$

with $j = 1, \ldots, m-1$, $h = 1, \ldots, \bar{h}$, where $\bar{h}$ is the smallest integer s.t. $\bar{h}\tilde{\beta} \equiv 0 \ (mod\ 2\pi)$ (for a detailed calculation of these generators, see [10]).

The generators $\mathbf{g}_{hj}$ of T thus evaluated can be represented as vectors in P originating from p_0. The set of reachable positions is the locus of points in the plane that can be reached by summing such vectors, i.e. the set of all points p such that the 2–dimensional Diophantine equation

$$\sum_{h=1}^{\bar{h}} \sum_{j=1}^{m-1} \gamma_{hj}\mathbf{g}_{hj} = p$$

has a solution with integers γ_{hj}.

If there exist no two generators such that any other generator can be written as a combination over the rationals of the two, than the reachable set is clearly dense. Otherwise, it is always possible to find two new vectors in the plane (the so–called "greatest common divisors" of the set of generators, see [12]) such that any reachable position can be written as an integer combination of the g.c.d. vectors. The set of reachable positions is discrete in the latter case, and furthermore it lies on a lattice whose description is given completely by the above analysis (see fig.8). Using this algebraic description of the generators, and the geometric properties of the polyhedra, the proof of theorems 2 and 3 easily follow.

4 Planning Algorithm

The theoretical analysis above summarized allows one to design a practical algorithm for planning manipulation of polyhedral parts by rolling. As already mentioned, however, some caution has to be taken in applying the results. In particular, although from the theory the discreteness of the reachable set appears to be an exception, this is the only practically relevant case. A first reason in fact is that any representation of the angles β_i and of the generators $\mathbf{g}_{hj}$ is forcedfully rational in a digital computer with finite precision. Secondly, and more stringently, as the description of the polyhedral part comes from a physical process of measurement or machining, it can only be known to within a tolerance. The numeric representation of such data has therefore to be chosen with comparable accuracy (usually much less than that available in modern computers).

These considerations imply that the only reasonable specification of a planning problem in this context is to give a desired face, position, and orientation, along with a tolerance for the latter two (see fig.8). Deciding whether reaching the goal within the tolerance is possible for the given part description and associated accuracy should be considered as an important part of any planning algorithm.

In this setting, an algorithm for finding a sequence of elementary tumbles that steers the polyhedron from configuration $(p_0\theta_0, i_0) \mapsto (p_f, \theta_f, 1)$ within a tolerance ϵ_p on positions and ϵ_θ on orientations, can be given as follows.

1. Measure the polyhedron parameters D_{ij} and α_{kl}, and provide their continued fraction expansion with reasonable accuracy;

2. Take face 1 on the plane, i.e., find a transit walk t_{i_0} such that $t_{i_0}^{-1} : (p_0\theta_0, i_0) \mapsto (p_1, \theta_1, 1)$;

3. Compute angles β_i and their g.c.d. $\tilde{\beta}$; the virtual rotation R_V and $\tilde{C}$; the plane development of the polyhedron on the plane and the $\hat{V}_i$'s;

4. Verify that the tolerance on orientations is admissible, i.e., that $\epsilon_\theta > \frac{|\tilde{\beta}|}{2}$;

5. Let $\delta\theta = \theta_f - \theta_1$ and k be the smallest integer s.t. $|k\beta_V - \delta\theta| < \epsilon_\theta$; then apply k times the virtual rotation, i.e. $R_V^k : (p_1, \theta_1, 1) \mapsto (p_2, \theta_2, 1)$;

6. Compute the generators $\mathbf{g}_{hj}$, and their g.c.d. $\tilde{\mathbf{g}}_1$, $\tilde{\mathbf{g}}_2$ using a lattice reduction algorithm (see [12]);

7. Verify that the tolerance on positions is admissible, i.e., that $\epsilon_p > \min\left\{||\frac{\tilde{\mathbf{g}}_1+\tilde{\mathbf{g}}_2}{2}||, ||\frac{\tilde{\mathbf{g}}_1-\tilde{\mathbf{g}}_2}{2}||\right\}$;

8. Let $\delta p = p_f - p_2$, and find the smallest integers $\tilde{k}_1$, $\tilde{k}_2$ such that $||\tilde{k}_1\tilde{\mathbf{g}}_1 + \tilde{k}_2\tilde{\mathbf{g}}_2 - p_d|| \leq \epsilon_p$;

9. Invert the g.c.d. algorithm to obtain integers k_{hj} such that $\sum_h \sum_j k_{hj}\mathbf{g}_{hj} = \tilde{k}_1\tilde{\mathbf{g}}_1 + \tilde{k}_2\tilde{\mathbf{g}}_2$;

If the admissibility checks hold true, the algorithm finds a solution to the planning problem in the form of a concatenation of walks, $k_{\bar{h},m-1}\mathbf{g}_{\bar{h},m-1} \circ \ldots \circ k_{11}\mathbf{g}_{11} \circ R_V^k \circ t_{i_0}^{-1} : (p_0, \theta_0, i_0) \mapsto (p_3, \theta_2, 1)$, with p_3 and θ_2 within the prescribed tolerance. The resulting path may be rather complex. Once converted in terms of the sequence of faces, positions, and orientations to be followed by the polyhedron, the length of the walk can often be trimmed by deleting the largest subsequence comprised within two equal configurations. To further reduce the complication of manipulation maneuvers, the sequence resulting from the algorithm can be used as a feasible starting solution of a branch–and–bound algorithm for discrete optimization. As an example of application, manipulation by rolling of a 32–pin connector is reported in fig.9.

5 Conclusions

In this paper we have investigated the structure of the reachable set of a polyhedron rolling on a plane, and deduced a complete algorithm for planning such motions. Experimental work is under implementation to show the practicality of manipulation by rolling

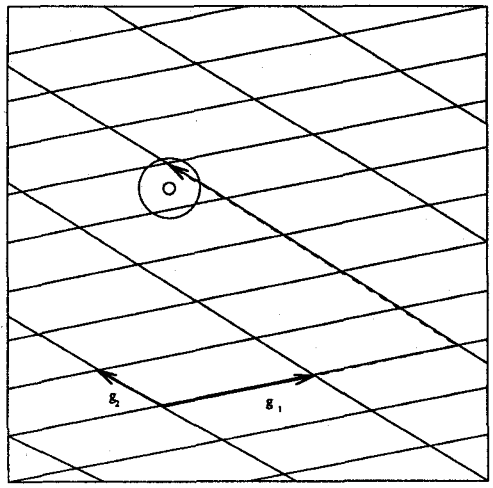

Figure 7: Lattice generated by $\mathbf{g}_1$, $\mathbf{g}_2$.

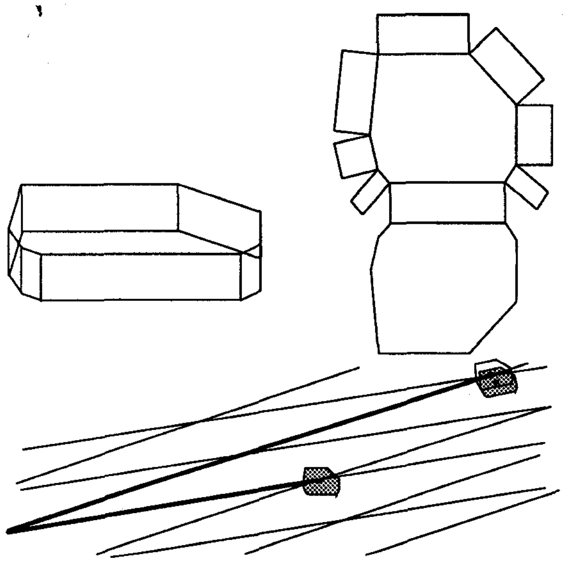

Figure 8: Polyhedral approximation of a 32–pin connector with its plane development. The part has 10 faces, 16 vertices and 24 edges. Lengths are measured with 1mm tolerance, ϵ_p = 1cm and $\epsilon_\theta = \frac{\pi}{30}$. The g.c.d. of defect angles is approximated to $\tilde{\beta} = \pi/21$. 315 generators are found by the algorithm. In the picture the manipulation between the two configurations $((0,0),0,1)$ and $((20,20),\frac{\pi}{21},1)$ is desired while configuration $((19.85,20.66),\frac{\pi}{21},1)$ is reached in 123 elementary tumbles.

parts. The interest of the concepts and tools developed however may not be confined to robot manipulation. In fact, the problem appears to have multiple cousins in the scientific literature at large: for instance in ergodic theory, automata theory, discrete gravitation theory (Regge's calculus), and billiards.

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