Topology and its Applications 27 (1987) 145-155
North-Holland

TOPOLOGICAL IMPROVEMENTS OF CATEGORIES OF STRUCTURED SETS

Horst HERRLICH
Feldhäuser Str. 69, 2804 Lilienthal, Fed. Rep. Germany

Received 15 May 1986

Categories of structured sets often fail to have some desirable properties. They may even fail to have any interesting decently behaved full subcategories. But, under some natural assumptions (and disregarding purely set theoretic problems concerning 'size'), it is always possible to embed them into nicely behaved topological categories, in particular each such category has:

(1) a topological hull (= Mac Neille completion),
(2) a cartesian closed topological hull (= Antoine-completion),
(3) a hereditary topological hull,
(4) a concrete quasitopos hull (= Wyler completion).

The purpose of this paper is to discuss these hulls and to provide several illuminating examples.

AMS (MOS) Subj. Class.: 54B30, 18B15, 18B30, 18D15, 54C35, 18B25

construct · concrete category · topological category · function space · cartesian closedness · quasitopos · Mac Neille completion · topological hull · injective hull · Cartesian closed topological hull · topological quasitopos hull

Introduction

The objects of this paper are categories of structured sets and structure-preserving maps (more formally: categories A supplied with a forgetful functor $U: A \to \mathbf{Set}$) satisfying the following condition:

(C) every constant map between A-objects is an A-morphism.

Such categories will be called *constructs*. Familiar constructs, such as **Top**, **Unif** and **Prox**, share an important categorical property: they are *topological*, i.e., they have sufficient initial and final structures. This property, even though being rather convenient, is considered by an increasing number of topologists and analysts (cf. the reference list B) not convenient enough. Additional convenience properties are required. The existence of a *quasitopos* structure is particularly desirable. This property splits naturally into two parts, each interesting in its own right: *cartesian closedness*, whose value has been recognized for more than 20 years, and *heredity*, whose usefulness has become apparent more recently. These properties will be described in Section 1.

Natural candidates for convenient constructs are full subconstructs and superconstructs of the classical constructs, in particular of **Top**. Unfortunately, constructs may fail to have any interesting convenient full subconstructs, e.g.:

(1) **Top** has only 2 full subconstructs, which are topological quasitopoi: they consists of all discrete spaces and, respectively, of all indiscrete spaces [24].

(2) $\mathbf{Top}_0$, the construct of T_0-spaces, has only one full subconstruct, which is topological: it consists of all discrete spaces.

(3) **Met**, the construct of metric spaces and non-expansive maps, has infinitely many topological (even quasitopoi) full subconstructs, but each of these contains only metric spaces whose underlying topologies are discrete.

It is the more surprising that (modulo some foundational problems, discussed in Section 3.2) *every* construct can be fully embedded as a finally-dense subcategory into some convenient construct: e.g., its *largest finally tight extension*, which is a concrete quasitopos with productive quotient maps [25]. Although extensions of constructs follow the rule:

(BB) 'bigger is better'; i.e. stronger convenience stipulations require bigger extensions,

they also follow the rule:

(SB) 'smaller is better'; i.e. smaller extensions generally preserve more structure of the original construct.

Hence it seems desirable to find, for a given construct $\boldsymbol{A}$ and a given convenience condition P, a smallest extension $P(\boldsymbol{A})$ of $\boldsymbol{A}$ satisfying P. Such an extension will be called a *P-hull* of $\boldsymbol{A}$. For the convenience properties, mentioned above, such hulls exist (modulo-foundational problems, see section 3.2) for any reasonable construct. The properties of these hulls and various examples will be presented in Section 2, their construction in Section 3. Finally, Section 4 indicates that the mentioned results are special cases of far more general results.

Background material, definitions, examples and references up to 1981 can be found in the survey article [23]. Most references given there will not be repeated here.

1. Convenience properties

1.1. Topological constructs

A construct $\boldsymbol{A}$ is called *topological* provided it satisfies the following equivalent conditions:

(1) $\boldsymbol{A}$ is initially complete, i.e., every source of the form $(f_i : X \to UA_i)_{i\in I}$ has an initial lift $(f_i : A \to A_i)_{i \in I}$,

(2) $\boldsymbol{A}$ is finally complete, i.e., every sink of the form $(f_i : UA_i \to X)_{i\in I}$ has a final lift $(f_i : A_i \to A)_{i\in I}$,

(3) $\boldsymbol{A}$ is an injective object in the quasicategory *Const* of constructs and concrete functors.

Most of the familiar constructs, consisting of 'topological objects', are topological, e.g., the constructs **Top, PrTop, PsTop, Conv, *k*-Top, Prox, Unif, Near, Mer, Bor** and **SynTop**, consisting of topological, pretopological, pseudotopological, convergence, compactly generated topological, proximity, uniform, nearness, merotopic, bornological and syntopogeneous spaces respectively. So are the constructs **Simp, Prost** and **Rere**, consisting of simplicial complexes, preordered sets and reflexive relations. Subconstructs of topological constructs, defined by separation axioms, like $\mathbf{Top}_0$, $\mathbf{Top}_1$, $\mathbf{Top}_2$, $\mathbf{Unif}_2$ etc., generally fail to be topological, but usually have familiar constructs as their topological hulls.

Topological constructs have many pleasant properties, e.g., they have concrete limits and concrete colimits, and their fibres are complete lattices.

Recent references: [20, 25].

1.2. Cartesian closed topological constructs

A topological construct $\boldsymbol{A}$ is *cartesian closed* provided it satisfies the following equivalent conditions:

(1) for every $\boldsymbol{A}$-object A the endofunctor $A\times \_ : \boldsymbol{A}\to\boldsymbol{A}$ has a right adjoint,

(2) $\boldsymbol{A}$ has decently behaved function spaces, i.e., for every pair (A, B) of $\boldsymbol{A}$-objects the set $\mathrm{Mor}_{\boldsymbol{A}}(A, B)$ can be supplied with the structure of an $\boldsymbol{A}$-object—usually denoted by B^A, and called a function space of a power object—such that for any $\boldsymbol{A}$-object C and any map $f: A\times C\to B$ the following conditions are equivalent:

(a) $f: A\times C\to B$ is an $\boldsymbol{A}$-morphism,

(b) $f^*: C\to B^A$, defined by $(f^*(c))(a)=f(a, c)$, is an $\boldsymbol{A}$-morphism,

(3) in $\boldsymbol{A}$ final epi-sinks are finitely productive, i.e., if $(f_i: A_i\to A)_{i\in I}$ and $(g_j: B_j\to B)_{j\in J}$ are final epi-sinks in $\boldsymbol{A}$, then so is $(f_i\times g_j: A_i\times B_j\to A\times B)_{(i,j)\in I+J}$,

(4) $\boldsymbol{A}$ is a injective object in the quasicategory $\mathbf{Const}_p$ of constructs with finite concrete products and finite product preserving concrete functors.

For small-fibred $\boldsymbol{A}$ these conditions are equivalent to:

(5) $\boldsymbol{A}$ has the following properties:

(a) in $\boldsymbol{A}$ quotient maps are finitely productive, i.e., if $f: A\to B$ and $g: C\to D$ are quotient maps in $\boldsymbol{A}$, then so is $f\times g: A\times C\to B\times D$,

(b) in $\boldsymbol{A}$ products commute with coproducts, i.e., for any $\boldsymbol{A}$-object A and any set-indexed family $(B_i)_I$ of $\boldsymbol{A}$-objects the natural morphism $\amalg(A\times B_i)\to A\times\amalg B_i$ is an isomorphism.

Of the familiar topological constructs, listed in 1.1, only **PsTop, Conv, *k*-Top, Bor, Simp, Prost** and **Rere** are cartesian closed.

Further examples: filter-generated merotopic spaces [16, 30] uniform limit spaces [31, 39], and Cauchy-spaces [15].

Recent references: [3, 11, 25, 28, 36, 37].

1.3. Hereditary topological constructs

A topological construct $\boldsymbol{A}$ is called *hereditary* provided it satisfies the following equivalent conditions:

(1) in $\boldsymbol{A}$ final sinks are hereditary, i.e., if $(f_i : A_i \to A)_{i\in I}$ is a final sink in $\boldsymbol{A}$, B is a subspace of A, B_i is the subspace of A_i with underlying set $f_i^{-1}[B]$, and $g_i : B_i \to B$ is the corresponding restriction of f_i, then $(g_i : B_i \to \to B)_{i\in I}$ is a final sink in $\boldsymbol{A}$ too;

(2) in $\boldsymbol{A}$ final epi-sinks are hereditary;

(3) in $\boldsymbol{A}$ partial morphisms are representable, i.e. every $\boldsymbol{A}$-object A can be embedded via the addition of a single point ∞_A into an $\boldsymbol{A}$-object $A^+ = A \cup \{\infty_A\}$ such that the following holds:

For every partial morphism $B \to A$, i.e., for every $\boldsymbol{A}$-morphism $f : C \to A$ from a subobject C of B into A, the unique function $f^+ : B \to A^+$, defined by

$$f^+(b) = \begin{cases} f(b), & \text{if } b \in C \\ \infty_A, & \text{if } b \notin C \end{cases},$$

is an $\boldsymbol{A}$-morphism;

(4) $\boldsymbol{A}$ is an injective object in the quasicategory $\mathbf{Const}_h$ of constructs with subspaces and subspace preserving concrete functors.

For small-fibred $\boldsymbol{A}$ these conditions are equivalent to:

(5) in $\boldsymbol{A}$ coproducts and quotients are hereditary.

Of the familiar topological constructs, listed in 1.1, only **PrTop, PsTop, Conv, Mer, Bor, Simp** and **Rere** are hereditary. The only full subconstructs of **Top**, which are hereditary topological, are those consisting of all discrete resp. of all indiscrete spaces [24].

1.4. *Concrete quasitopoi*

A topological construct $\boldsymbol{A}$ is a *quasitopos* provided it satisfies the following equivalent conditions:

(1) $\boldsymbol{A}$ is cartesian closed and in $\boldsymbol{A}$ partial morphisms are representable,

(2) $\boldsymbol{A}$ is cartesian closed and hereditary,

(3) in $\boldsymbol{A}$ final epi-sinks are universal, i.e., if $(f_i : A_i \to A)_{i\in I}$ is a final epi-sink in $\boldsymbol{A}$, f: B → A is an $\boldsymbol{A}$-morphism and for each $i \in I$ the diagram

$$\begin{tikzcd} B_i \arrow[r, "k_i"] \arrow[d, "g_i"'] & A_i \arrow[d, "f_i"] \\ B \arrow[r, "f"'] & A \end{tikzcd}$$

is a pullback in $\boldsymbol{A}$, then $(g_i : B_i \to B)_{i\in I}$ is a final epi-sink in $\boldsymbol{A}$,

(4) in $\boldsymbol{A}$ colimits are universal,

(5) for each $\boldsymbol{A}$-object A the comma category $\boldsymbol{A}/A$ is cartesian closed,

(6) $\boldsymbol{A}$ is an injective object in the quasicategory $\mathbf{Const}_{ph}$ of constructs with finite concrete products and subspaces and concrete functors preserving finite products and subspaces.

For small-fibred $\boldsymbol{A}$, these conditions are equivalent to:

(7) in $\boldsymbol{A}$ quotients and coproducts are universal.

Of the familiar topological constructs, listed in Section 1.1, only **PsTop, Conv, Bor, Simp** and **Rere** are quasitopoi.

Alternative names, which have been used for topological constructs, which are quasitopoi, are concrete quasitopoi [22], strongly topological categories [24], and topological universes [34].

Recent references: [2, 3, 4, 25]

2. Hulls

In this section various *extensions* of a construct $\boldsymbol{A}$, i.e., full concrete embeddings $E:\boldsymbol{A}\to P(\boldsymbol{A})$ are characterized. For simplicity we will assume that $\boldsymbol{A}$ is a full subconstruct of $P(\boldsymbol{A})$ and that $E:\boldsymbol{A}\to P(\boldsymbol{A})$ is the corresponding inclusion.

2.1. Topological hulls

Every construct $\boldsymbol{A}$ has (modulo foundational problems, see section 3.2) a *topological hull* (= *Mac Neille completion*) $T(\boldsymbol{A})$, characterized uniquely (up to isomorphism) by the following equivalent conditions:

(1) $T(\boldsymbol{A})$ is the smallest topological extension of $\boldsymbol{A}$,

(2) $T(\boldsymbol{A})$ is the largest initially and finally tight extension of $\boldsymbol{A}$,

(3) $T(\boldsymbol{A})$ is an initially and finally tight topological extension of $\boldsymbol{A}$,

(4) $T(\boldsymbol{A})$ is the injective hull of $\boldsymbol{A}$ in **Const**.

Examples of topological hulls: $T(\mathbf{Top}_0)=\mathbf{Top}$, $T(\mathbf{Unif}_2)=\mathbf{Unif}$, $T(\mathbf{Poset})=\mathbf{Prost}$, $T(\mathbf{Met})$ is the construct **PMet** of pseudometric spaces.

2.2. Cartesian closed topological hulls

Every construct $\boldsymbol{A}$ with finite concrete products has (modulo foundational problems, see section 3.2) a *cartesian closed topological hull* (= *Antoine completion*) $\mathrm{CT}(\boldsymbol{A})$, characterized uniquely (up to isomorphism) by the following equivalent conditions:

(1) $\mathrm{CT}(\boldsymbol{A})$ is the smallest finally tight cartesian closed topological extension of $\boldsymbol{A}$,

(2) $\mathrm{CT}(\boldsymbol{A})$ is a cartesian closed topological extension of $\boldsymbol{A}$, in which $\boldsymbol{A}$ is finally dense and powers B^A of $\boldsymbol{A}$-objects A, B are initially dense,

(3) $\mathrm{CT}(\boldsymbol{A})$ is the injective hull of $\boldsymbol{A}$ in $\mathbf{Const}_p$.

Examples of cartesian closed topological hulls: $\mathrm{CT}(\mathbf{Top})=\mathbf{EpiTop}$ [13, 32, 18], $\mathrm{CT}(\mathbf{PrTop})=\mathbf{PsTop}$ [18], $\mathrm{CT}(\mathbf{Tych})=c$-embedded convergence spaces [18, 19], $\mathrm{CT}(\mathbf{Unif})=$ bornological uniform spaces [7], $\mathrm{CT}(\mathbf{Prox})=$ coreflective hull of **Prox** in **Unif** [33], $\mathrm{CT}(\mathbf{HComp})=$ coreflective hull of **HComp** in **Unif** [9], $\mathrm{CT}(\mathbf{Poset})=$ **Prost**.

Recent references: [9, 10, 11, 12, 14, 28, 37].

2.3. Hereditary topological hulls

Every construct $\boldsymbol{A}$ with subspaces has (modulo foundational problems, see section 3.2) a *hereditary topological hull* $\mathrm{HT}(\boldsymbol{A})$, characterized uniquely (up to isomorphism) by the following equivalent conditions:

(1) $\mathrm{HT}(\boldsymbol{A})$ is the smallest finally tight hereditary topological extension of $\boldsymbol{A}$,

(2) $\mathrm{HT}(\boldsymbol{A})$ is the injective hull of $\boldsymbol{A}$ in $\mathbf{Const}_h$.

Examples of hereditary topological hulls: HT(**Top**) = **PrTop**, HT(**Poset**) = **Rere**, HT(**Met**) is the construct **Dist** of distance spaces (obtained by dropping the triangle-inequality in the definition of a pseudometric space).

Details will appear in the author's "Hereditary topological constructs", Proc. Sixth Prague Topol. Symp. 1986.

2.4. Concrete quasitopos hulls

Every construct $\boldsymbol{A}$ with finite concrete products and subspaces has (modulo foundational problems, see Section 3.2) a *concrete quasitopos hull* (= *Wyler-completion*) $\mathrm{CQ}(\boldsymbol{A})$, characterized uniquely (up to isomorphism) by the following equivalent conditions:

(1) $\mathrm{CQ}(\boldsymbol{A})$ is the smallest finally tight concrete quasitopos extension of $\boldsymbol{A}$,

(2) $\mathrm{CQ}(\boldsymbol{A})$ is a concrete quasitopos extension of $\boldsymbol{A}$, in which $\boldsymbol{A}$ is finally dense and objects of the form $(B^A)^+$, with A and B being $\boldsymbol{A}$-objects, are initially dense,

(3) $\mathrm{CQ}(\boldsymbol{A})$ is the injective hull of $\boldsymbol{A}$ is $\mathbf{Const}_{ph}$.

Examples of concrete quasitopos hulls: CQ(**Top**) = **PsTop** [38], CQ(**Poset**) = **Rere**, CQ(**Met**) = **Dist**. CQ(**Unif**) is the construct of submetrizable bornological merotopic spaces [8]. CQ (the construct of finite simplicial complexes) = **Simp**. CQ(STOP) = construct of superspaces [40].

Recent references: [2, 3, 4].

3. Constructions

In section 3.1 constructions of the hulls will be provided. The foundational problems, raised by these constructions, will be discussed briefly in section 3.2.

3.1. Construction of the hulls

Each of the hulls under discussion is a finally tight extension. It can therefore be obtained as a full subconstruct of the *largest finally tight extension*. This can be described as follows: If $\boldsymbol{A}$ is a construct, the objects of $\mathrm{Max}(\boldsymbol{A})$ are pairs (X, S), consisting of a set X and a structured sink S, i.e., a class of pairs (A, a), consisting of an $\boldsymbol{A}$-object A and a function $a: UA \to X$, subject to the following conditions:

(1) If $(A, a) \in S$ and $b: B \to A$ is an $\boldsymbol{A}$-morphism, then $(B, a \cdot b) \in S$,

(2) if A is an $\boldsymbol{A}$-object and $a: UA \to X$ is a constant map, then $(A, a) \in S$.

Morphisms in $\operatorname{Max}(\boldsymbol{A})$ from (X, S) to (Y, T) are functions $f: X \to Y$, satisfying the condition:

(M) $(A, a) \in S$ implies $(A, f \cdot a) \in T$.

The construct $\boldsymbol{A}$ can be regarded as a full finally dense subconstruct of $\operatorname{Max}(\boldsymbol{A})$ via the embedding $E: \boldsymbol{A} \to \operatorname{Max}(\boldsymbol{A})$, defined by $EA = (UA, \{(B, b) \mid b: B \to A \in \operatorname{Mor} \boldsymbol{A}\})$. Each of the four hulls of $\boldsymbol{A}$ can be obtained as the full subconstruct of $\operatorname{Max}(\boldsymbol{A})$, consisting of those objects (X, S) for which S can be expressed as an intersection $S = \bigcap_{i \in I} S(i)$ of some particular $S(i)$'s. These will be described, for each of the 4 hulls, as follows:

(1) The topological hull: if $i = (b, B)$ is a pair, consisting of an $\boldsymbol{A}$-object B and a map $b: X \to UB$, then

$$S(i) = \{(A, a) \mid b \cdot a: A \to B \in \operatorname{Mor} \boldsymbol{A}\}.$$

(2) The cartesian closed topological hull: if $i = (B, C, b)$ is a triple, consisting of $\boldsymbol{A}$-objects B and C and a map $b: UB \times X \to UC$, such that for each $x \in X$ the restriction $b(-, x): B \to C$ is an $\boldsymbol{A}$-morphism, then

$$S(i) = \{(A, a) \mid b \circ (\mathrm{id}_B \times a): B \times A \to C \in \operatorname{Mor} \boldsymbol{A}\}.$$

(3) The hereditary topological hull: if $i = (Y, B, b)$ is a triple, consisting of a subset Y of X, an $\boldsymbol{A}$-object B and a map $b: Y \to UB$, then

$$S(i) = \{(A, a) \mid b \circ a_Y: A_Y \to B \in \operatorname{Mor} \boldsymbol{A}\},$$

where A_Y is the subspace of A with underlying set $a^{-1}[Y]$ and $a_Y: UA_Y \to Y$ is the corresponding restriction of $a: UA \to X$.

(4) The concrete quasitopos hull: if $i = (Y, B, C, b)$ is a quadruple consisting of a subset Y of X, $\boldsymbol{A}$-objects B and C and a map $b: UB \times Y \to UC$, such that for each $y \in Y$ the restriction $b(-, y): B \to C$ is an $\boldsymbol{A}$-morphism, then

$$S(i) = \{(A, a) \mid b \circ (\mathrm{id}_B \times a_Y): B \times A_Y \to C \in \operatorname{Mor} \boldsymbol{A}\}$$

where A_Y and a_Y are defined as in (3).

Recent references: [2, 9, 10, 38]

3.2. *Foundational problems*

As is well known, such concepts as the 'set of all sets' or the 'category of all categories' can easily lead to contradictions. There are various ways to avoid these.

Most familiar, perhaps, is the distinction between sets and (proper) classes resp. between classes and (proper) conglomerates, leading to such concepts as the 'class of all sets' and the 'quasicategory of all categories'.

If $\boldsymbol{A}$ is a construct and $P(\boldsymbol{A})$ is one of the hulls of $\boldsymbol{A}$, discussed before, then $P(\boldsymbol{A})$ usually fails to be a construct: in fact if $\boldsymbol{A}$ is not small (the objects of $\boldsymbol{A}$ form a proper class) then the objects of $P(\boldsymbol{A})$ are proper classes too; hence the collection of $P(\boldsymbol{A})$-objects is a proper conglomerate; hence $P(\boldsymbol{A})$ is a quasiconstruct only. If this quasiconstruct is isomorphic to some construct, it will be called a *legitimate quasiconstruct.* In this case, there are no real problems. But sometimes $P(\boldsymbol{A})$ is 'too big' to be legitimate; then it is called *illegitimate.* Sufficient (and necessary) conditions for a construct $\boldsymbol{A}$ to have a legitimate hull, are exhibited:

(a) for the topological hull in [5],
(b) for the cartesian closed topological hull in [6],
(c) for the concrete quasitopos hull in [2].

It may be worth mentioning that

(1) there are constructs whose topological hull is illegitimate [5],
(2) there are topological constructs, whose cartesian closed topological hull is illegitimate [6],
(3) Spanier's quasispaces form an illegitimate construct [27].

There are other ways to deal with the above mentioned problem (e.g., by means of a chain of Grothendieck universes), but none, to the author's knowledge, can 'explain away' the problem.

4. Generalizations

The above results are (particularly interesting) special cases of far more general results. If the category of sets is replaced by a category $\boldsymbol{X}$, and instead of constructs *concrete categories* over $\boldsymbol{X}$, i.e., pairs $(\boldsymbol{A}, U)$ consisting of a category $\boldsymbol{A}$ and a forgetful functor $U : \boldsymbol{A} \to \boldsymbol{X}$, are considered, then the above constructions and most of the results remain valid. Naturally, in the cartesian closed case $\boldsymbol{X}$ should be required to be cartesian closed, in the quasitopos case $\boldsymbol{X}$ should be required to be a quasitopos, and in the hereditary case $\boldsymbol{X}$ should be required to carry a suitable factorization structure for morphisms. The general results not only imply the above results (of interest primarily to topologists and analysts) but also familiar results such as the characterization of (a) complete lattices as injective objects in **Poset**, (b) Mac Neille completions of partially ordered sets as injective hulls [17], and (c) locales as injective objects in the category of semilattices [21, 29].

In the general situation condition (C), saying that all constant maps are morphisms, is unnatural, hence dropped. It could have been omitted here too. This has not been done since condition (C) is responsible for the 'familiarity' of the concepts and hulls under discussion, e.g. it guarantees [25] that cartesian closedness is equivalent to the existence of decently behaved function spaces (condition (1) in 1.2), a condition topologists and analysts are primarily interested in; also it is responsible for the hulls of familiar constructs to be not too exotic.

References A

[1] J. Adámek, Theory of Mathematical Structures (Reidel, Boston, 1983).
[2] J. Adámek, Classification of concrete categories, Houston J. Math. 12 (1986) 305-326.
[3] J. Adámek and H. Herrlich, Cartesian closed categories, quasitopoi and topological universes, Comment. Math. Univ. Carolinae 27 (1986) 235-257.
[4] J. Adámek and H. Herrlich, A characterization of concrete quasitopoi by injectivity, to appear.
[5] J. Adámek, H. Herrlich and G.E. Strecker, Least and largest initial completions, Comment. Math. Univ. Carolinae 20 (1979) 43-77.
[6] J. Adámek and V. Koubek, Cartesian closed initial completions, Topology Appl. 11 (1980) 1-16.
[7] J. Adámek and J. Reiterman, Cartesian closed hull of the category of uniform spaces, Topology Appl. 19 (1985) 261-276.
[8] J. Adámek and J. Reiterman, The quasitopos hull of the category of uniform spaces, Topology Appl. 27 (1987) (this issue) 97-104.
[9] J. Adámek, J. Reiterman and G.E. Strecker, Realization of cartesian closed topological hulls, Manuscr. Math. 53 (1985) 1-33.
[10] J. Adámek and G.E. Strecker, Construction of cartesian closed topological hulls, Comment. Math. Univ. Carolinae 32 (1981) 235-254.
[11] I.W. Alderton, Initially structured categories: subcategories, supercategories and cartesian closedness, Thesis, Univ. South Africa, 1984.
[12] I.W. Alderton, Cartesian closedness and the Mac Neille completion of an initially structured category, Quaest. Math. 8 (1985) 63-78.
[13] P. Antoine, Etude élémentaire des catégories d'ensembles structurés, Bull. Soc. Math. Belgique 18 (1966) 142-164 and 387-414.
[14] H.L. Bentley and H. Herrlich, The coreflective hull of the contigual spaces in the category of merotopic spaces, Lecture Notes Math. 915 (1982) 16-25.
[15] H.L. Bentley, H. Herrlich and E. Lowen-Colebunders, The category of Cauchy spaces is cartesian closed, Topology Appl. 27 (1987) (this issue) 105-112.
[16] H.L. Bentley, H. Herrlich and W.A. Robertson, Convenient categories for topologists, Comment. Math. Univ. Carolinae 17 (1976) 207-227.
[17] B. Banaschewski, G. Bruns, Categorical characterization of the Mac Neille completion, Archiv Math. 18 (1967) 369-377.
[18] G. Bourdaud, Espaces d'Antoine et semi-espaces d'Antoine, Cahiers Top. Geom. Diff. 16 (1975) 107-134.
[19] G. Bourdaud, Some cartesian closed topological categories, Lect. Notes Math. 540 (1976) 93-108.
[20] G.C.L. Brümmer, Topological categories, Topology Appl. 18 (1984) 27-41.
[21] G. Bruns and H. Lakser, Injective hulls of semilattices, Canad. Math. Bull. 13 (1970) 115-118.
[22] E.J. Dubuc, Concrete quasitopoi, Lect. Notes Math. 753 (1979) 239-254.
[23] H. Herrlich, Categorical topology 1971-1981, Gen. Topol. Rel. Mod. Analysis and Algebra, Proc. 5th Prague Topol. Symp. 1981 (Heldermann, 1983) 279-383.
[24] H. Herrlich, Are there convenient subcategories of Top? Topology Appl. 15 (1983) 263-271.
[25] H. Herrlich, Universal topology, Cat. Topol., Proc. Conf. Toledo, Ohio 1983 (Heldermann, 1984) 223-281.
[26] H. Herrlich and L.D. Nel, Cartesian closed topological hulls, Proc. AMS 62 (1977) 215-222.
[27] H. Herrlich and M. Rajagopalan, The quasicategory of quasispaces is illegitimate, Archiv Math. 40 (1983) 364-366.
[28] H. Herrlich and G.E. Strecker, Cartesian closed topological hulls as injective hulls, Quaest. Math. 9 (1986) 263-280.
[29] A. Horn and N. Kimura, The category of semilattices, Algebra Univ. 1 (1971) 26-38.

[30] M. Katětov, On continuity structures and spaces of mappings, Comment. Math. Univ. Carolinae 6 (1965) 257-278.
[31] R.S. Lee, The category of uniform convergence spaces is cartesian closed, Bull. Austral. Math. Soc. 15 (1976) 461-465.
[32] A. Machado, Espaces de Antoine et pseudo-topologies, Cahiers Topol. Geom. Diff. 14 (1973) 309-327.
[33] L.D. Nel, Cartesian closed coreflective hulls, Quaestiones Math. 2 (1977) 269-283.
[34] L.D. Nel, Topological universes and smooth Gelfand-Naimark duality, Contemporary Math. 30 (1984) 244-276.
[35] F. Schwarz, Funktionenräume und exponentiale Objekte in punktetrennend initialen Kategorien, Thesis Univ. Bremen 1983.
[36] F. Schwarz, Product compatible reflectors and exponentiability, Categorical Topology, Proc. Conf. Toledo, Ohio 1983 (Heldermann, 1984) 505-522.
[37] G.E. Strecker, On cartesian closed topological hulls, Categorical Topology Proc. Conf. Toledo, Ohio 1983 (Heldermann, 1984) 523-539.
[38] O. Wyler, Are there topoi in topology? Lecture Notes Math. 540 (1976) 699-719.
[39] O. Wyler, Function spaces in topological categories, Lecture Notes Math. 917 (1979) 411-418.
[40] O. Wyler, Supertopological spaces and function superspaces, Internat. Confer. Categorical Topology, L'Aquila, 1986.

References B

Convenience requirements in topology and analysis

E. Binz, Bemerkungen zu limitierten Funktionenalgebren, Math. Ann. 175 (1968) 169-184.
E. Binz, Continuous convergence on $C(X)$, Lecture Notes Math. 469 (1975).
E. Binz and H.H. Keller, Funktionenräume in der Kategorie der Limesräume, Ann. Acad. Sci-Fenn. Sec. AI 383 (1966) 1-21.
R. Brown, Ten topologies for $X \times Y$, Quart. J. Math. Oxford 14 (2) (1963) 303-319.
R. Brown, Function spaces and product topologies, Quart. J. Math. Oxford 15 (2) (1964) 238-250.
R. Brown, A convenient category of topological spaces; historical note, Univ. of Wales Pure Math. Preprint 86.4, 1986.
R. Brown, Development and prospects for categories, which are convenient for topology, Internat. Confer. Categorical Topology, L'Aquila, 1986.
C.H. Cook and R.H. Fischer, On equicontinuity and continuous convergence, Math. Ann. 159 (1965) 94-104.
E.J. Dubuc and H. Porta, Convenient categories of topological algebras and their duality theory, J. Pure Appl. Algebra 1 (1971) 281-316.
G.A. Edgar, A cartesian closed category for topology, Topology Appl. 6 (1976) 65-72.
A. Ehresmann (Bastiani), Applications différentiables et variétés différentiables de dimension infinie, J. d'Analyse Math. 13 (1964) 1-114.
H.R. Fischer, Limesräume, Math. Ann. 137 (1959) 269-303.
A. Frölicher, Kompakt erzeugte Räume und Limesräume, Math. Z. 129 (1972) 57-63.
A. Frölicher, Catégories cartésienne fermées engendrées par des monoids, Cahiers Top. Géom. Diff. 21 (1980) 367-375.
A. Frölicher, Smooth structures, Lecture Notes Math. 962 (1982) 69-81.
A. Frölicher and W. Bucher, Calculus in vector spaces without norm, Lecture Notes Math. 30 (1966).
A. Frölicher, B. Gisin and A. Kriegl, General differentiation theory, Categ. Theor. Meth. Geom., Proc. Aarhus, 1983, pp. 126-153.
A. Frölicher and A. Kriegl, Convergence vector spaces for analysis, Convergence Struct. Blechyně 1984 (Akademie Verlag, Berlin, 1985) 115-125.
W. Gähler, Grundstrukturen der Analysis I, II (Akademie Verlag, Berlin, 1977/1978).
H. Hogbe-Nlend, Bornologies and Functional Analysis (North-Holland, Amsterdam, 1977).

S.S. Hong and L.D. Nel, Duality theorems for algebras in convenient categories, Math. Z. 166 (1979) 131-136.
H.H. Keller, Über Probleme, die bei einer Differentialrechnung in topologischen Vektorräumen auftreten, Nevanlinna-Festband (Springer, Berlin, 1966).
A. Kriegl, Die richtigen Räume für Analysis im Unendlich-Dimensionalen, Monatsh. Math. 94 (1982) 108-124.
A. Kriegl, Eine kartesisch abgeschlossene Kategorie glatter Abbildungen zwischen beliebigen lokalkonvexen Vektorräumen, Monatsh. Math. 95 (1983) 287-309.
A. Machado, Quasi-variétés complexes, Cahiers Top. Géom. Diff. 11 (1970) 231-279.
L.D. Nel, Convenient topological algebra and reflexive objects, Lecture Notes Math. 719 (1979) 259-276.
L.D. Nel, Universal topological algebra needs closed topological categories, Topology Appl. 12 (1981) 321-330.
L.D. Nel, Enriched algebraic categories with applications in Functional Analysis, Lecture Notes Math. 915 (1982) 247-259.
L.D. Nel, Topological universes and smooth Gelfand-Naimark duality, Contemp. Math. 30 (1984) 244-276.
L.D. Nel, Upgrading functional analytic categories, Proc. Confer. Categorical Topology Toledo 1983 (Heldermann, 1984) 408-424.
L.D. Nel and A. Kriegl, A convenient setting for holomorphy, preprint.
K. Seip, Differential calculus and cartesian closedness, Lecture Notes Math. 540 (1976) 578-604.
K. Seip, A convenient setting for differential calculus, J. Pure Appl. Alg. 14 (1979) 73-100.
K. Seip, A convenient setting for smooth manifolds, J. Pure Appl. Alg. 21 (1981) 279-305.
E. Spanier, Quasi-topologies, Duke Math. J. 30 (1963) 1-14.
N.E. Steenrod, A convenient category of topological spaces, Michigan Math. J. 14 (1967) 133-152.
R.M. Vogt, Convenient categories of topological spaces for homology theory, Archiv Math. 22 (1971) 545-555.
J. Wloka, Limesräume und Distributionen, Math. Ann. 152 (1963) 351-400.