On the Parallel Between Mathematics and Morals*

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Would that morals were like the laws of number and logic: eternal truths that absolutely constrain all possible behaviours. Then, the problems of ethics would be settled on a calm and rational basis, once and for all. Tribal differences would vanish, behaviour would conform naturally to ethical norms, and evildoing would become as rare as arithmetical errors.[1]

Or perhaps things would not be so simple. One can after all add up debts and write down the wrong answer, by mistake or design. The laws of mathematics, like those of ethics, are not gods or any other kind of causal agents. The forms, unfortunately, cannot defend themselves, as they do not have a causal action on the physical world. Neither ethical nor mathematical truths and ideals can fight tanks, or assaults by postmodernist rhetoric (though again, neither can they be liquidated by those enemies). They depend on human minds attuned to them to act on their behalf—to implement those ideals and teach them to the next generation. Given that there are more motives to make ethical than arithmetical errors, perhaps evil would persist.

The forms do however have the capacity to engender love of themselves, in a rightly disposed mind. That is why Plato required a training in mathematics for those who would undertake the rule of the State.[2] Insight into the necessities of mathematics is apt for training the mind to love the necessities of ethics, and hence moti-

* I am grateful to Jean Curthoys for very extensive discussions.

[1] 'Were the nature of human actions as distinctly known as the nature of *quantity* in geometrical figures, the strength of *avarice* and *ambition*, which is sustained by the erroneous opinions of the vulgar as touching the nature of *right* and *wrong*, would presently faint and languish.' (Hobbes, epistle dedicatory to *De Cive,* in T. Hobbes, *Man and Citizen*, B. Gert (ed.), (Garden City, N.Y.: Anchor Books, 1972, 91).) On Locke's ambitions in the same direction, see W. Youngren, 'Founding English ethics: Locke, mathematics and the innateness question', *Eighteenth-Century Life,* **16** (1992), 12–45; J. Gibson, 'Locke's theory of mathematical knowledge and of a possible science of ethics', *Mind,* **5** (1896), 38–59.

[2] Plato, *Republic,* bk VII.

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vates the ruler to make this world conform to those necessities, to the degree that that is possible.

The necessities of mathematics also make good models of absolute objectivity, for those seeking examples of truths independent of the arbitrary and subjective judgments of individuals and tribes. Arguments for ethical relativism arising from the mere fact that ethical principles are held by people, and are not checkable by measurement or scientific observation, face the objection that mathematical truths do not have their objectivity impugned by similar considerations.

Or so it seemed to the ancients. How has the parallel between mathematics and ethics survived what we have learned about those subjects in the millennia since? It is argued that the parallel is clearer now than it was then, and that it stems from the central position of *equality* in both mathematics and ethics.

Sceptics and Relativists

By way of introduction, let us consider how the existence of established truths in mathematics impedes standard arguments for scepticism and relativism in ethics. Those arguments, it will appear, would be as destructive of mathematical truth as of ethical truth, if they had any force at all.

'I believe that it is now pretty generally accepted by professional philosophers that ultimate ethical principles must be arbitrary', wrote a typical linguistic analytic philosopher in 1957.[3] His only reason for this conclusion was that the regress of reasons must end in something unproved. But ultimate mathematical principles are not arbitrary. Though of course there is a true answer to the question, What theorems follow from this arbitrary choice of formal axioms?, that has no bearing on the truths of number theory or operations research or calculus, which are about definite subject matters. There is no support for mathematical or ethical relativism from general considerations about axiomatisation.

Arguments for ethical relativism that arise from definite premises are of two kinds. Both of them are undermined by the parallel with mathematics, since they ought to apply to mathematics as easily as to ethics.

The first arises from the very possibility of ethical disagreement

[3] B. Medlin, 'Ultimate principles and ethical egoism', *Australasian Journal of Philosophy,* **35** (1957), 111–8.

among embodied believers, while the second arises from actual disagreement on ethics among individuals or tribes.

The first of these argues that simply because your belief is your belief, and my belief is mine, arising causally in each case from some combination of brain chemistry and indoctrination, there cannot be any fact of the matter as to which is right. This argument no doubt exists less in the higher reaches of philosophical debate than in the recesses of the undergraduate mind. A classic statement opens E. O. Wilson's *Sociobiology*:

> … self-knowledge is constrained and shaped by the emotional and control centers in the hypothalamus and limbic system of the brain. These centers flood our consciousness with all the emotions—hate, love, guilt, fear, and others—that are consulted by ethical philosophers who wish to intuit the standards of good and evil. What, we are then compelled to ask, made the hypothalamus and limbic system? They evolved by natural selection. That simple biological statement must be pursued to explain ethics and ethical philosophers, if not epistemology and epistemologists, at all depths.[4]

The problem with this argument, obviously, is that it proves too much, since it applies to any putative objective knowledge at all. It is an argument of the form 'We have eyes, therefore we cannot see',[5] and is just as clearly invalid. One would like to instance science as an example of knowledge that all would agree cannot be undermined in this way. Unfortunately, the 'Strong Program in the Sociology of Scientific Knowledge' attempts to show scientific opinions *are* relative, using exactly this argument.[6] So again one can retreat to the last bastion of reason, mathematics, and explain how a causal story does not in itself undermine the objectivity of the results of the causal process.

Take an electronic calculator. Why does the calculator show 4 when you punch in 2+2? On the one hand, there is a causal story

[4] E. O. Wilson, *Sociobiology: The New Synthesis* (Cambridge, Mass: Belknap Press, 1975), 3; cf. J. Ladd, editor's introduction to *Ethical Relativism* (Belmont: Wadsworth, 1973), 5; similar from a Wittgensteinian point of view in J. Lear, 'Ethics, mathematics and relativism', *Mind,* **92** (1983), 38–60.

[5] A. Olding, 'Religion as smorgasbord', *Quadrant,* **42** (5) (May, 1998), 73–5; further in J. Franklin, 'Stove's discovery of the worst argument in the world', *Philosophy,* **77** (4) (2002), 615–24.

[6] D. Bloor, *Knowledge and Social Imagery* (London: Routledge & Kegan Paul, 1976), esp. 5, 7, 9.

about the wiring inside, which explains why 4 is displayed. But the explanation cannot avoid mention of the fact that 2+2 *is* 4. On the contrary, the wiring is set up exactly to implement the laws of arithmetic, which are true in the abstract. The causal apparatus is designed specifically to be in tune with or track the world of abstract truths. If it succeeds, the causal and abstract stories co-operate, and the explanation of the outcome requires both. For all that the relativist argument being considered here has said, the same may be true of brains and ethical truths.

The second common argument for ethical relativism arises from the *actual* diversity of morals among different tribes.[7] The fact that there are so many ways of behaving that are enforced by one tribe while forbidden by another, it is argued, shows that there is no fixed place on which an objectivist view of morality can take a stand.

The parallel between ethics and mathematics suggests two ways of attacking the standard arguments for ethical relativism, one based on the difference between outcomes and basic principles, and the other based on differences between tribes in mathematical beliefs.

Mathematics makes a clear distinction between basic principles and the deductions made from them, or their consequences in different circumstances. The mathematical laws of planetary motion are exactly the same for Mercury and the Moon, but the laws prescribe different orbits for the two bodies, since they are in different places and have different forces acting on them. For the same reason, basic ethical principles of respect for persons will prescribe different actions and customs for a small tribe at subsistence level from those suitable for a complex welfare state. Since the invariance and objectivity of basic ethical principles prescribes a diversity of outcomes, the onus is on someone arguing from cultural diversity to show that the observed diversity cannot be explained by the interaction of universal principles with diverse circumstances. As has been observed by several critics, that task has rarely been seriously attempted.[8]

Is it true that there is diversity between tribes in ethics but not in mathematics? Is there a diversity of mathematical beliefs among

[7] E. Hatch, *Culture and Morality: The Relativity of Values in Anthropology* (New York: Columbia University Press, 1983); F. Snare, 'The empirical bases of moral skepticism', *American Philosophical Quarterly,* **21** (1984), 215–35.

[8] S. E. Asch, *Social Psychology* (Englewood Cliffs: Prentice-Hall, 1952), ch. 13; K. Duncker, 'Ethical relativity? An enquiry into the psychology of ethics', *Mind,* **48** (1939), 39–57.

tribes, and if so, how does it bear on the objectivity of mathematics? It is something of a myth that there are tribes who have no numbers beyond 4, or 2,[9] but many tribes are very vague about large numbers, and others have counting systems difficult to convey in our terminology.[10] Whether such tribes should be said to have mathematical beliefs incompatible with ours, such as the belief that there are no numbers greater than 40, is hard to say. Certainly, having a mental world in which the possibility of numbers greater than 40 cannot arise is close to having a tacit belief that there are no numbers greater than 40. We do not normally take such beliefs or quasi-beliefs to be any reason to doubt the objectivity of our own mathematical beliefs. Instead, we explain them away by saying condescendingly that the natives had no need to consider our concepts, but if they had they would have found themselves reaching the same conclusions as we have. We take it that our study of mathematics has allowed us to understand why the natives' perspective is limited, and why the opening of their minds would cause them to agree with us.

But the same reasoning is applicable in the ethical case. Typically, the ways in which 'primitive' morality differs from our own is in its lack of universalism. There seems to be a universal prohibition on lying to anyone within one's circle of concern, but the prohibition often does not extend to lying to slaves, enemies or foreigners.[11] The benevolence extended to kin is in various ways not extended to the deformed, other tribes and so on. That is however exactly the kind of error that is explained by a later and deeper perspective—the perspective of human equality. Just as we can not only disagree with Nazis about the inferiority of Jews and Slavs, but see they had no relevant evidence for those errors, so we can see that being a slave or a member of another tribe cannot possibly be relevant to moral equality. Likewise, if tribal legal custom includes punishment for crimes that applies to the kin of the perpetrator, we believe our understanding of personal identity and personal responsibility shows what is wrong with it. The idea of moral progress is possible because critical scrutiny of moral ideas is possible both from outside a society and from within it.[12]

[9] J. W. Harris, 'Australian Aboriginal and Islander mathematics', *Australian Aboriginal Studies,* 1987 no. 2, 29–37.

[10] J. Mimica, *Intimations of Infinity: the mythopoeia of the Iqwaye counting system and number* (Oxford: Berg, 1988).

[11] F. Snare, 'The diversity of morals', *Mind,* **89** (1980), 353–69.

[12] M. M. Moody-Adams, 'The idea of moral progress', *Metaphilosophy,* **30** (1999), 168–85.

The parallel with mathematics should, indeed, give the moral objectivist the confidence not to worry about the diversity of morals among tribes. Relativist anthropologists spoke as if the objectivist ought to be dismayed by diversity, and should forever be trying to minimise apparent differences between tribes. But the discovery of diversity was one of the sources of objectivism itself, from imperialist horror at suttee to the rhetoric of 'crimes against humanity' at the Nuremberg trials.[13] The objectivist wishes, in many cases, to highlight cultural diversity in morals, in order to emphasise how seriously some tribes have gone wrong.

If ethics is to follow mathematics into absolutist territory, it needs to make clear what principles are taken as fundamental, what are derived by deduction, and how the principles are to be known. It is argued that the key to answering these questions lies in the notion of equality of intrinsic worth. Before developing that line of reasoning, it is desirable to recall some ideas on equality in mathematics—well-known ideas, but ones often obscured by typical philosophers' views of mathematics, based as they are on experience with formal logics.

Equality in Mathematics

Bertrand Russell analyses 'there are two dogs' as 'There is a dog A and a dog B and A ≠ B'. In this analysis, the concept 'two' has disappeared, analysed in purely logical terms—where equality is counted as a term of logic. The example conveys the main philosophical idea behind Russell and Whitehead's project of reducing mathematics to logic. Expressed in less linguistic and more metaphysical terms, the idea is that number arises from numerical distinctness (the non-identity of dogs A and B) coupled with their equality in some repeatable respect (being dogs). Once there is a 'count' universal, like dog, whose nature is to structure its instances discretely (in contrast to 'stuff' universals like water), that universal necessarily gives rises to numbers.

The emphasis here is more on *in*equality, or numerical distinctness, that on equality. Equality comes into its own in analysing the relations between numbers that lie at the basis of arithmetic. What is it for 1 + 1 to equal 2? One apple plus one apple make two apples

[13] R. Macklin, 'Universality of the Nuremberg code', in *The Nazi Doctors and the Nuremberg Code*, G. Annas and M. Grodin (eds) (New York: Oxford University Press, 1992), 240–57.

because of their equality in being apples. The apples are numerically distinct, and though not identical in all respects are identical in being apples. That is enough for there to be necessarily two apples, whenever there is an apple and another one.

Let us take just one example of the vast superstructure of pure mathematical truths that rests on these foundations, an example particularly revealing of the role of equality. As the following diagram makes clear, the number of different pairs in four objects is 6.

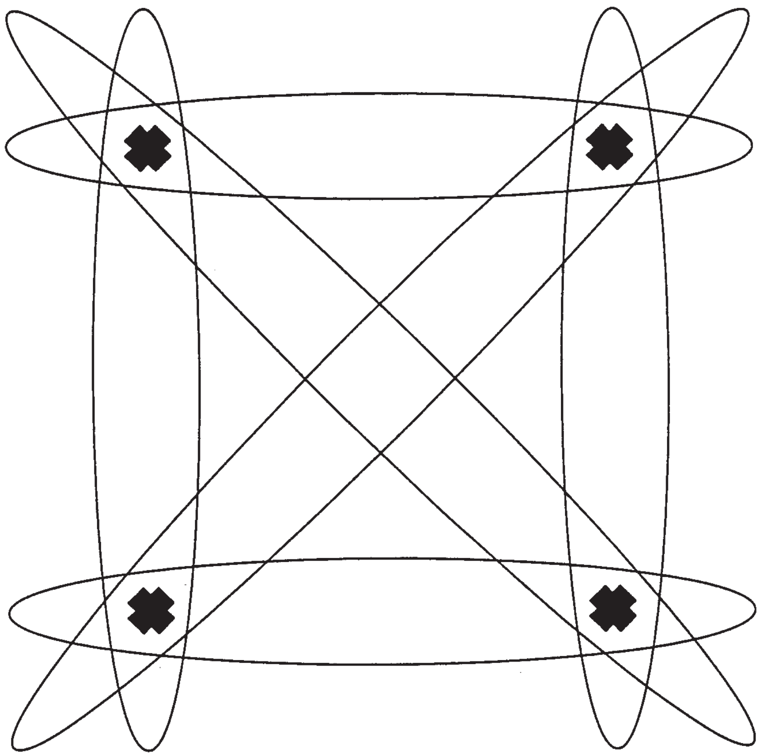

Fig. 1. There are 6 different pairs in 4 objects.

Nothing is required for this truth over and above the distinctness of the four objects, and their equality simply as objects.

So much for pure mathematics. One example from applied mathematics will show the crucial role of equality in making pure mathematical facts applicable to real world situations. As is well known, complicated questions about the probability of events in games with dice and cards are solved by counting the numbers of equiprobable outcomes. The probability of two dice giving a total of 2 (that is, of both showing 1) is 1/36, while the probability of their giving a total of 3 (that is, one of them showing 1 and the other 2) is 2/36. The reason is that the first event consists of one of the 36 basic equiprobable outcomes of two dice (1 and 1), while the second event consists of two of them (1 and 2, and 2 and 1). It is the equiprobability of the basic outcomes that reduces problems in probability to the pure mathematics of counting. The equiprobability of the basic outcomes, it is agreed, results from a symmetry argument. Outcomes such as the 36 possible falls of two dice are equiprobable because there is in some sense a symmetry between them. Debate has been heated as to what this symmetry consists in—is it the physical symmetry of the dice? The equality of the long run observed relative frequency of outcomes? Our equal ignorance of the outcomes? These are fair questions, but the calculation of outcomes does not depend on answering them. Provided the equiprobability of the basic outcomes is granted, they can be counted to give the probabilities of combinations of them.

The example is typical of a wide range of symmetry arguments in modern science, where equality—of directions of pressure, of the effects of weights on a balance beam, of equal and opposite reactions, of light beams going back and forth—is what allows mathematics to gain purchase and solve physical problems.[14]

Equality in Ethics

The definite article in the title of Alan Donagan's book, *The Theory of Morality,* is important. He argues that there is a coherent theory underlying the general moral outlook and behaviour of all (normal) people, though it is not necessarily consciously expressed. *Rules* of ethics are not basic, nor are rights, or virtues. Instead, these are all generated by a more fundamental assumption, that persons are valuable in themselves. Thus, the reason why murder is wrong is not anything to do with the co-ordination of society or the maximisation of happiness, much less the command of a deity or the exercise of a virtue, but the fact that murder results in the destruction of something intrinsically valuable, a human life. He writes:

[14] M. G. Doncel, (ed.), *Symmetries in Physics (1600–1980)* (Barcelona: Universitat Autonoma de Barcelona, 1987); J. Rosen, *Symmetry in Science* (New York: Springer, 1995); A. Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics* (Princeton: Princeton University Press, 1999).

> I take the fundamental principle of that part of traditional morality which is independent of any theological presupposition to have been expressed in the scriptural commandment, 'Thou shalt love thy neighbour as thyself', understanding one's neighbour to be any human being, and love to be, not a matter of feeling, but of acting in ways in which human beings as such can choose to act. The philosophical sense of this commandment was correctly expressed by Kant in his formula that one act so that one treats humanity always as an end and never as a means.[15]

All moral rules, he maintains, even very detailed ones about specific cases, should be deducible from this general principle, with some thought. Thus it is possible to say why the prohibition against murder might be reconsidered in the case of capital punishment: the destruction of the life of one person is balanced against the destruction of life of the criminal's victims, actual or potential; that is a consideration of the same nature as the one that led to the prohibition of murder in general. Rights arise in the same way: a right to life is simply the wrongness of destruction of a life, seen from the point of view of the person living the life.

Donagan argues further that it is possible to say exactly what it is about humans that makes them valuable. It is their rationality. He defines rationality rather narrowly, as 'a capacity to perform acts whose contents belong to the domain of logic'.[16] He is less than clear on why this aspect of human nature alone is the one that confers worth. There are indeed alternative theories: for some it is the possession or immortal souls that confers worth, for others, consciousness, for still others, the capacity of humans to undergo complex experiences of fulfilment, disappointment and sorrow.[17] Others suggest it is merely the ability to have interests.[18] There is something to be said for all these views, at least *prima facie*. Which is right is an important question. But it is a question similar to the ones above about the foundations of probability. Our grasp of the equality of worth is more solid than our grasp of what properties, if any, of humans are the foundation of that worth, just as our grasp of the

[15] A. Donagan, *The Theory of Morality* (Chicago: Chicago University Press, 1977), 65–6; similar in N. Brown, *The Worth of Persons* (Sydney: Catholic Institute of Sydney, 1983).

[16] Donagan, *Theory*, 235.

[17] H. Frankfurt, 'The moral irrelevance of equality', *Public Affairs Quarterly* **14** (2000), 87–103, at 101.

[18] P. Singer, *Practical Ethics,* 2nd edn, (Cambridge: Cambridge University Press, 1993), 12–13.

equality of probabilities is not undermined by our confusions about what probability is.

Deductions from the principle of equality

An example of how deductions from equality work and apply to real cases can perhaps best be seen in the tradition of equality before the law, where theory has been honed by long experience of applications to cases. 'Our equality of birth by nature impels us to seek equality under the law', and Western law since ancient times has made serious efforts to implement that principle, including the removal of legal institutions incompatible with it, such as slavery.[19] Equality is still a fundamental value of the law, called upon in cases where legally established but unjust practices need to be set aside. Such was the case in the *Mabo* decision, where the Australian High Court held that the doctrine of *terra nullius*, according to which Australia was unoccupied at the time of white settlement, incorporated an injustice. The deeper value of the law that was held to be sufficient to overturn centuries of unjust precedents was the principle of equality. Equality required that the rights of aborigines to land could not be regarded as of no moment. One of the *Mabo* judges had written more explicitly in an earlier case:

> At the heart of that obligation [to act judicially] is the duty of a court to extend to the parties before it equal justice, that is to say, to treat them fairly and impartially as equals before the law and to refrain from discrimination on irrelevant or irrational grounds.[20]

It does not follow, and it is not true, that pure deductions from the abstract principle of equality can solve all questions in ethics. If human life had been simpler than it is, then the implications of equality might have been straightforward. For example, if food had been the only necessity of human life, and all other goods comparatively unimportant, then a fundamental equality would have implied equal rights to food. But human nature is more complicated than that of the leech, and it has been credibly maintained that the goods proper to human nature are not only diverse but incommensurable.[21] It is not surprising that ethical discussion has a good

[19] Plato, *Menexenus* 239a; J. M. Kelly, *A Short History of Western Legal Theory* (Oxford: Clarendon, 1992), 29–30, 104–5, 146–8.

[20] W. Deane in *Leeth* v. *The Commonwealth*, *Commonwealth Law Reports* **174** (1991-2), 486–7; cf. *CLR* **168** (1980), 522.

[21] J. Finnis, *Natural Law and Natural Rights* (Oxford: Clarendon, 1980), 87–97.

deal of room for play in disagreements about the relative value of outcomes and of different human goods. Psychiatric, biological and economic evidence can be relevant to those disputes. Long experience in life can increase ethical awareness, because it can deepen understanding of human nature and of the different circumstances it can face.

For these reasons, the implications for action of a general principle of equal worth, or an equal right to consideration, may not be identical actions. The circumstances of people matter, and enter into the calculation. An equal right of children to a fair share of educational resources, for example, will require different actions in the cases of a musical or mathematical prodigy, a well-adjusted child of average intelligence, and an intellectually retarded child. All have rights to education, but the plans must be tailored to each child's abilities to profit from teaching, and one plan may cost more than another. Equality of consideration also admits in general the consideration of morally relevant qualities in which people may differ, such as desert.[22]

That is well recognized in law. Another of the *Mabo* judges writes that 'equality' means more than a purely formal requirement that there be no irrelevant discriminations among litigants. The High Court, she says, has been embedding in constitutional interpretation a theory of equality 'not dissimilar to that propounded by Aristotle.' This theory, as she explains it, involves an active taking into account of relevant differences, so that true equality between persons is preserved; it suggests, for example, the provision of legal aid and interpreter services in court, to prevent discrimination by default.[23]

It might seem, then, that the principle of equality is so qualified in practice as to be close to vacuous. That is not true. To adapt a principle to circumstances is not to qualify it, but to work out its implications, in combination with other premises. The inability of the abstract principle of human equality to resolve complex disputes does not mean it plays little role. As Amartya Sen remarks in discussing the 'equality of what?' question, if someone disputes an egalitarianism of economic outcomes with a theory of the equality of libertarian freedoms, the plausibility of both sides of the debate depends on their connections to a more basic equality of concern. If there were not some credibility to the contentions that equality of

[22] L. Pojman, 'Equality and desert', *Philosophy,* **72** (1997), 549–70.

[23] M. Gaudron, 'Equality before the law with particular reference to Aborigines', *Judicial Review,* **1** (1992–4), 81–9; also H. J. McCloskey, 'A right to equality?', *Canadian Journal of Philosophy,* **6** (1976), 625–42.

basic concern implied equality of economic outcomes and also equality of freedoms, then the dispute would not be able to get under way.[24] It is natural to wish to decide for equality of outcomes, or resources, or opportunities, or initial positions in order to get down as soon as possible to the business of issuing policy prescriptions, but that avoids the hard work of discovering what the implications of basic equality are, as well as giving up any place from which to argue against those who make a different choice.

Similarly with the lifeboat cases that are staples of undergraduate teaching on the topic of equality. The stress of having to consider who should leave a lifeboat in which not all can survive is itself testament to the strength of our commitment to equality, and there is always a strong vote for the proposal that all should stay in the lifeboat and hope for the best. It is also possible to keep to a strict equality by deciding who is to go by lot. Even if we do decide that (other things being equal) the old should go first on the grounds that they have less future to lose, a certain equality of consideration is preserved, in that the decision is proportional to the loss to be sustained, not proportional to any alleged superiority of personal worth or quality.

Similar considerations apply in the practical disciplines in which moral philosophy shades off into casuistry, applied ethics, law and accountancy.[25] Although the complexity of real life makes for many 'hard cases' in these fields, appeals to equality of consideration are always very powerful. And that does not mean merely that equality is weighted heavily in comparison with other considerations. It means that any other consideration, such as skin colour or age or wealth, is by default of absolutely no weight, and the moral relevance of any consideration must be established in the face of the strong presumption against its relevance. Further, such a consideration, if relevant to one person, must be equally relevant to another; for example, if intellectual disability tells against one person's chance of gaining an academic position, it must tell equally against another's. Since both equality and the importance of the various goods proper to humans are well known to us humans, ethical discussion can proceed without being either vacuous or a matter of mere assertion.

The case is the same as in (applied) mathematics. All the molecules going over a waterfall are subject to the same

[24] A. Sen, *Inequality Reexamined* (Oxford: Clarendon, 1992), ch. 1.

[25] J. Franklin, 'Accountancy as computational causistics', *Business and Professional Ethics Journal,* **17** (4) (Winter, 1998), 21–37.

mathematical equation of motion. Their different destinations are not a qualification of that law, but a result of its working out.

Mathematical attempts to mimic ethics

It is a strange fact that whereas objectivist ethics has tended to avoid mathematics, reductive attempts to replace ethics by something else have been highly mathematical. Modern game theorists, utilitarians, and Rawls in his theory of justice have been full of mathematical models and in-principle calculations. The reason this is strange is that these theories are obviously intended to generate a system of behaviour or social arrangements that in large part mimics that recommended by naïve or folk objectivist ethics. If these theories are mathematical, why is objectivist ethics not equally so?

Let us examine what is really assumed in these models, and ask whether their axioms admit, or perhaps require, an objectivist interpretation.

Game theory is normally introduced with the classic scenario of the Prisoners' Dilemma. Two prisoners, in fact guilty of collaborating in a crime, are interrogated separately. The interrogator makes each an offer: parole if you confess and the other does not; 1 year's gaol if neither confesses, 10 years' gaol if both confess; life if you do not confess and the other does.

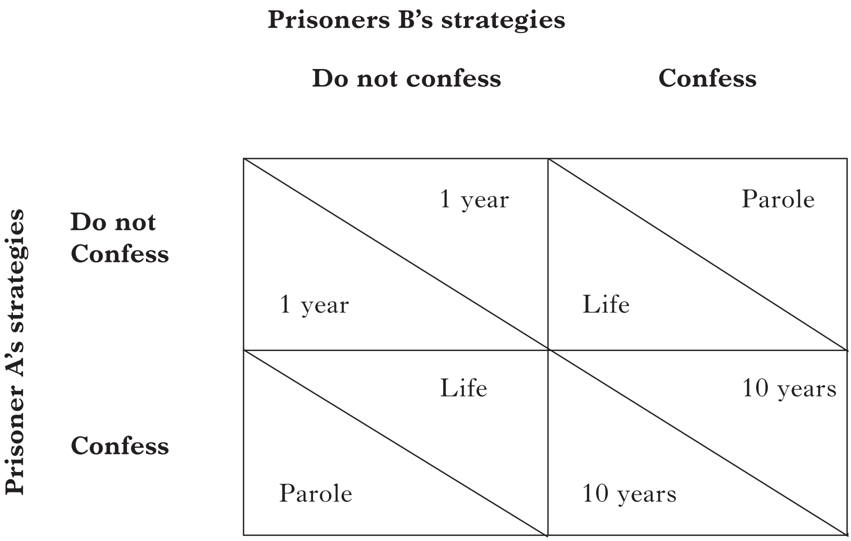

Fig 2. Payoff matrix for Prisoner's Dilemma.

The ordering of severity of the payoffs is designed to create a conflict between the self-interest of each prisoner, and what would be better for the pair of them: each is under pressure to 'save his own skin' by confessing, but knows that the other is under the same pressure, and that if they both confess, they do worse that if both refuse to confess. There is no definite best strategy in the single game, but most of the interest in the topic revolves around iterated prisoners' dilemma, where a similar game is played many times and each prisoner can observe the other's past behaviour. The best strategy is then 'tit-for-tat': co-operate (with the other prisoner, that is, do not confess) in the first round, then do as the other did in the previous round. This strategy gains the benefits of co-operation, without exposing the player to the costs of gullibility. The original applications of the game were to scientific questions, analysing co-operative behaviour in business and showing how altruism was compatible with the Darwinian theory of evolution. But it was not long before popularisers of sociobiology and some philosophers began to draw ethical conclusions. The philosophical significance was normally taken to be in favour of ethical egoism: altruism is explained away as 'really' self-interested action, on the part of either the individual or his/her 'selfish gene'.[26]

But another interpretation of Prisoner's Dilemma games is possible, arising from the observation that it was merely symmetry between the players that set up the dilemma, and we may decide for ourselves what the nature of the symmetry is. Can we read it as a symmetry of moral worth? Suppose we are in the position of the prisoners' guard, who is secretly in sympathy with their cause. He cannot change the punishments, but he can hint to them how to play. For him, the scale of punishments describes the proportions of two (equal) human lives of positive worth that will be lost in the different game outcomes. His view of the game is genuinely ethical, and he will, for example, wish to avoid having both prisoners confess, as that is to their mutual detriment. On this objective reading of the punishments, the game is still in existence, along with any general conclusions that arise from the mathematics.

The ability of mathematical models to produce structures that mimic ethics is, then, evidence neither for the thesis that ethics should be replaced, nor for the thesis that objectivist ethics is better off without mathematics. On the contrary, the natural tendency to

[26] R. Wright, *The Moral Animal: Evolutionary Psychology and Everyday Life* (New York: Vintage, 1995); J. L. Mackie, 'The law of the jungle', *Philosophy,* **53** (1978), 455–64.

regard the symmetries at the bottom of the mathematical models as equalities of ethical worth calls for a mathematical perspective on ethics.

Let us follow this through in another abstract model that to some degree promises to replace standard ethical theory, Rawls' theory of justice. Rawls regards distributive justice as dealing with 'the way in which the major social institutions distribute fundamental rights and duties and determine the division of advantages from social cooperation.'[27] He proposes to deduce just distributive arrangements from some assumptions about an 'initial position', in which individuals must choose principles from behind a 'veil of ignorance', which allows them self-interest and knowledge of general facts about human nature, but no knowledge of what position in society they will be born into. Rawls' model of deduction is a Euclidean one. 'We should strive for a kind of moral geometry', he writes, 'with all the rigor which this name connotes.'[28] Fundamental to the principles is equality: they are 'the principles that free and rational persons concerned to further their own interests would accept in an initial position of equality as defining the fundamental terms of their association'.[29]

What is the nature of the equality of persons in the initial position? Is it ethical equality or not? Officially, it is not. The persons in the initial position have self-interest, but their attitude to others is neither benevolent nor envious. The veil of ignorance includes ignorance even about the 'conception of good' that one will turn out to have. The appeal of Rawls's position has proved to be exactly his derivation of just distributions from non-moral postulates.[30] The reason why Rawls is able to operate without an assumption of benevolence is that it is replaced by ignorance: the self-interested actor in the initial position is forced to care about all people, because he does not know which of them will be him. As Rawls puts it, 'the

[27] J. Rawls, *A Theory of Justice* (revised edn, Oxford: Oxford University Press, 1999), 6.

[28] Rawls, *Theory of Justice*, 105.

[29] Rawls, *Theory of Justice*, 10.

[30] 'The reason why Rawls adopts the motivational postulates he does is actually very simple, that without them there can be no "moral geometry". Once we allow the actors in the original position to have substantive moral notions, we have to say that in the absence of self-interested biases people would agree on this or that principle, which is not deduction but assertion.' B. Barry, *The Liberal Theory of Justice: A Critical Examination of the Principal Doctrines in A Theory of Justice by John Rawls* (Oxford: Clarendon, 1973), 15.

combination of mutual disinterest and the veil of ignorance achieves much the same purpose as benevolence.'[31]

As with game theory, one can ask what the result would be if the equality concerned were to be read ethically. Suppose someone in the initial position were to argue that concern for others was justified not only because he might turn out to be them, but because they are morally similar to him and hence deserving of the same consideration. If, as Rawls says, mutual disinterest and ignorance achieves the same purpose as benevolence, then benevolence will produce the same results as mutual disinterest and ignorance. That is, all the deductions about just distributions that follow from the first principles will still be true.

Further, if one asks *why* all the actors in the initial position should be given an equal vote in choosing arrangements, Rawls answers with ethical language: 'Obviously the purpose of these conditions is to represent equality between human beings as moral persons, as creatures having a conception of their good and capable of a sense of justice.'[32] A non-ethical reading of Rawls therefore involves the logical strain of actors who have a conception of their own good and concern for themselves, but no concern for the good of those identical to themselves.

As in the case of game theory, the most natural interpretation of the mathematical model proves, on examination, to be one based on the objective equality of the worth of persons.

Knowing the principles

It is necessary to distinguish two ways in which mathematics and ethics parallel each other: their access to basic principles, and the way in which those basic principles imply, or cash out in, more detailed and complex consequences. When it comes to the way basic principles imply more complicated ones, mathematics and ethics are not in principle different to any body of knowledge that is sufficiently structured to be organised as a set of logical consequences of a small number of axioms. It is true that the fundamental role of equality in both mathematics and ethics gives them a commonality that other sciences may not share. Even there, a part of physics notably dependent on symmetry principles, such as the statics of balances or fluid dynamics, will look very similar (indeed, those parts of physics are often thought of as applied mathematics).

[31] Rawls, *Theory of Justice*, 128.
[32] Rawls, *Theory of Justice,* 17.

Knowledge of the axioms themselves is another matter. Empirical sciences, it is generally agreed, cannot get their principles except empirically. The value of the constant of gravitation is a brute fact, and there is nothing for it but to 'get out in the wet' and measure it. Mathematical and ethical principles do not seem to admit the same sort of impediment to complete understanding. If we gain knowledge of $2 \times 3 = 3 \times 2$ not by rote but by understanding the diagram then we have fulfilled the Aristotelian ideal of

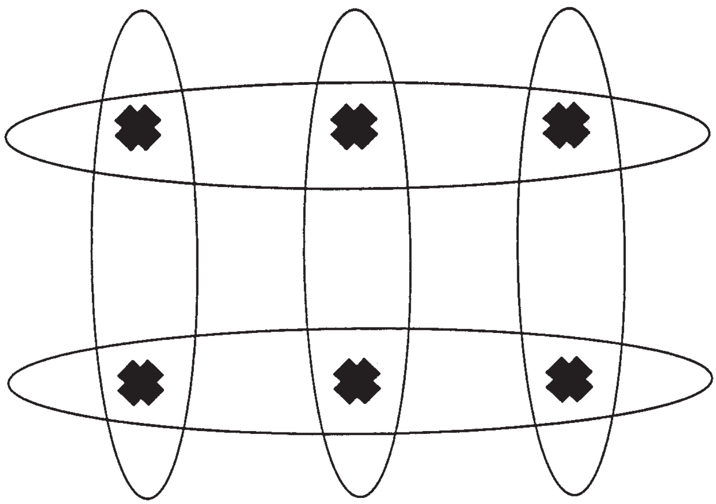

Fig. 3. Why $2 \times 3 = 3 \times 2$.

complete and certain knowledge through understanding the reason why things must be so. Any knowledge of the preciousness of human nature is of the same sort: we have at least one human nature, our own, open to our knowledge, and there is no impediment to knowing the value it has.

This way of speaking may tend to suggest a Platonist or Kantian epistemology of an access to a disembodied world of *a priori* certain truths. Kant agrees with Plato that one must think that way to truly safeguard the necessity of the principles. 'We are also at once reminded', he writes, 'that moral principles are not based on properties of human nature, but must subsist a priori of themselves, while from such principles practical rules must be capable of being deduced for every rational nature, and accordingly for that of man. Such a metaphysic of morals, completely isolated, not mixed with any anthropology, theology, physics, or hyperphysics, and still less with occult qualities (which we might call hypophysical), is not only an indispensable substratum of all sound theoretical knowledge of duties, but is at the same time a desideratum of the highest importance to the actual fulfilment of their precepts.'[33]

That is going too far. The parallel between mathematics and ethics allows us to see that we may have the necessity of principles without needing to detach ourselves from this world. For that is what mathematics has. The impossibility of tiling my bathroom floor with pentagonal tiles is a necessity at once mathematical and directly applicable to the real world.[34] Experience is not irrelevant to the knowledge of mathematical truths either, despite their necessity. Experience is necessary to come to know the concepts used in those truths, such as numbers. As Piaget's and later experiments on children show, the concept of number 'condenses' out of simpler notions of the density and the size of a group: the child needs to gain some experience with the stability of the number of a group of objects when they are spread out, bunched or otherwise rearranged, before it has a grasp of the number concept that will go into such propositions as of $2 \times 3 = 3 \times 2$.[35] Experience can deepen an understanding of the principles, but does not undermine whatever confidence we have in them. When we put 2 rabbits and another 2 rabbits in a box and later find 5 rabbits in there, it is our confidence in the truth of $2 + 2 = 4$ that makes us conclude they've bred, while the discovery of non-Euclidean geometry led to the conclusion that the question 'What geometry does space have?' is empirical, not mathematical.

The parallel with ethics in the deepening of understanding of principles is sometimes obscured by the caricature of mathematics, common among philosophers and logicians, as a series of mechanical though ingenious chains of deductions from simple premises. The philosophy of mathematics once took seriously the position of (one version of) logicism, or 'if-thenism', which held that one could choose mathematical axioms arbitrarily, and all there was to mathematics was seeing what followed from what axioms. That position proved untenable on various technical grounds, though its ghost has not entirely departed from the less-informed discussions in the

[33] I. Kant, *Groundwork for the Metaphysics of Morals*, 2nd section.

[34] J. Franklin, 'Mathematical necessity and reality', *Australasian Journal of Philosophy,* **67** (1989), 286–94; J. Franklin, 'The formal sciences discover the philosophers' stone', *Studies in History and Philosophy of Science,* **25** (1994), 513–33.

[35] J. Piaget, *The Child's Conception of Number* (London: Routledge and Paul, 1952); O. Frydman and P. Bryant, 'Children's understanding of multiplicative relationships in the construction of quantitative equivalence', *Journal of Experimental Child Psychology,* **58** (1994), 489–509.

subject. A classic example that shows the limitations of that point of view comes from Euclid's definition of a circle, as a plane figure 'such that all straight lines drawn from a certain point within the figure to the circumference are equal'. That is not an arbitrary definition, or an abbreviation. A circle at first glance is not given with reference to its centre—it is more likely something 'equally round all the way around'. Understanding that Euclid's definition applies to the same object requires an act of imaginative insight. The genius of the definition lies in its suitability for use in proofs of the kind Euclid gives immediately afterwards, proofs which would be very difficult with the more obvious phenomenological definition of a circle.[36] The same applies to the great analyses of continuity and of symmetry achieved in nineteenth century mathematics. It takes considerable thought to appreciate Cauchy and Riemann's definition of the function $f(x)$ being continuous, or 'having no gap in its graph' at $x = a$ by the formula (containing only logical and arithmetical, as opposed to geometrical, concepts):

$$\forall \varepsilon > 0\ \exists \delta > 0\ \forall\ x \text{ if } |x - a| < \delta \text{ then } |f(x) - f(a)| < \varepsilon$$

Such a definition (like Euclid's of the circle, or the definition of symmetry by abstract groups,[37] or the Turing machine definition of computability) is not subject to proof, only to an appreciation of its rightness, deeper or not according to the reader's depth of mathematical understanding.

Similarly with moral concepts. Experience is needed to form the concepts that go into them, in particular, the experiences that allow a pre-school child to form a theory of other minds: that other people have autonomous minds that have thoughts and wishes like one's own, but possibly not identical to one's own. That allows the child (at least one living in an appropriately supportive culture) to develop concepts of fairness, by recognizing that those other minds are not relevantly morally different to one's own, when it comes to getting what they deserve.[38] Piaget describes how very young

[36] B. Lonergan, *Insight: A Study of Human Understanding,* 3rd edn, (New York: Philosophical Library, 1970), 7–11.

[37] H. Weyl, *Symmetry* (Princeton, 1952).

[38] J. Piaget *et al., The Moral Judgment of the Child* (London: Routledge & Kegan Paul, 1932); R. Duska and M. Whelan, *Moral Development: A Guide to Piaget and Kohlberg* (New York: Paulist Press, 1975); C. C. Helwig, 'Children's conceptions of fair government and freedom of speech', *Child Development,* **69** (1998), 518–31; R. L. Campbell and J. C. Christopher, 'Moral development theory: a critique of its Kantian presuppositions', *Developmental Review,* **16** (1996), 1–47.

children believe that what is right is simply what is forbidden by adult authority. But at a later stage they develop a sense of fairness based on a sense of equality — initially a rather simplistic one:

> Some children are playing ball in a courtyard. When the ball goes out of bounds and rolls down the road one of the boys goes of his own free will to fetch it several times. After that he is the only one they ask to go and fetch it. What do you think of that?
> Wal (6) '*It isn't fair.* —Why? —*Because another boy should go.*'
> Schma (7) '*It's not fair, because they should have asked the others, and each in turn.*'

But the simple 'same for each' standard of equality soon comes to have added to it a capacity to take into account differences in the individuals, which may require differences in how they are treated in order to make the treatment fair.

> Two boys were running races. One was big, the other small. Should they both have started from the same place, or should the little one have started nearer?
> Bri (6) '*The little boy must have a start because the big boy can run faster than the little one.*'

(Here again, the adaptation to differing circumstances is not a qualification of the principle of equality, but an implication of it.) Piaget identifies 'three great periods in the development of the sense of justice in the child. One period, lasting up to the age of 7–8, during which justice is subordinated to adult authority; a period contained approximately between 8–11, and which is that of progressive equalitarianism; and finally a period which sets in towards 11–12, and during which purely equalitarian justice is tempered by considerations of equity.'[39]

On the contrast between mathematics and morals

Of course mathematics and ethics have important contrasts too, because of their different subject matters.

The contrast is perhaps most clearly brought out through the role played by the emotions in moral epistemology. Someone who does not have an immediate reaction of horror to photographs of the death camps seems to lack a necessary insight into the worth of

[39] J. Piaget *et al.*, *The Moral Judgment of the Child*, trans. M. Gabain (New York: Free Press, 1965), 309–15.

persons. Raimond Gaita asks us to imagine a tutorial in which one of its members had been a victim of a terrible evil of which all the other members were aware. What if the tutor asked the class to consider whether our sense of the terribleness of evil were not an illusion? 'Everyone would be outraged if their tutor were not serious and struck by unbelieving horror if he was.'[40] It would not be helpful to try to recast that reaction as a deliverance of 'reason', if reason is a term designed to contrast with 'emotion'. Gaita rightly complains of 'a distinction between reason and emotion that distorts our understanding of one of the most important facts about the ethical—that we often learn by being moved by what others say or do.'[41] Our ability to acquire moral knowledge by immediate emotional empathy with other humans is why serious novels can deepen our moral understanding—for example, when Pasternak in *Doctor Zhivago* has the fully developed character of Lara disappear into the Gulag, it is the empathy the reader has developed with the character that points up the moral horror of a political system that treats people like vermin. A fundamental demand of humans to be recognised as human by others is one of the 'needs of the soul', in Simone Weil's words;[42] it is prior (in knowledge) to any speculations about what features of human nature may generate it, or any identification of rights. And without at least some of that initial emotional attunement to the irreducible worth of humans, there can be no meaning to discussions of human nature or rights.

None of that applies to mathematics. It is clear why there can be autistic mathematicians but not autistic novelists or moral theorists.

Moral philosophy is preeminently a field requiring mature discernment of its practitioners. A person must come to base his judgments on his own understanding, not on the dictates of external authority. (That of course no more implies a relativism about values than the fact that one uses one's own mind to decide on the truth of mathematical theorems rather than accepting the authority of a teacher implies a relativism about the propositions of mathematics.) The development of discernment must be based on experience, and it is certainly true that the kind of experience required is different from the experience that leads to mathematical maturity. Moral experience is more personal, both in the sense of involving more of

[40] R. Gaita, *Good and Evil: An Absolute Conception* (Basingstoke: Macmillan, 1991), 319.

[41] R. Gaita, 'Reflections on the euthanasia debate', *Eureka Street,* **5** (10) (Dec, 1995), 22-7.

[42] S. Weil, *The Need for Roots* (London: Routledge & Kegan Paul, 1952), 3.

one's own personality, and requiring insight into the depth of the personality of others. Though there can be child saints, there are no child prodigies in moral philosophy or law, since long experience in the human world is necessary for maturity in those fields. Mathematics is different, naturally. It deals with a more impersonal subject matter, and there can be prodigies in mathematics as there are in chess, since solving difficult puzzles can be a genuine mathematical advance.

Nevertheless, the differences should not be exaggerated, especially when considering the kinds of experience that induce mathematical and ethical insight. The sense of 'revelation' that some report about ethical insights is harder to remember in basic mathematics, since the original insights of number and geometry occur at an age covered by infantile amnesia. But that experience can be seen at least from the outside in the children studied by such experiments as Piaget's. At a later age, though raw puzzle-solving power is admired in young mathematicians, there is also such a thing as mathematical maturity, often required for admission into higher courses and much prized among the leaders of the profession who determine which questions will be considered 'interesting'. So, despite its lack of emotional and interpersonal content, mathematical experience does have its subtleties and relation to the maturing stages of the human person.

In any case, such contrasts as do exist between mathematics and ethics are not of such a nature as to detract from the parallels, in their objectivity and in the foundational role of equality.

Conclusion

In 1930, when there was less nervousness than today about expressing robustly objectivist views on ethics, W. D. Ross wrote, in *The Right and the Good:*

> That an act, *qua* fulfilling a promise, or *qua* effecting a just distribution of good, or *qua* returning services rendered, or *qua* promoting the virtue or insight of the agent, is *prima facie* right … is self-evident just like a mathematical axiom, or the validity of a form of inference, is evident. The moral order expressed in these propositions is just as much part of the fundamental nature of the universe (and, we may add, of any possible universe in which there were moral agents at all) as is the spatial or numerical structure expressed in the axioms of geometry or arithmetic. In our confidence that these propositions are true there is involved the

> same trust in our reason that is involved in our confidence in mathematics; and we should have no justification for trusting it in the latter sphere and distrusting it in the former. In both cases we are dealing with propositions that cannot be proved, but that just as certainly need no proof.[43]

A great deal of suspicion has flowed through the Western mind since then, and the Zeitgeist has whispered many insinuations about how sophisticated moderns understand historical conditioning and are not taken in by objectivist claims. To mathematicians, it has been water off a duck's back. It should be the same for moral philosophers, and for the same reasons.

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[43] W. D. Ross, *The Right and the Good* (Oxford: Clarendon, 1930), 29–30; further 32–4.