Uncertainty and Trade Elasticities *

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Abstract

Incomplete information makes trade more elastic. When firms face uncertainty about demand, selection into exporting occurs based on productivity alone. In contrast, with complete information firms can condition export decisions on both productivity and demand. The difference in the information available to firms alters the value of trade at the extensive margin. We show that the identification of trade elasticities with incomplete information requires quantity data, while trade elasticities in complete information economies require sales data. Using Brazilian export data, we quantify trade elasticities in models with and without uncertainty, and find that the elasticities are larger under uncertainty. This gap increases when demand is more uncertain.

Keywords: Uncertainty, firm size distribution, extensive margin, trade elasticities.

JEL: F12, F13.

1 Introduction

Welfare implications for standard trade theory, most recently developed in Arkolakis, Costinot, and Rodríguez-Clare (2012) and Melitz and Redding (2015), show that partial elasticities of trade with respect to variable trade costs are key parameters for evaluating the welfare gains from trade. While these implications are derived from a broad class of models in which firms have complete information about their economic environment, a growing branch of the

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trade literature has demonstrated that models with uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior.[1] However, normative implications of this alternative information structure for measurements of the trade elasticities, and therefore welfare gains from trade, are not yet well understood.[2] In this paper, we develop trade elasticity expressions for trade models with demand uncertainty and quantitatively show that demand uncertainty alters the size of these key trade elasticities that ultimately determine the welfare gains from trade.

We show that the distinction between firm *productivity* versus *profitability* is key for identifying trade elasticities in economic environments with and without uncertainty. To do so, we consider two economic environments, the first of which is a stylized version of Jovanovic's (1982) learning model as extended by Timoshenko (2015b) to a trade context. In this model, firms make export decisions after observing their firm-specific productivity but before observing their firm-specific demand in foreign markets. Hence, firms make export quantity and participation decisions based on *productivity* alone. Export sales thereafter depend on productivity, through export quantity decisions, and the realization of demand shocks in foreign markets. Therefore, standard empirical strategies that identify trade elasticities from export sales data are inappropriate in this framework, as export sales data contain information that does not directly influence how a firm responds to changes in trade costs. Instead, export quantity data does contain the necessary information about productivity to identify trade elasticities.

The second economic environment we consider is a standard trade model with complete information (Melitz (2003), Bernard, Redding, and Shott (2010), Arkolakis et al. (2012), Melitz and Redding (2015)), in which firms observe both productivity and demand prior to making export decisions. In contrast to an economy with uncertainty, in this model export decisions are based on a combination of a firm's productivity and demand, defined as a firm's *profitability*. The appropriate information about profitability is embedded in export sales data, which therefore identifies trade elasticities in a complete information environment.

We quantify the magnitude of the trade elasticity under different information environments and find that trade is more elastic in the presence of demand uncertainty. For quantification, we adapt the structural elasticity estimation approach in Bas, Mayer, and Thoenig

[1]These papers incorporate Jovanovic (1982) learning mechanism into the Melitz (2003) model, which features monopolistically competitive exporters that are heterogeneous in productivity and learn about their unobserved idiosyncratic demand in foreign markets. See Arkolakis, Papageorgiou, and Timoshenko (2018) for implications for firm growth as a function of age and size, Timoshenko (2015b) for implication for firm product switching behavior, and Bastos, Dias, and Timoshenko (Forthcoming) for implications for firm input and output pricing behavior.

[2]A notable exception is Arkolakis, Papageorgiou, and Timoshenko (2018), who characterize constrained efficiency of a model in which firms learn about demand, but do not engage in international trade.

(2017) to our model with demand uncertainty and to the model with complete information. We discipline the complete information model's profitability distribution with empirical export sales data and discipline the demand uncertainty model's productivity distribution with data on quantities exported. We apply this estimation strategy to Brazilian export data and find that demand uncertainty amplifies an increase in export sales arising from new entrants in response to a decline in variable trade costs - the extensive margin response - relative to the model with complete information. We further find that the amplification effect is larger when an export destination exhibits larger demand uncertainty, as measured by the variance of demand shocks realizations.

This paper shows that the information structure faced by firms is crucially important for measuring the extensive margin response to a decline in trade costs. In countries or industries in which exporters face high demand uncertainty, by assuming away information asymmetries, estimates of the partial elasticity of trade with respect to variable trade costs will likely understate the true magnitude of extensive margin adjustments, and therefore, the extent of welfare gains.

Our work contributes to the growing literature on decomposing trade elasticities. Chaney (2008) shows that the partial elasticity of trade can be decomposed into an intensive and an extensive margin of adjustment. Melitz and Redding (2015) further show that the extensive margin of adjustment crucially depends on the distributional assumptions with respect to the sources of firm-level heterogeneity. Sager and Timoshenko (2017) characterize a flexible distribution that well describes firm-level heterogeneity and find the extensive margin trade elasticity to be small. This paper demonstrates that selection into exporting (and hence the extensive margin of trade elasticity), depends on the information structure faced by firms.

Our work also contributes to a literature on measuring trade elasticities. Imbs and Mejean (Forthcoming) find that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production. Furthermore, Imbs and Mejean (2015) document that elasticities computed using industry-level data are often larger than those using aggregated data. This paper demonstrates that firms' information sets affect trade elasticity measurement and documents an amplification effect on trade elasticities attributed to uncertainty faced by firms in foreign markets.

A related strand of literature estimates trade elasticities based on complete information models of trade. Eaton and Kortum (2002) and Simonovska and Waugh (2014) use the aggregate trade flows and prices data to estimate trade elasticities. In contrast, Caliendo and Parro (2015) rely on trade flows and tariffs data. We contribute to this literature by providing an alternative method based on a structural model of trade to compute trade elasticities and show how an assumption about the information structure faced by firms alters elasticity estimates.

Our paper also relates to the literature on information asymmetries in trade. Two highly related papers are by Timoshenko (2015a) and Dickstein and Morales (2016). Timoshenko (2015a) finds that past continuous export history predicts current export choice. Dickstein and Morales (2016) extends this work and demonstrates that firm-level information such as lagged sales and industry averages predict exporting, but only for large firms (e.g., in terms of productivity or domestic sales). The authors find that small firms do not seem to make decisions based on this additional information. While these papers uncover a large set of factors that predict export decisions, our paper abstracts from such possible correlations in order to highlight as clearly as possible the interaction between information and trade elasticities.[3]

The rest of the paper is organized as follows. Section 2 presents the theoretical framework, contrasts the elasticity implications between an environment with and without uncertainty, and describes a method to estimate trade elasticities based on a model with demand uncertainty. Section 3 presents elasticity estimation results. Section 4 concludes. Appendix A provides a detailed description of the demand uncertainty model and complete information model. Appendix B provides proofs to all Propositions. Appendix C demonstrates that our results are robust to the alternative firm-level choice variable under uncertainty.

2 Theoretical Framework

2.1 Economic Environment

In this section we consider a model with heterogeneous firms that export products in markets characterized by monopolistic competition. This environment is similar to that in Melitz (2003), and we assume exogenous entry as in Chaney (2008). We further introduce information asymmetries by constructing a stylized version of the learning model in Jovanovic (1982) as was embedded into a trade model in Timoshenko (2015b) to a trade context. All derivations are relegated to Appendix A.

[3] Furthermore, Bergin and Lin (2012) show that the entry of new varieties increases at the time of the announcement of the future implementation of the European Monetary Union, suggesting that changes in the information available to firms have immediate consequences for firms' decisions. Lewis (2014) studies the effect of exchange rate uncertainly on trade; Allen (2014) shows that information frictions help to explain price variation across locations; Fillat and Garetto (2015) show that aggregate demand fluctuations can explain variation in stock market returns between multinational and non-multinational firms; Handley and Limao (2015) show that when trade policy is uncertain, there is less entry into foreign markets.

2.1.1 Demand

There are N countries and K sectors, such that each country is indexed by j and each sector is indexed by k. Each country is populated by a mass of L_j identical consumers whose preferences are represented by a nested constant elasticity of substitution utility function given by

$$U_j = \prod_{k=1}^{K} \left[\left(\int_{\omega \in \Omega_{ijk}} \left(e^{\theta_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k - 1}{\epsilon_k}} d\omega \right)^{\frac{\epsilon_k}{\epsilon_k - 1}} \right]^{\mu_k}, \tag{1}$$

where Ω_{ijk} is the set of varieties in sector k consumed in country j originating from country i, $c_{ijk}(\omega)$ is the consumption of variety $\omega \in \Omega_{ijk}$, ϵ_k is the elasticity of substitution across varieties within sector k, $\theta_{ijk}(\omega)$ is the demand shock for variety $\omega \in \Omega_{ijk}$, and μ_k is the Cobb-Douglas utility parameter for goods in sector k such that $\sum_{k=1}^{K} \mu_k = 1$.

Each consumer owns a share of domestic firms and is endowed with one unit of labor that is inelastically supplied to the market. Cost minimization yields a standard expression for the optimal demand for variety $\omega \in \Omega_{ijk}$, given by

$$c_{ijk}(\omega) = e^{\theta_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega \in \Omega_{ijk}$, Y_{jk} is total expenditures in country j on varieties from sector k, and P_{jk} is the aggregate price index in country j in sector k.[4]

2.1.2 Supply

Each variety $\omega \in \Omega_{ijk}$ is supplied by a monopolistically competitive firm. Each firm can potentially supply one variety of a product from each sector. Upon entry, a firm is endowed with an idiosyncratic labor productivity level e^{φ} and a set of idiosyncratic product-destination specific demand shocks $\{\theta_{ijk}\}$.[5] Productivity and demand shocks are drawn from independent distributions. Denote by $g^{\varphi}_{ijk}(.)$ the distribution from which firms draw productivity, φ, and by $g^{\theta}_{ijk}(.)$ the distribution from which firms draw demand shock, θ_{ijk}. Firms from country i face fixed costs, f_{ijk}, and variable costs, τ_{ij}, of selling output to country j. Fixed and variable costs are denominated in units of labor.

[4] Note that $Y_{jk} = \mu_k Y_j$, where Y_j is aggregate income in country j.

[5] Following the finding of Foster, Haltiwanger, and Syverson (2008) who document that idiosyncratic firm-level demand shocks rather than productivity account for a greater variation in sales across firms, we focus on the demand shocks that are firm specific. Each firm from country i draws a separate demand shock for each destination j and industry k. The ijk subscript on θ_{ijk} therefore indicates that idiosyncratic firm-level demand shocks are origin-destination-sector specific. The superscript does not refer to an aggregate origin-destination-sector level shock that is common across firms. We abstract from such potential aggregate shocks since they do not affect properties of the distribution of quantities or sales across firms, which is the focus of our analysis.

We assume that firms do not possess complete information when making export decisions. In particular, while firms always observe their productivity shock, φ, they do not observe their demand shocks, θ_{ijk}, when making export decisions. Therefore, firms choose a quantity to export to each destination market before knowing destination-specific demand shocks for their product. Firms choose export quantities to maximize *expected* profits, subject to consumer demand (2) and prior beliefs about demand, $E_\theta(\exp(\theta_{ijk}/\epsilon_k))$.[6] Henceforth, the subscript of the expectation operator indicates that the expectation is taken with respect to the distribution of the random variable indicated in the subscript.

The firm's decision problem yields an expression for the optimal quantity exported, given by

$$q_{ijk}(\varphi) \quad = \quad B^q_{ijk} \cdot e^{\epsilon_k \varphi}. \tag{3}$$

where B^q_{ijk} is an origin-destination-industry fixed effect.[7] The corresponding productivity entry threshold is given by

$$e^{(\epsilon_k - 1)\varphi^*_{ijk}} \quad = \quad \frac{B^\varphi_{ijk}}{\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}, \tag{4}$$

where B^φ_{ijk} is an origin-destination-industry fixed effect. Once all goods are supplied to markets, demand shocks are realized and prices clear the goods markets for each variety. A firm's realized export sales are given by

$$r_{ijk}(\theta_{ijk}, \varphi) \quad = \quad B^r_{ijk} \cdot e^{(\epsilon_k - 1)\varphi + \frac{\theta_{ijk}}{\epsilon_k}}, \tag{5}$$

where B^r_{ijk} is an origin-destination-industry fixed effect.

2.1.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} \quad \equiv \quad M_{ijk} \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\theta, \varphi) h_{ijk}(\theta) \frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi > \varphi^*_{ijk})} d\theta d\varphi,$$

[6] We assume that prior beliefs are the same across firms and equal the population mean. This assumption does not fundamentally change our results. In principle, we could expand the set of shocks on which a firm bases its quantity decision to any idiosyncratic shocks including demand expectations which could be arbitrarily correlated with firm productivity. What is important is that some idiosyncratic information is known to a firm before making decisions, and some idiosyncratic information is revealed to that firm after those decisions have been made.

[7] We refer the reader to Appendix A.1.1 for a derivation of and full expression for the origin-destination-industry fixed effects found in equations (3), (4) and (5).

where M_{ijk} is the mass of firms exporting from country i to country j in industry k, and φ^*_{ijk} is the productivity entry threshold. The partial elasticity of trade flows with respect to the variable trade costs is given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = \underbrace{(1-\epsilon_k)}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left(\underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\gamma_{ijk}(\phi^*_{ijk})}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right), \tag{6}$$

where $\phi_k = (\epsilon_k - 1)\varphi$ is a rescaled productivity, and the extensive margin is given by

$$\gamma_{ijk}(\phi^*_{ijk}) = \frac{e^{\phi^*_{ijk}} g^{\phi}_{ijk}(\phi^*_{ijk})}{\int_{\phi^*_{ijk}}^{+\infty} e^{\phi} g^{\phi}_{ijk}(\phi) d\phi}. \tag{7}$$

Equation (6) decomposes the partial trade elasticity into intensive and extensive margin components, and equation (7) shows that the extensive margin of the partial trade elasticity is governed by the rescaled productivity entry threshold, ϕ^*_{ijk}, and the rescaled productivity distribution, $g^{\phi}_{ijk}(\cdot)$.

2.2 Complete Information Environment

In a complete information environment, firms observe both the productivity and demand shocks upon entry.[8] Firms' decisions therefore depend on a single profitability parameter defined by $z_{ijk} \equiv (\epsilon_k - 1)\varphi + \theta_{ijk} = \phi_k + \theta_{ijk}$. Productivity and demand shocks therefore simultaneously determine selection into exporting through their impact on the export entry threshold, the export sales distribution and the partial trade elasticity. In particular, under complete information, the single *profitability* entry threshold, z^*_{ijk} is given by

$$e^{z^*_{ijk}} = B^{\varphi}_{ijk}. \tag{8}$$

Subsequently, a firm's export sales are given by

$$r^{CI}_{ijk}(z_{ijk}) = B_{ijk} e^{z_{ijk}}, \tag{9}$$

and the partial trade elasticity is given by

$$\eta^{CI}_{ijk} = \underbrace{(1-\epsilon_k)}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left(\underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\gamma^{CI}_{ijk}(z^*_{ijk})}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right), \tag{10}$$

[8]Appendix A.2 contains a formal description of the complete information economy.

where the superscript 'CI' stands for the 'Complete Information' environment, and the extensive margin is given by

$$\gamma_{ijk}^{CI}(z_{ijk}^{*}) \quad = \quad \frac{g_{ijk}^{z}(z_{ijk}^{*})e^{z_{ijk}^{*}}}{\int_{z_{ijk}^{*}}^{+\infty} e^{z} g_{ijk}^{z}(z)dz}, \tag{11}$$

where $g_{ijk}^{z}(.)$ is the distribution of firm profitability.

2.3 Properties of the Partial Trade Elasticity

In comparing the expressions for the partial trade elasticity between the incomplete versus complete information environments, equations (6) versus (10) clearly indicate that the two elasticities differ solely along the extensive margin dimension. In this section we establish a set of properties of the extensive margin component of the partial trade elasticity that will enable us to show that (under some mild conditions) trade is more elastic under incomplete information.

Observe from equations (7) and (11) that the extensive margin component of the partial trade elasticity admits the same functional form in both information environments and can be simply written as

$$\gamma(x) \quad \equiv \quad \frac{e^{x}g(x)}{\int_{x}^{+\infty} e^{z}g(z)dz}. \tag{12}$$

Here we drop all the subscripts and focus solely on the functional form of the extensive margin and its properties. Observe that given a distribution $g(.)$, the extensive margin $\gamma(x)$ is a function of the entry threshold denoted by x. Similarly, for a given threshold x, the extensive margin $\gamma(x)$ is a function of the distribution $g(.)$. To proceed we make the following assumptions about distribution $g(x)$.

Assumption 1 (A1) *The probability density function $g(x)$ has the following properties:*

(i) $x \in \mathbb{R}$ *is the support of the distribution,*

(ii) $E(e^{x}) \equiv \int_{-\infty}^{+\infty} e^{z}g(z)dz$ *exists and is finite, and*

(iii) the function $\log\left(\int_{x}^{+\infty} e^{z}g(z)dz\right)$ *is concave in* x.

Assumption (iii), that the log of the conditional expectation of profitability is a concave function of the threshold value, is important for understanding what assumptions on uncertainty make the extensive margin more elastic. Assumption (iii) requires that the upper tail of the distribution $g(x)$ not have too much mass. Without such a restriction, total sales of marginal firms relative to average sales could become very small as the threshold increases, and the extensive margin elasticity might not be monotonically increasing. Monotonicity will allow us to characterize the effect of uncertainty on the extensive margin elasticity. Accordingly, Proposition 1 below establishes two novel properties of the extensive margin $\gamma(x)$.

Proposition 1 *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

(i) $\gamma(x) \equiv [e^x g(x)]/\int_x^{+\infty} e^z g(z)dz$ *is an increasing function of x.*

(ii) Denote the extensive margin elasticity associated with $g(x)$ as $\gamma(x)$. Let $\tilde{g}(x)$ be a mean preserving spread of $g(x)$, with extensive margin elasticity $\tilde{\gamma}(x)$. There exists x^* such that $\tilde{\gamma}(x) < \gamma(x)$ for all $x > x^*$, $\tilde{\gamma}(x) = \gamma(x)$ if $x = x^*$, and $\tilde{\gamma}(x) > \gamma(x)$ for all $x < x^*$.

To provide some intuition for why Proposition 1 holds, it is instructive to normalize the numerator and denominator of $\gamma(x)$ by $\int_{-\infty}^{+\infty} e^z g(z)dz$ as follows,

$$\gamma(x) \quad = \quad \frac{e^x g(x)/\int_{-\infty}^{+\infty} e^z g(z)dz}{\int_x^{+\infty} e^z g(z)dz/\int_{-\infty}^{+\infty} e^z g(z)dz}.$$

Define $h(x) = e^x g(x)/\int_{-\infty}^{+\infty} e^z g(z)dz$. Notice that $\int_{-\infty}^{+\infty} h(x)dx = 1$ and $h(x) \geq 0$. Hence, $h(x)$ is the probability density function. The corresponding cumulative distribution function is then given by

$$H(x) = \int_{-\infty}^{x} e^z g(z)dz \Big/ \int_{-\infty}^{+\infty} e^z g(z)dz. \tag{13}$$

The corresponding survival function is given by $1 - H(x) = \int_x^{+\infty} e^z g(z)dz/\int_{-\infty}^{+\infty} e^z g(z)dz$. With this change in notation, the extensive margin $\gamma(x)$ is given by

$$\gamma(x) = \frac{h(x)}{1 - H(x)}. \tag{14}$$

Hence, the extensive margin of the partial trade elasticity is a hazard rate associated with a random variable X distributed according to $h(x)$. As such, $\gamma(x)$ inherits properties of the hazard rate function the first of which is being a monotonically increasing function. Notice that part (iii) of A1 ensures that distribution $h(x)$ has a log-concave survival function. Log-concavity of the survival function ensures that the corresponding hazard rate is increasing: Notice from equation (14) that $\gamma'(x) = -d^2 \log(1 - H(x))/dx^2$ which is positive if and only if $\log(1 - H(x))$ is concave.

Part (ii) of Proposition 1 shows that the extensive margin elasticity as a function of threshold values $x \in \mathbb{R}$ exhibits a single crossing property. The single crossing property establishes that the extensive margin elasticity function associated with the cumulative distribution function of the mean preserving spread, $\tilde{\gamma}(x)$, only crosses the extensive margin elasticity function associated with the less dispersed cumulative distribution function , $\gamma(x)$, once from above. Notice that equation (14) implies that $\gamma(x) = -d\log(1 - H(x))/dx$. As a result, as we show in the proof of Proposition 1 in Appendix B, $\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$. By equation (13), the cumulative distribution functions $H(x)$ and $\tilde{H}(x)$ are defined by distributions $g(x)$ and $\tilde{g}(x)$ respectively. By part (ii) of Proposition 1, $\tilde{g}(x)$ is a mean preserving spread of $g(x)$. Hence $G(x)$ crosses $\tilde{G}(x)$ once from below. As a result, this single-crossing property is also preserved when defining a distribution according to transformation in equation (13). Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x)$ from below once, and for all values of x sufficiently large we know $\gamma(x) > \tilde{\gamma}(x)$.

A corollary of part (ii) of Proposition 1 is that the single crossing property of the extensive margin holds for any affine transformation of the abscissa for either of the functions.

Corollary 1 *Let $g(x)$ be a probability density function satisfying A1. $\forall a \in \mathbb{R}$ there exists $x^*(a)$ such that $\gamma(x) > \tilde{\gamma}(x+a)$ if $x > x^*(a)$; $\gamma(x) = \tilde{\gamma}(x+a)$ if $x = x^*(a)$, and $\gamma(x) < \tilde{\gamma}(x+a)$ if $x < x^*(a)$.*

In the next subsection we use insights from the discussion herein to show that the extensive margin of the partial trade elasticity is larger under incomplete information.

2.3.1 Comparison of the Partial Trade Elasticities

Observe from equations (6), (7), (10), and (11) that the difference in the partial trade elasticity between the two information environments arises from the extensive margin. Further, the difference in the extensive margin arises from two separate channels: the entry threshold, ϕ^*_{ijk} versus z^*_{ijk}, and the distribution of the corresponding shock, $g^{\phi}_{ijk}(.)$ versus $g^{z}_{ijk}(.)$.

To compare the extensive margin elasticities between the two information environments, $\gamma_{ijk}(x)$ versus $\gamma^{CI}_{ijk}(x)$, let us first focus on the distribution that define those elasticities, $g^{\phi}_{ijk}(.)$ versus $g^{z}_{ijk}(.)$. Assume that both distributions satisfy Assumption 1. It is worth noting that distributions that are commonly used in the trade literature, such as a Normal distribution satisfy A1. Furthermore, the Double EMG distribution that we will use in Section 3.2.1 also satisfies A1.

Recall that profitability shock z is defined as $z = \phi + \theta$, where ϕ and θ are independent and are drawn from the probability density functions $g^{\phi}_{ijk}(.)$ and $g^{\theta}_{ijk}(.)$ respectively. Without the loss of generality we can assume that the mean of θ equals zero. In this case, $g^{z}_{ijk}(.)$ is a mean-preserving spread of $g^{\phi}_{ijk}(.)$. Hence, by part (ii) of Proposition 1, for a sufficiently high entry threshold, $\gamma_{ijk}(x) > \gamma^{CI}_{ijk}(x)$, and therefore trade is more elastic under incomplete information. We will now show that this property holds even if the entry thresholds are different.

From equations (4) and (8), the entry threshold ϕ^*_{ijk} under incomplete information versus z^*_{ijk} under complete information are related as follows: $\phi^*_{ijk} = z^*_{ijk} - \log\left(E_\theta\left(\exp(\theta_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. Therefore, the proper comparison of elasticities involves comparing $\gamma_{ijk}(x)$ and $\gamma^{CI}_{ijk}(x+a)$, where constant $a \equiv \log\left(E_\theta\left(\exp(\theta_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. By Corollary 1, for ϕ^*_{ijk} high enough, $\gamma_{ijk}(\phi^*_{ijk}) > \gamma^{CI}_{ijk}(\phi^*_{ijk}+a) = \gamma^{CI}_{ijk}(z^*_{ijk})$, i.e. trade is more elastic under complete information.

To summarize, as follows from our discussion, trade is more elastic under complete information because the distribution of the shock that determines the partial trade elasticity is more dispersed under complete information than under uncertainty. Under uncertainty, trade elasticity is determined by the distribution of productivity, while under complete information trade elasticity is determined by the distribution of the profitability shock, a mean preserving spread of productivity. As a result, the associated hazard rate, consequently the partial trade elasticity, is lower under complete information than under uncertainty.

We next demonstrate that the information environment faced by firms affects not only the magnitude of the partial trade elasticity, but also the type of data that are needed to identify the partial trade elasticity.

2.4 Identification of the Partial Trade Elasticity

Comparing equations (6) and (10) makes clear that the information environment directly impacts the identification of the extensive margin of the partial trade elasticity. Intuitively, the information available to firms at the time of making export decisions determines how responsive their decisions are to changes in variable trade costs. Therefore, quantifying the partial trade elasticity in the complete and incomplete information economies requires different data for proper identification.

As can be seen from equations (10) and (11), under complete information the trade elasticity is identified by the profitability distribution, $g^z_{ijk}(.)$, and the profitability entry threshold, z^*_{ijk}. Profitability, is exactly the information that determines firm entry decisions by equation (8) and subsequently the equilibrium the distribution of firm sales by equation (9). Hence, export sales data contain the information about profitability that is necessary to identify the partial trade elasticity under complete information environment.

As can be seen from equations (6) and (7), under incomplete information the partial trade elasticity is identified by the rescaled productivity distribution, $g^\phi_{ijk}(.)$, and the rescale productivity entry threshold, ϕ^*_{ijk}. Hence, the partial trade elasticity is governed by productivity, which, under incomplete information, is the only information available to the firms at the time at which they make export decisions. As can be seen from equations (3) and (5), the model implies that productivity can be identified from data on export quantities, but not export sales. Equation (3) shows that, conditional on variables common to all firms (contained in B^q_{ijk}), a firm's export quantity decision is entirely governed by its firm-level productivity shock, φ. Since firms only observe their productivity, and productivity is the only idiosyncratic information upon which firms base their export decisions, productivity alone determines a firm's production response to a potential change in variable trade costs. Therefore, equation (3) shows that such productivity information is contained exactly in the export quantity data.

This is in contrast to the information embedded in the export sales data. Equation (5) shows that, conditional on aggregate variables that are common across all firms (B^r_{ijk}), a firm's export sales depend on both the firm's known productivity, φ, and the firm's subsequent idiosyncratic demand realization, θ_{ijk}. The idiosyncratic firm-level demand shocks only affect the realized distribution of sales across firms, but play no role in a firm's decision making process and therefore do not affect the firm's response to a change in variable trade costs. As a result, the model implies that, relative to export quantity data, export sales data contain additional information that does not directly influence firms export decisions.

Note that productivity and demand are two standard interpretations of model ingredients relative to the data. Following the trade literature, the model's productivity and demand shocks stand in for any variation that allows the model to be consistent with the data on sales and quantities. What is important for our paper is that, in environments with uncertainty, productivity shocks stand in for any information that firms possess at the time of making export decisions while demand shocks stand in for any information that determines sales and that is revealed after exporting goods.

To summarize, quantity data identifies the extensive margin contribution to the partial trade elasticity by enabling inference about the productivity distribution and productivity entry threshold. In contrast, it is only appropriate to identify the partial trade elasticity using export sales data when firms possess complete information about their demand. In the next section we adopt an estimation approach suggested by Bas, Mayer, and Thoenig (2017) and extended by Sager and Timoshenko (2017). Relative to these papers, we extend the approach to an environment with demand uncertainty.

2.5 Estimation Approach

In this section we detail our approach to estimating partial trade elasticities in the presence of demand uncertainty. As shown in equation (6), in an environment with uncertainty, selection occurs based on the productivity alone. Hence the extensive margin of trade elasticity depends on the productivity entry threshold and the distribution of the productivity draws. Both can be recovered using the data on the distribution of export quantity as we now describe.

Consider the following change of notation. Let $\tilde{\varphi} \equiv \epsilon_k \varphi$ and denote by $g^{\tilde{\varphi}}(.)$ the probability distribution function of $\tilde{\varphi}$. Given the change in notation, $g^{\tilde{\varphi}}(.)$ is just the distribution of φ, $g^{\varphi}_{ijk}(.)$ scaled by the elasticity of substitution, ϵ_k.

Given this change in notation, the partial trade elasticity can be expressed as

$$\eta_{ijk} = (1-\epsilon_k)\left(1 + \frac{\epsilon_k}{\epsilon_k - 1} \frac{g^{\tilde{\varphi}}(\tilde{\varphi}^*) e^{\frac{\epsilon_k - 1}{\epsilon_k}\tilde{\varphi}^*}}{\int_{\tilde{\varphi}^*}^{+\infty} e^{\frac{\epsilon_k - 1}{\epsilon_k}\varphi} g^{\tilde{\varphi}}_{ijk}(\varphi) d\varphi}\right). \tag{15}$$

The distribution $g^{\tilde{\varphi}}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. Taking the logarithm of equation (3) yields

$$\log q_{ijk} = \log B^q_{ijk} + \tilde{\varphi}. \tag{16}$$

Observe that the distribution of log-export quantity is given by the distribution of $\tilde{\varphi}$ scaled by a constant. Hence, parameters of $g^{\tilde{\varphi}}_{ijk}(.)$ can be recovered from fitting the distribution to the empirical distribution of log-export quantity.

Given the scaled productivity distribution, $g^{\tilde{\varphi}}_{ijk}(.)$, we follow Bas et al. (2017) in recovering the scaled productivity threshold, $\tilde{\varphi}^*_{ijk}$, by matching the model-implied average-to-minimum ratio to that in our quantity data. The model-implied average-to-minimum ratio of export quantities given by

$$\text{Average-to-Minimum Ratio} = e^{-\tilde{\varphi}^*_{ijk}} \int_{\tilde{\varphi}^*_{ijk}}^{+\infty} \frac{e^{\varphi} g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi > \tilde{\varphi}^*_{ijk})} d\varphi. \tag{17}$$

In the Section 3 we apply the described elasticity estimation approach to quantify the partial trade elasticity.

3 Quantifying Trade Elasticities

3.1 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[9] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

We proxy the theoretical notion of export quantity with an empirical measure of export

[9]For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

weight.[10] The properties of export weight differ substantially across industries. Hence, we further conduct our analysis at the destination-year-industry level where we define an industry as a 6-digit HS code.

We define an observation to be a distribution of export quantity across firms for a given destination-year-industry triplet, and focus on observations where at least 100 firms export.[11]

The final sample consists of 190 destination-year-industry observations, and covers 14 destinations and 35 industries. Table 1 provides summary statistics of log-export quantities and log-sales distributions in our sample.

3.2 Parameter Estimates

3.2.1 The Export Quantity Distribution

To recover the partial trade elasticity we proceed by, first, assuming that the productivity is drawn from a Double EMG distribution, $DEMG(m, \upsilon^2, \xi_L, \xi_R)$. The resulting log-export quantity distribution, $g_{ijk}^{\tilde{\varphi}}(.)$, then also follows a Double EMG distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$ with parameters scaled by the elasticity of substitution, ϵ_k, and described by the following cumulative distribution function:[12]

$$\begin{aligned} G(\varphi) = \Phi\left(\frac{\varphi-\mu}{\sigma}\right) \quad &- \quad \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(\varphi-\mu)+\frac{\sigma^2}{2}\lambda_R^2} \Phi\left(\frac{\varphi-\mu}{\sigma}-\lambda_R\sigma\right) \\ &+ \quad \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(\varphi-\mu)+\frac{\sigma^2}{2}\lambda_L^2} \Phi\left(-\frac{\varphi-\mu}{\sigma}-\lambda_L\sigma\right), \end{aligned} \tag{18}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.[13]

The Double EMG distribution provides a very flexible generalization of common distributional assumptions used in the literature. From equation (18), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2017) and Fernandes et al. (2015). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution. By assuming the Double EMG distribution we, therefore, allow the data to recover the best fit of distribution between the Exponential, Normal, Double Exponential or the

[10]Export weight is used as a measure of export quantity in a number of studies including Bastos et al. (Forthcoming).

[11]The thresholds of 100 firms ensures that an empirical distribution can be accurately described by percentiles. This threshold is also consistent with the literate. See Fernandes et al. (2015), Sager and Timoshenko (2017).

[12]The parameters of the productivity versus log-export quantity are related as follows: $\mu = \epsilon_k m$, $\sigma^2 = \epsilon_k^2 \upsilon^2$, $\lambda_L = \xi_L/\epsilon_k$, and $\lambda_R = \xi_R/\epsilon_k$.

[13]For notational compactness we drop the ijk subscripts in this section.

corresponding convolutions.[14]

For each destination-year-industry observation, we choose distribution parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$ so that the percentiles of the theoretical log-quantity distribution match the percentiles of the empirical log-quantity distribution. We follow Sager and Timoshenko (2017) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(q_i^{data} - q_i(\mu, \sigma^2, \lambda_L, \lambda_R)\right)^2,$$

where q_i^{data} is the i-th percentile of the empirical quantity distribution for a given destination-year-industry, $q_i(\mu, \sigma^2, \lambda_L, \lambda_R)$ is the model implied i-th quantity percentile for given parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$, and N_P is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical quantity distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Table 2 summaries distribution parameter estimates across 190 observations. As can be seen from the Table, the average sample value of σ is 1.97, rejecting a common assumption of Exponentially or Double Exponentially distributed productivity shocks. Furthermore, as can be inferred from the values of the left and right tail parameters, λ_L and λ_R, distributions exhibit substantial heterogeneity in the fatness of both tails. The value of the right tail parameter, λ_R varies between 0.34 and 20.61, with about 72 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. This finding is consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[15] Furthermore, we also find that distributions exhibit fatness in the left tail ($\lambda_L < 2$) in approximately 38 percent of observations.[16]

3.2.2 Entry Threshold

Next, we use the fitted distribution to recover the productivity entry thresholds. For each destination-year-industry observation we solve equation (17) for the productivity entry threshold using the data on the corresponding average-to-minimum ratio of export quantity and the distribution parameter estimates.

Figure 1 provides a scatter plot of the entry threshold estimates and the corresponding

[14]See Sager and Timoshenko (2017) for a more thorough characterization of the Double EMG distribution.
[15]See Axtell (2001) and di Giovanni et al. (2011).
[16] Sager and Timoshenko (2017) document fat left tails in the context of export sales distributions.

average-to-minimum ratios of log-export quantity. Each dot in the Figure corresponds to a destination-year-industry observation. The values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution. Hence, the values represent deviations from the mean of the distribution. Figure 1 shows that the greater is the deviation, i.e. the more negative is the value on the y-axis, the lower is the entry threshold. A lower entry threshold relative to the mean implies a smaller size of a marginal exporter relative to an average exporter.

3.3 Trade Elasticities

3.3.1 Elasticity Estimates

Given estimated distribution parameters and entry thresholds from Section 3.2, we compute the partial trade elasticity, η_{ijk}, and the extensive margin contribution to the trade elasticity from equation (15). Notice that computation requires estimates of the elasticity of substitution across varieties, ϵ_k. We use estimated elasticities of substitution from Soderbery (2015), which refines estimates in Feenstra (1994) and Broda and Weinstein (2006).[17] Table 3 summarizes average values for and heterogeneity in elasticity estimates.

From Table 3, observe that an average contribution of the extensive margin to trade elasticity is 0.04. In the context of the volume of aggregate trade flows, this magnitude can be understood as follows. Suppose, for example, that a decline in trade costs leads to an increase in trade flows by a million dollars. For an average observation, the new exporters would account for approximately \$38,000 out of a million dollars of the newly created trade.[18]

3.3.2 Comparison to Complete Information

To compare the estimates of trade elasticity between the two information environments, we first re-estimate the partial trade elasticity under the assumption of complete information. As discussed in Section 2.2, in a model with complete information the partial trade elasticity depends on the distribution of export sales. Hence, we re-fit the Double EMG distribution to match the distribution of log-export sales, and further use the average-to-minimum ratio of export sales to impute the value of the profitability entry threshold. Panel A in Table 3 provides summary statistics of the elasticity estimates.

[17] Soderbery (2015) estimates the elasticity of substitution values at the HS-10 digit level using the U.S. import data. To use Soderbery (2015) estimates aggregate the elasticities to the HS-6 digit level equally weighing corresponding HS-10 sub-categories for each HS-6 category.

[18] Sager and Timoshenko (2017) show that this magnitude is a result of an abundance of small exporters in export sales distributions. Other frequently used trade data sets exclude these small firms and, hence, generate much higher extensive margin elasticities.

Result 1: *Under demand uncertainty, the extensive margin contribution to the trade elasticity is larger relative to the complete information environment.*

As can be seen from Panel A in Table 3, the complete information economy yields lower values for the extensive margin elasticity.[19] In a complete information environment, the average contribution of the extensive margin is smaller by two orders of magnitude relative to a model with demand uncertainty. Panel B in Table 3 compares the elasticity estimates across the same observations. In particular, it reports summary statistics of the ratio of the quantity implied trade elasticity relative to the sales implied trade elasticity. We call this ratio the *amplification effect* because demand uncertainty produces a higher contribution of the extensive margin to trade, an order of magnitude of 10^4.

To motivate this magnitude, consider the following example. Suppose trade increases by a million dollars due to a decline in trade costs. Then, a trade elasticity estimate from a complete information model would attribute approximately \$170 out of a million dollars of new trade to trade generated by entrants. In a model with incomplete information, \$38,000 out of a million dollars can be attributed to trade by entrants. Hence, complete information dampens the (already small) contribution of new exporters to trade. Conversely a model with uncertainty amplifies the contribution of the extensive margin to trade.[20]

3.3.3 Role of Demand Uncertainty

The magnitude of the uncertainty amplification effect is tightly linked to the extent of variation arising from the demand shocks. Substituting equation (3) into equation (5) and taking the logarithm we obtain

$$\log r_{ijk} \quad = \quad \frac{\epsilon_k - 1}{\epsilon_k} \log q_{ijk} + FE_{jk} + \frac{\theta_{ijk}}{\epsilon_k}, \tag{19}$$

where $FE_{jk} = \log\left(Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k - 1}{\epsilon_k}}\right)$. Notice that the distribution of the demand shocks generates a wedge between the distributions of log-export sales and log-export quantity. This wedge is larger when the variance of demand shocks is higher. If the variance of the demand shocks is zero, then the distributions of log-export sales and log-export quantity would coincide, yielding no amplification effect. As the variance of the demand shock rises, the distributions

[19] In both models, however, the average partial trade elasticity is around 4 as a result of the overall small contribution of the extensive margin to that elasticity.

[20] While the extensive margin contribution to the partial trade elasticity is larger in the complete information than uncertainty economy, note that the magnitudes are partially generated by our generally small estimates of the extensive margin contribution. The magnitude of the estimates is a feature of our data, which includes the universe of customs declarations and therefore contains smaller firms than most standard datasets. See Sager and Timoshenko (2017) for a discussion of the potential biases that may contaminate estimation on truncated data.

of log-export sales and log-export quantities are more dissimilar. Hence we would expect a larger amplification effect.

Given equation (19), we measure the extent of demand variation in a given destination-year-industry as the difference between the variance of log-export sales and the variance of log-export quantities. Assuming for simplicity that the demand shocks are uncorrelated with log-export quantities and applying the variance operator to both side of equation (19), we obtain

$$V\left(\frac{\theta_{jk}}{\epsilon_k}\right) \quad = \quad V(\log r_{jk}) - \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^2 V(\log q_{jk}). \tag{20}$$

We first compute the variance of log-export sales, $V(\log r_{jk})$, and the variance of log-export quantity, $V(\log q_{jk})$, across firms within a given destination-year-industry observation.We then use equation (20) to back out the value of the variance of the demand shocks, $V\left(\theta_{jk}/\epsilon_k\right)$ for each destination-year-industry observation.

Result 2: *The difference in the trade elasticity estimates between environments with demand uncertainty and complete information is larger in more uncertain economies.*

Figure 2 depicts the relationship between the variance of the demand shocks and the amplification effect. The x-axis measures the variance of the demand shocks, while the y-axis measures the ratio of the export quantity implied relative to the export sales implies estimate of the extensive margin elasticity. The figure confirms that the difference in elasticity estimates between the complete information and uncertainty economies increases with an increase in demand uncertainty. In the data, exporters do not have full information about product demand in destination markets and introducing uncertainty into the model leads to a larger extensive margin adjustment.

4 Conclusion

Recently, models of learning along the lines of Jovanovic (1982) have been embedded in trade models with heterogeneous firms to analyze firm behavior such as growth (Arkolakis et al., 2018), export participation (Timoshenko, 2015a), product switching (Timoshenko, 2015b), and pricing decisions (Bastos et al., Forthcoming). This paper is the first to examine the impact of information structure on measuring the partial trade elasticity with respect to variable trade costs.

In this paper, we study the implications of demand uncertainty for the partial trade elasticity. We introduce uncertainty with respect to product demand to an otherwise standard new trade model with heterogeneous firms, as in Melitz (2003). With demand uncertainty, firms must choose how much of their product to export prior to observing the destination specific demand shock. As a result, firms make export decisions based on their productivity and, hence, selection into exporting and the extensive margin of adjustment are driven by firms' productivity.

In a model with complete information, firms know their product demand in destination markets. Firms can choose how much of their product to export with complete information about their profitability. Profitability is a measure of productivity *and* demand that characterizes idiosyncratic profit across firms.

We quantify the effect of uncertainty by comparing the trade elasticity in model environments with and without product demand uncertainty. To compute the trade elasticity, we reformulate the structural estimation approach in Bas, Mayer, and Thoenig (2017) for use in an environment with incomplete information. We discipline the distribution of productivity separately from that of profitability by using Brazilian microdata on export quantities and export sales. Upon measuring trade elasticities, we find that under demand uncertainty the extensive margin adjustments to changes in variable trade costs is larger relative to the complete information economy. Furthermore, the effect is stronger in economies with higher demand uncertainty (e.g., higher variance in sales distributions relative to variance in quantity distributions).

This paper shows that the information structure faced by firms is important for measuring the extensive margin response to a decline in trade costs. In countries or industries in which exporters face high demand uncertainty, by assuming away information asymmetries, trade elasticity estimates will likely understate the true magnitude of extensive margin adjustments.

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Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across destination-year-industry observations over 1997-2000.

Statistic	Mean	Standard Deviation	Min	Max
Panel A: Properties of log-quantity				
Standard Deviation	2.46	0.48	1.24	3.38
Skewness	0.09	0.40	-1.00	0.96
Interquartile Range	3.48	0.82	1.88	5.50
Kelly Skew	0.02	0.13	-0.36	0.27
Panel B: Properties of log-sales				
Standard Deviation	2.28	0.41	1.30	3.19
Skewness	-0.13	0.27	-0.88	0.57
Interquartile Range	3.09	0.62	1.74	4.47
Kelly Skew	-0.04	0.11	-0.32	0.28

Note: the statistics are reported across 190 destination-year-industry observations where at least 100 firms export. An industry is defined as a 6-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Double EMG distribution parameter estimates.

Statistic	Mean	Standard Deviation	Min	Max
σ	1.97	0.63	0.02	3.21
λ_L	7.91	6.33	0.40	26.26
λ_R	4.03	5.65	0.34	20.61

[a] The summary statistics are reported across 190 destination-year-industry observations. An industry is defined as a 6-digit HS code.

Table 3: Trade elasticity estimates.

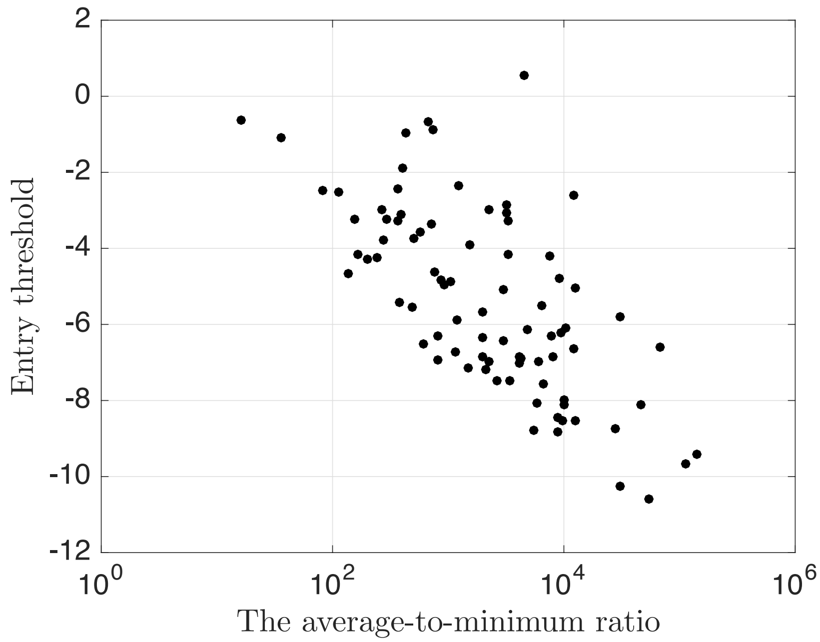
	Extensive Margin Elasticity		Partial Trade Elasticity, η_{ijk}	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: Estimates of trade elasticity				
Quantity based[a]	0.04	0.13	3.88	3.79
Sales based[b]	$1.7 \cdot 10^{-4}$	$8.8 \cdot 10^{-4}$	3.82	3.86
Panel B: Amplification effect				
Amplification effect[c]	$2.2 \cdot 10^{4}$	$8.9 \cdot 10^{4}$	1.04	0.13

[a] The quanity based measure of trade elasticity is based on a model with demand uncertainty. The summary statistics are reported across 84 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[b] The sales based measure of the trade elasticity is based on a model with complete information. The summary statistics are reported across 124 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

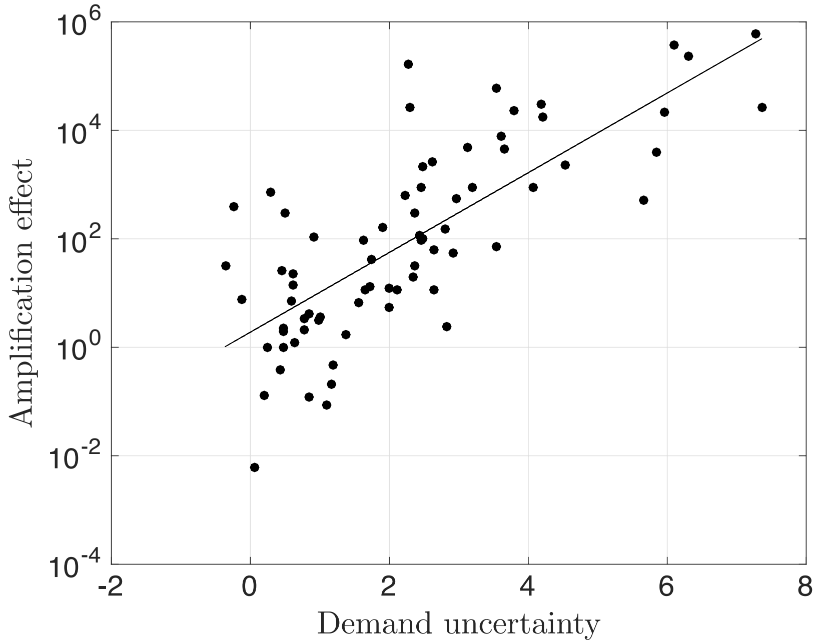
[c] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 73 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.

Figure 1: The entry threshold and average-to-minimum ratio.



Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of export quantity for observation with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a destination-year-industry observation. Values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution.

Figure 2: Amplification effect and demand uncertainty.



Notes: The figure depicts a scatter plot of the amplification effect and demand uncertainty. The amplification effect is defined as the ratio of the extensive margin elasticity estimates between the quantity based and the sales based measures. Demand uncertainty is defined as the variance of the demand shocks estimated using equation (20). Each dot corresponds to a destination-year-industry observation. The solid line is an OLS best fit line.

A Theoretical Appendix

A.1 A Model with Demand Uncertainty

In this section we provide derivations for the theoretical results in Section 2. We consider a monopolistically competitive environment as in Melitz (2003) with exogenous entry as in Chaney (2008). We further introduce information asymmetries by constructing a stylized version of the learning model in Timoshenko (2015b).

A.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E[\pi(\varphi)] = \max_{q_{ijk}} E_{\theta_{ijk}} \left(p_{ijk} q_{ijk} - \frac{w_i \tau_{ij}}{e^{\varphi}} q_{ijk} \right) - w_i f_{ijk}, \tag{21}$$

subject to the demand equation (2). The expectation over the demand draw, θ_{ijk}, is given by the distribution from which the demand parameter is drawn, $h_{ijk}(.)$. Substituting equation equation (2) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{q_{ijk}(\varphi)} q_{ijk}(\varphi)^{\frac{\epsilon_k - 1}{\epsilon_k}} E_\theta \left(e^{\frac{\theta_{ijk}}{\epsilon_k}} \right) Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k - 1}{\epsilon_k}} - \frac{w_i \tau_{ij}}{e^{\varphi}} q_{ijk}(\varphi) - w_i f_{ijk}. \tag{22}$$

The first order conditions with respect to quantity yield the optimal quantity,

$$q_{ijk}(\varphi) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k} e^{\epsilon_k \varphi} \left(E_\theta \left(e^{\frac{\theta_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{23}$$

A firm's realized revenue is then given by

$$r_{ijk}(\theta_{ijk}, \varphi) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k - 1} e^{(\epsilon_k - 1)\varphi + \frac{\theta_{ijk}}{\epsilon_k}} \left(E_\theta \left(e^{\frac{\theta_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1} (\tau_{ij} w_i)^{1 - \epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1},$$

or equivalently

$$r_{ijk}(\theta_{ijk}, \varphi) \quad = \quad B_{ijk}^r e^{(\epsilon_k - 1)\varphi + \frac{\theta_{ijk}}{\epsilon_k}},$$

were $B_{ijk}^r = \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k - 1} \left(E_\theta \left(e^{\frac{\theta_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1} (\tau_{ij} w_i)^{1 - \epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$.

A.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, φ^*_{ijk}, is a solution to the zero-expected profit condition given by

$$E_\theta[\pi(\varphi^*_{ijk})] = 0. \tag{24}$$

Substituting equation (23) into equation (22) and solving equation (24) for φ^*_{ijk} yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}, \tag{25}$$

or equivalently

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} = \frac{B^\varphi_{ijk}}{\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}},$$

where $B^\varphi_{ijk} = (\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}) \,/\, \left(\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\right)$.

A.1.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} = M_{ijk}\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} r_{ijk}(\theta,\varphi)g^\theta_{ijk}(\theta)\frac{g^\varphi_{ijk}(\varphi)}{Prob_{ijk}(\varphi > \varphi^*_{ijk})}d\theta d\varphi, \tag{26}$$

where M_{ijk} is the mass of firms exporting from country i to country j in industry k. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} = J_i \times Prob_{ijk}(\varphi > \varphi^*_{ijk}), \tag{27}$$

where J_i is the exogenous mass of entrants. Equation (26) can then be simplified as follows:

$$\begin{aligned} X_{ijk} &= J_i\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} q_{ijk}(\varphi)p_{ijk}(\theta,\varphi)g^\theta_{ijk}(\theta)g^\varphi_{ijk}(\varphi)d\theta d\varphi = \\ &= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{e^\varphi E_\theta\left(e^{\frac{\theta}{\epsilon_k}}\right)}\left(\int_{-\infty}^{+\infty} e^{\frac{\theta_{ijk}}{\epsilon_k}} g^\theta_{ijk}(\theta)d\theta\right) g^\varphi_{ijk}(\varphi)d\varphi = \\ &= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{e^\varphi}g^\varphi_{ijk}(\varphi)d\varphi = \end{aligned}$$

$$= \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\varphi_{ijk}^*}^{+\infty} e^{(\epsilon_k - 1)\varphi} g_{ijk}^{\varphi}(\varphi) d\varphi.$$

Define $\phi_k = (\epsilon_k - 1)\varphi$ with the corresponding probability density function denoted by $g_{ijk}^{\phi}(.)$ and $\phi_{ijk}^* = (\epsilon_k - 1)\varphi_{ijk}^*$. Then, the aggregate trade flow can be equivalently written as

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\phi_{ijk}^*}^{+\infty} e^{\phi} g_{ijk}^{\phi}(\phi) d\phi. \tag{28}$$

Differentiate the logarithm of equation (28) with respect to $\log \tau_{ij}$ to obtain

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} \quad = \quad (1 - \epsilon_k) - \frac{e^{\phi_{ijk}^*} g_{ijk}^{\phi}(\phi_{ijk}^*)}{\int_{\phi_{ijk}^*}^{+\infty} e^{\phi} g_{ijk}^{\phi}(\phi) d\phi} \frac{\partial \phi_{ijk}^*}{\partial \log \tau_{ijk}}. \tag{29}$$

Differentiate equation (25) with respect to τ_{ij} to obtain

$$\frac{\partial \phi_{ijk}^*}{\partial \log \tau_{ij}} \quad = \quad \epsilon_k - 1. \tag{30}$$

Combine equations (29) and (30) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} \quad = \quad (1 - \epsilon_k)\left(1 + \frac{g_{ijk}^{\phi}(\phi_{ijk}^*) e^{\phi_{ijk}^*}}{\int_{\phi_{ijk}^*}^{+\infty} e^{\phi} g_{ijk}^{\phi}(\phi) d\phi}\right). \tag{31}$$

A.1.4 Estimation Approach

Consider the following change of notation: let $\tilde{\varphi} \equiv \epsilon_k \varphi$. Denote by $g^{\tilde{\varphi}}(.)$ the probability distribution function of $\tilde{\varphi}$. Given the change in notation, $g^{\tilde{\varphi}}(.)$ is the distribution of φ, $g_{ijk}^{\varphi}(.)$, scaled by the elasticity of substitution, ϵ_k.

With the change in notation, equations (25) and (28) can be written as

$$e^{\frac{\epsilon_k - 1}{\epsilon_k}\tilde{\varphi}^*} \quad = \quad \frac{\epsilon_k w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}} \tag{32}$$

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\tilde{\varphi}_{ijk}^*}^{+\infty} e^{\frac{\epsilon_k - 1}{\epsilon_k}\phi} g^{\tilde{\varphi}}(\varphi) d\varphi \tag{33}$$

Differentiating equation (32) and (33) with respect to τ_{ij}, the partial trade elasticity can be expressed as

$$\eta_{ijk} = (1-\epsilon_k)\left(1+\frac{\epsilon_k}{\epsilon_k-1}\frac{g^{\tilde{\varphi}}_{ijk}(\tilde{\varphi}^*_{ijk})e^{\frac{\epsilon_k-1}{\epsilon_k}\tilde{\varphi}^*_{ijk}}}{\int_{\tilde{\varphi}^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k-1}{\epsilon_k}\varphi} g^{\tilde{\varphi}}_{ijk}(\varphi)d\varphi}\right).$$

The distribution $g^{\tilde{\varphi}}_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. From equation (23), the optimal quantity can we written as

$$q_{ijk}(\tilde{\varphi}) = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} e^{\tilde{\varphi}}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{34}$$

Hence, the distribution of log-export quantity is given by the distribution of $\tilde{\varphi}$. Given the distribution of $g^{\tilde{\varphi}}_{ijk}(.)$, the scaled productivity entry threshold, $\tilde{\varphi}^*_{ijk}$, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (34) the average export quantity, $\tilde{q}_{ijk}$, and the minimum export quantity, $q^{\min}_{ijk}$, are given by

$$\begin{aligned}
\tilde{q}_{ijk} &= \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}\int_{\tilde{\varphi}^*_{ijk}}^{+\infty}\frac{e^{\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi>\tilde{\varphi}^*_{ijk})}d\varphi\\
q^{\min}_{ijk} &= \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}e^{\tilde{\varphi}^*_{ijk}}.
\end{aligned}$$

Hence, the average-to-minimum ratio, $\tilde{q}_{ijk}/q^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-\tilde{\varphi}^*_{ijk}}\int_{\tilde{\varphi}^*_{ijk}}^{+\infty}\frac{e^{\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi>\tilde{\varphi}^*_{ijk})}d\varphi. \tag{35}$$

A.2 A Model with Complete Information

In this section, for comparison purposes, we develop theoretical results in a model with complete information. The information structure only affects the supply side of the economy. Hence, on the demand side, the utility of a representative consumers is still given by equation (1), and the demand for a given variety is given by equation (2).

A.2.1 Supply

In contrast to a model with uncertainty, in a model with complete information firms make their market participation and quantity decisions after observing their productivity and demand shocks.

For each destination and industry firms maximize profits given by

$$\max_{q_{ijk}} p_{ijk}q_{ijk} - \frac{w_i\tau_{ij}}{e^{\varphi}}q_{ijk} - w_i f_{ijk}, \tag{36}$$

subject to the demand equation (2). The first order conditions with respect to quantity yield the optimal quantity being given by

$$q_{ijk}(\theta_{ijk},\varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}. \tag{37}$$

Notice that in contrast to equation (23), in a complete information environment the quantity choice is determined by a combination of a supply and a demand shocks, i.e. by a firm's profitability. Using equations (2) and (37), a firm's optimal sales are further given by

$$r_{ijk}(\theta_{ijk},\varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k-1} e^{(\epsilon_k-1)\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}, \tag{38}$$

or equivalently

$$\begin{aligned} r_{ijk}(\theta_{ijk},\varphi) &= B_{ijk}e^{(\epsilon_k-1)\varphi+\theta_{ijk}} \\ r_{ijk}^{CI}(z_{ijk}) &= B_{ijk}e^{z_{ijk}}, \end{aligned}$$

where $B_{ijk} = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}$, $z_{ijk} = (\epsilon_k - 1)\varphi + \theta_{ijk}$, and 'CI' stands for Complete Information.

A.2.2 Entry

Given the optimal profits, firms enter the market as long as the profit is positive. Hence, the optimal any demand draw, θ_{ijk}, productivity entry threshold, $\varphi^*_{ijk}(\theta_{ijk})$, is a solution to the zero-profit condition given by

$$\pi(\varphi^*_{ijk}(\theta_{ijk})) = 0. \tag{39}$$

Substituting equation (37) into equation (36) and solving equation (39) for $\varphi^*_{ijk}(\theta_{ijk})$ yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}(\theta_{ijk})} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}e^{\theta_{ijk}}}. \tag{40}$$

Notice that in contrast to the incomplete information environment discussed in Section A.1.1 and equation (25), the productivity entry threshold depends on the realized value of demand parameter, θ_{ijk}. Firms with a higher demand parameter have a lower productivity entry threshold.

Equation (40) can be viewed as defining an entry boundary in the space of (θ_{ijk},φ) or as defining the profitability entry threshold z^*_{ijk}. The profitability entry thresholds is given by the sum of $(\epsilon_k - 1)\varphi_{ijk}$ and θ_{ijk} such that equation (40) holds:

$$e^{z^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1}} \equiv B^{\varphi}_{ijk}. \tag{41}$$

Hence, in a model with complete information, selection into exporting occurs based on profitability rather than productivity as is the case in a model with uncertainty.

A.2.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is given by

$$\begin{aligned} X_{ijk} &= J_i \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} q_{ijk}(\theta, \varphi) p_{ijk}(\theta, \varphi) g^{\theta}_{ijk}(\theta) g^{\varphi}_{ijk}(\varphi) d\theta d\varphi \qquad (42) \\ &= J_i \left(\frac{\epsilon_k}{\epsilon_k - 1} \frac{\tau_{ij} w_i}{P_{jk}}\right)^{1-\epsilon_k} Y_{jk} \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} e^{(\epsilon_k - 1)\varphi + \theta_{ijk}} g^{\theta}_{ijk}(\theta) g^{\varphi}_{ijk}(\varphi) d\theta d\varphi \end{aligned}$$

Define $z_{ijk} = (\epsilon_k - 1)\varphi + \theta_{ijk}$. From equation (41) the entry into exporting occurs when $z_{ijk} > z^*_{ijk}$. Using this change of variables, equation (42) can be be written as

$$X_{ijk} = J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz, \tag{43}$$

where $g^z_{ijk}(.)$ is the distribution of profitability z_{ijk}.

Compare the expressions for the aggregate trade flow between the two information environments, equation (28) versus equation (43). Notice, that in the incomplete information environment, the aggregate trade flows are determined by the distribution of *productivity*, $g^{\varphi}_{ijk}(\varphi)$, while in the complete information environment the aggregate trade flows are determined by the distribution of *profitability*, $g^z_{ijk}(z)$.

Following the same differentiation steps as in Section A.2.3, the partial elasticity of trade flows with respect to the variable trade costs is given by

$$\begin{aligned} \eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} &= (1 - \epsilon_k) \left(1 + \frac{g^z_{ijk}(z^*_{ijk}) e^{z^*_{ijk}}}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz}\right) = \\ &= (1 - \epsilon_k) \left(1 + \frac{g^z_{ijk}(z^*_{ijk})}{Prob^z_{ijk}(z > z^*_{ijk})} \left(\frac{\tilde{r}_{ijk}}{r^{\min}_{ijk}}\right)^{-1}\right). \end{aligned}$$

The last equality hold due to equation (45) below.

A.2.4 Estimation Approach

The distribution $g^z_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export sales. From equation (38), the optimal sales can we written as

$$r_{ijk}(z_{ijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} e^{z_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{44}$$

Hence, the distribution of log-export sales is given by the distribution of z_{ijk}. Given the distribution of $g^z_{ijk}(.)$, the profitability entry threshold, z^*_{ijk}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (34) the average export sales, $\tilde{r}_{ijk}$, and the minimum export sales, $r^{\min}_{ijk}$, are given by

$$\begin{aligned} \tilde{r}_{ijk} &= \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^*_{ijk}}^{+\infty} e^z \frac{g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz \\ r^{\min}_{ijk} &= \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{z^*_{ijk}}. \end{aligned}$$

Hence, the average-to-minimum ratio, $\tilde{r}_{ijk}/r^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-z^*_{ijk}} \int_{z^*_{ijk}}^{+\infty} \frac{e^z g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz. \tag{45}$$

To contrast the two information environments, notice that while equations for estimating the entry thresholds are similar, equation (35) versus (45), different data are used for estimation. In the environment with demand uncertainty the relevant distributions and entry thresholds are identified from the empirical export quantity distributions, while in the complete information framework, log export sales identify the necessary parameters.

B Proofs of Propositions

Proposition 1 *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

(i) $\gamma(x) \equiv [e^x g(x)]/\int_x^{+\infty} e^z g(z)dz$ is an increasing function of x.

(ii) Denote the extensive margin elasticity associated with $g(x)$ as $\gamma(x)$. Let $\tilde{g}(x)$ be a mean preserving spread of $g(x)$, with extensive margin elasticity $\tilde{\gamma}(x)$. There exists x^* such that $\tilde{\gamma}(x) < \gamma(x)$ for all $x > x^*$, $\tilde{\gamma}(x) = \gamma(x)$ if $x = x^*$, and $\tilde{\gamma}(x) > \gamma(x)$ for all $x < x^*$.

Proof of Proposition 1

Part (i) First, define $h(x) = (e^x g(x))/E$, where $E = \int_{-\infty}^{+\infty} e^x g(x)dx$. Notice that $h(x)$ is positive for all x and that $\int_{-\infty}^{+\infty} h(x)dx = 1$. Hence, $h(x)$ is a probability density function.

The corresponding cumulative density function is given by $H(x) = \int_{-\infty}^{x} e^z g(z)dz/E$. The corresponding survival function is given by $1 - H(x) = \int_{x}^{+\infty} e^z g(z)dz/E$.

Next, function $\gamma(x)$ can then be written as

$$\gamma(x) \quad = \quad \frac{e^x g(x)}{\int_x^{+\infty} e^z g(z)dz} = \frac{h(x)}{1 - H(x)}.$$

Hence, $\gamma(x)$ is a hazard rate associated with the distribution $H(x)$. By Theorem 10 in Rinne (2014), the hazard rate $\gamma(x)$ is monotonically increasing in x if and only if its logarithmic survival function, $\log(1 - H(x))$, is concave. Notice that by part (iii) of A1, $\log(1 - H(x))$ is a concave function of x. Hence, $\gamma(x)$ is increasing in x. For completeness, we reproduce the proof of this result below.

Notice that

$$\gamma(x) = -\frac{d\log(1 - H(x))}{dx}.$$

Hence,

$$\frac{d\gamma(x)}{dx} = -\frac{d^2\log(1 - H(x))}{dx^2}.$$

Since $\log(1 - H(x))$ is a concave function of x, $d^2\log(1 - H(x))/dx^2 < 0$. Therefore, $d\gamma(x)/dx > 0$.

Part (ii) Function $\tilde{\gamma}(x)$ is given by

$$\tilde{\gamma}(x) \quad = \quad \frac{e^x \tilde{g}(x)}{\int_x^{+\infty} e^z \tilde{g}(z)dz} = \frac{\tilde{h}(x)}{1 - \tilde{H}(x)},$$

where $\tilde{g}(.)$ is a mean preserving spread of $g(.)$, $\tilde{h}(x) = [e^x \tilde{g}(x)]/\int_{-\infty}^{+\infty} e^x \tilde{g}(x)dx$, and $\tilde{H}(x)$ is the corresponding cumulative distribution function.

$\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$ as follows for the following set of equivalent inequalities:

$$
\begin{aligned}
\gamma(x) &> \tilde{\gamma}(x) \\
-\frac{d\log(1-H(x))}{dx} &> -\frac{d\log(1-\tilde{H}(x))}{dx} \\
-d\log(1-H(x)) &> -d\log(1-\tilde{H}(x)) \\
d\log(1-H(x)) &< d\log(1-\tilde{H}(x)) \\
\int d\log(1-H(x)) &< \int d\log(1-\tilde{H}(x)) \\
\log(1-H(x)) &< \log(1-\tilde{H}(x)) \\
(1-H(x)) &< (1-\tilde{H}(x)) \\
-H(x) &< -\tilde{H}(x) \\
H(x) &> \tilde{H}(x).
\end{aligned}
$$

We will now show in three steps that $H(x)$ crosses $\tilde{H}(x)$ once from below, and therefore there exists x^* such that $H(x) > \tilde{H}(x)$ holds for $x > x^*$, and therefore (ii) holds.

Step 1: Denote by X and $\tilde{X}$ random variables distributed according to $g(x)$ and $\tilde{g}(x)$ respectively. Since $\tilde{g}(x)$ is a mean preserving spread of $g(x)$, it holds that $\tilde{X} = X + \hat{X}$, where $\hat{X}$ is distributed according to $\hat{g}(x)$ with mean zero, and $\hat{X}$ is independent from X. Hence, $\tilde{g}(.)$ is a convolution of $g(.)$ and $\hat{g}(.)$ and can be written as

$$
\tilde{g}(x) = \int_{-\infty}^{+\infty} g(x-u)\hat{g}(u)du.
$$

Step 2: Denote by X^h, $\tilde{X}^h$, $\hat{X}^h$ random variables distributed according to $h(x)$, $\tilde{h}(x)$, and $\hat{h}(x)$ respectively, where $\hat{h}(x) = [e^x\hat{g}(x)]/\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx$. Similarly, it can be show that $\tilde{h}(.)$ is a convolution of $h(.)$ and $\hat{h}(.)$:

$$
\begin{aligned}
\int_{-\infty}^{+\infty} h(x-u)\hat{h}(u)du &= \frac{\int_{-\infty}^{+\infty} e^{x-u}g(x-u)e^u\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{\int_{-\infty}^{+\infty} e^x g(x-u)\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{e^x\tilde{g}(x)}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \tilde{h}(x).
\end{aligned}
$$

Thus, it hold that $\tilde{X}^h = X^h + \hat{X}^h$, where $\tilde{X}^h$ and $\hat{X}^h$ are independent.

Step 3: Consider a random variable $\bar{X} = X^h + \hat{X}^h - E(\hat{X}^h)$ with the cumulative distribution function denoted by $\bar{H}(x)$. $\bar{X}$ is a mean preserving spread of X^h and therefore the two corresponding cumulative distribution functions satisfy the single-crossing property whereby $H(x) = \bar{H}(x)$ if $x = E(X^h)$; $H(x) < \bar{H}(x)$ for $x < E(X^h)$, and $H(x) > \bar{H}(x)$ for $x > E(X^h)$.

Next, notice that $\tilde{X}^h = \bar{X} + E(\hat{X}^h)$. Therefore the cumulative distribution function of $\tilde{X}^h$ is a shift of the cumulative distribution function of $\bar{X}$ along the x-axis, namely $\tilde{H}(x) = \bar{H}(x - E(\hat{X}^h))$. Hence $\tilde{H}(x)$ preserves the same single-crossing property with respect to $H(x)$. Namely $\exists x^*$ that that $H(x) = \tilde{H}(x)$ if $x = x^*$; $H(x) < \tilde{H}(x)$ for $x < x^*$, and $H(x) > \tilde{H}(x)$ for $x > x^*$. ■

Corollary 1 *Let $g(x)$ be a probability density function satisfying A1. $\forall a \in \mathbb{R}$ there exists $x^*(a)$ such that $\gamma(x) > \tilde{\gamma}(x+a)$ if $x > x^*(a)$; $\gamma(x) = \tilde{\gamma}(x+a)$ if $x = x^*(a)$, and $\gamma(x) < \tilde{\gamma}(x+a)$ if $x < x^*(a)$.*

Proof of Corollary 1

Notice that part (ii) of Proposition 1 implies a single crossing property of $\gamma(.)$ and $\tilde{\gamma}(.)$. This property is preserved under an affine transformation of the abscissa for either of the functions. Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x + a)$ from above for some $x^*(a)$.■

C Robustness

In this section we demonstrate the robustness of our theoretical and quantitative results to the way we choose to model a firm's decision under uncertainty. In the main text we assume that in a model with uncertainty, firms choose export quantities before demand shocks are realized. This assumption is consistent with the majority of the literature on learning such as Timoshenko (2015b), Arkolakis et al. (2018), Berman et al. (Forthcoming). In contrast to this literature, in this section we assume that firms choose prices before demand shocks are realized. Below, we present an alternative representation of the model with uncertainty in line with this assumption. In this model, given that firms choose prices, the price data contain information necessary to identify the partial elasticity of trade flows with respect to variable trade costs. We subsequently quantify trade elasticities according to this insight. Our quantitative results are unaffected by the change in the firm's choice variable.

The intuition for this equivalence lies in the fact that the price and quality are inversely related through sales. In a model with uncertainty where firms choose quantities, the empirical distribution of quantity identifies the underlying theoretical distribution of productivities. In a model with uncertainty where firm choose prices, the optimal price equals to the inverse of productivity. Therefore, theoretical productivity distribution is identified by the empirical distribution of the inverse of the prices, which is proportional to the empirical distribution of quantities through the equation quantity = sales/price.

C.1 Alternative Model of Demand Uncertainty

The economic environment and demand are the same as in Section 2.

$$c_{ijk}(\omega) = e^{\theta_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{46}$$

C.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E[\pi(\varphi)] = \max_{p_{ijk}} E_{\theta_{ijk}} \left(p_{ijk} q_{ijk} - \frac{w_i \tau_{ij}}{e^{\varphi}} q_{ijk} \right) - w_i f_{ijk},$$

subject to the demand equation (46). The expectation over the demand draw, θ_{ijk}, is given by the distribution from which the demand parameter is drawn, $h_{ijk}(.)$. Substituting equation equation (46) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{p_{ijk}(\varphi)} p_{ijk}(\varphi)^{1-\epsilon_k} E\left(e^{\theta_{ijk}}\right) Y_{jk} P_{jk}^{\epsilon_k - 1} - \frac{w_i \tau_{ij}}{e^{\varphi}} E\left(e^{\theta_{ijk}}\right) p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} - w_i f_{ijk}.$$

The first order conditions with respect to price yield the optimal price,

$$p_{ijk}(\varphi) \quad = \quad \left(\frac{\epsilon_k}{\epsilon_k - 1} \right) \frac{w_i \tau_{ij}}{e^{\varphi}}. \tag{47}$$

A firm's realized revenue is then given by

$$r_{ijk}(\theta_{ijk}, \varphi) \quad = \quad e^{\theta_{ijk}(\omega)} \left(\frac{\epsilon_k}{\epsilon_k - 1} \frac{w_i \tau_{ij}}{e^{\varphi}} \right)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}.$$

C.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, φ^*_{ijk}, is a solution to the zero-expected profit condition given by

$$E[\pi(\varphi^*_{ijk})] \quad = \quad 0,$$

and is given by

$$e^{(\epsilon_k - 1)\varphi^*_{ijk}} \quad = \quad \frac{\epsilon_k^{\epsilon_k} w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{(\epsilon_k - 1)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} E\left(e^{\theta_{ijk}}\right)}. \tag{48}$$

C.1.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} = M_{ijk} \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\theta, \varphi) h_{ijk}(\theta) \frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi > \varphi^*_{ijk})} d\theta d\varphi, \tag{49}$$

where M_{ijk} is the mass of firms exporting from country i to country j in industry k. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} = J_i \times Prob_{ijk}(\varphi > \varphi^*_{ijk}),$$

where J_i is the exogenous mass of entrants. Equation (49) can then be simplified as follows:

$$\begin{aligned} X_{ijk} &= J_i \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\varphi) h_{ijk}(\theta) g_{ijk}(\varphi) d\theta d\varphi = \\ &= J_i \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k - 1} E\left(e^{\theta_{ijk}}\right) (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k - 1)\varphi} g_{ijk}(\varphi) d\varphi. \end{aligned} \tag{50}$$

Differentiate equation (50) with respect to τ_{ij} to obtain

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = (1 - \epsilon_k) \frac{X_{ijk}}{\tau_{ijk}} - \frac{X_{ijk}}{\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k - 1)\varphi} g_{ijk}(\varphi) d\varphi} e^{(\epsilon_k - 1)\varphi^*_{ijk}} g_{ijk}(\varphi^*_{ijk}) \frac{\partial \varphi^*_{ijk}}{\partial \tau_{ijk}}. \tag{51}$$

Differentiate equation (48) with respect to τ_{ij} to obtain

$$\frac{\partial \varphi^*_{ijk}}{\partial \tau_{ij}} = \frac{1}{\tau_{ij}}. \tag{52}$$

Combine equations (51) and (52) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = (1 - \epsilon_k) \left(1 + \frac{g_{ijk}(\varphi^*_{ijk}) e^{(\epsilon_k - 1)\varphi^*_{ijk}}}{(\epsilon_k - 1) \int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k - 1)\varphi} g_{ijk}(\varphi) d\varphi} \right). \tag{53}$$

C.1.4 Estimation Approach

From equation (47), the distribution $g_{ijk}(.)$ can be directly recovered from the empirical distribution of the logarithm of the inverse of export price as follows:

$$\log\left(1/p_{ijk}(\varphi_{ijk})\right) = B^p_{ijk} + \varphi_{ijk}. \tag{54}$$

Hence, the distribution of the logarithm of the inverse of export price is given by the distribution of φ_{ijk}. Given the distribution of $g_{ijk}(.)$, the productivity entry threshold, φ^*_{ijk}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of the inverse of export prices. From equation (47) the average of the inverse of export price, $\widetilde{1/p}_{ijk}$, and the minimum of the inverse of export price , $(1/p)^{\min}_{ijk}$, are given by

$$\begin{aligned}\widetilde{1/p}_{ijk} &= \frac{\epsilon_k - 1}{\epsilon_k}(\tau_{ij}w_i)^{-1}\int_{\varphi^*_{ijk}}^{+\infty}\frac{e^{\varphi}g_{ijk}(\varphi)}{Prob^{\varphi}_{ijk}(\varphi > \varphi^*_{ijk})}d\varphi \\ (1/p)^{\min}_{ijk} &= \frac{\epsilon_k - 1}{\epsilon_k}(\tau_{ij}w_i)^{-1}e^{\varphi^*_{ijk}}.\end{aligned}$$

Hence, the average-to-minimum ratio, $\widetilde{1/p}_{ijk}/(1/p)^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-\varphi^*_{ijk}}\int_{\varphi^*_{ijk}}^{+\infty}\frac{e^{\varphi}g_{ijk}(\varphi)}{Prob^{\varphi}_{ijk}(\varphi > \varphi^*_{ijk})}d\varphi. \tag{55}$$

C.2 Trade Elasticity Estimates

Table C1 replicates results in Table 3 and shows that the quantitative magnitude of the trade elasticities and the amplification effect remains robust to the alternative firm-level choice variable under uncertainty.

Table C1: Trade elasticity estimates, the log of the inverse of prices.

	Extensive Margin Elasticity		Partial Trade Elasticity, η_{ijk}	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: Estimates of trade elasticity				
Price based [a]	0.03	0.13	2.88	3.51
Sales based[b]	$1.7 \cdot 10^{-4}$	$8.8 \cdot 10^{-4}$	3.82	3.86
Panel B: Amplification effect				
Amplification effect [c]	$1.1 \cdot 10^{4}$	$5.1 \cdot 10^{4}$	1.03	0.15

[a] The summary statistics are reported across 109 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[b] The sales based measure of the trade elasticity is based on a model with complete information. The summary statistics are reported across 124 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[c] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 77 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.