S&P/JPX JGB VIX Index

White Paper

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Scope of the Document

This document explains the design and implementation of the S&P/JPX Japanese Government Bond Volatility Index (JGB VIX). The index measures the fair market value of constant 30-day forward volatility of 10-year JGB futures prices implied by options on 10-year JGB futures. The index differs from at-the-money (ATM) implied volatilities as it incorporates additional information contained in the skew of out-of-the-money (OTM) option prices, and represents a purely directional model-free and “strikeless” volatility measure.

Acknowledgements

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Background: JGBs, JGB Futures, and Options on JGB Futures

Japanese Government bonds and Treasury bills, with a combined notional outstanding of just under a quadrillion yen as of March 2014, constitutes the second largest sovereign debt market after that of the United States. A large domestic investor base accounts for more than 90% of notional holdings, with Bank of Japan, Japan Post Bank, regional and major banks, life insurers, and pension funds being among the biggest creditors. Listed derivatives on JGBs, namely futures and future options, are primarily traded on the Osaka Exchange (OSE) after merging with the Tokyo Stock Exchange. Volatility of JGBs, and in turn JGB futures prices, likely has important implications for a wide range of investors.

The Formula

The JGB VIX calculation is based on a simplified option-implied pricing formula for a hypothetical variance swap contract on JGB futures:[1]

$$\sigma_t = \sqrt{\frac{1}{(T-t)}\left[\frac{2}{P_t(T)}\left(\sum_{K_i<\hat{K}} Put_t(K_i)\frac{\Delta K_i}{K_i^2} + \sum_{K_i\geq\hat{K}} Call_t(K_i)\frac{\Delta K_i}{K_i^2}\right) - \left(\frac{F-\hat{K}}{\hat{K}}\right)^2\right]}$$

where the variables are defined as

t	time of calculation
T	forward volatility horizon
F	time t price of a JGB futures contract expiring at or after T
$Put_t(K_i)$	time t price of a JGB futures OTM put option struck at K_i and expiring at T
$Call_t(K_i)$	time t price of a JGB futures OTM call option struck at K_i and expiring at T
$\hat{K}$	first available strike less than or equal to F
K_i	i[th] highest out-of-the-money OTM strike
ΔK_i	calculated as $K_i - K_{i-1}$ for the highest strike K_i; $K_{i+1} - K_i$ for the lowest strike K_i; and $(K_{i+1} - K_{i-1})$ / 2 otherwise
$P_t(T)$	time t price of a zero coupon JGB maturing at T

and the quantity inside the square-root approximately represents the fair time t strike of a variance swap on F expiring at T, as discussed in the Appendix.

[1] See Appendix for a discussion.

Implementation

Horizon: JGB VIX is designed to measure a rolling 30-day implied volatility of 10-year JGB futures.

Data: Futures and options data are obtained from daily settlement prices published by the OSE, and JGB yield data are obtained from Reuters.

Step 1: Choose option maturities

Except on days when one series of options has exactly 30 days to expiry, a time-weighted average of the expected variances implied by the two option maturities is used to infer the 30-day expected volatility.

The OSE lists options in a quarterly March-June-September-December cycle plus one or two closest serial months, and each option expires on the last trading day of the month prior to its expiry month. The near- and next-month options are used to interpolate a constant 30-day volatility. For example, on Friday, 21 Jun 2013, the index is calculated based on options expiring on 28 Jun 2013 ("near -term") and 31 Jul 2013 ("next-term").

The two option series at times reference different underlying futures contract months; for practical purposes, the pricing implication of this mismatch is ignored.[2]

Step 2: Choose option strikes

For each maturity, the strike that minimizes the difference between the call and put prices is chosen as a proxy for the ATM strike. OTM call (put) options struck above (below) the ATM strike are included up to and including the first strike for which the settlement price is 0 or 0.01, if any.

Continuing the example from above, assume the underlying future expiring on 20 Sep 2013 settled at 142.10 on 21 Jun 2013 and the settlement prices for the near- and next-term options were as follows:

[2] See Appendix for a discussion.

Strike	Near Call	Near Put	Next Call	Next Put
148			0.01	5.91
147.5			0.01	5.41
147			0.02	4.91
146.5			0.02	4.42
146			0.02	3.92
145.5			0.03	3.42
145			0.04	2.94
144.5	0.01	2.4	0.06	2.46
144	0.01	1.91	0.11	2.01
143.5	0.05	1.45	0.2	1.6
143	0.13	1.03	0.36	1.26
142.5	0.28	0.68	0.59	0.99
142	0.55	0.45	0.9	0.8
141.5	0.89	0.29	1.24	0.64
141	1.28	0.18	1.65	0.55
140.5	1.71	0.11	2.04	0.44
140	2.17	0.07	2.49	0.39
139.5	2.64	0.04	2.94	0.35
139	3.12	0.02	3.4	0.3
138.5	3.61	0.01	3.86	0.26
138	4.11	0.01	4.32	0.22
137.5			4.78	0.18
137			5.25	0.15

In this example, the 142 strike minimizes the difference between the put and call settlement prices for both near and next options, and is therefore defined as the ATM strike for both series.[3] This strike comes close to the value of the underlying futures price settled at 142.10. Options to be included in the index calculation are highlighted in light green. The settlement price of the ATM strike is set as the average of the call and put prices, i.e. 0.50 and 0.85 for the near- and next-term options, respectively.

Note that the number of strikes included varies between puts and calls as well as between the two series. These numbers change through time as market conditions change.

Step 3: Calculate near- and next-term variances

The option-implied variance is the quantity inside the square-root function of the volatility formula:

$$\sigma^2 = \frac{1}{(T-t)}\left[\frac{2}{P_t(T)}\left(\sum_{K_i<\hat{K}} Put_t(K_i)\frac{\Delta K_i}{K_i^2} + \sum_{K_i\geq\hat{K}} Call_t(K_i)\frac{\Delta K_i}{K_i^2}\right) - \left(\frac{F-\hat{K}}{\hat{K}}\right)^2\right]$$

Further continuing the example, the near- and next-term options expire in 0.01917 (=7/365) and 0.10958 (=40/365) years, respectively, based on an ACT/365 day count convention.

Each option chosen in Step 2 is assigned the weight $\frac{\Delta K_i}{K_i^2}$ ("strike weight"):

[3] Note that the two option maturities will not always have the same ATM strikes. A steeply upward- or downward-sloping volatility term structure is more likely to cause the ATM strikes to be different.

	Near Term Options				Next-Term Options			
Strike	Price	delta K	Strike Weight	Price x Weight	Price	delta K	Strike Weight	Price x Weight
147.5					0.01	0.5	2.29819E-05	2.29819E-07
147					0.02	0.5	2.31385E-05	4.6277E-07
146.5					0.02	0.5	2.32967E-05	4.65934E-07
146					0.02	0.5	2.34566E-05	4.69131E-07
145.5					0.03	0.5	2.3618E-05	7.08541E-07
145					0.04	0.5	2.37812E-05	9.51249E-07
144.5					0.06	0.5	2.39461E-05	1.43676E-06
144	0.01	0.5	2.41127E-05	2.41127E-07	0.11	0.5	2.41127E-05	2.65239E-06
143.5	0.05	0.5	2.4281E-05	1.21405E-06	0.2	0.5	2.4281E-05	4.8562E-06
143	0.13	0.5	2.44511E-05	3.17864E-06	0.36	0.5	2.44511E-05	8.80239E-06
142.5	0.28	0.5	2.4623E-05	6.89443E-06	0.59	0.5	2.4623E-05	1.45275E-05
142	0.5	0.5	2.47967E-05	1.23983E-05	0.85	0.5	2.47967E-05	2.10772E-05
141.5	0.29	0.5	2.49722E-05	7.24194E-06	0.64	0.5	2.49722E-05	1.59822E-05
141	0.18	0.5	2.51496E-05	4.52694E-06	0.55	0.5	2.51496E-05	1.38323E-05
140.5	0.11	0.5	2.5329E-05	2.78619E-06	0.44	0.5	2.5329E-05	1.11447E-05
140	0.07	0.5	2.55102E-05	1.78571E-06	0.39	0.5	2.55102E-05	9.94898E-06
139.5	0.04	0.5	2.56934E-05	1.02774E-06	0.35	0.5	2.56934E-05	8.99269E-06
139	0.02	0.5	2.58786E-05	5.17572E-07	0.3	0.5	2.58786E-05	7.76357E-06
138.5	0.01	0.5	2.60658E-05	2.60658E-07	0.26	0.5	2.60658E-05	6.7771E-06
138					0.22	0.5	2.6255E-05	5.7761E-06
137.5					0.18	0.5	2.64463E-05	4.76033E-06
137					0.15	0.5	2.66397E-05	3.99595E-06
			Sum	4.20733E-05			Sum	0.000145614

For the 7- and 40-day zero coupon (ZC) JGB prices, we assume a flat front-end curve and use the one-month zero-coupon bond yield of 0.0007 published by Reuters to calculate both as follows:

$P(7)$ = 7-day ZC JGB price = exp (- 0.0007 * 0.01917) = 0.9999865

$P(40)$ = 40-day ZC JGB price = exp (- 0.0007 * 0.10958) = 0.9999233

Putting it all together:

$$\sigma^2_{near} = \frac{1}{(7/365)}\left[\frac{2}{0.9999865}(0.0000420733) - \left(\frac{142.10-142}{142}\right)^2\right] = 0.00436184$$

$$\sigma^2_{next} = \frac{1}{(40/365)}\left[\frac{2}{0.9999233}(0.000145614) - \left(\frac{142.10-142}{142}\right)^2\right] = 0.00265313$$

Step 4: Calculate JGB VIX based on the 30-day time-weighted sum of the variances

The 30-day variance is calculated as the time-weighted sum of the near- and next-term variances calculated in Step 3.

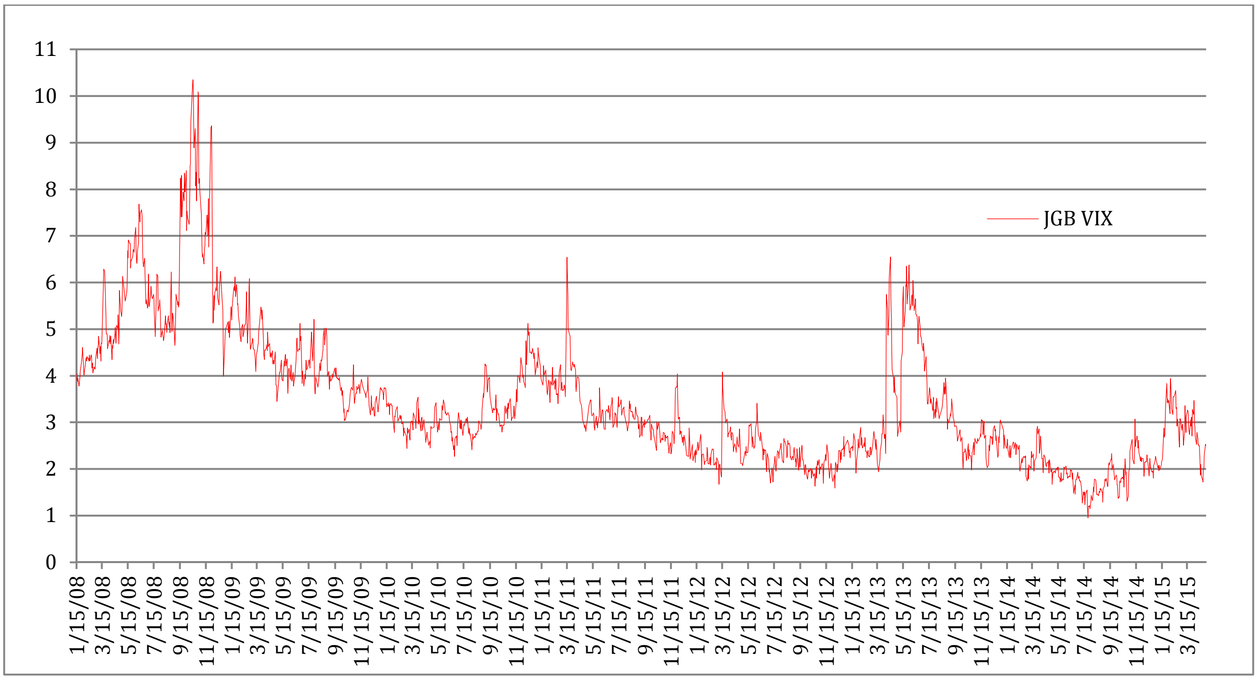
$$JGB\text{–}VIX_t = 100 \times \sqrt{\left[T_{near}\sigma^2_{near}\left(\frac{N_{next}-30}{N_{next}-N_{near}}\right) + T_{next}\sigma^2_{next}\left(\frac{30-N_{near}}{N_{next}-N_{near}}\right)\right] \times \frac{365}{30}}$$

where N_{near} and N_{next} denote the number of days until near- and next-term expiry, respectively.

Finally:

$$JGB\text{–}VIX_t = 100 \times \sqrt{\left[\frac{7}{365} 0.00436184 \left(\frac{40-30}{40-7}\right) + \frac{40}{365} 0.00265313 \left(\frac{30-7}{40-7}\right)\right] \times \frac{365}{30}} = 5.26$$

Historical daily JGB VIX values since 15 Jan 2008[4]



Appendix: Theory and Implementation

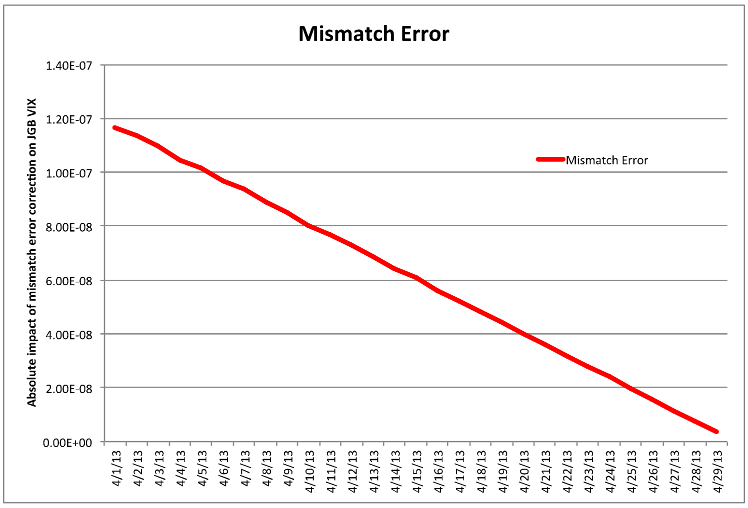
A mathematically rigorous treatment of variance swaps on government bond futures and the theoretically precise pricing formula can be found in Mele and Obayashi (2014, a). According to the theoretical variance pricing formula, the fair market value of a government bond variance swap is the expectation of future realized variance taken under the so-called *forward probability* (see, e.g., Jamshidian, 1989), not under the *risk-neutral probability* as in the standard equity variance case that assumes constant interest rates (e.g. CBOE VIX®). This feature of the fair value arises from the obvious fact that interest rates cannot be taken to be constant in the context of fixed income security valuation. While it may appear subtle, an understanding of this property is of fundamental importance when pricing derivatives that may become traded on the index such as futures and options.

[4] We use a constant 1m JGB zero yield of 30 basis points to calculate historical daily JGB VIX values for all days before 5 April 2010, which is the day when Reuters data become available.

Four approximations are made to the theoretical variance pricing formula for practical purposes to arrive at the JGB VIX formula:

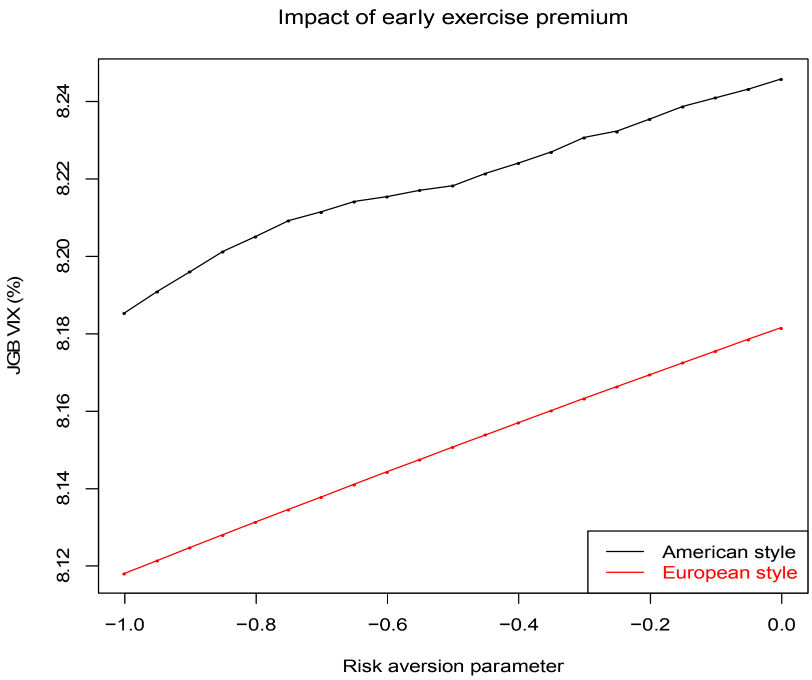
Approximation 1: Maturity mismatch between options and futures

JGB VIX measures the option-implied volatility of JGB futures, and the JGB VIX formula is mathematically precise only when the maturities of the options and the underlying futures are the same. When the maturity of options being used to price the variance precedes that of the underlying futures, theory dictates that one must adjust for this maturity mismatch. Fortunately, for short horizons such as in the case of JGB VIX, model-based estimates of this error based on Vasicek (1977) dynamics suggest that the numerical impact of the maturity mismatch is negligible and therefore can be ignored for practical purposes.



Approximation 2: American- vs. European-style options

The JGB VIX formula's derivation relies on the assumption that the options may be exercised only upon expiry, i.e. European-style. In practice, however, OSE's JGB future options may be exercised at any time, i.e. American-style, and therefore carry an early exercise premium. Using the Longstaff Schwartz (2001)-based American-to-European price transformation proposed in Mele and Obayashi (2014, b), the pricing error is estimated to be just about 1% of the index level. While the estimated impact is greater than the maturity mismatch, it is still small enough to justify the use of American options with the JGB VIX formula. For details regarding the estimation procedure and the chart below, see Mele and Obayashi (2014, b).



Approximation 3: Finite strike ranges and discrete increments

The mathematical derivation assumes the availability of option prices for a continuum of strikes ranging from zero to infinity. In practice, only finite sets of options are listed, which leads to the discrete strike-weighted sums in the JGB VIX formula. This is an unavoidable reality of any options market, but it is straightforward to get a sense of the approximation error simply by varying the range and increment of strikes used to calculate the index.

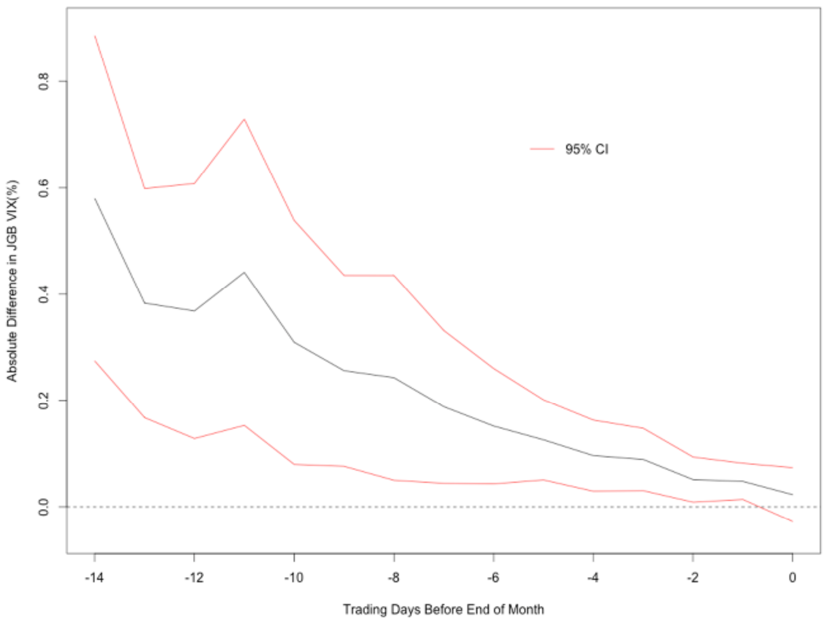
Approximation 4: Missing at-the-money strike

The term $\left(\frac{F-\hat{K}}{\hat{K}}\right)^2$ adjusts for the case in which the price of an option struck exactly at-the-money is not observable as explained in full detail by Mele and Obayashi (2014, b).

Interpolation vs. Extrapolation and the "Early Roll" Rule

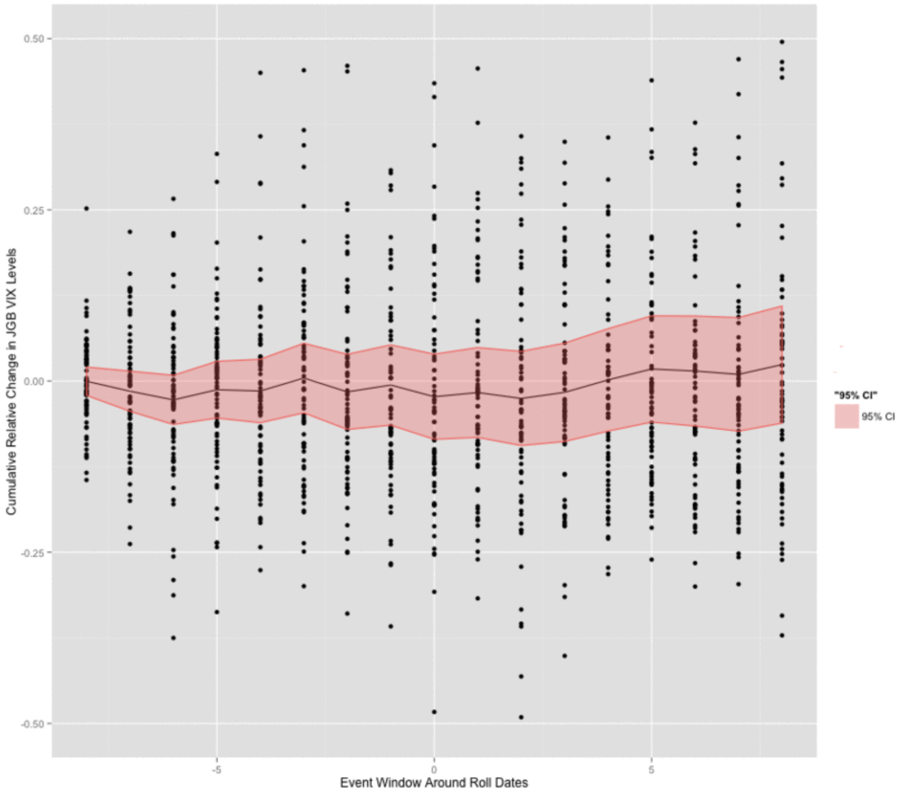
Before CBOE began incorporating weekly SPX options in its VIX calculations, the VIX Methodology specified the use of 2nd and 3rd month options to extrapolate the 30-day volatility when the 1st month option is within one week to expiry, which was intended to reduce the potential impact of any idiosyncratic behavior of super short-dated options. The calculation methodology for the CBOE/CBOT 10-Year Treasury Note Volatility Index (TYVIX), the US counterpart to the JGB VIX, adopts this feature.

The listing schedule of options on JGB futures is such that the first three serial months are available for trading in March, June, September, and December, but only the first two serial months are listed during the other eight months of a year, which precludes the incorporation of an early roll rule. Nevertheless we are able to compare the difference between versions with and without the early roll one month in each quarter. The figure below shows the average and 95% confidence band of the interpolated version minus the extrapolated version over the 15 trading days going into the end of March, June, September, and December since 15 Jan 2008.



One can see that the interpolated version is on average greater than the extrapolated version with statistical significance, implying a steeper volatility slope between the 2nd and 3rd months than between the 1st and 2nd months, and the difference diminishes into month-end. One may reasonably interpret this to mean that the impact on index values of any irregular price action of near-month options within five days to expiry is sufficiently suppressed by the fact that most of the time-weight is on the 2nd month option whether we interpolate or extrapolate into month ends.

Looking at this from another angle, we look for the presence of any systematic patterns of JGB VIX, i.e. interpolation-only, around expiry of the near-month option using a standard event study framework. The figure below shows the average and 95% confidence band of cumulative log change of JGB VIX 10 days before and after month-ends, and we find no statistically significant patterns. This further supports the notion that the absence of the early roll feature from the JGB VIX methodology does not generate any significant biases or quirks in the index's behavior.



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