A REVEALED PREFERENCE RANKING OF U.S. COLLEGES AND UNIVERSITIES*

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We present a method of ranking U.S. undergraduate programs based on students' revealed preferences. When a student chooses a college among those that have admitted him, that college "wins" his "tournament." Our method efficiently integrates the information from thousands of such tournaments. We implement the method using data from a national sample of high-achieving students. We demonstrate that this ranking method has strong theoretical properties, eliminating incentives for colleges to adopt strategic, inefficient admissions policies to improve their rankings. We also show empirically that our ranking is (1) not vulnerable to strategic manipulation; (2) similar regardless of whether we control for variables, such as net cost, that vary among a college's admits; (3) similar regardless of whether we account for students selecting where to apply, including Early Decision. We exemplify multiple rankings for different types of students who have preferences that vary systematically. *JEL* Codes: I2, I23, C35, D11.

I. Introduction

In this article, we propose a new ranking of U.S. undergraduate programs based on students' revealed preferences—that is, the colleges students prefer when they can choose among them. The result is a ranking of colleges based on their desirability. Our procedure builds on existing methods for binary paired comparison rankings that have been studied extensively in statistics and applied to the ranking of players in tournaments, such as chess and tennis. Intuitively, when a student makes his matriculation

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decision among colleges that have admitted him, he identifies a single college as the winner of a competition, which we describe as "a matriculation tournament." Our method efficiently integrates the information from thousands of matriculation tournaments to produce a ranking of colleges ordered by student preferences as revealed by their college choices. Our ranking method is both rigorous and straightforward in motivation, making it easy to communicate what information the resulting ranking contains and does not contain.

The two existing measures of students' revealed preference that are in common use are a college's admissions rate—the percentage of the college's applicants that it admits—and its matriculation rate or "yield"—the percentage of admitted students who matriculate at the college. These measures play an important role in the rating formulas of *U.S. News and World Report* (USNWR) and other college guides. The influential nature of such guides gives colleges an incentive to improve their admissions rates and yield, not just by actually becoming more desirable but by superficial manipulation. Anecdotal evidence suggests that colleges manipulate their admissions rate by encouraging noncompetitive students to submit applications (Toor 2000) and that colleges improve their yield by deliberating rejecting applicants whose qualifications are so good that they will likely get into more desirable colleges and therefore not matriculate (Golden 2001). Incentives to improve their yield may also cause colleges to favor Early Decision applicants who precommit themselves to matriculate if admitted (Fallows 2001; Avery, Fairbanks, and Zeckhauser 2003).[1]

Using a theoretical model, we demonstrate a number of positive properties of our revealed preference ranking method. In particular, we show that it is not vulnerable to the aforementioned manipulations that affect the admissions rate or yield. For example, our method removes the incentive to reject overqualified applicants. This is because although overqualified applicants are unlikely to enroll, a college gains tremendously in our ranking when they do enroll and it loses only trivially in our

1. There are other reasons a college may favor Early Decision applicants, such as revenue and enrollment management. Early Decision programs are different from Early Action programs, in which students do not precommit. Instead, in Early Action, colleges commit to admitting a student early on, before having seen most of their applicant pool.

ranking if the overqualified applicants enroll instead at a much higher ranked institution.

To show how our method works, we construct ranking examples using 3,240 high-achieving students whom we surveyed specifically for this study. To produce definitive rankings—if, for instance, colleges guides wanted to use our ranking—we would need a fully representative sample of students, which we could construct using administrative data. However, the purpose of this article is to explore the properties of our ranking method, and—for this purpose—the survey data have important advantages over administrative data. First, the survey data allow us to show that our ranking is, as predicted by theory, resistant to manipulation by colleges. Second, the survey data are useful for showing whether it is important to account for the potentially confounding effects of tuition discounts and other factors that might make a college "win" when it would otherwise lose. Third, the survey data allow us to demonstrate how multiple rankings can be constructed when different types of students—for instance, students from different regions of the country—have systematically different ideas about what makes a college desirable.

Finally, because the survey data include the students' own reports of the colleges they prefer *regardless* of whether they actually applied to them, we can investigate the consequences of students' self-selecting into applications. To clarify, the issue is not self-selection per se. We show, using theory, that some types of self-selection are innocuous: students applying to a random subset of colleges, students failing to apply to colleges that will predictably deny them admission, students failing to apply to colleges where they are highly overqualified. An issue arises when selection is strategic in the sense that a student who prefers college A to college B applies to college B but not to college A, even though he has a reasonable chance of admission at both schools. In practice, such selection is most likely to arise in the context of Early Decision.

Our revealed preference ranking exercise is analogous to other multiple comparison problems in which we need produce a complete ranking of objects—athletes, teams, vertically differentiated goods—when an exhaustive set of direct comparisons is not available.[2]

2. For example, in a Swiss system tournament, every competitor competes against only a few other individuals rather than against every other competitor.

Although analogous to some ranking exercises, our revealed preference ranking should be differentiated from college rankings such as that of USNWR, which purports to reduce college "goodness" or "quality" to a single index via a formula that gives various weights to college characteristics, such as average test scores, per student resources, faculty qualifications, graduation rate, class size, and alumni giving. Because USNWR purports to measure quality, it needs to choose the weight for each college characteristic. In contrast, the revealed preference rankings implicitly weight college characteristics by the degree to which students care about them. Also, whereas rankings like USNWR assume that college quality is unidimensional, we do not impose such unidimensionality.[3] To the extent that students' preferences exhibit substantial agreement in their preferences, our method will reflect this. However, our method applies equally well to situations in which students preferences are extremely idiosyncratic (random preferences, for instance, will produce a ranking so imprecise as to be no ranking at all) and situations in which different types of students perceive desirability in systematically different ways. In this article, we initially show a ranking that assumes desirability is unidimensional, but we subsequently show rankings specific to various types of students.[4]

See David (1988) for a summary of the rich body of work on multiple comparison modeling, which mainly focuses on paired comparison models, where each tournament contains only two players. See Zermelo (1929), Good (1955), Elo (1978), and Glickman (1993, 1999, 2001) for application of paired comparison models to player ratings based on chess tournament results. Although this study is the first to use statistical comparison models to rank colleges, statistical models have been used to identify the characteristics of colleges that are most attractive to students and to identify the characteristics of students that are most attractive to colleges. See, for example, Manski and Wise (1983), Avery and Hoxby (2004), and Long (2004).

3. There are several concerns with USNWR-type rankings. The first is that the weights on observable college characteristics are ad hoc. The second is that there are many college characteristics that USNWR does not consider—perhaps because they are not readily measured. Notice that this is a not a problem for revealed preference rankings: students prefer colleges based on whatever they observe, not what the statistician can measure. Third, USNWR has an incentive to create fresh interest every year to sell publications. Such an incentive probably explains why the magazine's weighting formula is opaque and changed annually. Fourth, the revealed preference measures used by USNWR (the admissions rate and yield) are highly manipulable by colleges, as we show in this article.

4. We show the unidimensional ranking first because we find that, in practice, multiple rankings are not needed for very elite colleges.

This article's purpose is exploring the ranking method, not producing a definitive ranking. Nevertheless, the rankings we construct herein are informative for the approximately 100 colleges we attempt to rank. These colleges are precisely the ones that most interest high-achieving students. Because the students in our sample exhibit substantial agreement in their preferences, our ranking of them is reasonably precise despite the modest size of our sample.

The article proceeds as follows. Section II describes the data, and Section III describes our base model. Our baseline ranking is presented in Section IV. We show rankings for specific types of students in Section V. In Section VI, we analyze incentives for strategic manipulation. In Section VIII, we take up the issue of students self-selecting into college applications—using Early Decision as the concrete example. We conclude in Section IX.

II. Data

To construct revealed preference rankings, we use data from the College Admissions Project survey of high school seniors in the college graduating class of 2004.[5] We designed the survey to gather data on students with very high college aptitude who are likely to gain admission to the colleges with a national or broad regional draw. Because these students represent a very small share of the population of U.S. students, nationally representative surveys, such as the National Educational Longitudinal Survey, do not provide enough information for an analysis of their behavior. The focus of the College Admissions Project on students with very strong academic credentials ensures that we have a sufficient number of tournaments among colleges with a national draw to construct a revealed preference ranking among them. In addition, by designing the survey ourselves, we ensured that we had data on parents' alma maters, each student's net cost at each college that admitted him or her, and the colleges a student preferred regardless of whether he applied. Such variables are usually omitted by surveys.

If we used administrative data, rather than survey data, we could produce definitive rankings of many more colleges—although many of them could probably only be included in regional

5. See Avery and Hoxby (2000) for additional detail.

or other specialized rankings (see Section V). Administrative data would not, however, allow us to explore many of the issues we explore herein.

II.A. Survey Design

To find students who were appropriate candidates for the survey, we worked with counselors from 510 U.S. high schools that had a record of sending several students to selective colleges each year.[6] Each counselor selected 10 students at random from the top of the senior class as measured by grade point average. Counselors at public schools selected students at random from the top 10% of the senior class, and counselors at private schools (which tend to be smaller and have higher mean college aptitude) selected students at random from the top 20% of the senior class.[7] The counselors distributed the surveys to students, collected the completed surveys, and returned them to us for coding.[8] Students were tracked using a randomly assigned number.

Survey participants completed a first questionnaire in the fall of their senior year. The fall questionnaire asked for information on the student's background, credentials, and college applications. The majority of the questions were taken directly from the Common Application, which is accepted by many colleges in lieu of their proprietary application forms. Each student listed up to 10 colleges that he or she preferred, in order of preference, regardless of whether he was applying there. In addition, each student listed the colleges and graduate schools attended by his or her parents and siblings.

Students completed a second questionnaire in the spring, after the deadline for them to accept an offer of admission. The spring questionnaire asked for information about the student's

6. High schools were identified using experts (thanked in our acknowledgments) and published guides to secondary schools, such as Peterson's (1999) and *Newsweek*'s annual list.

7. The counselors were given detailed instructions for random sampling from the top 20, 30, 40, or 50 students in the senior class depending on the size of the school. For example, a counselor from a public school with 157 students was asked to select 10 students at random from the top 20 students in the senior class, with the suggestion that the counselor select students ranked numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19.

8. The exception was the parent survey, which parents mailed directly to us in an addressed, postage-paid envelope so that they would not have to give possibly sensitive financial information to the high school counselor.

admission outcomes, financial aid offers, scholarship offers, and matriculation decision. We obtained detailed information on aid, grants, scholarships, and loans so that we could compute each student's net cost at each school. On a third questionnaire distributed to a parent of each survey participant, we collected information on parents' income range (see Table I for the income categories.)

We matched the survey to colleges' own data on their tuition, room and board, location, and other college characteristics for the school year that corresponded to the survey participants' freshmen year.[9]

The College Admissions Project survey produced a response rate of approximately 65%, including full information for 3,240 students from 396 high schools.[10] The final sample contains students from 43 states plus the District of Columbia.[11] Although the sample was constructed to include students from every region of the country, it is intentionally representative of applicants to highly selective colleges and therefore nonrepresentative of U.S. high school students as a whole. Also, because the students are drawn from schools that send several students to selective colleges each year, the students in the sample are probably slightly better informed than the typical high-aptitude applicant.

II.B. Sample Statistics

The summary statistics shown in Tables I and II demonstrate that students in the sample are high achieving. The average (combined verbal and math) SAT score among participants was 1357, which put the average student in the sample at the 90th percentile of all SAT takers.[12] About 5% of the students won a National Merit Scholarship; 20% of them won a portable outside

9. See Avery and Hoxby (2004) for a complete description of administrative data sources.

10. The most common reasons for failure to return the survey were changes of high school administration, an illness contracted by the counselor, and other administrative problems that were unrelated to the college admissions outcomes of students who had been selected to participate.

11. The states missing from the sample are Alaska, Delaware, Iowa, Mississippi, North Dakota, South Dakota, and West Virginia.

12. We converted American College Test (ACT) scores to SAT scores using the cross-walk provided by the College Board. We converted all college admissions scores into national percentile scores using the national distribution of SAT scores for the freshman class of 2000–2001.

TABLE I

Description of the Students in the College Admission Project Data

Variable	Mean	Std. Dev.	Minimum	Maximum
Male	0.41	0.49	0.00	1.00
White, non-Hispanic	0.73	0.44	0.00	1.00
Black, non-Hispanic	0.04	0.18	0.00	1.00
Asian	0.16	0.36	0.00	1.00
Hispanic	0.04	0.19	0.00	1.00
Native American	0.00	0.03	0.00	1.00
Other race/ethnicity	0.04	0.19	0.00	1.00
Parents are married	0.83	0.38	0.00	1.00
Sibling(s) enrolled in college	0.23	0.42	0.00	1.00
Parents' income (dollars)	119,929	65,518	9,186	240,000
Expected family contribution (dollars)	27,653	16,524	0	120,000
Applied for financial aid?	0.59	0.49	0.00	1.00
National Merit Scholarship winner	0.05	0.22	0.00	1.00
Student's combined SAT score (points)	1357	139	780	1600
Student's SAT score (national percentiles)	90.4	12.3	12.0	100.0
Median SAT score at *most* selective college to which student was admitted (national percentiles)	86.4	10.4	33.5	98.0
Median SAT score at *least* selective college to which student was admitted (national percentiles)	73.8	14.6	14.3	97.0
Student's high school was private	0.45	0.50	0.00	1.00

Notes. The data source is the College Admissions Project data set. Unless otherwise noted, all variables are shares.

scholarship; and 46% of them won a merit-based grant from at least one college. Forty-five percent of the students attended private school, and their parents' income averaged $155,570 in 2009 dollars.[13] However, 76% of the sample had incomes below the cut-off where a family is considered for aid by selective private colleges, and 59% of the students applied for need-based financial aid. Among survey participants, 73% were white, 16% Asian, 3.5% black, and 3.8% Hispanic.

Table II shows descriptive statistics on the colleges where the students applied, were admitted, and matriculated. It shows that the survey participants made the sort of application and matriculation decisions we would expect. The mean college to which they applied had a median SAT score at the 83rd percentile; the mean

13. Dollars are adjusted in 2009 dollars using the Consumer Price Index-U. Avery and Hoxby (2004) provide further detail about the construction of the aid variables and how, in some cases, parents' income was estimated based on their Expected Family Contribution, a federal financial aid measure.

TABLE II

DESCRIPTION OF THE COLLEGES IN THE COLLEGE ADMISSION PROJECT DATA

	Colleges at Which Students					
	Applied		Were Admitted		Matriculated	
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Matriculated at this college	0.18	0.39	0.28	0.45	1.00	0.00
Admitted to this college	0.66	0.47	1.00	0.00	1.00	0.00
Grants from this college (dollars)	1778	4933	2720	5870	4029	7051
Loans from this college (dollars)	413	1856	641	2282	1020	2722
Work study amount from this college (dollars)	111	483	172	593	296	768
Father is an alumnus of this college	0.03	0.17	0.04	0.20	0.07	0.25
Mother is an alumna of this college	0.02	0.14	0.03	0.17	0.04	0.19
Sibling attended or attends this college	0.04	0.19	0.05	0.21	0.08	0.28
College is public	0.2631	0.4403	0.3325	0.4711	0.2843	0.4512
College's median SAT score (national percentiles)	83.8816	12.0390	80.5947	12.5188	83.4215	12.5494
In-state tuition (dollars)	18,181	9,199	16,435	9,594	17,432	9,513
Out-of-state tuition (dollars)	20,498	5,891	19,294	6,191	19,841	6,371
Tuition that applies to this student (dollars)	19,277	7,965	17,671	8,492	18,340	8,599
College is in-state	0.2666	0.4422	0.3270	0.4691	0.3368	0.4727
Distance between student's high school and this college (miles)	673	873	597	809	576	827

Notes. The data source is the College Admissions Project data set. Unless otherwise noted, all variables are shares.

college to which they were admitted had median SAT score at the 81st percentile. Nearly half of the students (47.5%) applied to at least one Ivy League college. These statistics suggest that students applied to a portfolio of colleges that included both safety schools and colleges that were something of a reach.

Students matriculated at colleges that were more selective, on average, than the colleges to which they were admitted: the median SAT score of matriculation colleges was at the 83.4th percentile, as opposed to the 81st percentile for colleges to which students were admitted. Thus, although students included safety schools in their portfolios, they infrequently matriculated at them. One measure of the high college aptitude of the survey participants is the list of colleges at which the largest numbers of participants matriculated. Seventeen institutions enrolled at least 50 students from the sample: Harvard, Yale, University of Pennsylvania, Stanford, Brown, Cornell, University of Virginia, Columbia, University of California–Berkeley, Northwestern, Princeton, Duke, University of Illinois, New York University, University of Michigan, Dartmouth, and Georgetown.

III. A Model of College Admissions

In this section, we present a model of college admissions. It is important to explain the role that the model plays in this article. The model demonstrates how revealed preferences evince themselves in a world where colleges maximize plausible objective functions in the admissions process. However, our revealed preference method does *not* require such maximization: it would produce the same ranking if we only assume that students reveal which colleges they prefer by choosing them. The primary benefit of the model is that it clarifies colleges' incentives to manipulate admissions decisions and the students' incentives to self-select into applications. Thus, the model helps us pinpoint how to test whether the ranking is sensitive to manipulation and/or self-selection.

III.A. The Basic Model

There are I students and J colleges. We focus on the case where I is very large so that asymptotic results apply to each ranking method studied. Each college seeks to enroll a proportion

π of all students (in expectation), but there are not enough students to fill all the colleges: $\pi > 1 \backslash I$.

Students differ in college aptitude (hereafter, just "aptitude") and (unidimensional) preferences among colleges. Aptitude z_i is distributed according to a continuous density function f which takes values on (0, 1) with associated cumulative distribution function F. A student's aptitude z_i is observable to any college to which he applies. In the base model, we assume that all students apply to all colleges. We relax this assumption later, allowing each student to observe a signal of his aptitude and to vary his (probabilistic) choice of applications in response to the observed signal.

Student i has a separate utility value, u_{ij} for attending each college j, where

$$u_{ij} = \theta_j + \varepsilon_{ij} \tag{1}$$

θ_j represents our unidimensional index of the desirability of college j, and ε_{ij} is a (mean zero) idiosyncratic noise term. We index the colleges so that $\theta_1 \geq \theta_2 \geq \ldots \geq \theta_J$. We assume that θ_1 and θ_2 are sufficiently large that each college is the first choice for some proportion of students greater than π. This ensures that at least these two colleges are selective—that is, that they do not admit all applicants—when all students apply to all colleges.

We build on the Bradley and Terry (1952) and Luce (1959) models of choice by assuming that the ε_{ij}'s follow an extreme value distribution.[14] Under these conditions, the model can be rewritten as a conditional logit model, sometimes called McFadden's choice model. (See Maddala [1983] and Anderson, de Palma, and Thisse [1992] for further mathematical background.) Specifically, if student i is admitted to the specific colleges in set S_i, then his probability of matriculating at college j is given by

$$p_{ij*} = \frac{\exp(\theta_{j*})}{\sum_{j \in s_i} \exp(\theta_j)}, j^* \in S_i. \tag{2}$$

14. The main alternative to the assumption of an extreme value distribution for potentially observed desirability is a normal distribution (Thurstone 1927; Mosteller 1951). Previous research indicates that the distinction between extreme value and normal distributions has little effect on the results of paired comparison analysis (see Stern 1992 for further discussion). However, the choice of the extreme value distribution eases computational efficiency, and we adopt it for that reason.

DEFINITION. A *revealed preference ranking* of colleges RP_j is given by the estimated θ values from a given set of matriculation data: $RP_j = \hat{\theta}_j$. This ranking can be viewed as either ordinal (where colleges with higher estimated θ's have higher ordinal rankings) or cardinal (substituting the estimated θ's in equation [2] to produce estimated matriculation probabilities for the choice among a given set of colleges).

Throughout this article, we estimate the θ values as the posterior modes for each college's θ_j based on maximizing the product of multinomial logit probabilities derived from equation (2). Section IV provides additional detail about the numerical methods we use to implement our ranking method.

III.B. Admission Rules

An admission rule for college *j* identifies the students who are admitted to that college as a function of the aptitudes of individual students, because these aptitudes are observable to the colleges. Colleges only discriminate among students based on aptitudes.

DEFINITION. An *admission rule* for college *j* is a function A_j: [0,1] → {0,1}, where $A_j(z) = 1$ indicates that an applicant with aptitude z is admitted by college *j* and $A_j(z) = 0$ indicates that an applicant with aptitude z is rejected by college *j*.

DEFINITION. A combination of admissions rules $A = (A_1, A_2, \ldots A_J)$ is *feasible* if some colleges enroll π students, other colleges enroll fewer than π students, and all colleges that enroll fewer than π students admit all applicants. Any set of admission rules for colleges other than *j*, A_{-j}, is *possibly feasible* if there is some admission rule for *j*, a_j*, such that (A_{-j}, a_j*) produces a set of feasible admission rules.

That is, a set of admissions rules is feasible if every college that can fill its class does so, and every college that cannot fill its class is necessarily nonselective, as its primary goal is then to enroll as many students as possible.

PROPOSITION 1. Given any set of possibly feasible admissions rules for all but one college *j** and any admissions rule for *j** that fills its class, the revealed preference ranking produces consistent estimates of θ_j for every college.

Proof. See Online Appendix.

The combination of admissions rules in any Nash equilibrium of the admissions game must be feasible. Thus, Proposition 1 implies that in a Nash equilibrium each college expects to receive the same (asymptotic) revealed preference ranking for any feasible admissions rule it might adopt given the admissions rules of the other colleges. Intuitively, suppose that college B wishes to avoid competition with college A and rejects all students that (it anticipates) will be admitted to college A. But it could still be compared and ranked relative to college A via a triangulation method because colleges A and B necessarily compete with the nonselective colleges that admit all students. Thus, the relative performance of colleges A and B in competition with nonselective colleges induces consistent estimates of θ_A and θ_B even if these colleges never compete against each other directly.

III.C. Accounting for College Characteristics that Vary across Students

For the purpose of empirical estimation, we generalize equation (2) to incorporate a number of college characteristics that vary across students and that (are likely to) systematically influence the student's matriculation choice. Examples of such characteristics are: the "net price" charged after accounting for institutional aid (as opposed to the "list price"), the college's distance from the student's home, and whether the college is the alma mater of one of the student's parents. This generalization of the model does not directly affect the logic or results of Propositions 1 through 4 but would complicate exposition of the proofs of those results.

To understand this point, consider college C, which offers large tuition discounts to a few admits, small discounts to a slightly larger number of admits, and no discount to most admits. Furthermore, suppose that when C does not offer an admit a tuition discount, it always loses in pairwise competition with college A, and it loses in pairwise competition with college B with a 0.5 probability. Suppose that the small tuition discount moderately improves college C's chances of winning an admit in pairwise competition with either A or B and that the large tuition discount greatly improves C's chances of winning an admit in pairwise competition with either A or B.

Our revealed preference rankings are based on the decisions of the *average* admit to each college—in this example, the ranking of college C is based on an appropriately weighted combination of the cases just listed. If we did not know how the net price offered by C varies across the students it admits, we might find the win-loss record among colleges A, B, and C somewhat confusing. Knowing the tuition discounts, we can correctly infer that certain students were more likely to choose college C because they were offered tuition discounts that the average admit does not experience.

In more abstract terms, we derive better estimates of the θ_j's (which reflect colleges' average characteristics) by simultaneously estimating the effects of individually-varying characteristics.

Now let each student i have a separate utility value, u_{ij}, for attending each college j:

$$u_{ij} = \theta_j + \sum x_{ij}\delta_i + \varepsilon_{ij}$$

As before, θ_j represents the unidimensional desirability of college j, and ε_{ij} is an idiosyncratic noise term. The vector $x_{ij} = (x_{1j}, x_{2j}, \ldots x_{Kj})$ represents K mean zero characteristics of college j that can vary among students and the vector $\boldsymbol{\delta} = (\delta_1, \delta_2, \ldots \delta_K)$ represents linear weights that translate those characteristics into utility values.[15] The probabilities for the matriculation model become:

$$p_{ij*} = \frac{\exp(\theta_{j*} + x'_{ij}\delta)}{\sum_{j \in S_i} \exp(\theta_j + x'_{ij}\delta)}, j^* \in S_i. \tag{3}$$

In practice, we find that estimating $\boldsymbol{\delta}$ along with the θ_j's has only a small effect on our ranking. We discuss the implications of this finding later.

In our empirical analysis, we include 10 covariates for each college: four related to cost (tuition, grants, loans, and work-study funding), three related to geography (distance, a "home state" indicator, and a "home region" indicator), and three for family

15. Each characteristic is presented in mean zero form so that we obtain the college's desirability at its average level in the data from our estimation procedure.

connections (indicators for father, mother, or sibling having attended the college).

IV. The Baseline Ranking (Assumes Desirability Is Unidimensional)

IV.A. Estimation

The revealed preference ranking method summarizes each college's desirability as the posterior mode of that college's θ_j based on maximizing the product of multinomial logit probabilities derived from equation (3). The posterior modes of the θ_j's can be computed with a Newton-Raphson algorithm for multinomial logit models as implemented in Stata. Asymptotic convergence of the estimated θ_j's follows from Ford (1957). In addition to using this maximum likelihood method, we fit our models using Markov chain Monte Carlo (MCMC) simulation from the posterior distribution to obtain summaries of the desirability parameters for colleges. The two methods produce identical rankings, so the advantage of the MCMC method is that it generates quantities of interest that are more complex than simple estimates of the desirability parameter.[16] In particular, if we want to compare (say) colleges 15 and 20, we can extract pairs of values from the simulated posterior distribution, then compute the proportion of pairs in which the estimate for θ_{15} is greater than θ_{20}.

To conduct the MCMC simulation, we assume a locally uniform but proper prior distribution that factors into independent densities and that consists of the following components:

$$\begin{aligned} &\theta_i \sim N(0, \sigma^2) \\ &1/\sigma^2 \sim Gamma(0.1, 0.1) \\ &\delta_k \sim N(0, 100) \text{ for } k = 1, 2, \ldots K, \end{aligned} \tag{4}$$

where δ_k indexes the kth element of the vector $\boldsymbol{\delta}$.

We use the program BUGS (Spiegelhalter et al. 1996) to implement the MCMC algorithm. We set initial values of all

16. More precisely, maximum likelihood and MCMC produce identical rankings for the schools we attempt to rank. Another advantage of MCMC is that it produces indicators of statistical significance that do not rely on asymptotic properties of the likelihood function, such as symmetry around the maxima, that may not hold with limited information.We do not attempt to rank schools for which we have only a few matriculation tournaments.

parameters equal to the prior mean values and then simulate values from the conditional posterior distributions of each model parameter. We repeat this process until the distributions of values for individual parameters stabilize. The values simulated beyond this point can be viewed as coming from the posterior distribution. For each model, we use a "burn-in period" of 10,000 iterations to reach the stationary distribution and then summarize parameters on the basis of every fifth iteration of a subsequent 30,000 iterations to reduce the autocorrelation in successive parameter draws.[17] This process produces 6,000 values per parameter on which to calculate parameter summaries.

IV.B. Results from the Baseline Model

This section presents the revealed preference ranking results that result from applying our estimation procedure to the data from the College Admissions Project, under the assumption that college desirability is unidimensional. In our baseline ranking, we include the 110 colleges that competed in matriculation tournaments in at least six of the nine census divisions of the United States. We do not rank the military academies. A small college may fail to appear in the rankings simply because, in our sample, such a small number of students apply to it that it does not meet the six division criterion even if it would meet this criterion if we had the entire population of applicants.

The six division criterion for inclusion in the baseline ranking is designed to serve as a proxy for ensuring that (1) there is sufficient information in student matriculation decisions to compare the college to others, and (2) the ranking of each college is based on a diverse set of applicants, not those who are concentrated in a particular part of the country. Although the six-division cut-off for inclusion in the baseline ranking is somewhat arbitrary, descriptive statistics suggest that it is reasonably successful at achieving the two criteria. The mean college in the baseline ranking competed in 73 matriculation tournaments, and the median college competed in 57.

Table III presents the baseline revealed preference ranking of colleges and universities with a national draw. For each college, we show its mean desirability among students based on the point estimate of the θ value for that college.

17. Based on trace plots from our data analyses, 10,000 iterations was sufficient to reach the stationary distribution.

TABLE III

A Revealed Preference Ranking of Colleges Based on Matriculation Decisions

Rank Based on Matriculation (with Covariates)	College Name	Theta	Implied Prob. of "Winning" vs. College Listed... 1 Row Below	10 Rows Below	Rank Based on Matriculation (no Covariates)
1	Harvard University	9.13	0.59	0.93	1
2	Caltech	8.77	0.56	0.92	3
3	Yale University	8.52	0.59	0.92	2
4	MIT	8.16	0.51	0.89	5
5	Stanford University	8.11	0.52	0.90	4
6	Princeton University	8.02	0.73	0.90	6
7	Brown University	7.01	0.56	0.78	7
8	Columbia University	6.77	0.54	0.73	8
9	Amherst College	6.61	0.51	0.71	9
10	Dartmouth	6.57	0.52	0.72	10
11	Wellesley College	6.51	0.53	0.71	12
12	University of Pennsylvania	6.39	0.56	0.71	11
13	University of Notre Dame	6.13	0.51	0.70	16
14	Swarthmore College	6.07	0.55	0.69	13
15	Cornell University	5.87	0.53	0.67	17
16	Georgetown University	5.77	0.50	0.64	15
17	Rice University	5.75	0.50	0.64	19
18	Williams College	5.75	0.51	0.66	14
19	Duke University	5.72	0.52	0.65	18
20	University of Virginia	5.65	0.51	0.67	21
21	Brigham Young University	5.61	0.53	0.68	20
22	Wesleyan University	5.48	0.55	0.67	24
23	Northwestern University	5.30	0.51	0.64	23
24	Pomona College	5.27	0.52	0.65	22
25	Georgia Institute of Technology	5.17	0.50	0.63	30
26	Middlebury College	5.17	0.50	0.64	27
27	U. of California–Berkeley	5.17	0.51	0.64	25
28	University of Chicago	5.11	0.51	0.63	29
29	Johns Hopkins University	5.08	0.54	0.63	26
30	U. of Southern California	4.92	0.52	0.60	32
31	Furman University	4.86	0.52	0.60	28
32	U. of North Carolina–Chapel Hill	4.77	0.52	0.58	34

TABLE III (CONTINUED)

Rank Based on Matriculation (with Covariates)	College Name	Theta	Implied Prob. of "Winning" vs. College Listed... 1 Row Below	10 Rows Below	Rank Based on Matriculation (no Covariates)
33	Barnard College	4.70	0.51	0.57	35
34	Oberlin College	4.67	0.51	0.57	38
35	Carleton College	4.63	0.51	0.56	31
36	Vanderbilt University	4.61	0.51	0.56	36
37	Davidson College	4.58	0.50	0.55	42
38	UCLA	4.57	0.50	0.58	44
39	University of Texas at Austin	4.56	0.51	0.57	33
40	University of Florida	4.53	0.52	0.57	40
41	New York University	4.46	0.51	0.55	46
42	Tufts University	4.43	0.51	0.57	50
43	Washington and Lee University	4.41	0.51	0.57	45
44	Vassar College	4.39	0.50	0.58	47
45	Grinnell College	4.38	0.50	0.57	43
46	University of Michigan	4.37	0.50	0.58	48
47	U. Illinois Urbana-Champaign	4.36	0.53	0.59	41
48	Carnegie Mellon University	4.26	0.50	0.56	59
49	U. of Maryland–College Park	4.26	0.50	0.57	51
50	College of William and Mary	4.25	0.50	0.57	62
51	Bowdoin College	4.25	0.52	0.57	53
52	Wake Forest University	4.16	0.51	0.56	54
53	Claremont Mckenna College	4.14	0.51	0.56	39
54	Macalester College	4.08	0.50	0.55	65
55	Colgate University	4.07	0.51	0.56	55
56	Smith College	4.05	0.51	0.57	49
57	Boston College	4.04	0.50	0.57	56
58	University of Miami	4.02	0.50	0.58	37
59	Mount Holyoke College	4.01	0.50	0.58	60
60	Haverford College	3.99	0.51	0.60	63
61	Bates College	3.96	0.50	0.60	52
62	Connecticut College	3.95	0.51	0.60	61
63	Kenyon College	3.92	0.51	0.59	57

TABLE III (CONTINUED)

Rank Based on Matriculation (with Covariates)	College Name	Theta	Implied Prob. of "Winning" vs. College Listed… 1 Row Below	10 Rows Below	Rank Based on Matriculation (no Covariates)
64	Emory University	3.88	0.51	0.59	66
65	Washington University	3.86	0.51	0.60	64
66	Occidental College	3.83	0.52	0.62	68
67	Bryn Mawr College	3.77	0.52	0.61	67
68	Southern Methodist University	3.70	0.50	0.59	58
69	Lehigh University	3.69	0.53	0.59	71
70	Holy Cross College	3.59	0.51	0.58	69
71	Reed College	3.57	0.50	0.58	78
72	Rensselaer Polytechnic Institute	3.55	0.50	0.57	77
73	Florida State University	3.55	0.51	0.57	73
74	Colby College	3.50	0.51	0.56	72
75	UC–Santa Barbara	3.45	0.53	0.56	83
76	Miami U.–Oxford Campus	3.34	0.50	0.54	75
77	George Washington University	3.34	0.50	0.57	79
78	Fordham University	3.33	0.50	0.57	74
79	Dickinson College	3.33	0.51	0.59	70
80	Sarah Lawrence College	3.28	0.51	0.57	80
81	Catholic University of America	3.26	0.50	0.58	91
82	Bucknell University	3.26	0.50	0.58	85
83	U. of Colorado at Boulder	3.26	0.51	0.60	81
84	U. of Wisconsin–Madison	3.24	0.51	0.61	84
85	Arizona State University	3.22	0.51	0.61	86
86	Wheaton College	3.17	0.53	0.61	76
87	Trinity College	3.07	0.51	0.59	82
88	Rose-Hulman Inst. of Tech.	3.04	0.51	0.59	99
89	U. of California–Santa Cruz	2.99	0.50	0.58	94
90	Boston University	2.98	0.51	0.65	88
91	U. of California–San Diego	2.96	0.50	0.66	98

TABLE III (CONTINUED)

Rank Based on Matriculation (with Covariates)	College Name	Theta	Implied Prob. of "Winning" vs. College Listed... 1 Row Below	10 Rows Below	Rank Based on Matriculation (no Covariates)
92	Tulane University	2.94	0.52	0.66	90
93	University of Richmond	2.86	0.51	0.67	97
94	Case Western Reserve Univ.	2.80	0.51	0.71	89
95	Colorado College	2.76	0.51	0.70	95
96	Indiana Univ.–Bloomington	2.71	0.50	0.72	93
97	Penn State–University Park	2.71	0.51	0.73	92
98	American University	2.68	0.51	0.74	87
99	Hamilton College	2.65	0.57	0.73	96
100	University of Washington	2.36	0.52	0.70	100
101	University of Rochester	2.30	0.51	0.91	101
102	Michigan State University	2.27	0.53		108
103	Lewis & Clark College	2.16	0.56		104
104	Clark University	1.92	0.51		102
105	Skidmore College	1.90	0.53		103
106	Purdue University	1.76	0.52		110
107	Colorado State University	1.70	0.51		107
108	Syracuse University	1.65	0.50		106
109	University of Vermont	1.63	0.53		105
110	Scripps College	1.50	0.82		109

Estimates of other parameters:		
Tuition (in thousands of dollars, in-state or out-of-state, whichever applies)	–0.021	(0.019)
Grants (in thousands of dollars)	0.087	(0.007)
Loans (in thousands of dollars)	0.098	(0.017)
Work-study (in thousands of dollars)	0.050	(0.007)
Indicator: Is dad's college	0.481	(0.158)
Indicator: Is mom's college	0.050	(0.202)
Indicator: Is a sibling's college	0.592	(0.135)
Indicator: College in home state	0.074	(0.132)
Indicator: College in home region	0.005	(0.108)
Distance from home (thousands of miles)	0.035	(0.067)

Notes. Estimates of parameters in equation (3) by maximum likelihood as described in the text. Numbers in parentheses are standard errors. Test statistics for parameter estimates without a standard error shown may be found in Online Appendix Table A.1.

All of the top 20 colleges in the baseline ranking, except for the University of Virginia, are private institutions. About four-fifths of the top 20 are universities—the exceptions being Amherst, Wellesley, Williams, and Swarthmore. The next 20 institutions are, however, a mix of public and private, small and large, colleges and universities. They are also more geographically diverse. They include private schools from Midwestern and Southern states: University of Chicago, Furman, Carleton, Davidson, Northwestern, Oberlin, Vanderbilt. There are also several public universities: the University of California–Berkeley, the University of California–Los Angeles, Georgia Institute of Technology, the University of Texas at Austin, and the University of North Carolina at Chapel Hill. The colleges ranked from 41 to 106 include a good number of states' "flagship" universities, numerous liberal arts colleges, several private universities, and a few more institutes of technology.[18]

Although the estimated θ's are not based largely on head-to-head tournaments (they are based on multiplayer tournaments and on inferences drawn from triangulated tournaments), the probability that any college will be picked over another college in a head-to-head matriculation tournament (in which the student is admitted only to two schools) can be approximated[19] by substituting the point estimates of the θs into the following expression:

$$Prob(j \rightarrow j') = \frac{\exp(\theta_j)}{\exp(\theta_j + \exp(\theta_{j'}),} \tag{5}$$

where → denotes the relation "is ranked higher than."

Columns (4) and (5) of Table III provide the translation for a student who is admitted to only that college and college ranked one placed lower (column (4)) and for a student who is admitted to only that college and the college ranked 10 places

18. The students in our sample who had a Florida resident as a parent were the first cohort to receive Florida A-Plus Scholarships, which allowed them to attend public universities in Florida for free. The initiation of the scholarships generated considerable excitement and may have raised the ranking of public universities in Florida, such as Florida State, among students in our sample.

19. A more principled estimate of the probability would involve averaging the quantity in equation (5) over the posterior distribution of the θ's. Rather than perform this computation analytically, the analysis could take advantage of the MCMC simulation by evaluating the probability expression for each draw of the θ's, and then average this set of values to obtain the approximate posterior mean of the preference probability.

lower (column (5)). Other comparisons can be computed from the estimated θ's. The top row of Table III shows, for instance, that if a students were choosing between (only) Harvard and California Institute of Technology (Cal Tech), her probability of matriculating at Harvard is predicted to be 59%. If the student were choosing between (only) Harvard and Wellesley, the college listed 10 places below Harvard, her probability of matriculating at Harvard is predicted to be 93%.

The estimates in columns (1) through (5) of Table III are computed in a procedure that simultaneously estimates the effect of college characteristics, such as distance and net cost, that vary among students who are admitted to the same school. Column (6) of Table III shows the ordinal rankings that result if our ranking procedure ignores the variation in these characteristics. There is little difference in the results of the two procedures. Some schools move up or down a few places—for instance, Yale and Cal Tech trade places, Stanford and MIT trade places—but the rankings are very similar. In fact, there is 99% correlation between the rankings that do and do not account for the college characteristics that vary among students. This finding does not necessarily suggest that students are indifferent to the individually varying characteristics, such as net cost. We think that a more likely interpretation of the finding is that similarly desirable colleges match one another's offers in an overall way even if the exact details of the offers differ. For instance, a student who is offered a grant by one college may be offered a generously subsidized loan by a similar desirable second college.

We do not attach standard errors to the estimated θ's in Table III because the test of interest is whether any two colleges' rankings are statistically distinct. Instead, we present Table A.1 in the Online Appendix. It shows the percentage of posterior draws in MCMC simulation in which one college's estimated θ is higher than another's. These percentages are the Bayesian analog of paired significance tests. For instance, in 70% of the draws from the posterior distribution, Harvard's estmated θ was higher than Cal Tech's and, in 98% of the draws, Harvard's estimated θ was higher than Yale's. For all other colleges, Harvard's ranking was higher in at least 99% of the draws. It is important not to confuse these significance test analogs with the estimated probability that a college "wins" in a head-to-head competition. For example, Harvard's ranking is higher than Princeton's in more than 99% of draws, but the probability that a student choosing between (only) Harvard and Princeton would matriculate at Harvard is only 75%. In other words, what Table A.1 shows is our confidence about the relative position of the θ's.

As a rule, the lower one goes in the revealed preference ranking, the less distinct is a college's desirability from that of its immediate neighbors in the ranking. Among the top 10 colleges, we generally enjoy confidence of about 75% that a college is ranked higher than the college listed one or two below it. This confidence falls to about 65% for colleges ranked 11 to 20 and falls further to 55% to 60% for colleges ranked 21 to 40. This is not surprising: in many ordinal rankings, cardinal desirability is more bunched the lower one goes in the ranking. That is, there may be less consensus among students about colleges' desirability as we move from the best-known colleges to those that are less widely known. However, in our case, there is another, independent reason the distinctness of colleges' desirability falls off. By the nature of our sample, the data are thickest for the most selective colleges. We did a simple test to determine the degree to which data thickness by itself was responsible for the fall off in confidence: we randomly selected only 20 observations per college. With these data, we found that about two-thirds of the drop-off in confidence disappeared. That is, if our data were equally representative for all colleges, our confidence about the exact rank order would still fall, but it would probably fall only about a third as fast as it does.

IV.C. Comparing Measures of Revealed Preference

One might wonder whether computing the revealed preference ranking is worth the effort. Perhaps, in practice, it produces a ranking very similar to that produced by the readily available admissions rate or yield? If this were so, college guides might reasonably stick with the admissions rate and yield, even if they fully recognized the superior properties of the revealed preference ranking. Table IV shows that this is not so. It is based on the same admissions rate and yield data that are fed into rating formulas like the one used by USNWR.[20]

The leftmost column of Table IV shows the top 20 colleges ranked by the revealed preference method. The middle column shows what these colleges' rankings would be if the admissions

20. The data source is the Common Data Set (2000).

TABLE IV

A Comparison of the Revealed Preference Ranking of Colleges and Rankings Based on the Crude Admissions and Matriculation Rates

	Rank Based on:		
	Revealed Preference (based on Matriculation Tournaments)	Admissions Rate	Matriculation Rate
Harvard	1	4	139
Caltech	2	9	854
Yale	3	12	309
MIT	4	13	422
Stanford	5	7	297
Princeton	6	5	266
Brown	7	14	561
Columbia	8	6	438
Amherst	9	19	916
Dartmouth	10	20	647
Wellesley	11	23	492
U Penn	12	104	794
U Notre Dame	13	58	459
Swarthmore	14	28	1,016
Cornell	15	45	649
Georgetown	16	22	703
Rice	17	25	996
Williams	18	29	797
Duke	19	32	859
U Virginia	20	76	630

Notes. Leftmost column shows baseline rank from Table III. The admissions and matriculation rates are based on the Common Data Set (2000), used by most college guidebooks.

rate were used instead. Half of the top 20 colleges would be ranked outside of the top 20 by the admissions rate. Thus, the admissions rate is an inaccurate indicator of a college's desirability.

Yield turns out to be a *very* inaccurate measure of desirability. The rightmost column of Table IV shows what the top 20 colleges' rankings would be if yield were used. All of the top 20 colleges would be ranked outside the top 100. This occurs because there are many nonselective colleges that have yields close to 100%. A nonselective college is one that will accept any student who has the equivalent of a high school diploma. Applying to one of these colleges is, essentially, the same as enrolling. A student goes to the registrar and signs up. Students like this do not hold matriculation tournaments. Even if they were very high achieving (and they tend not to be) and appeared in our data, they would not contribute much to the revealed preference ranking because they consider so few colleges.

V. Revealed Preference Rankings when Colleges' Desirability Is Multidimensional

So far, we have assumed that colleges' desirability is unidimensional. However, it is plausible that different types of students judge desirability in systemically different ways. In this section, we divide students into types based on the two criteria most requested by college experts with whom we have discussed this article: region of the country and "math/science" versus "humanities" interests. We compute rankings for each type of student, but we note that such rankings are relatively noisy with our survey data because we base the estimated θ on a modest number of observations.

Table V shows the baseline ranking and separate rankings for students who (1) intend to major in the humanities and (2) intend to major in math-oriented areas (engineering, math, computer science, and the physical sciences). There is substantial consistency across these rankings: four colleges (Harvard, Yale, Stanford, and Princeton) are included in the top six colleges in the baseline revealed preference ranking and in both alternate rankings. There are, however, a few notable differences across these rankings. Cooper Union for the Advancement of Science and Harvey Mudd, schools that specialized in the hard sciences, are ranked more highly among students who are math-oriented. Similarly, humanities-oriented students find Cal Tech and MIT so undesirable that they fall out of the top 25 institutions. These same students are so oriented to liberal arts colleges that 12 of them appear in their top 25 whereas only 7 appear there in the baseline ranking.

Online Appendix Table A.2 shows the rankings we obtain for students based on the census division of their high schools. Some of these estimates are very imprecise because the sample for each region is small. In fact, some colleges can be ranked only in some regions—for instance, neither Cal Tech nor Stanford is rankable in Division 6. Also, owing to the small samples, we merely group schools outside of the top 30 in each subgroup ranking. Nevertheless, the results shown in Table A.2 are informative. In the

TABLE V

IMPROVEMENT IN RANKING THAT COLLEGE CAN GAIN BY STRATEGICALLY REJECTING APPLICANTS FOR WHOM IT WOULD HAVE TO COMPETE WITH A COLLEGE THAT IS SIGNIFICANTLY MORE HIGHLY RANKED

	Other College Is Significantly More Highly Ranked if Its Estimated θ_j Is Statistically Significantly Greater than the College's Own θ_j with Confidence of...	
	95%	90%
Mean improvement	0	0.1
Median improvement	0	0
Standard deviation of improvement	0.7	1.2
1st percentile of improvement	–3	–5
10th percentile of improvement	0	–1
90th percentile of improvement	0	1
99th percentile of improvement	2	3

Notes. The simulation is carried out on all 110 colleges that are ranked in the baseline ranking, shown in Table III. The table shows the summary statistics from the simulation. The tests of statistical significance are one-sided—that is, tests that the other college's θ is greater than the college's own θ.

ranking of the top 10 institutions, each region essentially reproduces the baseline ranking. Though there are some conspicuous examples of regional preferences (for example, Southern students find Southern colleges such as University of the South, Clemson, and Rhodes more desirable than do other students), we find that even for colleges ranked 31 to 60, the regional rankings are not very different than the baseline rankings. Regional favorites never represent more than 10% of the top 30 in a given region, and many colleges are ranked in this range from 31 to 60 in almost every region.

The substantial similarities between the baseline rankings and type-specific rankings in Table V and Table A.2 suggest that most elite colleges' desirability is not misstated if we assume that it is unidimensional. However, the differences we observe between the baseline and type-specific rankings suggest that the assumption of unidimensional desirability becomes implausible as we move away from the most elite colleges. The differences also suggest that the baseline ranking is fairly uninformative about colleges that have a specialized mission intended to appeal only to a certain type of students. For example, Brigham Young appears in the top 10 in the subgroup rankings for Division 8 (the home of most Mormons), but does not appear high up in any other division's rankings. If Brigham Young is intended to attract only Mormon students, then its ranking among non-Mormon students contains little content.

VI. Incentives for Manipulation by Colleges

We return to the case in which desirability is unidimensional to focus on incentives for manipulation by colleges. First, we extend the theoretical model to study the Nash equilibrium in admissions rules as a function of the ranking system, then we conduct a series of simulations to verify that the properties of the Nash equilibrium hold in our empirical application.

VI.A. Theoretical Model of Incentives for Manipulation by Colleges

We extend our model by analyzing how the colleges behave if their desirability is judged based on their admissions rate or yield. We compare this to how they behave if their desirability is judged based on the revealed preference ranking.

As discussed in Section I, the two measures of a college's desirability that are commonly used in current rating methods are the admissions rate and yield, where the yield W_j for college j is given by the ratio

$$W_j = (\text{number of students enrolling at college } j)/(\text{number admitted to college } j) \tag{6}$$

Among colleges that fill their classes, there is a one-to-one correspondence between yield and admissions rate, so we only include yield in our definition of a more standard ranking method.

Definition. A *selectivity-based ranking* is an additively separable function of average aptitude and yield: $R_j(Z_j,\ W_j) = (1-\beta)G(W_j) + \beta W_j$ where G is a (weakly) increasing function in Z_i that takes values from 0 to 1 and β is a constant with $0 < \beta \leq 1$.

The selectivity-based ranking is meant to capture the real trade-off that colleges face when their published ranking is based on their yield or admissions rate: colleges are torn between the benefits of competing for the highest-aptitude students and the benefits of maximizing yield to improve their rankings.

Clearly, we do not know colleges' objective functions, but it is reasonable to postulate that colleges care about some or all of the following: filling their class, the aptitude of students who enroll, and their rankings. For concreteness, we assume that a college that fills its class has an objective function given by a weighted average of the average aptitude of students who enroll (Z_j) and its cardinal rankings (R_j and RP_j):

$$U_j(Z_j, R_j, RP_j) = (1 - \alpha_1 - \alpha_2)Z_j + \alpha_1 R_j + \alpha_2 RP_j. \quad (7)$$

This assumption is fairly unrestrictive because any of the weights may equal zero. That is, we assume that the parameters α_1 and α_2 range between 0 and 1, subject only to the restriction $\alpha_1 + \alpha_2 \leq 1$.

In addition, we assume for concreteness that a college's first priority is to fill its class in expectation. Thus, we let the utility for any college with expected enrollment $\pi' \neq \pi$ be: $-|\pi - \pi'|$. This element of the utility functions ensures that each college fill its class if possible and otherwise will attempt to enroll as close to proportion π of all students (in expectation) as possible.

We consider the Nash equilibrium of the admissions game where colleges choose admission rules simultaneously to maximize their utility values in equation (4). In equilibrium, no college will enroll an expected proportion of students greater than π, and any college that enrolls an expected proportion of students less than π will admit all applicants.

DEFINITION. A *threshold admissions rule* for college j consists of an aptitude cutoff z_j^* such that college j admits any applicant with aptitude of at least z_j^* and no applicant with aptitude less than that: $A_j(z) = 1$ if $z \geq z_j^*$ and $A_i(z) = 0$ if $z < z_j^*$. Note that if a college admits all applicants, it is still following a threshold admissions rule with an aptitude cutoff of 0.

PROPOSITION 2. If colleges do not care about their selectivity based rankings ($\alpha_1 = 0$), then there is a unique Nash equilibrium with all colleges adopting threshold admissions rules.

Proof. See Online Appendix.

Proposition 2 formalizes the intuition for threshold admissions rules. Even if a student with aptitude close to 1 is unlikely to enroll, there is no cost and some potential benefit to admitting this student if the college does not care about its yield. Note that even though colleges may care about their revealed preference rankings ($\alpha_2 > 0$), in equilibrium, college j recognizes that its choice of admissions rule a_j has no effect on RP_j, and so it acts as if $\alpha_2 = 0$.

When all colleges adopt threshold admission rules, students in the highest range of abilities are admitted by all colleges, students in the next highest range of abilities are admitted by colleges 2 through J, and so on. Thus, a college that values yield has a clear incentive to act strategically by avoid competition for the most popular students—that is, rejecting high-aptitude students. Proposition 3 formalizes this intuition.

PROPOSITION 3. For any selectivity-based ranking rule, there is no Nash equilibrium where all colleges adopt threshold admissions rule for α_1 and β sufficiently large.

Proof. See Online Appendix.

A college can improve its yield by deviating from a threshold admission rule to admit additional low-aptitude students at the expense of high-aptitude students. This deviation causes the given college to compete more frequently against colleges with lower θ's and less frequently against colleges with higher θ's. However, in a revealed preference ranking system, a college's losses in matriculation tournaments against colleges with lower θ's are disproportionately costly to its ranking, so that there is no net gain nor loss in ranking from strategic selection of opponents.[21]

In the context of the current model, there is a *direct* relationship between θ_j and yield W_j if all colleges act adopt threshold admission rules. Thus, there is a clear but flawed logic for using yield as a factor in existing ranking methods. If all colleges use threshold admission rules, then the combination of yields for all colleges is a sufficient statistic for the combination of underlying

21. Many ranking systems for college football and college basketball now attempt to adjust their team ratings for "strength of schedule"—rewarding teams for scheduling difficult opposition. However, the adjustment methods are frequently ad hoc and are frequently altered from year to year in response to criticism. (See "Sorting through All the Scenarios," http://sports.espn.go.com/ncf/news/story?page=roadtobcs/1202 (last accessed July 1, 2012) for a discussion of this point in the context of the Bowl Championship Series system for football bowl pairings.) By contrast, the revealed preference ranking system adjusts for the strength of competition in matriculation tournaments in a way that maintains the asymptotic properties of the rankings.

desirability parameters—that is, the θ_i's. That is, if all colleges are nonstrategic, the revealed preference ranking and selectivity-based rankings are essentially equivalent. But the advantage of the revealed preference ranking is that it is robust to strategic behavior by individual colleges, whereas a selectivity-based ranking is not.

VI.B. Empirical Tests of Incentives for Manipulation by Colleges

We now conduct simulations to test whether a college could improve its revealed preference ranking by strategically rejecting applicants for whom it would have to compete with at least one other college that is significantly more highly ranked. For conciseness, we call these "tough tournaments." This strategy is important because Propositions 2 and 3 indicate that a a college can improve college can improve its selectivity-based ranking but not its revealed preference ranking by avoiding tough tournaments.

To choose the other colleges that are significantly more highly ranked, we use a statistical test based on the colleges' estimated θ's from our baseline ranking. We consider another college to be significantly more highly ranked if its θ is greater than the college's own θ with 90% confidence in a one-sided test.

Table VI shows the results of this simulation for all 110 colleges that appear in our baseline ranking. Each college avoids all tough tournaments, as already defined. On average, a college that employs this strategy rejects 19% of the applicants whom it actually accepted.[22]

We find that when colleges practice this manipulation strategy, they do not improve their place on our revealed preference ranking. The median and mean improvement in ranking is 0. More than 80% of colleges change their revealed preference ranking by 0 or only 1 place.

Another test of whether the classic manipulation—avoiding tough tournaments—works is whether the actual probability of winning tough tournaments is lower than the model-based predicted probability of winning them. To see this, observe that if a college could benefit from strategically rejecting certain applicants, it would have to be the case that the revealed preference

22. We are bending over backward to make colleges' manipulations successful because we are assuming that they can figure out which students will be accepted by one or more much more highly ranked colleges. In reality, they can only guess which students will fit this profile.

TABLE VI

Revealed Preference-Based Rankings for Students with Tastes for Various Fields of Study

Baseline Ranking (Matriculation Tournaments with Covariates, Students with All Preferred Majors)	Ranking among Students Who Plan to Major in Engineering, Math, Computer Science, or the Physical Sciences	Ranking among Students Who Plan to Major in the Humanities
Harvard	Harvard	Yale
Caltech.	Caltech.	Stanford
Yale	Yale	Harvard
MIT	MIT	Princeton
Stanford	Stanford	Brown
Princeton	Princeton	Columbia
Brown	Wellesley College	Notre Dame
Columbia	Williams College	Amherst College
Amherst College	Dartmouth College	Univ. of Pennsylvania
Dartmouth College	Notre Dame	Dartmouth College
Wellesley College	Amherst College	Swarthmore College
Univ. of Pennsylvania	Brown	Georgetown
Notre Dame	Columbia University	Wellesley College
Swarthmore College	Swarthmore College	Pomona College
Cornell	Cornell	Duke
Georgetown	Univ. of Pennsylvania	St. John's College
Rice	Duke	Kalamazoo College
Williams College	Rice	Middlebury College
Duke	Cooper Union	University of the South
University of Virginia	Colgate	Claremont McKenna
Brigham Young	University of Chicago	Rice
Wesleyan	Harvey Mudd	Cornell
Northwestern	Georgia Inst. of Technology	Carleton College
Pomona College	Northwestern	Wesleyan
Georgia Inst. of Technology	University of Virginia	Northwestern

Notes. Leftmost column shows baseline rank from Table III. The remaining columns show rankings based on estimating the parameters in equation (3) by maximum likelihood for students who are (middle column) math-oriented and (rightmost column) humanities-oriented.

model overestimates the probability that these applicants matriculate at the college. Put another way, if an applicant's actual probability of matriculating is equal to his model-based predicted probability of matriculating, then a college's ranking is unaffected by its tournament for him. Only if an applicant's actual probability of matriculating is lower than his model-based predicted probability of matriculating can a college improve its ranking by strategically rejecting him and thereby avoiding his tournament.

Online Appendix Table A.3 shows the results of this test. To construct Table A.3, we randomly split our sample of tournaments in half. We use the first half of the sample to estimate the model. We then use the estimated coefficients to predict win probabilities for the second half of the sample. Using just the second half of the sample, we compare the model-based predicted win probabilities to the empirical win probabilities. We repeat this exercise 50 times, choosing a new random sample split with each iteration.

Consider the first two rows of Table A.3. The left column shows that colleges ranked 21 to 30 (in our baseline ranking) have a 28% predicted probability of winning a matriculation tournament if that tournament contains no college that is significantly more highly ranked. The same colleges have only a 2% predicted probability of winning a matriculation tournament if that tournament does contain a college that is significantly more highly ranked. (We are using a 90% one-sided test to choose which colleges' θ's are statistically significantly higher.) These predicted probabilities show that the revealed preference ranking is working: a college is more likely to win a tournament if that tournament contains no school that has a better revealed preference ranking. More important, note the tiny or nonexistent differences between the predicted and empirical probabilities of winning tournaments. The colleges have a 28% empirical probability and a 28% predicted probability of winning a matriculation tournament that contains no college that is significantly more highly ranked. The colleges have a 4% empirical probability of winning a matriculation tournament that does contain a college that is significantly more highly ranked. This is very similar to the 2% the model predicts, and it indicates that the college could not improve its ranking by avoiding such tournaments.

The remaining rows of the table show results for colleges initially ranked 31 to 40, 41 to 50, and so on. Very consistently, there are only tiny or nonexistent differences between the model's predicted probabilities and empirical probabilities. This is true even for the small share (7–20%) of tournaments that are most relevant for manipulation—those that contain a significantly more highly ranked college.

Summing up, the tests indicate that, unlike selectivity-based rankings, the revealed preference ranking cannot be manipulated by colleges strategically rejecting applicants who will force them to compete in tough tournaments.

VII. Self-Selection of Applications and Revealed Preference Rankings

In practice, students only apply to a limited number of colleges. In this section, we consider the two forms of self-selection that are most obvious in reality: (1) students' self-selecting into applications based on their admissions chances, and (2) students strategically limiting their applications in return for a greater probability of admission. In practice, the latter behavior is most likely to occur when a college offers Early Decision, an admissions program that requires students to precommit to enroll if admitted. Avery, Fairbanks, and Zeckhauser (2003) and Avery and Levin (2010) show that because the commitment benefits colleges, they increase students' admission chances if they participate in the program.

Other forms of self-selection are, of course, possible. Chade, Lewis, and Smith (2011) and Lien (2009) study the choice of an optimal portfolio of applications.[23] Despite assuming that all students have the same preferences over colleges, they find that the problem is so complex that Nash equilibria exhibit systematic nonmonotonicities. We could not incorporate models such as theirs into our analysis without imposing unrealistic simplifying assumptions.

VII.A. *Application Choices Based on Admissions Chances*

Extending our theoretical analysis, we now demonstrate that our revealed preference ranking still produces consistent rankings and desirable incentives for colleges when students self-select into applications based on their chance of admission.

We allow students to observe a signal of their own aptitude, which in turn provides information about their admissions chances at each college. Our focus in the model is how college rankings affect the admissions rules chosen by colleges, so we continue to require optimal strategies for colleges. However, we allow students to use probabilistic application rules that incorporate both equilibrium and nonequilibrium behavior.[24]

23. Fu (2011) uses National Longitudinal Survey data to study the choices of students to apply to colleges of different levels of selectivity but does not assess the choices of students to apply to more than one college within a single level of selectivity.

24. Formal equilibrium analysis with limited applications per student would require an exogenous cost of application (as in Chade, Lewis, and Smith 2011) or an

Specifically, we assume that student i receives (imprecise) signal s_i of his aptitude z_i. We further assume that s_i takes values on [0,1] and that s_i and z_i are affiliated so that higher values of s_i are probabilistically linked with higher values of z_i. Finally, we assume that these aptitude signals are uncorrelated with idiosyncratic preferences.[25]

We allow students to vary their choices of applications probabilistically as a function of the observed signal s_i. Define $\hat{J}$ to be the set of all nonempty subsets of colleges, containing a total of $2^J - 1$ elements. We assume that there is a probability density function ρ: $(\hat{J}x[0,1]) \rightarrow [0,1]$ such that a student with aptitude signal s_i applies (only) to the colleges in subset $\hat{j} \in \hat{J}$ with probability $\rho(\hat{j}, s_i)$.

The definition of ρ allows for a variety of intuitive rules for applications, including:

(1) Each student applies to all colleges.
(2) Each student classifies colleges into the categories such as reach, match, and safety based on aptitude signals s_i, and then applies to fixed positive numbers J_R, J_M, and J_S of colleges chosen randomly from these three groups.[26]
(3) Each student makes independent decisions about whether to apply to each college (J decisions in all), applying with higher probabilities to colleges that seem most appropriate given aptitude signal s_i.

Propositions 1 and 2 will hold for functions ρ that satisfy a "connectedness condition" that allows for comparison of all colleges in the revealed preference method. Formally, Ford

exogenously fixed limit on number of applications (as in Lien 2009). There is some empirical evidence that students apply to fewer colleges on average than would be optimal based on reasonable estimates for application costs (Pallais 2009). Thus, it may be appropriate to allow for nonoptimal choices of applications in empirical analysis—as we do with our general definition of ρ.

25. Our empirical analysis of selection effects in Sections VI and VII can also be viewed as covering the case where ability signals and preferences are correlated.

26. The College Board specifically uses the terms *reach*, *match*, and *safety* in its advice to students, recommending that students apply to at least one college in each category; http://professionals.collegeboard.com/guidance/applications/how-many (last accessed July 1, 2012).

(1957) identifies the following sufficient condition for consistency of the revealed preference estimates of θ_i.[27]

CONDITION (P) (FORD). (1) For each college-student pair, there is a positive ex ante probability that the student applies to that college and a positive probability that the college admits that student conditional on application. (2) For any partition of colleges into two nonempty subsets S1 and S2, there is a positive probability that any student rejects an offer of admission to a college in S2 to enroll in a college in S1 *and* there is a positive probability that any student rejects an offer of admission to a college in S1 to enroll in a college in S2.

This is a relatively weak condition, but it is not satisfied for some extreme choices of ρ—for example, if (1) students never apply to colleges outside of their home region, or (2) students with high-valued and low-valued signals apply to mutually exclusive sets of colleges. Proposition 4 identifies one modest restriction on ρ that satisfies the connectedness condition and thus maintains the results of Propositions 1 and 2.

PROPOSITION 4. Suppose that for each pair of colleges j_1 and j_2 and each signal s, there is positive probability that a student applies to both j_1 and j_2. Then the results of Propositions 1 and 2 continue to hold given the application density function ρ.[28]

27. Ford (1957) actually proved that maximum-likelihood estimation provides a unique solution in a finite sample of binary comparisons (that is, where each student is admitted to exactly two colleges) given a variant of condition (P) that is appropriate to finite samples.

28. Proposition 3 will continue to hold for functions ρ such that, conditional on admission, higher-aptitude applicants are less likely than lower-aptitude applicants to matriculate at one (or more) specific colleges. Intuitively, we expect this property to hold for most ρ's because threshold admission rules tend to provide higher-aptitude applicants with more admission options than lower-aptitude applicants. Empirically, in our data, we find a significant negative relationship between SAT scores—one obvious measure of aptitude—and matriculation rates for admitted students. At Ivy League colleges, for example, a 10-point increase in SAT verbal or math score is associated with a 1% point decline in the conditional probability of enrolling given admission to a particular college. This finding suggests that Proposition 3 is relevant in practice. Note, however, Proposition 3 does not hold for all functions ρ. For example, if each student applies to one selective college and one nonselective college that admits all students, then each admitted student at a

Proof. See Online Appendix.

VII.B. Early Decision and Strategic Self-Selection of Applications

In this section, we use the phenomenon of Early Decision to assess what happens when students strategically limit their applications in return for a higher probability of admission at the college(s) to which they do apply.

For a revealed preference ranking, the problem with Early Decision is that it causes student matriculation tournaments to generate ambiguous information. When a student is admitted via Early Decision, we lose all information about the other colleges where the student would otherwise have applied; indeed, we do not even know if the student would have enrolled at the Early Decision college if other options were available. It is quite possible that a student who prefers college A to college B applies Early Decision to college B to increase his or her probability of admission there, even if he or she has a reasonable chance of admission at college A.

The loss of information is potentially substantial for the six Early Decision colleges ranked in the top 10 in the baseline rankings (Amherst, Columbia, Dartmouth, Princeton, Stanford, and Yale), an average of approximately 30% of admitted students in the College Admissions Project survey were admitted through Early Decision.[29]

Fortunately, our fall survey asked students to list the colleges they were considering in preference order. We hope that the answers supply us with information on the matriculation tournament each student would have held in the absence of incentives to strategically limit his applications. We can therefore cross-validate the rankings produced by the tournaments we observe. We do this in two ways. First, we treat students' ordered listings as pseudo–matriculation tournaments in the college ranked first is assumed to be the one at which the student would have matriculated. All other colleges are treated as

particular selective college is equally likely to enroll regardless of that student's ability. In this instance, the yield for each selective college is independent of its admissions rule.

29. Three of these colleges (Princeton, Stanford, and Yale) no longer offer Early Decision.

tournament losers (even though the student might not have been admitted to all of them). We compute a ranking based on the pseudo–matriculation tournaments for all students, including those for whom we observe an actual tournament. Second, we use all of the information in students' preference orderings by estimating a rank-ordered logistic (Plackett-Luce or exploded logit) model. This is an extension of the model already described. The likelihood of observing a certain ordering of, say, 10 colleges is modeled as the product of the probability that the college ranked first would have won in a tournament with the 9 others, the probability that the college ranked second would have won in a tournament with the 8 others, and so on to the probability that the college ranked 9th would have won in a tournament with the college ranked 10th.

That is, the probability that student i ranks his menu of m_i colleges in order 1, 2,... m_i is given by:

$$Prob(1 \rightarrow 2 \rightarrow ... \rightarrow m_i) = \prod_{j=1}^{m_i} \frac{exp(\theta_{i=j})}{\sum_{r=1}^{m_i} exp(\theta_r)}, j \in S_i, \tag{8}$$

where $Prob(1 \rightarrow 2 \rightarrow \ldots \rightarrow m_i)$ is the probability of observing the event $1 \rightarrow 2 \rightarrow \ldots \rightarrow m_i$ among the possible permutations of the colleges, and r indexes the colleges ranked equal to or lower than college j.

The rank-ordered logistic estimates are likely to be an upper bound on the ranking of Early Decision schools. Some students who applied to a college through Early Decision listed only that college in their preference list. These students were either very confident about being admitted at their truly most preferred college or were engaging in self-protective psychological behavior in which they convinced themselves that the college where they would be committed to enroll (if admitted) was the only one they liked. In estimation procedures based on actual or pseudo–matriculation tournaments, these students have no influence. In the rank-ordered logistic model, however, these students do have influence and we treat their (sole listed) Early Decision college as though it was their truly most preferred—even though we suspect that this method inflates the ranking of the Early Decision colleges. (Self-protective psychological behavior may also cause a student to list a college first when it is not truly his most preferred college. In this case, our estimates based on pseudo–matriculation tournaments are also biased in favor of Early Decision colleges.)

TABLE VII

PREFERENCE-BASED RANKINGS FOR A VARIETY OF SPECIFICATIONS

College Name	Rank Based On: Matriculation Tournaments with Covariates (baseline)	Highest Listing in Preference Ordering (All planned applications)	Rank Ordered Logit on Preference Ordering (All planned applications)
Harvard University	1	1	1
Caltech	2	7	6
Yale University	3	4	2
MIT	4	3	8
Stanford University	5	2	3
Princeton University	6	5	4
Brown University	7	6	5
Columbia University	8	8	7
Amherst College	9	13	10
Dartmouth College	10	11	9
Wellesley College	11	33	29
University of Pennsylvania	12	12	11
University of Notre Dame	13	14	18
Swarthmore College	14	10	17
Cornell University	15	15	14
Georgetown University	16	9	13
Rice University	17	21	20
Williams College	18	22	15
Duke University	19	16	12
University of Virginia	20	25	23
Brigham Young University	21	47	53
Wesleyan University	22	24	22
Northwestern University	23	17	19
Pomona College	24	20	26
Georgia Inst. of Technology	25	43	41
Middlebury College	26	29	16
UC Berkeley	27	42	30
University of Chicago	28	19	25
Johns Hopkins University	29	30	21
U. of Southern California	30	37	58
Furman University	31	49	71
U. North Carolina at Chapel Hill	32	39	38
Barnard College	33	26	27
Oberlin College	34	45	51
Carleton College	35	23	34
Vanderbilt University	36	55	59
Davidson College	37	31	37
UCLA	38	44	54
University of Texas–Austin	39	53	79
University of Florida	40	84	105
New York University	41	36	31

TABLE VII (CONTINUED)

College Name	Rank Based On: Matriculation Tournaments with Covariates (baseline)	Highest Listing in Preference Ordering (All planned applications)	Rank Ordered Logit on Preference Ordering (All planned applications)
Tufts University	42	40	32
Washington and Lee	43	61	67
Vassar College	44	28	49
Grinnell College	45	112	45
University of Michigan	46	46	50
U. Illinois Urbana-Champaign	47	41	76
Carnegie Mellon University	48	27	28
U. of Maryland–College Park	49	86	102
College of William and Mary	50	69	64
Bowdoin College	51	62	33
Wake Forest University	52	58	47
Claremont Mckenna College	53	85	36
Macalester College	54	70	43
Colgate University	55	83	60
Smith College	56	57	40
Boston College	57	59	55
University of Miami	58	73	82
Mount Holyoke College	59	82	56
Haverford College	60	32	24
Bates College	61	74	35
Connecticut College	62	75	70
Kenyon College	63	65	85
Emory University	64	35	42
Washington University	65	50	46
Occidental College	66	96	90
Bryn Mawr College	67	51	63
Southern Methodist University	68	90	113
Lehigh University	69	79	75
Holy Cross College	70	52	48
Reed College	71	18	44
Rensselaer Polytechnic Institute	72	56	74
Florida State University	73	99	112
Colby College	74	64	39
UC: Santa Barbara	75	106	108
Miami University: Oxford	76	94	93
George Washington University	77	89	77
Fordham University	78	113	85
Dickinson College	79	63	89
Sarah Lawrence College	80	34	66
Catholic University of America	81	111	109
Bucknell University	82	88	87
U. of Colorado at Boulder	83	101	88
U. of Wisconsin at Madison	84	71	69

TABLE VII (CONTINUED)

College Name	Rank Based On: Matriculation Tournaments with Covariates (baseline)	Rank Based On: Highest Listing in Preference Ordering (All planned applications)	Rank Based On: Rank Ordered Logit on Preference Ordering (All planned applications)
Arizona State University	85	91	98
Wheaton College	86	48	61
Trinity College	87	72	68
Rose-Hulman Inst. of Tech.	88	54	73
UC–Santa Cruz	89	104	111
Boston University	90	76	83
UC–San Diego	91	81	86
Tulane University	92	93	80
University of Richmond	93	60	65
Case Western Reserve	94	95	81
Colorado College	95	68	57
Indiana Univ.–Bloomington	96	98	101
Penn State–University Park	97	87	96
American University	98	100	99
Hamilton College	99	97	72
University of Washington	100	80	95
University of Rochester	101	67	92
Michigan State University	102	107	114
Lewis & Clark College	103	109	91
Clark University	104	110	103
Skidmore College	105	77	78
Purdue University	106	66	106
Colorado State University	107	103	100
Syracuse University	108	105	97
University of Vermont	109	92	104
Scripps College	110	38	52
Corr: column (1) and this column		0.82	0.83
Corr: column (3) and this column		0.82	0.88

Notes. Leftmost column shows baseline rank from Table III. The middle column shows ranking based on estimating equation (3) using students' preference orderings as pseudo matriculation tournaments (see text). The rightmost column shows ranking based on estimating equation (8), a rank-ordered logistic model, using students' preference orderings.

Our results, presented in Table VII, show that revealed preference rankings based on data that include preference orderings are quite similar to the baseline rankings: the correlation between these rankings is 0.83. As expected, the rankings based on the preference orderings produce more favorable results than the baseline rankings for colleges (such as Princeton) that offered Early Decision. At the same time, both sets of rankings in Table VII produce the identical set of top eight colleges as the baseline rankings. This gives us confidence that students strategically selecting applications, in particular as induced by Early Decision programs, has relatively little effect on the revealed preference rankings.

VIII. Conclusion

This article proposes a new revealed preference method for ranking colleges and implements this method using data from a national sample of high-achieving students. We demonstrate that this ranking method has strong properties, both as a theoretical matter and as an empirical matter. Our revealed preference ranking method eliminates incentives for colleges to adopt strategic and inefficient admissions policies to improve their rankings. Our baseline ranking, which assumes that all students judge colleges' desirability similarly, is fairly precise for elite colleges, even with only a tiny share of the observations that would be available if we used administrative data. The ranking is not sensitive to whether we control for college characteristics that vary among students, such as net cost or distance from home. Students self-selecting into applications based on their chance of admission has no effect on the ranking, and students strategically selecting into Early Decision causes the ranking to misrepresent colleges' desirability only very slightly. When we construct multiple rankings for students of different types—for instance, humanities-oriented and math-oriented—we find that the multiple rankings are very similar to the baseline ranking but exhibit differences we would expect. For instance, math-oriented students construct desirability in such a way that they prefer institutes of technology more than humanities-oriented students do.

Supplementary Material

An Online Appendix for this article can be found at QJE online (qje.oxfordjournals.org).

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