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Decision Field Theory: A Dynamic–Cognitive Approach to Decision Making in an Uncertain Environment

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Decision field theory provides for a mathematical foundation leading to a dynamic, stochastic theory of decision behavior in an uncertain environment. This theory is used to explain (a) violations of stochastic dominance, (b) violations of strong stochastic transitivity, (c) violations of independence between alternatives, (d) serial position effects on preference, (e) speed–accuracy tradeoff effects in decision making, (f) the inverse relation between choice probability and decision time, (g) changes in the direction of preference under time pressure, (h) slower decision times for avoidance as compared with approach conflicts, and (i) preference reversals between choice and selling price measures of preference. The proposed theory is compared with 4 other theories of decision making under uncertainty.

Beginning with von Neumann and Morgenstern's (1947) classic expected utility theory, steady progress has been made in the development of formal theories of decision making under risk and uncertainty. For rational theorists, the goal has been to formulate a logical foundation for representing the preferences of an ideal decision maker (e.g., Machina, 1982; Savage, 1954; Wakker, 1989a). For behavioral scientists, the goal has been to identify the behavioral principles that human preferences actually obey (e.g., Edwards, 1962; Kahneman & Tversky, 1979; Luce & Fishburn, 1991).

The goal of the present theoretical endeavor differs from both of these goals. Our purpose is to understand the motivational and cognitive mechanisms that guide the deliberation process involved in decisions under uncertainty. Deliberation is a time-consuming and effortful cognitive process that involves an extensive amount of information seeking, weighing of consequences, and conflict resolution (cf. James, 1950, pp. 528–529). This deliberation process is manifested by indecisiveness, vacillation, inconsistency, lengthy deliberation, and distress (Janis & Mann, 1977; Svenson, 1992). Notably absent in previous theoretical accounts is any mention about this deliberation process. Earlier theories do not provide an explanation for why preferences waver over time nor do they provide a mechanism for determining how long deliberation lasts. This omission applies to previous theories of decision making cast from the mold of expected utility theory.

The new contributions of the proposed theory are characterized in Table 1, which provides a classification of theories according to two attributes: deterministic versus probabilistic and static versus dynamic. *Deterministic* theories postulate a binary preference relation that is either true or false for any pair of actions. *Probabilistic* theories postulate a probability function that maps each pair of actions into the closed interval [0, 1]. *Static* theories assume that the preference relation (for deterministic models) or the probability function (for probabilistic models) is independent of the length of deliberation time. *Dynamic* theories specify how the preference relation or probability function changes as a function of deliberation time.[1] For the last 45 years, the deterministic–static view has dominated research on decision making under uncertainty. The purpose of this article is to build on this previous work by extending these theories into the stochastic–dynamic category.

The remainder of this article is organized as follows. In the following section, we establish why it is necessary to build a theory of decision making on a theoretical foundation that is stochastic rather than deterministic and dynamic rather than static. In the section on the basic assumptions of decision field theory, the theory is presented in a series of seven stages, beginning with the familiar deterministic–static approach and ending with the new stochastic–dynamic approach. The fourth section provides a crucible for testing decision field theory against other major contemporary theories of decision making under uncertainty. The concluding section summarizes the advan-

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[1] It is useful to distinguish dynamic decision models from dynamic decision tasks. The latter refers to tasks that involve a sequence of decisions in which choices and outcomes available at later stages depend on choices and outcomes that occur at earlier stages. Static decision tasks involve only a single stage—one decision followed by one outcome. Dynamic decision models can be applied to both types of tasks. However, most of the psychological research has been concerned with static decision tasks. Consequently, this article is limited to static decision tasks. In future work, we plan to extend the theory to dynamic decision tasks as more research on this important topic becomes available.

tages of decision field theory vis-à-vis the deterministic–static approach.

Two Fundamental Properties of Human Decision-Making Behavior

Before presenting the new stochastic–dynamic approach to decision making, we review the main empirical reasons for shifting away from the currently dominant deterministic–static approach. This is done by reviewing empirical evidence that contradicts theories based on deterministic or static assumptions.

Variability of Preferences: Determinism Versus Probabilism

Current deterministic theories of decision making begin with the foundational assumption that the choice between two actions, A and B, is determined by a binary preference relation, denoted $\geq_p$, which is either true or false for any pair of actions (cf. Fishburn, 1988). According to deterministic theories, action A will be chosen over action B if A $\geq_p$ B is true and B $\geq_p$ A is false. All of the remaining axioms of deterministic theories are defined in terms of this binary relation.

This all-or-none assumption was refuted over 40 years ago in one of the earliest investigations of risky decision making performed by Mosteller and Nogee (1951). In this classic experiment, subjects were given a choice between rejecting or accepting a poker hand that had a ⅓ chance of winning money and a ⅔ chance of losing money. The amount to win was manipulated and the amount to lose was fixed at 5¢. Mosteller and Nogee pointed out that if choice is determined by an all-or-none preference relation as posited by deterministic theories, then the relative frequency of accepting the poker hand should be zero for amounts to win below some critical value and then it should jump to 1.0 for amounts to win above that same critical value (i.e., the curve in Figure 1 should look like a step function rather than an S-shaped function).

On the contrary, Mosteller and Nogee (1951) found that the probability of choosing a monetary gamble was a gradually increasing S-shaped function of the amount to win, which was strikingly similar to the psychometric functions found in psychophysics. The S-shaped curve shown in Figure 1 is an example for a typical subject from the Mosteller and Nogee experiment, and similar results were later reported by Edwards (1955). On the basis of these results, Mosteller and Nogee (1951) concluded that "subjects are not so consistent about preference and indifference as postulated by von Neumann and Morgenstern but have a graded response, gradually increasing the frequency of risks taken as the value of the risk increases" (p. 404).

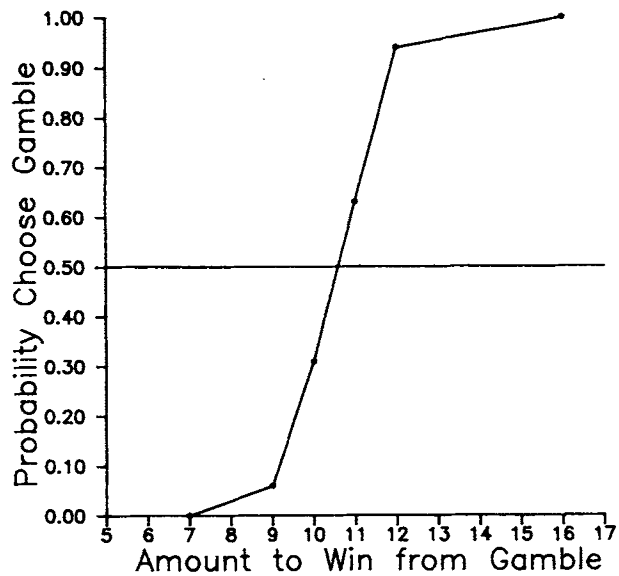

Figure 1. The probability of accepting a gamble plotted as a function of the amount to win by playing the gamble. (The gamble produced either the win indicated on the abscissa with probability [⅓] or a loss of 5¢ with probability [⅔]. The data are from a single subject in the Mosteller and Nogee [1951] experiment.)

Table 1
Categorization of Decision Theories

Category	Static	Dynamic
Deterministic	Expected utility	Dynamics of action
Probabilistic	Random utility	Decision field theory

Note. Prospect theory and rank-dependent utility theory are also included in the deterministic–static category. Thurstone's (1959) preferential choice model is an example of a random utility model. Random utility theories are summarized in Colonius (1984). Dynamics of action is a theory of motivation developed by Atkinson and Birch (1970). Affective balance theory (Grossberg & Gutowski, 1987) is also a member of the deterministic–dynamic category.

In view of this fact, some deterministic theorists define what may be called the *direction of preference* in terms of choice probability. Let *Pr*(A, B) symbolize the probability that action A is chosen over action B in a binary choice. It is assumed that *Pr*(A, B) $\geq$ .5 if and only if A $\geq_p$ B (see Luce & Suppes, 1965, p. 333). According to this definition, choice probability is primary, and the binary relation $\geq_p$ is a secondary measure derived from an arbitrary categorization of the continuous probability measure. However, observe that this derived measure of preference direction throws out all information about preference strength, that is, the magnitude of the choice probability.[2] As Goldstein and Busemeyer (1992) pointed out, it is difficult to conceive of any application in which an enormous change in probability from $.5 + 10^{-100}$ to 1.0 is ignored, whereas a minuscule change from $.5 - 10^{-100}$ to $.5 + 10^{-100}$ is crucial. Obviously, a theory that can account for both the direction and strength of preference is superior to one that can only account for the direction of preference.

[2] *Preference strength* can be defined in several ways. Here we define it as choice probability. Alternatively, it can be defined as the rated difference in preference between two gambles (e.g., see Mellers, Chang, Birnbaum, & Ordonez, 1992). These two definitions involve distinct psychological processes. For example, the probability of choosing $10 over $9.99 is 1.00, even though the rated difference in value is small.

Recognizing the importance of accounting for preference strength, determinists extended their theories by proposing the simple scalability hypothesis (e.g., Becker, DeGroot, & Marschak, 1963b): $Pr(A, B) = F[u(A), u(B)]$, where u is a utility function defined on a set of actions by a deterministic theory, and F is an increasing function of the first argument and a decreasing function of the second argument. However, all simple scalability models imply a property called *independence between alternatives* (Tversky & Russo, 1969): If $Pr(A, C) > Pr(B, C)$, then $Pr(A, D) > Pr(B, D)$. Systematic violations of this property have been observed for over 30 years. The most robust example is an interesting phenomenon named by Lee (1971) as the Myers effect, which is illustrated below (see also Busemeyer, 1979, 1985; Myers & Katz, 1962; Myers & Sadler, 1960; Myers, Suydam, & Gambino, 1965; Suydam & Myers, 1962). Figure 2 shows an example taken from Busemeyer (1979) in which subjects chose between a gamble and a certain value.[3] In this figure, A represents the gamble "win or lose 5¢ with equal probability," B represents the gamble "win or lose 50¢ with equal probability," C represents a certain loss of 1¢, and D represents a certain gain of 1¢. The probabilities of choosing A over C, B over C, A over D, and B over D are indicated by the height of each of the four bars from left to right, respectively, in Figure 2. According to the simple scalability hypothesis, that the first bar exceeds the second bar implies $u(A) > u(B)$. But this is contradicted by the fact that the fourth bar exceeds the third bar, which implies the opposite order $u(A) < u(B)$. Consequently, it is impossible to use the simple scalability hypothesis to explain actual choice probability behavior. This breakdown of simple scalability demonstrates the importance of considering the effect of the context produced by the pairing of two actions on choice probability. The section on the basic assumptions of decision field theory provides a detailed explanation.

In sum, the first unavoidable fact about human decision making is that preferences are inconsistent. We propose that this inconsistency arises from changes in preference over time and that this process of change must be rigorously specified so that it can be evaluated as a viable scientific explanation. Any psychological theory of decision making must be capable of predicting how choice probability changes as a function of the events and payoffs that define each pair of actions. This first fact rules out the deterministic derivatives of expected utility theory and points to the need for probabilistic accounts.[4]

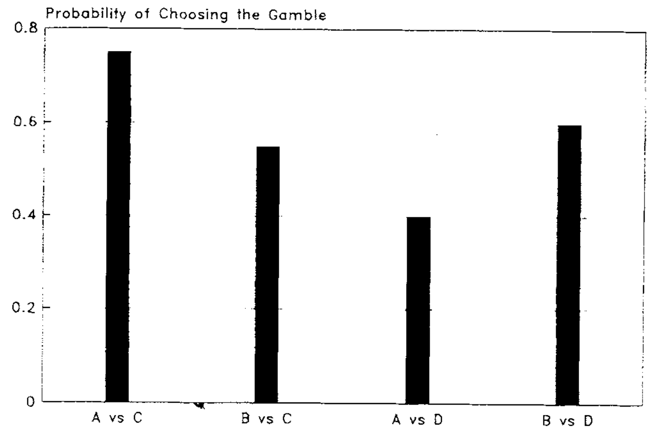

Figure 2. Violation of independence between alternatives. (Action A stands for win or lose 5¢ with equal probability; Action B stands for win or lose 50¢ with equal probability; Action C stands for lose 1¢ for sure; Action D stands for win 1¢ for sure. The probability of choosing A over C, B over C, A over D, and B over D are shown by the height of the four bars from left to right, respectively. The data are from Busemeyer [1979].)

Preference Strength and Deliberation Time: Statics Versus Dynamics

Over 50 years ago, Dashiell (1937) discovered that mean choice time is a decreasing function of magnitude of preference strength, which is strikingly similar to the chronometric functions found in psychophysics. Several years later, Mosteller and Nogee (1951) found that the mean time to choose a gamble systematically decreased as the probability of choosing that gamble increased. More recently, this inverse relation between choice probability and decision time was replicated by Petrusic and Jamieson (1978; see Figure 3), and on the basis of this systematic relationship, Jamieson and Petrusic (1977) concluded that decision time may be an efficient way to estimate preference strength.

Static theories are silent concerning these lawful relationships. Although static theorists might argue that deliberation time is a separate issue and that preference can be studied independent of deliberation time, these assumptions turn out to be empirically false.

Recently, researchers investigating decision making under time pressure have shown that choice probabilities systematically change as a function of the time limit set by the experimenter (Ben Zur & Breznitz, 1981; Busemeyer, 1985; Harrison, Holtgrave, & Rivero, 1990; Wallsten & Barton, 1982; Wright, 1974). In fact, the probability of choosing an action can be moved from below .50 to above .50 (or vice versa) simply by manipulating time pressure (Ben Zur & Breznitz, 1981; Busemeyer, 1985, Experiment 2; Svenson & Edland, 1987). Figure 4 shows an example taken from Goldstein and Busemeyer (1992).

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[3] In two recent unpublished experiments, we replicated the Myers effect when both alternatives were uncertain. Thus, the effect does not depend on the condition of certainty for one of the alternatives.

[4] Experimental evidence for inconsistency of preferences between simple gambles has been replicated many times since the classic experiments by Mosteller and Nogee (1951) and Edwards (1955). For example, recently Camerer (1989) reports that 32% of the subjects in his experiment reversed their preferences when the same pair of gambles was displayed on two different occasions; this occurred when the same pair of gambles was repeated within the same experimental session, with real monetary stakes, and without any outcome feedback. One might try to argue that this type of inconsistency results from some deterministic change in preference during the time period between repetitions of the same gamble. However, this argument is without scientific merit unless one can specify precisely how preferences change between presentations of the same pair of gambles. A stochastic account recognizes the fact that we can never know all of the factors that influence an individual's decision from one moment to the next. The best we can hope to do is to determine the probability distribution as a function of the few known factors that we can identify or observe.

In this case, the subject chose between (a) a certain loss of 3¢ or (b) a gamble in which the payoffs produced by the gamble were normally distributed with a mean equal to zero and a standard deviation equal to 50¢. The probability of choosing the gamble decreases from above .50 to below .50 simply by increasing the length of the time limit.

Apparently most decisions do involve time pressure. Collecting and analyzing information for a decision is a time-consuming process, and time is a valuable resource. Consequently, decision makers must frequently, if not always, limit the amount of time that they can spend on any given decision. As a matter of fact, it is commonly believed that the occasional irrational behavior exhibited by subjects in the laboratory is simply due to the use of heuristics that require little effort. Under the appropriate incentive conditions, it is argued, subjects could be induced to use more effortful decision procedures, and much of the paradoxical behavior would disappear. So even when there is no explicit time limit, there is implicit time pressure because of the cost of processing time. The overall picture that emerges from a large number of decision-making experiments (see Payne, Bettman, & Johnson, 1992) indicates that decision makers do in fact trade off accuracy (e.g., average payoff) for effort (e.g., decision time).

In sum, the second unavoidable fact about human decision making is that decisions take time, and the amount of time spent making a decision influences the final choice. A psychological theory of decision making must be capable of explaining that decision time is inversely related to choice probability as well as that deliberation time influences choice probability. These basic facts rule out static models of decision making, including random utility models, and point to the need for dynamic accounts of how preferences change over time.[5]

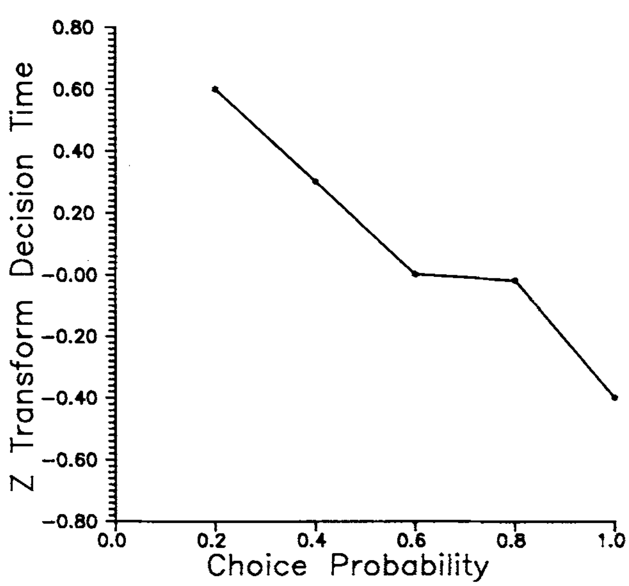

Figure 3. The mean time to choose an action plotted as a function of the probability that the action is chosen. (The response times are expressed in terms of a standardized (z-score) scale. The data are from Petrusic and Jamieson [1978].)

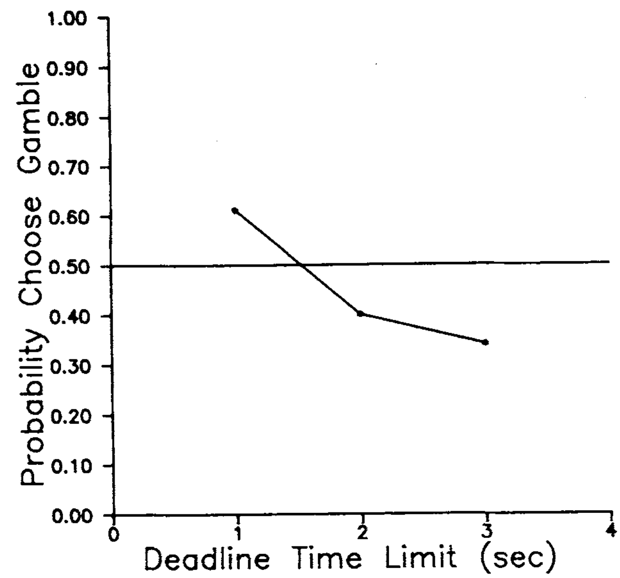

Figure 4. Effect of deadlines on choice probability. (The probability of choosing a gamble over a certain value is plotted as a function of the deadline time limit. The gamble produced a normal distribution of payoffs with a mean equal to zero and a standard deviation equal to 50¢. The certain value was equal to −3¢. Each point is based on 250 observations. These data are the same as those reported in Goldstein and Busemeyer [1992].)

Basic Assumptions of Decision Field Theory

Decision field theory[6] is built on psychological principles drawn from three different areas of psychology: (a) early motivational theories of approach–avoidance conflict (Anderson, 1962; Bower, 1959; Estes, 1960; Hull, 1938; Lewin, 1935; Miller, 1944), (b) later extensions of approach–avoidance ideas to decision making (Atkinson & Birch, 1970; Coombs & Avrunin, 1977, 1988), and (c) recent information-processing theories of choice response time (Edwards, 1965; Laming, 1968; Link & Heath, 1975; Ratcliff, 1978; Smith, 1992; Stone, 1960; Vickers, 1979). Although this presentation is focused on the application of decision field theory to decision making under uncertainty, it is a general theory with a much broader base of application. Decision field theory provides a common foundation for predicting (a) choice probability and the distribution of choice response times (Busemeyer & Townsend, 1992), (b) buying prices, selling prices, and cash equivalents (Busemeyer & Goldstein, 1992), and (c) approach–avoidance movement behavior (Townsend & Busemeyer, 1989). The theory was developed to encompass a number of fundamental properties of human de-

[5] Marley (1989) has recently developed a dynamic version of random utility theory. However, this theory has difficulty explaining the basic results of Petrusic and Jamieson (1978).

[6] The field part of this theory's name derives primarily from Lewin's (1935) use of the term, which placed great emphasis on qualitative field topological relations that objects bore to each other and to the decision maker.

cision behavior, including the probabilistic and dynamic properties reviewed earlier.

Before presenting the new theory, it is helpful to first introduce the experimental paradigm used to investigate decision making under risk or uncertainty. Although it is possible to apply decision field theory to complex real-life decisions, these decisions do not allow the necessary experimental control to discriminate among competing theories. The scientific support for all decision-making theories is based primarily on simple, but highly controlled, experimental tasks that are designed to investigate important properties of the decision process. Figure 5 illustrates a typical example of a laboratory decision problem. On each trial, the decision maker must choose between two hypothetical medical treatments. A_L symbolizes the action shown on the left, and A_R symbolizes the action shown on the right. Furthermore, on each trial one of two uncertain events may occur; S_1 denotes the presence of one disease state, and S_2 denotes the presence of another disease state. The payoff produced by taking action A_i(i = L or R) when event S_j(j = 1 or 2) occurs is denoted y_{ij}. In Figure 5, for example, if the treatment on the left is taken (A_L) and the second disease is present (S_2), then $y_{L2} = -200$ is the payoff. Typically, monetary payoffs are used in laboratory experiments. The deliberation time period lasts from the start of the decision problem until the final action is taken, after which the payoff is delivered and the trial ends.

Information about the probabilities of the events S_j can be given in two different ways (cf. Luce & Raiffa, 1957, p. 13). For decisions under *risk*, the event probabilities are directly stated in the problem (e.g., the subject is told that disease S_2 occurs with probability .25 under the current pattern of symptoms). For decisions under *uncertainty*, the decision maker must learn and infer the event probabilities from past experience. For example, the probability of a disease given a particular symptom pattern may be estimated from past experience with previous trials in which the same symptom pattern was displayed. In real life, we rarely receive direct information about event probabilities, and it is more common to have this information conveyed through experience with past outcomes from similar decisions. Decision field theory was developed for this more natural type of uncertain decision problem.

Only two uncertain events are shown in Figure 5, S_1 and S_2. This simple example is important because a large number of experiments have used this two-event case. Initially, the theory is presented by using this simple example, but in the Appendix

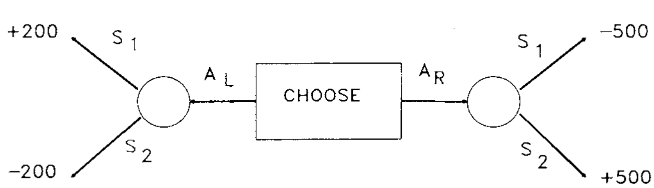

Figure 5. Prototypic choice problem for decisions in an uncertain environment. (The arrows exiting the box indicate the two actions [A_L vs. A_R]. The branches exiting each circle indicate the uncertain events [S_1 vs. S_2]. The payoffs are shown at the endpoints of each branch. The probabilities of the events are learned from experience.)

we provide the general formulas for computing model predictions for the general case with more than two uncertain events. Also, in the section Test of Competing Theories, we present an application of the theory to normally distributed payoffs.

Only two courses of action are available in Figure 5, A_R and A_L. It is straightforward to develop decision field theory for decisions with more than two alternatives (see Busemeyer & Townsend, 1989). However, most of the past research has focused on the two-alternative case, and, furthermore, most theories of decision making under uncertainty are limited to the paired comparison case. Therefore, only the two-alternative decision problem is considered in this article.

Rather than presenting the full theory all at once, it is easier to grasp the basic ideas by presenting them in a sequence of seven incremental stages. Note, however, that this is not the way the theory was originally conceived (cf. Busemeyer & Townsend, 1989). Decision field theory is a synthesis of two prior and independent lines of psychological theory (i.e., approach–avoidance theories of motivation and information-processing theories of choice response time). The sequence of stages presented below was mathematically derived from this synthesis. The main purpose of this organization is to present a clear understanding of the basic assumptions underlying this synthesis.

The first stage begins with the traditional type of theory that should be familiar to any researcher in the field of decision making. With each subsequent stage, a new and more general theory is formed by including a new processing assumption into the previous theory. Each new processing assumption is needed to represent a fundamental property of the deliberation process involved in decision making. The last stage completes the entire description of this application of decision field theory (see Table 2 for a schematic outline).

Stage 1: Deterministic Subjective Expected Utility (SEU) Theory

According to SEU theory, each distinct payoff produced by an action is assigned a subjective probability weight. The magnitude of this weight is a function of the event that determines whether this payoff will occur if the action is taken. For example, in Figure 5, the payoff +500 produced by action A_R (shown on the right) is assigned a weight that depends on the event S_2, denoted $w(S_2)$. From a cognitive view, this weight reflects the *amount of attention* given to event S_2 on each presentation of the choice problem.

The SEU of an action is a weighted average of the utilities of the payoffs produced by an action using the subjective probabilities as the weights. For the example shown in Figure 5, the SEU's for actions A_R and A_L are defined as follows:

$$v_R = w(S_1) \cdot u(-500) + w(S_2) \cdot u(+500), \quad (1a)$$

$$v_L = w(S_1) \cdot u(+200) + w(S_2) \cdot u(-200), \quad (1b)$$

where $u(y)$ is the utility of payoff y with $u(0) = 0$, and $w(S_j)$ is the subjective probability weight assigned to event S_j with $0 \le w(S_j) \le 1$.

The choice between actions A_R and A_L in Figure 5 is determined by the sign of the difference in SEUs for each action:

Table 2
Construction of Decision Field Theory in Seven Stages

Stage and theory	New parameter	New phenomenon
1: Deterministic SEU	d = Mean difference	Preference direction
2: Random SEU	σ^2 = variance of difference	Preference strength
3: Sequential SEU	θ = inhibitory threshold	Speed–accuracy trade-offs
4: Random walk	z = initial anchor point	Preference reversals with time pressure
5: Linear system	s = growth–decay rate	Serial position effects
6: Approach–avoidance	c = goal gradient	Time to approach is less than time to avoid
7: Decision field	h = time unit	Real time processing

Note. Lower stage theories are special cases of higher stage theories. SEU = subjective expected utility.

$$d = v_R - v_L. \tag{1c}$$

Action A_R is chosen when $d > 0$, and action A_L is chosen when $d < 0$.

Savage (1954) originally proposed a prescriptive version of SEU theory in which the subjective probabilities obeyed the laws of mathematical probability theory. Later, Edwards (1962) proposed a descriptive version of SEU theory with subjective probability weights that do not obey the laws of mathematical probability theory. Numerous other variations of descriptive SEU theory have followed that retain many of the same basic characteristics (e.g., Kahneman & Tversky's, 1979, prospect theory).

The descriptive versions of SEU theory have been used to explain important empirical violations of Savage's prescriptive theory, including those found by Allais (1953) and Ellsberg (1961). Decision field theory can account for these results using the same type of explanation as used by these descriptive SEU theories. Because these results fail to distinguish the present theory from earlier accounts, they are not analyzed in detail in this article.

A major problem with descriptive SEU theories (e.g., prospect theory) is their inability to account for the fundamental variability of human preference (e.g., see Figure 1). These theories can only be used to predict the direction of preference (i.e., whether the choice probability exceeds .50), and, consequently, they fail to provide any means for predicting preference strength (i.e., the magnitude of choice probability). Previous attempts to address this problem have been based on the simple scalability hypothesis. For example, Becker et al. (1963b) proposed that

$$Pr(A_R, A_L) = F(d) = F(v_R - v_L), \tag{1d}$$

where F is a cumulative distribution function. However, as we pointed out earlier (see Figure 2), violations of the independence property rule out the scalability hypothesis (Busemeyer, 1979, 1985; Myers & Katz, 1962; Myers & Sadler, 1960; Myers et al., 1965; Suydam & Myers, 1962). To overcome this major limitation, we turn to the second stage of Table 2.

Stage 2: Random SEU Theory

According to deterministic SEU theory, the decision maker uses exactly the same subjective probability weights on every repetition of a choice problem. Random SEU theory generalizes deterministic SEU theory by allowing the decision maker's attention to switch from one event to another across choice trials. For example, on one trial the decision maker may focus primarily on event S_1 in Figure 5, producing a preference favoring action A_L. On another trial with the same choice problem the decision maker may focus primarily on event S_2, producing a preference for action A_R. This variability in subjective probability weights causes the difference in subjective expected utilities to vary across choice trials, and this random difference is called a *valence difference.*

According to random SEU theory (Busemeyer, 1985), the attention weight for event S_j is a continuous random variable, denoted $W(S_j)$, which can change from trial to trial because of attentional fluctuations. Consequently, the SEU for each action is also a random variable, which is called the *valence of an action.* The valences for actions A_R and A_L shown in Figure 5 are defined as follows:

$$V_R = W(S_1) \cdot u(-500) + W(S_2) \cdot u(+500), \tag{2a}$$

$$V_L = W(S_1) \cdot u(+200) + W(S_2) \cdot u(-200). \tag{2b}$$

The difference between these two valences determines the preference state (P) on any trial:

$$P = V_R - V_L. \tag{2c}$$

Action A_R is chosen whenever $P > 0$; action A_L is chosen whenever $P < 0$.

The primary difference between SEU theory and random SEU theory can be seen by comparing Equations 1a and 2a. The attention weights $w(S_j)$ in SEU theory are fixed across trials, whereas the attention weights $W(S_j)$ in random SEU theory vary across trials. These changes in attention weights cause change in the valences, V_R and V_L, which cause change in the preference state, P, which finally causes choice to vary from trial to trial.

Given that the amount of attention allocated to each event fluctuates across trials, the previous subjective probability weight $w(S_j)$ appearing in Equation 1a is now interpreted as the average weight given to event S_j, that is, $w(S_j) = E[W(S_j)]$ (averaged across trials). On the basis of this process interpretation, it follows that the mean valence difference equals the average of the valence difference: $d = E[V_R - V_L] = E[V_R] - E[V_L] = v_R - v_L$, which is mathematically equal to the difference in SEUs for the deterministic model (i.e., Equation 1c). According to random SEU theory, however, the decision maker is never directly aware of the mean difference d. Only a sample estimate, the ephemeral preference state, P, is available on any trial. The residual, $\epsilon = P - d$, represents the trial-by-trial fluctuation around the mean. By using these definitions, the preference state on any trial can be expressed as

$$P = V_R - V_L = d + \epsilon. \tag{2d}$$

Thus, probability of choosing action A_R over A_L is $Pr(A_R, A_L) = Pr[P > 0] = Pr[\epsilon > -d]$. The above choice model is mathematically related to a class of choice models known as *random utility models* (see Colonius, 1984).

At this first stage of the theory, the distribution of the preference state, P, is derived from the distribution of the residual, ϵ, and the latter is derived from the joint distribution of the attention weights, $W(S_1)$ and $W(S_2)$, which is unknown. In the last stage of the theory, we are able to mathematically derive the distribution of the preference state, P, from the dynamic process. However, at this first stage, we are forced to make an ad hoc assumption regarding the distribution of the residual, ϵ. In accordance with previous random utility theories (e.g., Thurstone, 1959), we postulate that ϵ is normally distributed with zero mean and variance: $Var[\epsilon] = Var[V_R - V_L] = \sigma^2$. The parameter σ^2 is called the *variance of the valence difference.* In this case, the probability of choosing action A_R over A_L is

$$Pr(A_R, A_L) = Pr[\epsilon > -d] = F[(d/\sigma)], \tag{2e}$$

where F is the standard normal cumulative distribution function. In other words, choice probability is an increasing S-shaped function of the discriminability index, (d/σ).

Deterministic SEU theory is a special case of random SEU theory, which is obtained by setting the residual ϵ to zero (i.e., letting σ approach zero in Equation 2e). If the variance of the valence difference is nonzero but constant across all choice pairs (i.e., Thurstone's Case 5), then random SEU theory is within the simple scalability class of models (i.e., it is a special case of Equation 1d). To explain violations of the independence property (see Figure 2), it is necessary to allow the variance of the valence difference to change across choice pairs.

The reason that the variance of the valence difference changes across choice pairs is as follows. According to basic statistical theory, the variance of a difference between two random variables is

$$\sigma^2 = Var[V_R - V_L] = \sigma_R^2 + \sigma_L^2 - 2 \cdot \sigma_{RL}, \tag{2f}$$

where σ_R^2 is the variance of the valence for action A_R, σ_L^2 is the variance of the valence for action A_L, and σ_{RL} is the covariance between these two valences. Each of these three quantities are influenced by the payoffs and events associated with each pair of actions in the following manner. First, consider the variance σ_R^2, which represents the uncertainty of the valence produced by action A_R. Suppose that taking action A_R always produces exactly the same payoff for all events (i.e., A_R is a sure thing). In this extreme case, the same payoff is always anticipated for action A_R (i.e., there is no uncertainty produced by action A_R), and, consequently, $\sigma_R^2 = 0$. Now suppose that action A_R produces a wide range of possible payoffs with equal probability as in Figure 5. In this case, attention will switch from one possible payoff to another, producing variability in the valence for action A_R, and the magnitude of σ_R^2 will be determined by the range of payoffs produced by action A_R. In Figure 5, for example, the variance for action A_R is

$$\begin{aligned}\sigma_R^2 &= E[(V_R - v_R)^2] \\ &= w(S_1) \cdot [u(-500) - v_R]^2 + w(S_2) \cdot [u(+500) - v_R]^2. \end{aligned} \tag{2g}$$

The variance for action A_L is

$$\begin{aligned}\sigma_L^2 &= E[(V_L - v_L)^2] \\ &= w(S_1) \cdot [u(+200) - v_L]^2 + w(S_2) \cdot [u(-200) - v_L]^2. \end{aligned} \tag{2h}$$

In this example, if the events are equally likely, then the variance of the valence for action A_R is larger than that for action A_L. Now we are prepared to show how changes in the variance of the valence difference accounts for the Myers effect (see Figure 2).

The violation of independence shown in Figure 2 can be explained by the fact that the variance of gamble B (50^2) is much larger than that for gamble A (5^2). Table 3 shows the discriminability ratios (d/σ) produced by the four choice pairs illustrated in Figure 2. The two cells in the first row correspond to the first two bars in Figure 2. In this case, the mean difference favors the gamble over the certain value ($d = +1$), but the discriminability ratio decreases from +.2 to +.02 because of the increase in variance. The two cells in the second row correspond to the second two bars in Figure 2. In this case, the mean difference favors the certain value ($d = -1$), but the discriminability ratio now increases from −.2 to −.02 because of the increase in variance.

The covariance, σ_{RL}, between the two valences represents the similarity or dissimilarity of the payoffs produced by each action. Consider, for example, Figure 5. Note that the consequences produced by each action are mediated by a common set of events $\{S_1, S_2\}$. If event S_1 occurs, then action A_R produces a large negative payoff (−500), whereas action A_L produces a large positive payoff (+200). The opposite pattern occurs when event S_2 occurs: A_R produces a large positive payoff (+500), whereas A_L produces a large negative payoff (−200). This produces a strong negative correlation between the payoffs produced by actions A_R and A_L. The covariance for this example is

Table 3
Discriminability Ratios (d/σ) Corresponding to Figure 2

	Action	
Action	A	B
C	(+1/5) = +.20 >	(+1/50) = +.02
D	(−1/5) = −.20	< (−1/50) = −.02

Note. A = win or lose 5¢ with equal probability; B = win or lose 50¢ with equal probability; C = lose 1¢ for sure; D = win 1¢ for sure. d = mean valence difference favoring choice of the column action over the row action. The expected value of the column action is zero, so d = −(value of row action). σ = standard deviation of valence difference. The variance of the row action is zero and, consequently, so is the covariance term. The standard deviation of column action equals the magnitude of the gain or loss. Independence is violated because the rank order of the two columns changes across rows.

$$\sigma_{RL} = E[(V_R - v_R)\cdot(V_L - v_L)]$$
$$= w(S_1)\cdot[u(-500) - v_R]\cdot[u(200) - v_L]$$
$$+ w(S_2)\cdot[u(500) - v_R]\cdot[u(-200) - v_L]. \quad (2i)$$

Recall that the variance of the valence difference is negatively related to the covariance (see Equation 2f). Thus, increasing the similarity of the payoffs produced by two actions will increase the discriminability ratio magnitude. According to random SEU theory, this is another major reason why the independence property is violated (see Becker, DeGroot, & Marschak, 1963a).

As a final check on the psychological reasonableness of the variance parameter, consider the following extreme case. Suppose one was given a choice between a $1,000 prize and a $999 prize. Although the utility difference is relatively minute, any reasonable decision maker would always choose the dominant prize. This deterministic behavior is entirely consistent with random SEU theory. For in this simple case, there is no uncertainty, and according to Equations 2f through 2i, $\sigma^2 = 0$, so that the probability of choosing the $1,000 prize computed from Equation 2e is 1.0.

In sum, by incorporating a probabilistic valence process into the deterministic SEU theory, random SEU theory provides an explanation for both the direction and strength of preference. This same process is also used to explain violations of the independence between alternatives property. The explanatory power gained by incorporating this process requires the addition of only a single new parameter, σ^2 = the variance of the valence difference. However, as noted earlier, the basic limitation of this theory is that it fails to provide any mechanism for explaining the systematic relation between choice probability and decision time (e.g., see Figure 3). This limitation is surmounted in the next stage of Table 2.

Stage 3: Sequential SEU Theory

According to random SEU theory, choice is based on just a single sample of a valence difference on any trial. Sequential SEU theory generalizes this by allowing a sequence of one or more samples to be accumulated during the deliberation period of a trial. The basic idea is that attention may switch from one event to another within a single choice trial. At the beginning of the choice problem, the decision maker anticipates the payoff that would be produced by taking each action and compares these two anticipated payoffs. This initial comparison determines the preference state for the first sample, $P(1) = [V_R(1) - V_L(1)]$, but it does not necessarily lead to a decision. A few moments later, attention may shift to a new pair of anticipated payoffs, producing another sample valence difference, which is added to the previous preference state to produce a new preference state, $P(2) = P(1) + [V_R(2) - V_L(2)]$. As this deliberation process continues, the new preference state after $n > 2$ samples is

$$P(n) = P(n-1) + [V_R(n) - V_L(n)]$$
$$= \Sigma_k[V_R(k) - V_L(k)],\ k = 1, \ldots, n, \quad (3a)$$

where $P(n-1)$ is the previous preference state after $n-1$ samples, and $[V_R(n) - V_L(n)]$ is the new valence difference. This deliberation process continues until the preference state exceeds an inhibitory threshold criterion, θ. Positive preference states represent a momentary preference favoring action A_R, and this action is taken as soon as $P(n)$ exceeds θ. Negative preference states represent a momentary preference favoring action A_L, and this action is taken as soon as $-P(n)$ exceeds θ. The total number of samples needed to reach the threshold is a random variable, N, and decision time is an increasing function of N.

Figure 6 illustrates a sample path generated by a simulation of this sequential sampling process. The preference state is plotted as a function of the number of samples that have been processed within a single choice trial. The up-and-down vacillation in preference state reflects the fluctuations in valence difference across samples. For example, consider the fictitious medical decision illustrated in Figure 5. The decision maker may initially focus attention on the possibility that disease S_1 is present, which would produce payoffs that favor treatment A_L, but moments later the decision maker's attention may switch to the possibility that disease S_2 is present, which would produce payoffs favoring treatment A_R. The two horizontal lines above and below zero represent the inhibitory thresholds for each action. The vertical line at $N = 17$ samples indicates that the preference state for action A_R exceeded the threshold at that point, and a choice favoring action A_R was executed immediately after that sample.

The mean valence difference now represents the mean change in preference produced by each new sample: $d = E[V_R(n) - V_L(n)] = E[P(n) - P(n-1)] = v_R - v_L$. This mean difference is defined in exactly the same way as it was for random SEU theory, except that sampling occurs within a trial rather than

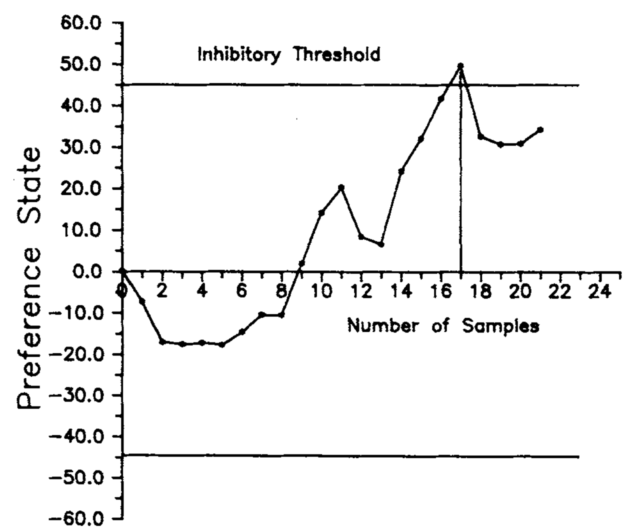

Figure 6. Sample path of a simulated sequential decision process. (The preference state for action A_R over A_L is plotted as a function of the number of samples. The sample path vacillates up and down because of fluctuations in the sign of the valence difference across samples. The horizontal lines above and below the neutral point represent the inhibitory threshold for choosing an action. The vertical line located at $N = 17$ samples indicates the point at which the sample path crosses the inhibitory bound, which satisfies the requirement for choosing the action on the right.)

across trials. The residual represents the change in preference state produced by the moment-to-moment fluctuations in attention during deliberation: $\epsilon(n) = [V_R(n) - V_L(n)] - (v_R - v_L) = [P(n) - P(n-1)] - d$. The variance of this residual, $Var[\epsilon(n)] = \sigma^2$, is determined in the same way as in random SEU theory (see Equations 2f through 2i). By using these definitions, the sequential sampling process (see Equation 3a) can be expressed in an equivalent form as

$$P(n) = P(n-1) + [d + \epsilon(n)]. \quad (3b)$$

The probability of choosing action A_R over A_L equals the probability that $\{P(n) > \theta\}$ occurs before $\{-P(n) > \theta\}$. Busemeyer (1985) developed a sequential sampling model for decision making under uncertainty by using normally distributed residuals. In this case, the probability of choosing action A_R over A_L (see Cox & Miller, 1965, for the derivation) is

$$Pr(A_R, A_L) = F[2 \cdot (d/\sigma) \cdot (\theta/\sigma)], \quad (3c)$$

where F is the standard logistic cumulative distribution function; that is, $F(x) = 1/[1 + \exp(-x)]$. In other words, choice probability is an increasing S-shaped function of the product of the discriminability index and the threshold criterion (measured in standard deviation units).

The primary difference between random SEU theory and sequential sampling theory can be seen by comparing Equation 2d with Equation 3b. Note that Equation 2d is just a special case of Equation 3b, with the number of samples restricted to just one sample. Sequential sampling theory allows the number of samples to increase as the threshold criterion θ increases. A large sample size provides a better estimate of the unknown mean difference within a single choice trial than a single sample. According to this idea, choosing between two uncertain courses of action is conducted in much the same way as a test of two hypotheses on the basis of a sequence of experimental observations (cf. DeGroot, 1970).

The practical consequence of this theoretical difference can be seen by comparing Equations 2e and 3c. According to both equations, choice probability is an increasing S-shaped function of the discriminability index (d/σ). The main difference is that sequential sampling theory also allows choice probability to depend on the threshold criterion θ. Holding discriminability constant, choice probability becomes more extreme as the threshold criterion increases. For example, consider a decision that is very difficult but very important. In this case, the discriminability index may be very low, but the threshold criterion may be very high, thereby producing a high probability of choosing the correct action (i.e., the action producing the larger SEU).

There is a cost produced by increasing the threshold: The mean number of samples required to reach the threshold, and consequently the decision time, increases as the threshold increases. The mean number of samples to reach the threshold (see Cox & Miller, 1965, for the derivation) is

$$E[N] = (\theta/d) \cdot [2 \cdot Pr(A_R, A_L) - 1]. \quad (3d)$$

To interpret this equation, suppose that the mean difference is positive ($d > 0$). As the threshold θ increases, $Pr(A_R, A_L)$ rapidly approaches 1.0, and the mean number of samples approaches (θ/d). The latter ratio is analogous to the familiar formula, time = (distance traveled)/(rate of travel), with N corresponding to time, θ corresponding to distance traveled, and d corresponding to rate of travel. Thus, increasing the threshold increases the probability of choosing the correct action (i.e., the larger SEU), but it also increases the time required to reach the criterion.

Consequently, the threshold criterion θ controls speed–accuracy or cost–benefit trade-offs in decision making. On the one hand, if the cost of prolonging the decision is low or the cost of making an incorrect decision is high, then a high threshold is selected. On the other hand, if the cost of prolonging the decision is high or the cost of making an incorrect decision is low, then a low threshold is selected.[7]

In sum, by including a sequential sampling process into random SEU theory, sequential SEU theory provides a mechanism for explaining the fundamental speed–accuracy or cost–benefit trade-off relations frequently observed in decision making. This additional explanatory power requires only one new parameter, θ = inhibitory threshold. But there is a problem with sequential SEU theory. If the mean difference is positive ($d > 0$), then the probability of choosing A_R over A_L is always predicted to be greater than .50 for all values of the threshold criterion θ (see Equation 3c). Therefore, sequential SEU theory fails to explain that choice probability can change from below .50 to above .50 (or vice versa) under time pressure (see Figure 4). The next stage in Table 2 overcomes this problem.

Stage 4: Random Walk SEU Theory

Sequential SEU theory was based on the assumption that the deliberation process always begins from a neutral point (i.e., $P(0) = 0$). However, prior knowledge or past experience with the same or a similar decision problem will bias the initial state. In particular, the decision maker may be able to recall a previous preference state from memory, and these recalled states will be influenced in the direction of the mean difference. Random walk SEU theory generalizes sequential SEU theory by allowing the initial preference state, $P(0)$, to start at some anchor point, z, biased by prior knowledge or past experience.[8]

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[7] The inhibitory threshold bound is also called the stopping criterion in sequential sampling theory. Previous applications of sequential sampling decision models in memory (Ratcliff, 1978) and perception (Link, 1992) have successfully used a constant stopping criterion for relatively fast decisions (i.e., in seconds). However, Busemeyer and Rapoport (1988) provided evidence indicating that the stopping criterion decreased over time for longer decisions (i.e., in minutes). This suggests that a constant inhibitory strength may provide an adequate approximation for short deliberation times, but the inhibitory strength may gradually decay toward zero for long deliberation times. Another possibility is that the inhibitory threshold varies randomly around some mean value rather than remaining fixed. However, this extension would not change the predictions of the theory, because the random deviations of the threshold around its mean would simply be added on to the residual, $\epsilon(n)$, in Equation 3b.

[8] The random walk SEU theory uses the following rule: Stop and choose A_R if and only if $z + \Sigma_n[V_R(n) - V_L(n)] \geq \theta$; stop and choose A_L if and only if $z + \Sigma_n[V_R(n) - V_L(n)] \leq -\theta$. This is equivalent to the following rule: Stop and choose A_R if and only if $\Sigma_n[V_R(n) - V_L(n)] \geq \theta - z$; stop and choose A_L if and only if $\Sigma_n[V_R(n) - V_L(n)] \leq -(\theta + z)$. A reviewer argued that we should have added another parameter by using the

According to the random walk SEU theory (Busemeyer, 1985), the preference state after n samples equals an anchor point plus the adjustments produced by the sequence of n samples ($n > 0$):

$$P(0) = z,$$
$$P(n) = P(n-1) + [V_R(n) - V_L(n)],$$
$$= P(n-1) + [d + \epsilon(n)],$$
$$= z + \Sigma_k[V_R(k) - V_L(k)],\ k = 1, \ldots, n, \quad (4)$$

where the mean difference, d, and the variance, σ^2, of the valence difference are defined in exactly the same way as they were for sequential SEU theory (see Equation 3b).

Figure 7 illustrates the theoretical choice probabilities for a hypothetical example in which a consumer is trying to decide whether to buy a standard brand versus a new brand of some consumer product. (See Equation A1 in the Appendix for the formula used to generate these predictions.) Suppose A_L is a highly familiar status quo brand (i.e., one that has been used for many years), and A_R is a new brand that, after careful investigation, is determined to be superior in quality. In this example, the initial state of preference is negative ($z < 0$, favoring the status quo), but the mean valence difference is positive ($d > 0$, favoring the new brand). The increasing curve in Figure 7 shows the probability that the new brand is chosen as a function of the duration of the time limit. The status quo tends to be chosen more frequently under a short time limit (i.e., low threshold), but the probability grows as a function of time so that the new brand tends to be preferred under longer time limits (i.e., high threshold). The decreasing curve shows how the probabilities change in the opposite direction when $z > 0$ and $d < 0$. This example illustrates how random walk SEU theory accounts for the fact that choice probability can change from below .50 to above .50 (or vice versa) simply by changing the length of deliberation time with a biased initial anchor point.

The assumption that the initial state is biased by past experience also provides one explanation for the inverse relation between choice probability and decision time (see Figure 3). The basic idea is simple: The further the initial state is from a threshold, the longer it takes to reach that threshold. If the mean difference is large and positive (i.e., $d > 0$), then the probability of choosing action A_R will be high, and the probability of choosing action A_L will be low. In addition, if the initial state is strongly biased in the same direction, then the mean time to choose A_R will be very short, and the mean time to choose A_L will be very long. This results in an inverse relation between choice probability and decision time. (See Equation A2 in the Appendix for the formula used to compute decision time for the random walk model.)

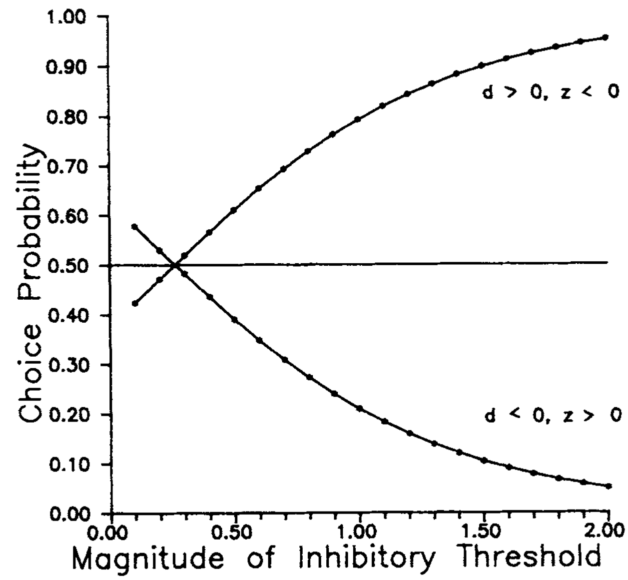

Figure 7. Predictions from random walk Subjective Expected Utility theory. (Probability of choosing the action on the right is plotted as a function of the inhibitory threshold magnitude. The increasing curve shows the probability when the mean difference is positive and the initial preference state is negative. The decreasing curve shows the probability when the mean difference is negative and the initial preference state is positive.)

In sum, the random walk SEU theory entails a total of four parameters: $\{d, \sigma, \theta, z\}$. (See the outline in Table 2.) First, the mean valence difference, d, is used to explain the direction of preference (as in SEU theory). Second, the variance of the valence difference, σ^2, is used to explain strength of preference (as in random SEU theory). Third, the threshold criterion, θ, is used to explain speed–accuracy trade-off effects (as in sequential SEU theory). Finally, the initial anchor point, z, is used to explain reversals in the direction of preference as a function of time pressure, and it also provides one explanation for the inverse relation between choice probability and decision time.

Nonetheless, there is a serious problem with the random walk SEU theory. The updating rule (see Equation 4) fails to provide any mechanism for explaining serial position effects on the final preference state.[9] According to the updating rule, the preference state is simply the initial state plus the sum of all the valence differences. Thus, the effect of a valence difference does not depend on whether it occurred early or late within the sequence. This assumption is contrary to the well established fact that primacy and recency effects are observed in both evaluative judgments (Hogarth & Einhorn, 1992) and decision making (Wallsten, in press; Wallsten & Barton, 1982). The next stage in Table 2 provides a simple solution to this problem.

following rule: Stop and choose A_R if and only if $k + \Sigma_n[V_R(n) - V_L(n)] \geq \theta_1$; stop and choose A_L if and only if $k + \Sigma_n[V_R(n) - V_L(n)] \leq -\theta_2$. The latter is equivalent to the rule: Stop and choose A_R if and only if $\Sigma_n[V_R(n) - V_L(n)] \geq \theta_1 - k$; stop and choose A_L if and only if $\Sigma_n[V_R(n) - V_L(n)] \leq -(\theta_2 + k)$. However, the stopping rule that we used can be equated with the stopping rule proposed by the reviewer by setting $\theta = [(\theta_1 - k) + (\theta_2 + k)]/2$ and $z = [(\theta_2 + k) - (\theta_1 - k)]/2$. Thus, the two rules make exactly the same predictions, and no advantage in terms of predictive power is gained by adding the redundant parameter.

[9] Serial position effects obtained in evaluative judgments are unrelated to serial position effects obtained in free recall. See Anderson and Hubert (1963) for more details.

Stage 5: Linear System SEU Theory

Linear system SEU theory relaxes the assumption that the effect of a valence difference is independent of its serial position. Instead, the impact of a valence difference may vary depending on whether it occurred early (i.e., a primacy effect) or late (i.e., a recency effect) within a sequence.

The simplest way to modify random walk SEU theory to incorporate serial position effects is the following linear updating rule:

$$\begin{aligned} P(n) &= (1 - s)\cdot P(n-1) + [V_R(n) - V_L(n)] \\ &= (1 - s)\cdot P(n-1) + [d + \epsilon(n)] \\ &= (1 - s)^n \cdot z + \Sigma_k (1 - s)^{n-k} \cdot [V_R(k) - V_L(k)], \\ &\quad k = 1, 2, \ldots, n, \end{aligned} \tag{5}$$

where the mean difference, d, and the variance of $\epsilon(n)$, σ^2, are defined in exactly the same way as they were for random walk SEU theory. According to this rule, the new preference state is a weighted compromise of the previous preference state and the new valence difference.

The new parameter, s, is called the growth–decay rate parameter. Note that random walk SEU theory is a special case, which is obtained by setting the growth–decay rate parameter, s, equal to zero. If the growth–decay rate is strictly between zero and one ($0 < s < 1$), then the linear updating rule (see Equation 5) produces recency effects so that the recent samples have greater impact. If the growth–decay rate is less than zero, then the linear updating rule (see Equation 5) produces primacy effects so that earlier samples have greater impact. Thus, primacy, recency, or no serial position effects can occur, depending on the value of the growth–decay rate parameter, s.

In sum, linear system SEU theory (see Equation 5) adds only one new parameter to random walk SEU theory, and it is the simplest possible way to incorporate serial position effects. Although more complex forms of serial position effects can be obtained by allowing the growth–decay rate to change as a function of time (Myung & Busemeyer, 1992) or by using nonlinear updating rules (Hogarth & Einhorn, 1992), the linear updating rule is more parsimonious, and at the present time it is adequate for explaining the main findings from research on decision making under uncertainty.

A significant problem with the linear system SEU theory is its failure to account for the fact that the time required to make a decision depends on the approach–avoidance nature of the conflict. According to the linear updating rule (see Equation 5), the preference state is only affected by the difference in valence, and it does not matter whether that difference came from two rewards or two punishments. Thus, approach–approach decisions are predicted to take just as long as avoidance–avoidance decisions, holding other factors constant. This basic property of the theory is contrary to the long-standing finding that avoidance–avoidance decisions produce longer mean deliberation times than do approach–approach decisions when the mean differences are held constant (Barker, 1942; Bockenholt, Albert, Aschenbrenner, & Schmalhofer, 1991; Busemeyer, 1985; Houston, Sherman, & Baker, 1991). This failure is rectified by the next stage.

Stage 6: Approach–Avoidance Theory

Up to this point, we have assumed that the average amount of attention given to a payoff depends solely on the event that determines whether the payoff will occur if an action is taken. For example, in Figure 5, the average weight given to the payoff (+500) of action A_R only depends on event S_2. Approach–avoidance theory generalizes this idea by allowing the average weight to be influenced by another dynamic variable called the *goal gradient* (Townsend & Busemeyer, 1989).

The goal gradient hypothesis originated in the early work on approach–avoidance conflicts by Lewin (1935), Hull (1938), and Miller (1944). The basic idea is that the attractiveness of a reward or the aversiveness of a punishment is a decreasing function of the distance from the point of commitment to an action. Experimental support for this hypothesis on the basis of animal research was reviewed by Miller (1959), and empirical support from human research is described in Epstein and Fenz (1965).

According to approach–avoidance theory, the consequences of an action become more salient as the preference state for that action approaches its threshold. If there is little or no possibility of taking an action, then its consequences are ignored; however, as the possibility of taking an action increases, then attention to its consequences increases. More specifically, the salience of a consequence produced by taking action A_R increases as the distance between $+P(n)$ and θ decreases. Similarly, the salience of a consequence produced by taking action A_L increases as the distance between $-P(n)$ and θ decreases.

Figure 8 illustrates two goal gradients, one for gains or rewards (i.e., the flatter gradient with a slope represented by the coefficient a) and another for losses or punishments (i.e., the steeper gradient represented by a coefficient b). Previous research indicates that the gradient for rewards tends to be flatter than that for punishments (see Miller, 1959), but this is not a

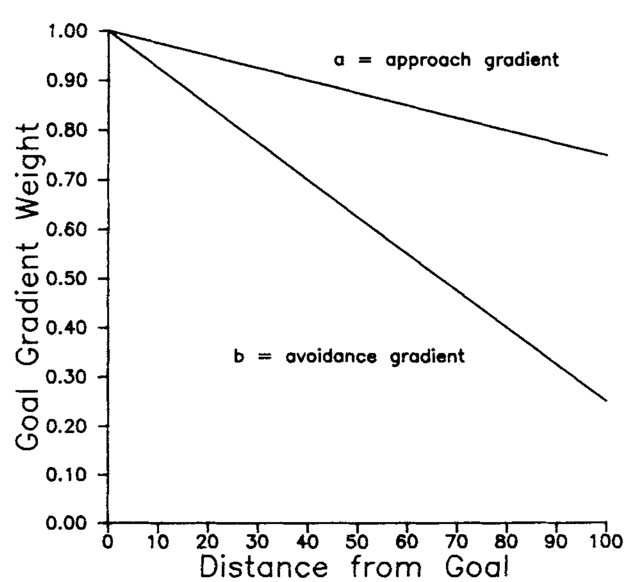

Figure 8. Approach–avoidance gradients. (The gradient weight is plotted as a function of the distance from the goal with a different line for the approach [flatter line] and avoidance [steeper line] gradients.)

necessary requirement of the theory. The horizontal axis represents distance from the threshold, and the vertical axis indicates the attention weight corresponding to this distance. As the distance increases, the goal gradient weight decreases. Although past research suggests that the gradients are exponential (cf. Shepard, 1987), linear approximations are used to maintain a simple and mathematically tractable theory.[10]

According to approach–avoidance theory, the average weight given to each payoff is a product of a goal gradient weight and an event weight. Consider the decision problem shown in Figure 5 again. The mean valence of each action is now a function of the current preference state because of the goal gradient:

$$\begin{aligned} v_R(n) &= E[V_R(n)\,|\,P(n)] \\ &= \{1 - b\cdot[\theta - P(n)]\}\cdot w(S_1)\cdot u(-500) \\ &\quad + \{1 - a\cdot[\theta - P(n)]\}\cdot w(S_2)\cdot u(+500), \end{aligned} \tag{6a}$$

$$\begin{aligned} v_L(n) &= E[V_L(n)\,|\,P(n)] \\ &= \{1 - a\cdot[\theta + P(n)]\}\cdot w(S_1)\cdot u(+200) \\ &\quad + \{1 - b\cdot[\theta + P(n)]\}\cdot w(S_2)\cdot u(-200), \end{aligned} \tag{6b}$$

where a and b are the slopes of the goal gradients for the gains and losses, respectively. It is helpful to compare Equations 1a and 6a. For example, the average attention weight assigned to the gain (+500) produced by choosing action A_R is now a product of two components: As in Equation 1a, one component is the average attention weight for event S_2, $w(S_2)$; the second new component is the goal gradient weight for a gain produced by action A_R, $\{1 - a\cdot[\theta - P(n)]\}$.

Recall that the mean valence difference is obtained by subtracting Equation 6b from Equation 6a. This results in a mean valence difference, $d(n)$, that is now a function of the current state of preference, $P(n)$. After some algebraic rearrangement, the mean valence difference can be expressed as follows:

$$d(n) = [v_R(n) - v_L(n)] = -c\cdot P(n) + \delta, \tag{6c}$$

where

$$c = b\cdot(v_{pR} + v_{pL}) - a\cdot(v_{rR} + v_{rL});$$

$$\delta = (v_{rR} - v_{rL})\cdot(1 - a\cdot\theta) + (v_{pR} - v_{pL})\cdot(1 - b\cdot\theta);$$

$$v_{rR} = w(S_2)\cdot u(+500) \text{ (the average gain for } A_R);$$

$$v_{pR} = w(S_1)\cdot u(-500) \text{ (the average loss for } A_R);$$

$$v_{rL} = w(S_1)\cdot u(+200) \text{ (the average gain for } A_L);$$

and

$$v_{pL} = w(S_2)\cdot u(-200) \text{ (the average loss for } A_L).$$

In the above equations, v_{rR} is the average gain and v_{pR} is the average loss for taking action A_R; v_{rL} is the average gain and v_{pL} is the average loss for taking action A_L.

Inserting this new derivation for the mean valence difference into the linear updating rule (see Equation 5) produces the following approach–avoidance updating rule for the preference state:

$$\begin{aligned} P(n) &= (1 - s)\cdot P(n-1) + [V_R(n) - V_L(n)] \\ &= (1 - s)\cdot P(n-1) + [d(n) + \epsilon(n)] \\ &= [1 - (s + c)]\cdot P(n-1) + [\delta + \epsilon(n)]. \end{aligned} \tag{6d}$$

According to Equation 6d, the new preference state $P(n)$ is now a weighted compromise of the previous preference state $P(n-1)$ and the new valence input $[\delta + \epsilon(n)]$. The mean valence input, δ, in Equation 6d is closely related to the mean difference, d, from the linear updating rule (see Equation 5). When the mean valence input is positive, $\delta > 0$, the preference state is driven in the positive direction on the average. When the mean valence input is negative, $\delta < 0$, then the preference state is driven in the negative direction on the average. The residual $\epsilon(n)$ has a mean of zero, and its variance, $Var[\epsilon(n)] = \sigma^2$, is defined in exactly the same way as it was for linear system SEU theory (see Equation 5).

Note that one of the original features of approach–avoidance theory was the distinction between approachable versus avoidable consequences, that is, rewards versus punishments (cf. Miller, 1944). This closely corresponds to the distinction between positively versus negatively framed outcomes that is made by more recent decision theorists (cf. Tversky & Kahneman, 1981). This analogy was recognized by Coombs and Avrunin (1988), and they have provided a detailed analysis of framing effects according to an approach–avoidance conceptual framework.

One criticism of earlier deterministic approach–avoidance models by Lewin (1935) and Miller (1944, 1959) was that the models were unclear about how avoidance–avoidance conflicts were resolved. It seemed that individuals would be compelled to vacillate back and forth forever and never reach a decision without some external force to help push them into action. The present approach–avoidance theory solves this problem by using a stochastic vacillation process. Although the present theory predicts that individuals will vacillate back and forth during deliberation, the preference state will always drift far enough to exceed the inhibitory threshold, θ, required to make a decision.

Technically, the process described by the approach–avoidance updating rule (see Equation 6d) is a Markov chain process. The preference states within the inhibitory threshold form a set of transient states. It is a well-known theorem of Markov chains (see Bhattacharya & Waymire, 1990) that the probability of leaving a set of transient states approaches 1.0 as the number of transitions increases to infinity.

The main new parameter in approach–avoidance theory, c, is called the goal gradient parameter. First, note that when $c = 0$ (i.e., the goal gradients are both flat), then the approach–avoidance theory reduces to the linear system SEU theory. Given

[10] For the relatively small amount of variation in preference state that is likely to occur in a laboratory decision task, a linear goal gradient can approximate an exponential gradient reasonably well, and the differences in the predictions produced by linear versus exponential gradients are small and unimportant. In particular, the basic prediction that avoidance–avoidance decisions take longer than approach–approach decisions remains unchanged by the use of linear versus exponential gradients.

that $c \neq 0$, the sign of the goal gradient depends on the approach–avoidance nature of the conflict. For avoidance–avoidance conflicts, c is positive, which causes the preference state to vacillate back and forth, and this in turn slows down the decision process. For approach–approach conflicts, c is negative, which causes the preference state to race toward the criterion, and this in turn speeds up the decision process. In sum, approach–avoidance theory accounts for the fact that avoidance-avoidance decisions take longer than approach–approach decisions (Barker, 1942; Bockenholt et al., 1991; Busemeyer, 1985; Houston et al., 1991) by including the goal gradient parameter, c. To establish this last conclusion in a more rigorous manner, we develop a real-time, rather than a discrete-time, decision process in the final stage of Table 2.

Stage 7: Decision Field Theory

The last four theories are all discrete-time dynamic processes, and, consequently, they cannot be used to make quantitative predictions for decision time. A real- or continuous-time process is needed for this purpose, which can be constructed in a simple way by introducing a time unit, h, into the theory. This time unit represents the amount of time used to process each sample valence difference. In other words, h represents the amount of time that it takes to retrieve and process one pair of anticipated consequences before shifting attention to another pair of consequences. The deliberation time, t, is then defined in terms of this time unit as $t = n \cdot h$, where n equals the number of samples that have been processed. Including the time unit into the approach–avoidance updating rule (see Equation 6d) produces the following linear stochastic difference equation (see Busemeyer & Townsend, 1992, for a more detailed explanation):

$$P(t) = (1 - s \cdot h) \cdot P(t - h) + [V_R(t) - V_L(t)]$$
$$= [1 - (s + c) \cdot h] \cdot P(t - h) + [\delta \cdot h + \epsilon(t)], \quad (7)$$

where $\epsilon(t)$ is the residual input with zero mean and $V[\epsilon(t)] = h \cdot \sigma^2$, and σ^2 is defined exactly as before by Equations 2f through 2i.

Note that the approach–avoidance theory is a special case in which the time unit is set equal to $h = 1$. However, it seems unrealistic to assume that each processing step requires exactly the same amount of time. To overcome this limitation, a more realistic continuous-time version of the model can be obtained by letting the time unit, h, approach zero in the limit (see Busemeyer & Townsend, 1992). In the latter case, the amount of attention allocated to each uncertain event is assumed to drift up and down in a continuous manner during deliberation like a continuously moving wave.

As the time unit, h, approaches zero, the preference state evolves in an approximately continuous manner over time. Furthermore, the distribution of the preference state at each time point, $P(t)$, can be mathematically derived from Equation 7 without making any assumptions regarding the distribution of the residual, $\epsilon(t)$, other than its mean and variance.[11] In this case, it has been proven that the distribution of $P(t)$ converges to the normal distribution as h approaches zero (see Bhattacharya & Waymire, 1990, chapter 5).

The left panel of Figure 9 shows the effect of the goal gradient parameter, c, on the probability of choosing A_R over A_L as a function of the mean valence input (for $s = 0$, $z = 0$, and $\delta > 0$, so that A_R tends to be favored over A_L). The three curves illustrate how this choice probability changes depending on the sign of the goal gradient parameter. For the top curve, c is positive, corresponding to an avoidance–avoidance situation. For the bottom curve, c is negative, corresponding to an approach–approach situation. The middle was obtained by setting c to zero. Increasing the goal gradient parameter tends to magnify the effect of the mean valence input on choice probability. (See Equation A3 in the Appendix for the formula used to calculate these predictions.)

The effect of the goal gradient parameter, c, on decision time is shown in the right panel of Figure 9. (See Equation A4 in the Appendix for the formula used to calculate these predictions.) The right-panel figure shows the mean decision time as a function of the mean valence input (using the same parameters as in the left-panel figure). The three curves illustrate how the mean choice time changes as a function of the sign of the goal gradient parameter. The important point is that holding the mean input constant, the avoidance–avoidance conflict (i.e., the top curve) is predicted to take longer on the average than the approach–approach conflict (i.e., the bottom curve). The effects shown in the right-panel figure hold for a wide range of parameter values and are not specific to the values used to generate this figure. This basic prediction of the model is supported in four different human decision-making experiments (Barker, 1942; Bockenholt et al., 1991; Busemeyer, 1985; Houston et al., 1991).

Summary of Decision Field Theory

Intuitively, the deliberation process can be summarized as follows. When confronted with a difficult personal decision, the decision maker tries to anticipate and evaluate all of the possible consequences produced by each course of action. For real decisions, a vast number of consequences may be considered, and these anticipated consequences are retrieved from a rich and complex associative memory process. Obviously, all of these consequences cannot be retrieved and evaluated all at once. Therefore, the decision maker must undergo a slow and time-consuming process of retrieving, comparing, and integrating the comparisons over time. No action is taken until the preference for one action becomes strong enough to goad the decision maker into action.

Decision field theory is an attempt to formalize this deliberation process. Of course, we do not wish to imply that decision makers consciously carry out the computations indicated by the linear stochastic difference equation (see Equation 7). Instead, these computations are assumed to be realized by an underlying neural system, and decision field theory is an ab-

[11] For the continuous-time model, it is not necessary to assume that the valence difference $[V_R(t) - V_L(t)]$ is independently sampled during deliberation to derive the prediction equations for decision field theory. The valence differences may be retrieved from an associative memory system with one association leading to another in a statistically dependent manner. See Busemeyer and Townsend (1992) for the details concerning the mathematical derivations of the predictions for the continuous-time model.

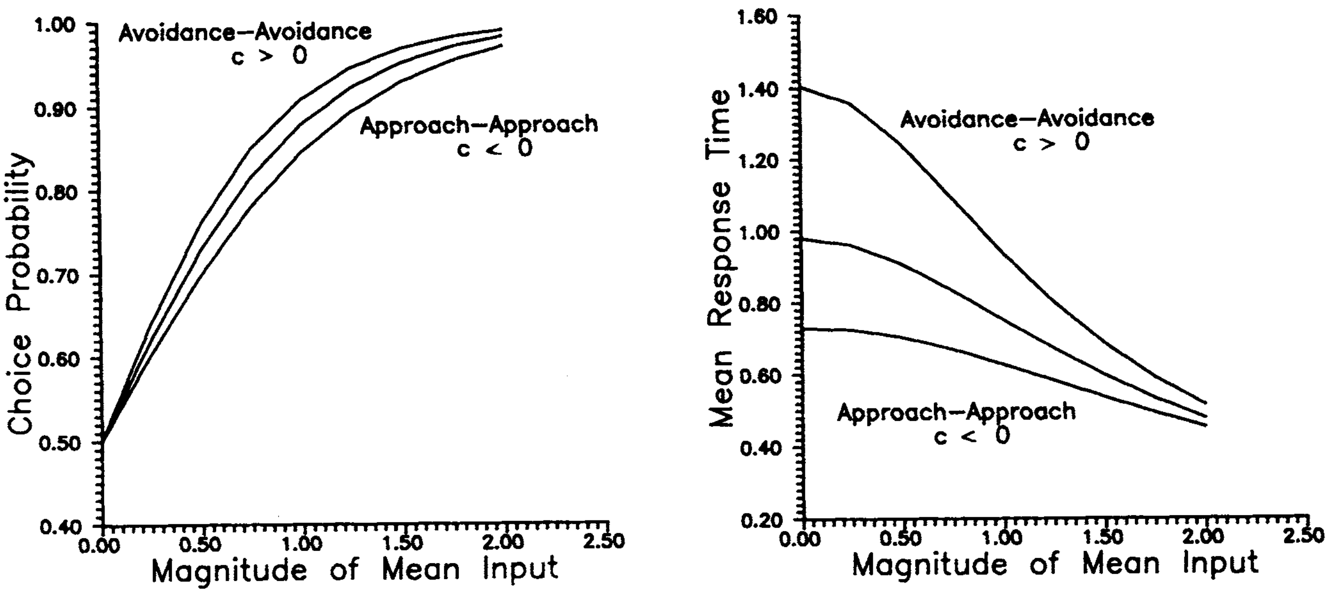

Figure 9. Predictions computed from decision field theory. (The left panel shows the probability of choosing action A_R plotted as a function of the mean input with a separate curve for each of three goal gradient parameters. The top curve was obtained with $c > 0$, the middle curve with $c = 0$, and the bottom curve with $c < 0$. The right panel shows the mean choice time plotted as a function of the mean input with a separate curve for each of three goal gradient parameters. The top curve was obtained with $c > 0$, the middle curve with $c = 0$, and the bottom curve with $c < 0$. The same parameters were used in both panels.)

stract representation of its essential dynamic properties (cf. Tuckwell, 1988).

A schematic overview of all of the main components of decision field theory is shown in Figure 10. Starting at the far left are the potential gains and losses involved in the example decision problem shown in Figure 5. The gains form the affective inputs to the *approach* subsystem, and the losses form the affective inputs to the *avoidance* subsystem. Each input is connected to

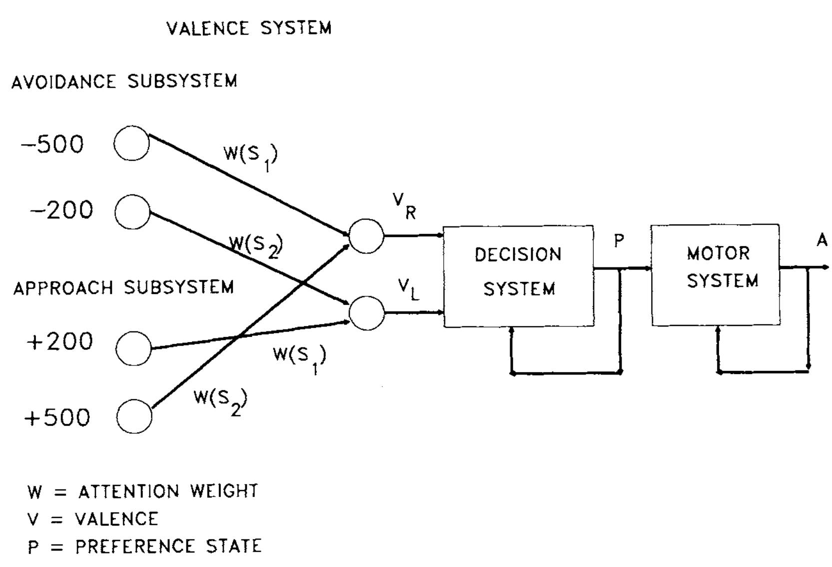

Figure 10. Outline of decision field theory. (The left panel indicates how the connection strengths [W_{ij}, connecting act i to consequence j] filter the input values to produce a valence for each act [V_i] as output. The right panel indicates how the valence [V] is input into a dynamic decision system that produces preference [P] as output, and this preference is then input into a dynamic motor system that produces actions [A] as output. S = uncertain event; R = right action; L = left action.)

an action by an *attention weight* (the *W*s in the figure), which represents the retrieval of an *association* between an action and a consequence. These weights change over time as a result of shifts in the decision maker's attention to the various consequences during deliberation. The *valence* of an action at any moment (the *V*s in the figure) represents the anticipated value of an action at that moment, and it is produced by the weighted sum of the input values connected to that action. The valence of each action is fed into a decision system that compares the valences and integrates these comparisons over time to produce a momentary *preference state, P.* Finally, this momentary preference state drives a motor system that inhibits responding until a *threshold* is exceeded, at which point the overt action (the A in the figure) is expressed.

Altogether, decision field theory entails a total of seven parameters: $\{\delta, \sigma, \theta, z, s, c, h\}$. (See the outline in Table 2.) First, the mean valence input, δ, is used to explain the direction of preference (as in SEU theory). Second, the variance of the valence input, σ^2, is used to explain strength of preference (as in random SEU theory). Third, the inhibitory threshold, θ, is used to explain speed–accuracy trade-off effects (as in sequential SEU theory). Fourth, the initial anchor point, z, is used to explain reversals in the direction of preference as a function of time pressure (as in random walk SEU theory). Fifth, the growth-decay rate, s, is used to account for serial position effects (as in linear system SEU theory). Sixth, the goal gradient parameter, c, is used to explain the effects of the approach–avoidance nature of a conflict on deliberation time. Finally, the time unit, h, is chosen to be as close to zero as needed to approximate a continuous-time process. Note that the first six stages in Table 2 are all special cases of decision field theory, because each of the earlier stages can be obtained from the last stage by putting the appropriate constraints on the parameters of decision field theory.

Table 4 summarizes the rules for mapping the basic experimental factors for decision making under uncertainty (i.e., uncertain events, payoffs, deadline time limits) on to the theoretical parameters of decision field theory for the prototypical problem shown in Figure 5. (The Appendix provides the general formulas for an arbitrary number of payoffs.) For example, the mean valence input, δ, and the goal gradient, c, are both determined by Equation 6c; the variance of the valence input, σ^2, is determined by Equations 2f through 2i; the inhibitory threshold is an increasing function of the deadline time limit; and the initial starting point, z, is biased in the direction of the mean input valence. In the next section, we compare decision field theory with other competing theories of decision making under uncertainty, using the mapping rules shown in Table 4 for all of these tests.

Tests of Competing Theories

In the previous sections, some general deficiencies with earlier theories were identified, and a number of qualitative areas of support for the assumptions of decision field theory were presented. This section provides a more detailed comparison of decision field theory with five different major theories of decision making. The primary method for empirically testing the theories is based on a *cross-validation* type of methodology. That is, parameters are estimated from one part of an experiment, and then these same parameters are applied to a separate part of the experiment and the predictions are evaluated. The structure of the valid theory should permit relatively accurate prediction without changing the parameters, whereas the invalid theories should tend to fail in these tests.

For each application described later, the predictions from decision field theory were computed from Equations A3 and A4 in the Appendix. Each data set was divided into two parts: The first part was used for parameter estimation, and the second part was used for cross-validation. The parameters were then estimated from the first part by finding values that minimized the sum of squared prediction errors. These same parameters were used to generate new predictions for the second part of the data set.[12]

For successful cross-validation, it is important to construct the most parsimonious model possible. With this objective in mind, we began with the lowest possible stage in Table 2 (Stage 3 is the simplest dynamic model) and moved to more complex stages only if it was necessary to obtain a satisfactory fit to the first part of the data set.

The experiments presented in this section were specifically designed to test basic properties of decision field theory, and they satisfy several stringent criteria. First, these experiments were designed to investigate an important aspect of either the probabilistic nature of preference (e.g., violations of independence between alternatives) or the dynamic nature of the deliberation process (e.g., the relation between choice probability and decision time). Second, the design of each experiment was sufficiently complex to permit theory testing on the basis of the cross-validation methodology mentioned earlier. Third, all of the results discussed in this section are highly reliable (being based on large sample sizes), and the findings of the original experiment have been replicated in one or more independent experiments. The results of these experiments should provide useful benchmarks for evaluating and comparing future theories concerned with the deliberation process of decision making.

Violations of Stochastic Dominance

Stochastic dominance is one of the main properties satisfied by current deterministic theories of decision making under uncertainty (e.g., see the review of rank dependent utility theories described in Wakker, 1989b). Technically, gamble A stochastically dominates gamble B if and only if the cumulative distribution of the payoffs produced by gamble A is always less than or equal to that for gamble B and the inequality is strict for at least one value.

For example, suppose one is given a choice between two ac-

[12] Decision field theory describes the probability that an individual will choose one action over another as a function of deliberation time. The parameters of the model may vary across individuals, and if they do, then ideally the model should be fit to individual data. However, the pattern of results reported in Tables 7–11 were consistent across individuals so that the average data were representative of the individual data, and fitting the prototypical subject was reasonable. Only group data were available for Tables 5 and 6.

Table 4
Rules for Mapping Experimental Factors Into Model Parameters by Using the Problem in Figure 5 as an Example

$$v_{rR} = w(S_2) \cdot u(+500), \text{ average gain for action on the right}$$
$$v_{pR} = w(S_1) \cdot u(-500), \text{ average loss for action on the right}$$
$$v_{rL} = w(S_1) \cdot u(+200), \text{ average gain for action on the left}$$
$$v_{pL} = w(S_2) \cdot u(-200), \text{ average loss for action on the left}$$
$$\sigma_R^2 = w(S_2)u(500)^2 + w(S_1)u(-500)^2 - (v_{rR} + v_{pR})^2$$
$$\sigma_L^2 = w(S_1)u(200)^2 + w(S_2)u(-200)^2 - (v_{rL} + v_{pL})^2$$
$$\sigma_{RL} = w(S_1) \cdot u(-500) \cdot u(200) + w(S_2) \cdot u(500) \cdot u(-200) - (v_{rR} + v_{pR})(v_{rL} + v_{pL})$$
$$\sigma^2 = \sigma_R^2 + \sigma_L^2 - 2 \cdot \sigma_{RL}, \text{ input variance}$$
$$\theta = f(L) \cdot \sigma, \text{ inhibitory threshold, where } f(L) \text{ is an increasing function of the time limit, } L$$
$$c = [b \cdot (v_{Rp} + v_{Lp}) - a \cdot (v_{Rr} + v_{Lr})], \text{ goal gradient parameter}$$
$$\delta = (v_{Rr} - v_{Lr})(1 - a \cdot \theta) + (v_{Rp} - v_{Lp})(1 - b \cdot \theta), \text{ mean input}$$
$$z = g(\delta) \cdot \theta, \text{ initial starting point, where } g(\delta) \text{ is an increasing function of the mean input, } \delta$$

Note. For simplicity, the mapping rules are presented for the two-event case. The general formulas for more than two events are given in the Appendix.

tions as in Figure 5. Also suppose the action on the right provides an even chance of winning \$4 and losing \$1; the action on the left provides an even chance of winning \$1 or losing \$1. In this case, the action on the right stochastically dominates the action on the left because (a) both actions have the same probability of losing \$1 and (b) both actions also have exactly the same probability of winning, but (c) one can win more with the action on the right. Thus, rank-dependent utility theory asserts that the action on the right should always be chosen to satisfy stochastic dominance.

The predictions from decision field theory for this situation depend on the correlational structure of the payoff matrix (see Equation 2f), and two different cases are shown in Table 5. In the positively correlated case, the valence difference never favors action A_L over A_R, independent of whether the decision maker attends to the event *heads* or *tails.* Therefore, according to decision field theory, the stochastically dominant action on the right should always be chosen. In the negatively correlated case, the sign of the valence difference changes depending on whether the decision maker attends to heads or to tails. On the average, the decision maker will attend to each event equally often, but moment-to-moment fluctuations in attention will produce variation in the sign of the valence difference, and there is some probability, albeit small, that the decision maker attends to the event tails frequently enough to drive his or her preference toward the threshold for taking the action on the left. Therefore, according to decision field theory, the stochastically dominated action on the left will be chosen occasionally.

Table 6 shows the results of two experiments, one by Katz (1964) and the other by Myers and Suydam (1964), that used payoff structures corresponding to the negatively correlated case in Table 5. In both experiments, subjects were first given 300 trials of training with uncertain decision problems similar to the prototypical problem shown in Figure 5. The columns labeled y_{R1} and y_{R2} indicate the monetary payoffs produced by choosing action A_R when either event S_1 or S_2 occurred, respectively. The columns labeled y_{L1} and y_{L2} indicate the monetary payoffs produced by choosing action A_L when either event S_1 or S_2 occurred, respectively. The columns labeled $Pr(S_1)$ and $Pr(S_2)$ indicate the probabilities of the two uncertain events for each condition, which were learned through 300 trials of training with outcome feedback. The column labeled *observed* shows the probability of choosing action A_R estimated after training, with each proportion based on over 2,000 observations pooled across subjects.

First, consider the symmetric payoff Conditions 1, 5, and 9 in Table 6. When the payoffs are symmetric, the probability of choosing each action is approximately .50. According to rank dependent utility theories, this implies that the weights for each event are equal in magnitude. Next consider Conditions 4 and 7. To satisfy stochastic dominance, the probability of choosing action A_R should equal 1.0, but instead the observed probability falls significantly below this extreme value. Similarly, for Conditions 2 and 3, stochastic dominance requires that the probability of choosing action A_R should equal zero, but on the contrary, the observed probability lies significantly above this extreme. Violations of stochastic dominance are found also for Conditions 12 and 16 in Table 6. Moreover, these results were replicated by Busemeyer (1982) with the choice probabilities estimated separately for each recently experienced event sequence. In sum, stochastic dominance is frequently violated, at least for uncertain decision problems in which the event probabilities are learned from experience.

These results are contrary to deterministic rank-dependent utility theories that are based on stochastic dominance.[13] Simple scalability theories are unable to explain the effect that changes in the correlational structure (see Table 5) have on choice probability.

According to decision field theory, these violations are due to fluctuations in attention to each event under the negative correlational payoff structure in Table 5, producing variation in the sign of the valence difference. The predictions shown in the last column of Table 6 were computed from decision field theory. Only two parameters were estimated from the 17 data points by using the mapping rules shown in Table 4. (Only Stage 3 of

[13] The following conclusion is derived from the rank-dependent utility model presented in Luce (1990). Define (y, S, x) as an action that yields y if event S occurs and yields x otherwise. Assume that $y >_p x$. If $(x, S, -x) \geq_p (-x, S, x)$ is observed, then the following preference order should be observed: $(y, S, -x) >_p (x, S, -x) \geq_p (-x, S, x)$.

Table 5
Two Cases for Testing Stochastic Dominance

	Positively correlated case		Negatively correlated case	
Action	Heads	Tails	Heads	Tails
Right	4	−1	4	−1
Left	1	−1	−1	1

Note. Each cell entry indicates the payoff produced by a row and column selection. Heads and tails are the two events from a flip of a fair coin.

Table 2 was needed for this data set. See the Appendix for the parameter values.) As can be seen in Table 6, decision field theory provides an accurate account of these results; the model accounts for 98% of the variance. However, a much stronger cross-validation test of decision field theory is described in the next application.

Violations of Independence Between Alternatives

Becker et al. (1963b) proposed the simple scalability hypothesis (e.g., Equation 1d) as a general method for extending deterministic models of risky decision making to account for the magnitude of choice probability. Since that time, the simple scalability hypothesis has been used implicitly by many current theorists as a rationale for generating choice probability predictions from deterministic models (e.g., Hogarth & Einhorn, 1990; Lopes, 1987). Recall that the main axiom of simple scalability theory is the independence property:

$$\text{If } Pr(A, C) > Pr(B, C), \text{ then } Pr(A, D) > Pr(B, D).$$

Decision field theory predicts that the independence property will be violated whenever the variance of the valence difference, σ^2, varies across choice pairs. This is expected to occur whenever the variance of each gamble is manipulated (see Table 3) or whenever the correlation between gambles is manipulated (see Table 5).

Table 7 shows the results of an experiment that systematically manipulated the variance of each gamble. This table contains the choice probabilities for 12 conditions taken from an experiment by Myers et al. (1965), which used experimental procedures similar to the experiments associated with Table 6. Table 7 is read in exactly the same way as Table 6, and each choice proportion in Table 7 was based on 750 observations.

The Myers effect can be seen by considering the choice probabilities obtained from Conditions 22–25. First, compare Conditions 22 and 23 in which A_L yields +1 for certain. In Condition 22, A_R yields +4 or −4 with equal probability, and in Condition 23, A_R yields −16 or +16 with equal probability. Note that increasing the payoff range increased the probability of choosing A_R. According to simple scalability theory, this implies that the utility of A_R increased when the payoff range increased.

Next, compare Conditions 24 and 25, and note that the only difference between these two conditions and Conditions 22 and 23 is the change in the certain value of A_L from +1 to −1. However, now the same increase in payoff range for A_R decreased the probability of choosing A_R. According to simple scalability theory, this implies that the utility of A_R decreased when the payoff range increased. But this contradicts the earlier conclusion and violates the independence between alternatives property implied by simple scalability theory.[14]

The violation of independence in Table 7 is not restricted to equal event probabilities. As can be seen by comparing Conditions 18–21 and Conditions 26–29 in Table 7, violations also occur with unequal event probabilities. Furthermore, this violation of independence (i.e., the Myers effect) has been replicated in numerous other experiments (Busemeyer, 1979, 1985; Katz, 1962; Myers & Katz, 1962; Myers & Sadler, 1960; see also Footnote 3).

The last column of Table 7 shows the predictions for all 12 conditions calculated from decision field theory by using exactly the same two parameters that were used to fit the results of Table 6 (i.e., a cross-validation test). As can be seen, decision field theory correctly predicted the independence violations for all three event probabilities in Table 7 without estimating any parameters from this table of the data. According to decision field theory, these violations of the independence property are caused by changes in the variance of the valence difference, σ^2, across choice pairs (see Table 3). However, this is not the only possible explanation, and the next two applications compare decision field theory with two other possible explanations.

Comparison With Probabilistic Regret Models

Myers et al. (1965) originally explained the violation of independence in terms of a regret ratio model. According to this model,

$$[Pr(A_R, A_L)/Pr(A_L, A_R)] = [ER(A_L)/ER(A_R)]^a, \quad (8a)$$

where $ER(A_L)$ and $ER(A_R)$ are the expected regrets corresponding to actions A_L and A_R, respectively. For decision problems involving only two uncertain events (similar to Figure 5), the expected regrets are defined as follows:

$$ER(A_L) = w(S_2) \cdot u(y_{R2} - y_{L2}), \text{ and}$$

$$ER(A_R) = w(S_1) \cdot u(y_{L1} - y_{R1}),$$

where it is assumed that $y_{R2} > y_{L2}$ and $y_{L1} > y_{R1}$ and $u(x)$ is an increasing function of x. If we take the logarithms of both sides of Equation 8a, then we obtain the following subtractive model for the logit score, L:

$$L = ln[Pr(A_R, A_L)/Pr(A_L, A_R)]$$
$$= a \cdot \ln[ER(A_L)] - a \cdot \ln[ER(A_R)]. \quad (8b)$$

Thus, if the choice probabilities are transformed into logit scores, L, then the logit score can be written as a subtractive combination of the regret produced by each action.

[14] The test of independence can be performed by setting A = action A_R under Conditions 22 and 24, B = action A_R under Conditions 23 and 25, C = action A_L under Conditions 22 and 23, and D = action A_L under Conditions 24 and 25. Then we have $Pr(A, C) < Pr(B, C)$ and $Pr(A, D) > Pr(B, D)$ in Table 7, violating the independence property. In other words, there is a crossover interaction effect of payoff range of A_R and the certain value of A_L on choice probability. This crossover interaction is the Myers effect.

Table 6
Choice Probabilities From Katz (1964) and Myers and Suydam (1964)

Condition	Payoff matrix y_{R1}	y_{R2}	y_{L1}	y_{L2}	Event probabilities $Pr(S_1)$	$Pr(S_2)$	Pr[choose A_R over A_L] Observed	Predicted
1	1	−1	−1	1	.5	.5	.49	.50
2	1	−2	−1	1	.5	.5	.31	.34
3	1	−4	−1	1	.5	.5	.25	.26
4	2	−1	−1	1	.5	.5	.65	.66
5	2	−2	−1	1	.5	.5	.50	.50
6	2	−4	−1	1	.5	.5	.37	.40
7	4	−1	−1	1	.5	.5	.71	.74
8	4	−2	−1	1	.5	.5	.58	.60
9	4	−4	−1	1	.5	.5	.51	.50
10	1	−1	−1	1	.6	.4	.62	.67
11	1	−4	−1	1	.6	.4	.36	.41
12	4	−1	−1	1	.6	.4	.81	.85
13	4	−4	−1	1	.6	.4	.70	.67
14	1	−1	−1	1	.8	.2	.93	.93
15	1	−4	−1	1	.8	.2	.82	.78
16	4	−1	−1	1	.8	.2	.94	.98
17	4	−4	−1	1	.8	.2	.94	.93

Note. Each proportion was based on at least 2,000 observations. Two parameters were used to fit the 17 data points, and 98% of the variance was predicted by the model.

Decision field theory predicts that the effects of the regrets for each action do not decompose into subtractive components in the logit transformation. On the contrary, nonadditive effects of the regret factors are expected to result from dividing the mean difference by the standard deviation of the valence difference. For example, when there are only two equally likely events, then the logit transformation applied to Equation 3c can be expressed in terms of expected regrets as

$$L = [ER(A_L) - ER(A_R)]/[ER(A_L) + ER(A_R)].$$

Busemeyer (1982) conducted two experiments to test the subtractive property of the regret ratio model, and statistically significant interactions were obtained from a majority of subjects in both experiments. The results from Busemeyer (1982, Experiment 1) are shown in Table 8. In this experiment, subjects were given 1,400 trials of training with uncertain decision problems similar to the prototypical problem shown in Figure 5, with two equally likely events, $P(S_1) = P(S_2) = .5$. The first two columns in Table 8 indicate the monetary payoffs, either y_{R1} or y_{R2}, produced by choosing action A_R when either event S_1 or S_2 occurred, respectively. The monetary payoffs for choosing action A_L are not shown because they were fixed at $y_{L1} = -1$ and $y_{L2} = +1$ for all conditions. Thus, the regret for A_L is determined by y_{R1} in the first column, and the regret for A_R is determined

Table 7
Choice Probabilities From Myers, Suydam, and Gambino (1965)

Condition	Payoff matrix y_{R1}	y_{R2}	y_{L1}	y_{L2}	Event probabilities $Pr(S_1)$	$Pr(S_2)$	Pr[choose A_R over A_L] Observed	Predicted
18	4	−4	1	1	.8	.2	.83	.73
19	16	−16	1	1	.8	.2	.93	.91
20	4	−4	−1	−1	.8	.2	.98	.98
21	16	−16	−1	−1	.8	.2	.88	.94
22	4	−4	1	1	.5	.5	.35	.22
23	16	−16	1	1	.5	.5	.43	.45
24	4	−4	−1	−1	.5	.5	.75	.78
25	16	−16	−1	−1	.5	.5	.60	.55
26	4	−4	1	1	.2	.8	.08	.02
27	16	−16	1	1	.2	.8	.10	.06
28	4	−4	−1	−1	.2	.8	.30	.27
29	16	−16	−1	−1	.2	.8	.15	.09

Note. Each proportion was based on 750 observations. The 12 predictions were based on exactly the same parameters estimated from the data shown in Table 6 (i.e., the predictions in this table are parameter free).

Table 8
Probability of Choosing A_R Over A_L From Experiment 1 of Busemeyer (1982)

Payoff matrix			Context cue value					
y_{R1}	y_{R2}	Source	−3	−2	−1	+1	+2	+3
1	−1	O	.25	.37	.46	.58	.64	.80
		P	.26	.37	.44	.56	.63	.74
1	−2	O	.07	.13	.15	.21	.35	.49
		P	.08	.14	.19	.28	.34	.43
1	−4	O	.05	.05	.07	.08	.16	.26
		P	.03	.06	.09	.14	.17	.23
2	−1	O	.52	.63	.76	.80	.85	.91
		P	.53	.63	.69	.79	.84	.91
2	−2	O	.25	.41	.48	.56	.67	.78
		P	.26	.37	.44	.56	.63	.74
2	−4	O	.07	.13	.17	.21	.34	.49
		P	.09	.15	.20	.30	.36	.45
4	−1	O	.71	.83	.90	.90	.91	.94
		P	.73	.80	.84	.90	.93	.96
4	−2	O	.45	.62	.72	.75	.80	.87
		P	.48	.58	.65	.75	.81	.89
4	−4	O	.26	.41	.47	.55	.69	.73
		P	.24	.34	.41	.53	.61	.71

Note. $y_{L1} = -1$ and $y_{L2} = 1$ for all conditions. O = observed proportion; P = predicted probability. Each proportion is based on 405, 945, and 2,025 observations for cues ±1, ±2, and ±3, respectively. A total of five parameters were used to fit 54 data points, and the percentage of variance predicted by the model equals 98%.

by y_{R2} in the second column of Table 8. The next eight columns contain the observed and predicted choice probabilities obtained in each of six different context cue conditions. Each context cue was an event sequence that preceded the current trial. Cues labeled +1, +2, or +3 indicate that a run of one, two, or three S_2 events preceded the current trial. Cues labeled −1, −2, or −3 indicate that a run of one, two, or three S_1 events preceded the current trial.

For example, consider the pair of rows corresponding to the payoff (1, −1) and the column corresponding to the context cue labeled +3. In this case, event S_2 occurred on all of the three immediately preceding trials, and the probability of choosing A_R equals .80. Apparently, subjects expected this run of three S_2 events to terminate with an S_1 event on the next trial. Now consider payoff condition (2, −1) and the context cue condition +3. Under this condition, action A_R stochastically dominates A_L. However, the probability of choosing action A_R was only .91, well below 1.0 required by deterministic rank-dependent utility theories. Violations of stochastic dominance are also obtained in payoff condition (4, −1) and context cue condition +3 and in payoff conditions (1, −2) and (1, −4) and context cue condition −3.

The predicted values in Table 8 were calculated from decision field theory. Only five parameters were estimated from the 54 choice probabilities by using the mapping rules shown in Table 4. (Only Stage 4 was needed. See the Appendix for the parameter values.) The model accounted for 98% of the variance in the choice proportions.

The critical interaction effects on the logit scores are shown in Table 9. Each row shows the observed and predicted interaction effect corresponding to the interaction contrast indicated by the far left column. The symbol L_{ij} indicates the mean logit score (averaged across context cues) obtained in the payoff condition with $y_{R1} = i$ and $-y_{R2} = j$. For example, the positive interaction contrast in the first row indicates that the increase in mean logit scores produced by changing y_{R1} from 1 to 2 when y_{R2} was fixed at −1 was larger than the increase produced by the same change in y_{R1} when y_{R2} was fixed at −2. As can be seen in Table 9, decision field theory accounts for the direction of the statistically significant interaction effects that violate the subtractive property implied by the regret ratio model. A cross-validation test for this experiment is described next.

Relation Between Choice Probability and Decision Time

Tversky (1972) developed a probabilistic choice theory called the elimination by aspects (EBA) theory to explain violations of the independence property. Although the EBA theory is considered a process theory of choice, Tversky (1972) did not develop any choice response time predictions. Subsequently, Marley (1981) overcame this limitation by extending EBA theory to account for both choice probability and choice response time. More recently, Busemeyer, Forsyth, and Nozawa (1988) derived the following critical property from the extended EBA theory: The mean time to make a decision is independent of the action that is eventually chosen. In other words, the extended EBA theory fails to account for the basic fact that the mean decision time for the more frequently chosen alternative is faster than that for the less frequently chosen alternative (see Figure 3).

In contrast, decision field theory generally does predict differences between conditional mean response times. According to this theory, the initial preference state, z, will be biased in the direction of mean input, δ, because of recall of preference states from previous choice trials. This initial bias causes the more favorable alternative in a pair to be chosen more quickly. A strong cross-validation test of the theory can be conducted by using the same parameters estimated from the choice probabilities in Table 8 to make predictions for the mean choice response time.

The choice speed data reported in Busemeyer (1982) provide a direct test of these two models, as is shown in Table 10. Speed = (1/latency), rather than latency, was analyzed to satisfy the

Table 9
Interaction Effects of Rewards and Punishments on Log Odds Scale From Busemeyer (1982)

Interaction	Observed	Predicted
$(L_{11} - L_{21}) - (L_{12} - L_{22})$	.36	.14
$(L_{12} - L_{22}) - (L_{14} - L_{24})$	−.56	−.25
$(L_{21} - L_{41}) - (L_{22} - L_{42})$	.03	.03
$(L_{22} - L_{42}) - (L_{24} - L_{44})$	.60	.14

Note. L_{ij} = mean logit score (averaged across cues) for the payoff condition with the reward set equal to $y_{R1} = i$ and the punishment set equal to $-y_{R2} = j$. Each contrast is based on 7,020 observations. The interaction effect was also significant in a second experiment reported in Busemeyer (1982).

Table 10
Differences in Choice Probabilities and Mean Choice Speeds From Experiment 1 of Busemeyer (1982)

(A)			(B)		
	$Pr(A_R, A_L) - Pr(A_L, A_R)$			$Pr(A_R, A_L) - Pr(A_L, A_R)$	
Gain for A_R	O	P	Loss for A_R	O	P
$y_{R1} = +1$	−.42	−.42	$y_{R2} = -1$	+.42	+.39
$y_{R1} = +4$	+.39	+.35	$y_{R2} = -4$	−.42	−.43

(C)			(D)		
	Speed(A_R) − speed(A_L)			Speed(A_R) − speed(A_L)	
Gain for A_R	O	P	Loss for A_R	O	P
$y_{R1} = +1$	−.33	−.28	$y_{R2} = -1$	+.06	+.24
$y_{R1} = +4$	+.16	+.21	$y_{R2} = -4$	−.30	−.29

Note. O = observed; P = predicted. Each contrast is based on 14,040 observations. The pattern of observed results shown in this table were replicated in a second independent experiment. No new parameters were used to generate the choice speed predictions.

assumptions of the statistical tests. (Latencies produce nonhomogenous variances, and the transformation to speed homogenizes the variance.) This table shows four separate interaction effects: Panel A shows the effect of manipulating the gain for A_R ($y_{R1} = +1$, $y_{R1} = +4$) on the mean differences in choice probabilities; Panel B shows the effect of manipulating the loss for A_R ($y_{R2} = -1$, $y_{R2} = -4$) on the mean differences in choice probabilities; Panel C shows the effect of the gain for A_R on the mean differences in choice speeds; and Panel D shows the effect of the loss for A_R on the mean differences in choice speeds. In all four cases, the mean difference is defined as (mean for A_R) − (mean for A_L). Increasing the gain caused the differences in choice probability and choice speed to change from negative to positive; whereas increasing the loss magnitude caused the differences to change from positive to negative. Both interactions shown in Panels C and D were statistically significant, and, furthermore, both interactions were replicated in two other experiments (Busemeyer, 1982, Experiment 2; Busemeyer, 1985, Experiment 1).

Contrary to the extended EBA theory, the differences between mean choice speed for each action were nonzero, and, furthermore, the sign of the differences in mean choice speed changed systematically in the same direction as the differences in choice probability. Note that decision field theory correctly predicts the interaction patterns for choice speed without estimating any new parameters from this part of the data. In conclusion, decision field theory successfully predicts this basic relationship between choice probability and choice speed, whereas EBA theory currently is unable to account for this fundamental relation.

Effects of Time Pressure on Decision Accuracy

Interest in the effects of time pressure on decision making has increased rapidly within the last few years (see Svenson & Maule, in press). One explanation for the effects of time pressure on decision making is that short deadlines force the decision maker to adopt a simple heuristic strategy that takes less time to execute but is less accurate (e.g., see Payne, Bettman, & Johnson, 1988). This implies that increasing time pressure always results in a decrease in decision accuracy (i.e., a decrease in the probability of choosing the action that produces the largest SEU).

Decision field theory provides an alternative explanation: Rather than switching decision strategies, decision accuracy is controlled by adjusting the inhibitory threshold bound, θ. If the discriminability ratio is very high, or the initial starting position is close to zero, then decision field theory predicts that increasing the inhibitory bound will increase decision accuracy. However, if the discriminability ratio is very low, and the initial starting position is biased in the correct direction by past experience, then decision field theory predicts that time pressure may improve decision accuracy by increasing the effect of the initial starting position on choice probability.

The results shown in Table 11 were taken from an experiment on uncertain decision making under time pressure by Busemeyer (1985). At the beginning of each choice trial, subjects were given a deadline time limit, and then they were asked to choose between a certain value (labeled here as action A_L) and an uncertain action (labeled here as action A_R). A known monetary payoff was delivered if the certain value was chosen. The monetary payoff for the uncertain action was randomly sampled from a normal distribution with a mean equal to zero. Subjects were given 360 trials of training to learn the distribution of payoffs produced by the uncertain alternative.

The probability of choosing action A_R following training for each of 18 conditions are presented in Table 11. The first column indicates the deadline time limit condition (1, 2, or 3 s), and the second column indicates the value of the certain alternative (−3¢, 0¢, +3¢). The next three columns contain the observed and predicted probabilities obtained when the standard deviation of the uncertain payoff equaled 5¢, and the last three columns contain the results obtained when the standard deviation of the uncertain payoff equaled 50¢. Each proportion is based on an average of 1,560 observations.

Two important interactions need explanation. The first interaction is analogous to the Myers effect—that is, the interaction between the certain value and the uncertain standard deviation on choice probability in Table 11. When the certain value was positive, increasing the standard deviation increased the probability of choosing the uncertain action; however, when the certain value was negative, increasing the standard deviation had the opposite effect. This crossover interaction is a violation of the independence property implied by simple scalability theories. The second interaction is the crossover interaction between the deadline time limit and the standard deviation of the uncertain action. Under the small standard deviation condition, increasing the time limit increased the probability of choosing the action with the largest expected value. But under the large standard deviation condition, increasing the time limit had the opposite effect, or, in other words, increasing the deliberation time decreased accuracy (where the correct action is the one producing the largest ex-

Table 11
Probability of Choosing the Uncertain Alternative From Experiment 1 of Busemeyer (1985)

		Uncertain standard (5¢)			Uncertain standard (50¢)		
		RW			RW		
Time limit (s)	Certain value	Observed	Predicted	DF predicted	Observed	Predicted	DF predicted
1	+3	.11	.09	.11	.27	.26	.30
1	0	.47	.50	.50	.48	.48	.50
1	−3	.88	.87	.88	.62	.61	.65
2	+3	.07	.08	.09	.31	.32	.32
2	0	.53	.51	.50	.52	.49	.50
2	−3	.91	.91	.93	.58	.58	.64
3	+3	.06	.06	.08	.36	.37	.34
3	0	.53	.52	.50	.50	.49	.50
3	−3	.94	.95	.96	.55	.56	.62

Note. Each proportion is based on 1,560 trials. The empirical pattern of results was replicated in a second independent experiment. Eleven parameters were used to fit the random walk subjective expected utility theory (RW), and four parameters were used to fit the decision field (DF) model. The percentage of variance predicted equals 99% for both models.

pected payoff). This interaction was replicated in a second experiment by Busemeyer (1985).

If decision makers change to a simple heuristic strategy under time pressure, then accuracy should decrease under time pressure for both the small and large variance conditions. That time pressure increased decision accuracy under the high variance condition is difficult to explain by using this view, because it violates the accuracy–effort trade-off relation assumed by cost–benefit theories of strategy selection (see Busemeyer, in press).

This counterintuitive result can be explained by decision field theory as follows. The initial starting position, z, is strongly biased in the direction of the mean input, δ, and this bias has its greatest effect when the threshold criterion, θ, is small (see Figure 7). Therefore, at short deadlines, accuracy should be relatively good even for the high-variance condition because of the effect of the initial starting position. However, the effect of the initial starting position rapidly diminishes, and the effect of the discriminability ratio rapidly increases as the threshold bound increases (see Figure 7). Therefore, at long deadlines, accuracy should be low for the high-variance condition because of the small discriminability ratio, but accuracy should be high for the low-variance condition because of the large discriminability ratio.

A quantitative test of this explanation was performed by estimating four parameters from the 18 data points in Table 11 by using the mapping rules shown in Table 4 (see the Appendix for the parameter values).[15] Stage 7 of Table 2 was needed for this application: (a) The mean input was set as $\delta = (0 - x)$, where 0 is the mean of the uncertain action (A_R) and x is the monetary value of certain alternative (A_L); (b) the variance of the input was set equal to the variance of the uncertain action; (c) the inhibitory bound was linearly related to the time limit; (d) the initial preference state was proportional to the mean input; and (e) the goal gradient parameter was proportional to the value of the certain alternative. (Note that negative certain values produce a larger avoidance component than positive certain values.) The predictions from this model are shown under the columns labeled DF (decision field) in Table 11. As can be seen, this model successfully accounts for both of the crucial interactions, and it also accounts for 99% of the variance in the choice probabilities. A cross-validation test of decision field theory for this data set is presented next.

Effect of Type of Conflict on Decision Time

One of the most interesting properties of the deliberation process is that longer deliberation seems to be needed for avoidance–avoidance decisions as compared with approach–approach decisions (Barker, 1942; Bockenholt et al., 1991; Busemeyer, 1985; Houston et al., 1991). According to decision field theory, this effect of the type of conflict on decision time results from a steeper goal gradient for the avoidance subsystem as compared with the approach subsystem. In other words, the goal gradient parameter, c, is assumed to increase from an approach condition to an avoidance conflict condition, and this

[15] An alternative way to model the effects of deadline time limits on decisions is to assume that the decision maker continues to accumulate information until the deadline is reached and then stops and chooses the action on the basis of the sign of the preference state at the deadline. This is called a fixed sample decision model, which was ruled out by Busemeyer (1985). There are two obvious problems with this model: (a) It produces a fixed or constant stopping time (equal to the deadline) with zero variance, and (b) the stopping time would not be affected by the value of the certain alternative. However, the observed decision times form a distribution with a mean located well before the deadline and a nonzero variance that is an increasing function of the mean. Furthermore, the mean decision time was inversely related to the value of the certain alternative. Decision field theory (with a constant inhibitory threshold as shown in Figure 6) satisfies these basic properties. It is plausible that the inhibitory boundaries decay toward zero as the deadline time limit approaches; however, this more complex model was not needed to account for the main results.

causes decision time to be longer for avoidance conflicts (see Figure 9, right panel).

Recall that if the goal gradient parameter is eliminated, then no differences in mean decision time are expected by changing from an approach condition to an avoidance conflict condition. In particular, decision field theory reduces to random walk SEU theory whenever $c + s = 0$, where c is the goal gradient parameter and s is the growth–decay rate parameter. Thus, an examination of the effect of type of conflict on decision time provides a critical test of the more complex decision field theory in comparison with the simpler random walk SEU theory.

The mean decision time data reported by Busemeyer (1985, Experiment 1) provide a cross-validation test of the decision field versus random walk SEU theories. The parameters of both models were estimated from the choice data shown in Table 11, and then these same parameters were used to make predictions for mean decision time.

The choice probability predictions produced by the decision field model for Table 11 have already been discussed. The random walk SEU theory also was fit to the choice data in Table 11 by using the following mapping of experimental factors to model parameters: (a) The mean input was estimated separately for each value of the certain action (recall that the mean of the uncertain action was fixed at zero in this experiment);[16] (b) the variance of the input was estimated separately for each standard deviation of the uncertain action; (c) the inhibitory bound was estimated for each time limit condition; and (d) the initial preference state was estimated separately for each value of the uncertain action. Altogether, 11 parameters were estimated from the 18 choice probabilities (see the Appendix for the specific values). The predictions are shown under the columns labeled RW (random walk) in Table 11. As can be seen, this model successfully accounts for both of the crucial interactions, and it accounted for over 99% of the variance in the choice probabilities.

The primary test of the two theories is based on the predictions for mean choice response time. Table 12 shows the results for the mean response time for each combination of deadline time limit and value of the certain alternative. Note that the mean response times consistently increase as the certain value changes from positive to zero to negative. The random walk SEU theory incorrectly predicts that the zero certain value always produces the longest mean response times, and it fails to predict much difference between the negative and positive certain value conditions. The slight difference that it does predict is produced by differences in the magnitude of the biased initial state parameter for positive and negative certain values. Also note that the random walk SEU theory was based on seven more parameters than the decision field model (see Footnote 16).

Decision field theory accounts for the effect of the sign of the certain value on mean response time. Observe that this is a parameter-free prediction, because it was predicted by a model whose parameters were estimated from the choice probability data. According to decision field theory, the slower response time for choice pairs containing a negative certain value was due to a steeper avoidance gradient under the avoidance conflict condition. In conclusion, the goal gradient parameter is needed to account for differences in decision times because of differences in the avoidance versus approach nature of conflict situations (Barker, 1942; Bockenholt et al., 1991; Busemeyer, 1985; Houston et al., 1991).

Preference Reversals Between Choice and Selling Prices

Choice may be the primary way to measure preference, but selling prices are also used to measure preference. Surprisingly, it turns out that these two measures of preference sometimes produce contradictory preference orders (see Slovic & Lichtenstein, 1983, for a review). Under certain conditions, subjects will choose one action more frequently than another action, but at the same time they will require a higher selling price for the less frequently chosen action. Decision field theory provides a way to link these discordant measures of preference together within a common theoretical framework. As this extension of the theory is covered in detail in another article (Busemeyer & Goldstein, 1992), only a brief mention of this application will be given here.

Busemeyer and Goldstein (1992) developed a dynamic and stochastic matching model for buying and selling prices from decision field theory. This unified approach to choice and pricing provides additional leverage for testing the theory because the joint distribution of choices and prices are explained by a common set of parameters. Furthermore, the dynamic and stochastic nature of the matching model provides predictions for the distribution of selling prices as a function of various information-processing factors, such as time, effort, and training. Busemeyer and Goldstein (1992) showed in detail how this matching model accounts for empirical findings from research on preference reversals, including the basic findings obtained by Lichtenstein and Slovic (1971), the effects of training found by Lindman (1971), the effects of multiple play found by Wedell and Bockenholt (1990), and the effects of elicitation procedure found by Bostic, Herrnstein, and Luce (1990).

Violations of Transitivity

One of the most important axioms of deterministic–static theories of decision making is the transitivity axiom: If $A \geq_p B$ and $B \geq_p C$, then $A \geq_p C$, for three arbitrary actions A, B, and C. All deterministic–static utility theories that assign a single real number to each action satisfy this axiom (e.g., SEU theory, prospect theory, rank-dependent utility theory).

Because of the variability of preferences, empirical tests of this axiom are actually based on the following probabilistic

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[16] Several other versions of random walk SEU theory were also fit. One version added a nonzero mean valence for the uncertain alternative for each variance condition. The same pattern of predictions for mean response time were produced by this 13-parameter model as were produced by the original 11-parameter random walk model. A leaner 5-parameter version of the random walk model was also fit, but the pattern of predictions for mean response time remain unchanged. Finally, a version of the random walk model was fit with the inhibitory bound independent of the standard deviation of the input, but this produced very poor mean response time predictions. Although this does not exhaust all of the possibilities, we could not find a version that was consistent with all of the results that we have reviewed.

Table 12
Mean Response Times From Experiment 1 of Busemeyer (1985)

Time limit (s)	Source								
	Observed			RW model			DF model		
	+3[a]	0[a]	−3[a]	+3[a]	0[a]	−3[a]	+3[a]	0[a]	−3[a]
1	.65	.67	.71	.69	.72	.70	.70	.73	.73
2	.77	.82	.83	.76	.80	.77	.74	.80	.81
3	.84	.89	.92	.86	.93	.87	.78	.88	.94
M	.76	.79	.82	.77	.82	.78	.74	.80	.83

Note. Each observation was based on 3,120 observations. The pattern of results was replicated in a second experiment. RW = random walk; DF = decision field. The predictions from each model were based on the parameters estimated from the choice data. No new parameters were used to predict the mean response times.
[a] The value of the certain alternative for each model was +3, 0, −3.

definitions of transitivity (see Luce & Suppes, 1965, p. 340). Assume that the following conditions are met: $Pr(A, B) \geq .5$ and $Pr(B, C) \geq .5$. Then strong stochastic transitivity (SST) satisfies $Pr(A, C) \geq \max[Pr(A, B), Pr(B, C)]$, and weak stochastic transitivity (WST) satisfies $Pr(A, C) \geq .5$. Naturally, SST implies WST.

Recall that the simple scalability hypothesis is the primary way to extend deterministic–static utility models to account for the magnitude of choice probability. All simple scalability theories satisfy SST (Tversky & Russo, 1969). This is due to the fact that all simple scalability models satisfy independence between alternatives, and SST is satisfied if and only if independence is satisfied (Tversky & Russo, 1969).

Decision field theory does not always satisfy SST. In fact, violations of SST are expected for precisely the same reasons as violations of independence between alternatives. For example, violations of SST are expected to occur whenever the similarity between pairs of actions is manipulated. (Note that the covariance term in Equation 2f is influenced by similarity.)

Empirically, violations of SST have been commonly observed when the similarity between a pair of actions is manipulated (Becker et al., 1963a; Mellers, Chang, Birnbaum, & Ordonez, 1992; Rumelhart & Greeno, 1971; Tversky, 1972). Violations of SST have also occurred when the variance of the payoffs was manipulated (Lindman, 1971).

Unlike violations of SST, violations of WST are rare. Moreover, they are only obtained under highly special conditions involving just noticeable differences in dimension values (Budescu & Weiss, 1987; Lindman & Lyons, 1978; Montgomery, 1977; Ranyard, 1977; Tversky, 1969; Zakay & Beer, 1992). For this reason, these violations are not treated as seriously as the more pervasive violations of SST.

Decision field theory is unable to account for these violations of WST. One could generalize the theory by replacing the linear stochastic difference equation (i.e., Equation 7) with a nonlinear equation (cf. Grossberg & Gutowski, 1987) and thereby account for violations of WST. However, theories that generally predict violations of WST fail to explain why WST is almost always satisfied. The key is to find a nonlinear theory that generally satisfies WST, but violates it under very special circumstances. Until this is achieved, the parsimony gained by using a theory that satisfies WST may outweigh the loss in predictive accuracy for an isolated condition.

Summary of Experimental Tests

Decision field theory was compared with five other major theories of decision making under uncertainty: rank-dependent utility, simple scalability theories, probabilistic regret theory, EBA theory, and a random walk choice model. For each comparison, we identified an important qualitative property that could be used to empirically discriminate between the two theories. For example, independence between alternatives was used to distinguish decision field theory from simple scalability theories. Furthermore, the qualitative properties that we selected for comparison were critical properties that rule out large classes of models (rather than specific cases). For example, violations of independence between alternatives rule out all of the various versions that fall into the large class of simple scalability models. For each comparison, the qualitative tests favored decision field theory over the comparison theory.

At first impression, one might wonder whether the success of decision field theory in these comparisons resulted from the use of more parameters. On the contrary, each of the comparison models had more parameters available to account for the data in Tables 6–12 than did the number used by decision field theory to account for these results. For example, rank-dependent utility theory allows one to estimate utility parameters for each payoff and a weight parameter for each event, but a new set of weights may be estimated separately for each rank ordering of the payoffs. This produces more parameters than the two parameters used by decision field theory. Despite the flexibility provided by the extra weight parameters for each rank ordering of payoffs, rank-dependent utility theory cannot account for the violations of stochastic dominance observed in Table 6. The probabilistic regret model involves five regret parameters plus additional subjective probability parameters to account for the results in Table 8 (more than the five parameters used by decision field theory), but it cannot account for the interaction effects shown in Table 9. The elimination by aspects model allows a separate parameter for every single choice pair shown in Table 8 (the ratio of the values of the unique aspects); nevertheless, it cannot account for the relation between choice probability and decision speed shown in Table 10. The random walk model was fit by using more parameters than was used in decision field theory, but it could not account for the effect of sign of the certain alternative on mean decision time in Table 12. Thus, the success of decision field theory over the comparison theories was not due to the use of more parameters.

Whenever a general theory is presented, the question of testability arises. To address this important issue, we have emphasized the importance of using a cross-validation model testing methodology. For example, in this article, we estimated parameters from the choice probability data and then used these same parameters to make new predictions for decision time. In Busemeyer and Goldstein (1992), we estimated parameters from the choice probability data and then used these same parameters to make new predictions for mean selling price. Ideally, all competing theories could be compared by using this powerful model-testing methodology. However, cross-validation methodology requires a theory that can make precise quantitative predictions for multiple measures of preference. At present, few theories of decision making under uncertainty fulfill this stringent requirement. Decision field theory is unique among decision making theories under uncertainty in the systematic use of cross-validation for testing its predictions.

Concluding Comments

Deterministic–static theories have dominated the field of decision making under uncertainty for the last 45 years. These theories have served a useful purpose by providing a rough first-order approximation to human decision behavior. However, these theories fail to provide any basis for explaining two of the most basic phenomena of human decision making: (a) the variability of preferences and (b) the systematic relation between preference and deliberation time. A higher fidelity, second-order approximation that captures these two fundamental properties of human decision behavior is long overdue.

Decision field theory provides a stochastic–dynamic alternative based on a description of the deliberation process that lies at the heart of human decision-making behavior. The main advantages of decision field theory include the following: (a) It accounts for a wider range of phenomena (see Table 2); (b) at the same time, it provides a more detailed process-orientated explanation of each phenomena; (c) its roots lie within a long tradition of motivation behavior in psychology (e.g., Lewin, 1935; Miller, 1959); and (d) its processing assumptions are more in keeping with modern approaches to cognition (e.g., Diederich, in press; Heath, 1992; Link, 1992; Ratcliff, 1978; Smith, 1992).

One of the most important ideas presented here is that the deliberation process involves an accumulation of information about the consequences of a decision, and the amount of attention allocated to the various consequences changes over time during deliberation. If this basic idea is correct, then there is a simple recipe for producing reversals in the direction of preference under time pressure manipulations: Present the decision maker with a choice between two alternatives, in which the first alternative has an advantage on the most prominent or salient dimension, but the second alternative has an advantage on all of the remaining dimensions. Under a short deadline time limit, only the most prominent dimension tends to be processed, and the first alternative should be chosen more frequently. Under longer deadlines, the most prominent dimension is still processed first, but many additional dimensions are also processed, so that the second alternative should be chosen more frequently. As this recipe indicates, further research examining the effects of attentional manipulations on deliberation will help illuminate the dynamics principles that guide the human decision process.

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Appendix

The equations for c, δ, and σ^2 can be generalized as follows when more than two payoffs are produced by each action.

$$c = b\cdot(v_{pR} + v_{pL}) - a\cdot(v_{rR} + v_{rL}).$$

$$\delta = (v_{rR} - v_{rL})\cdot(1 - a\cdot\theta) + (v_{pR} - v_{pL})\cdot(1 - b\cdot\theta).$$

$$\sigma^2 = \sigma_R^{\,2} + \sigma_L^{\,2} - 2\cdot\sigma_{RL}.$$

Define A as the subset of gains that can result from taking action A_R, and S_x, $x \in A$, is the event that produces x when action A_R is chosen.

$$v_{rR} = \sum_{x\in A} w(S_x)\cdot u(x).$$

Define B as the subset of losses that can result from taking action A_R, and S_x, $x \in B$, is the event that produces x when action A_R is chosen.

$$v_{pR} = \sum_{x\in B} w(S_x)\cdot u(x).$$

Define C as the subset of gains that can result from taking action A_L, and S_x, $x \in C$, is the event that produces x when action A_L is chosen.

$$v_{rL} = \sum_{x\in C} w(S_x)\cdot u(x).$$

Define D as the subset of losses that can result from taking action A_L, and S_x, $x \in D$, is the event that produces x when action A_L is chosen.

$$v_{pL} = \sum_{x\in D} w(S_x)\cdot u(x).$$

Define F as the subset of payoffs that can result from taking action A_R, and S_x, $x \in F$, is the event that produces x when action A_R is chosen.

$$\sigma_R^{\,2} = \sum_{x\in F} w(S_x)\cdot[u(x) - (v_{rR} + v_{pR})]^2.$$

Define G as the subset of payoffs that can result from taking action A_L, and S_x, $x \in G$, is the event that produces x when action A_L is chosen.

$$\sigma_L^{\,2} = \sum_{x\in G} w(S_x)\cdot[u(x) - (v_{rL} + v_{pL})]^2.$$

Define $S_{x,y}$ as the joint event that produces the payoff x for action A_R and the payoff y for action A_L.

$$\sigma_{RL} = \sum_{x\in F,\, y\in G} w(S_{x,y})\cdot[u(x) - (v_{rR} + v_{pR})]\cdot[u(y) - (v_{rL} + v_{pL})].$$

Link and Heath (1975) derived the equations for choice probability and the conditional mean response time for a general random walk model. These equations provide only approximate solutions to the discrete-time–discrete-state random walk model. However, they are exact for the continuous-time–continuous-state version of the random walk (i.e., the Wiener process). The probability of choosing action A_R over A_L for the Wiener process is

$$Pr(A_R, A_L) = \frac{\exp[4\cdot(d/\sigma)\cdot(\theta/\sigma)] - \exp[2\cdot(d/\sigma)\cdot(\theta - z)/\sigma]}{\exp[4\cdot(d/\sigma)\cdot(\theta/\sigma)] - 1}, \quad (A1)$$

where d is the mean valence difference, σ^2 is the variance of the valence difference, θ is the inhibitory threshold, and z is the initial starting position. The mean number of samples required to reach the threshold criterion for choosing A_R for the Wiener process is

$$E[N\,|\,\text{choose } A_R] = (1/d)\cdot\{(2\theta)\cdot\coth[4(d/\sigma)(\theta/\sigma)] - (\theta + z)\cdot\coth[2(d/\sigma)(\theta + z)/\sigma]\}, \quad (A2)$$

where coth(x) is the hyperbolic cotangent function. The mean time to choose A_L can be obtained from Equation A2 by replacing d with $-d$ and z with $-z$.

Busemeyer and Townsend (1992) derived the probability that action A_R is chosen over action A_L for both the discrete- ($h > 0$) and continuous- ($h \rightarrow 0$) time versions of decision field theory. The equations for the continuous-time process are given below. The probability of choosing action A_R over A_L for decision field theory is

$$Pr(A_R, A_L) = S(z)/S(\theta), \quad (A3)$$

$$S(x) = \int_{-\theta}^{x} \exp\{[(c + s)\cdot y^2 - 2\cdot\delta\cdot y]/\sigma^2\}dy,$$

and c is the goal gradient parameter, s is the growth–decay rate parameter, δ is the mean valence input, σ^2 is the variance of the valence input, θ is the inhibitory threshold, and z is the initial starting position. The mean choice time conditioned on the choice of action A_R for decision field theory is

$$E[T\,|\,\text{choose } A_R] = 2\cdot\{S(z)\cdot H_1(z) + [S(\theta) - S(z)]\cdot H_2(z)\}/S(z), \quad (A4)$$

where

$$H_1(z) = \int_{z}^{\theta} [S(\theta) - S(x)]\cdot f(x)\cdot S(x)/S(\theta)\cdot dx,$$

$$H_2(z) = \int_{-\theta}^{z} [S(x) - S(-\theta)]\cdot f(x)\cdot S(x)/S(\theta)\cdot dx,$$

$$f(x) = 1/\{\sigma^2\cdot\exp[((c + s)\cdot x^2 - 2\cdot\delta\cdot x)/\sigma^2]\},$$

and S(x) is defined in Equation A3. The mean time conditioned on the choice of A_L is obtained by substituting $-\delta$ for δ and $-z$ for z in Equation A4.

The continuous-time model requires the integral of the normal density function, which can only be done by using a series expansion. For this reason, the discrete-time formulas may be more convenient. For very small time units, the two formulas agree very closely, and in the limit they agree exactly (see Busemeyer & Townsend, 1992).

The predictions from decision field theory were generated from the discrete-time equations presented in Busemeyer and Townsend (1992) because these equations are easier to implement than the continuous equations. The time unit was fixed at h = .000625, as this produced results that matched the continuous-time model up to the first two decimal places.

Predictions for Table 6

Two parameters (indicated by italics) were estimated from the first 17 choice proportions in Table 6: $u(4) = -u(-4) = \mathit{2.76}$ (subjective values for payoffs +4 and −4), and $\theta = \mathit{1.70}\cdot\sigma$ (the inhibitory threshold). The remaining parameters were either fixed or derived from the estimates: $u(y) = -u(-y) = y$, y = 1, 2 (the values for payoffs ±1 and ±2); $w(S_i) = Pr(S_i)$ (the actual event probability from Table 6); $(s + c) = 0$ (zero growth–decay plus goal gradient); and $z = 0$ (no bias). The mean input, δ, and the variance of the input, σ^2, were computed from the formulas shown in Table 4. This model accounted for 98% of the variance in the 17 proportions. Slightly better fits were obtained by including a positive growth–decay rate parameter, but this extra parameter did not change the basic pattern of predictions, and so we retained the simpler model.

The same parameters used in Table 6 were then used again to predict the probabilities for Table 7. The payoffs (+16, −16) never appeared in Table 6, and so we fixed $u(16) = -u(-16) = 16$ to show that the model predicts violations of independence without estimating any new parameters. The fit to the 12 data points in Table 7 is improved by estimating $u(16)$ and $u(-16)$, but this did not change the basic pattern of predictions.

Predictions for Table 8

Five parameters (indicated by italics) were estimated from the 54 choice proportions in Table 8: $u(4) = 2.90$, $u(-4) = -2.97$ (values for payoffs +4 and −4); $w(S_1) = .5 + .72 \cdot (j/6)$, $j = \pm1, \pm2, \pm3$ (weight for S_1 at each context cue); $w(S_2) = 1 - w(S_1)$; $\theta = .49 \cdot \sigma$ (inhibitory threshold); and $z = \tanh(.92 \cdot [u(y_{R1}) - u(y_{R2})] \cdot \theta$ (initial preference state). The remaining parameters were either fixed or derived from the following estimates: $u(y) = u(-y) = y$, $y = 1, 2$ (the values for payoffs ±1, ±2), and $(c + s) = 0$ (zero growth–decay plus goal gradient parameter). The mean input, δ, and the variance of the input, σ^2, were computed from the formulas given in Table 4. The model accounted for 98% of the variance in the 54 proportions. Slightly better fits were obtained with a positive growth-rate parameter, but this extra parameter did not change the basic pattern of predictions, and so the simpler model was retained. The main difference between these parameters and those estimated from the data in Table 6 is the inclusion of an initial bias, which is assumed to be an increasing function of the mean input (averaged over context cues). This is probably due to the fact that Busemeyer (1982) used more extensive training than did the experiments reported in Table 6, which would allow a stronger bias to build up from past experience with each payoff matrix.

The same parameters used in Table 8 were then used again to predict the mean response times for each action in Table 10. The mean choice response time for each action was computed from Equation 5b in Busemeyer and Townsend (1992), but Equation A4 produces the same pattern of results.

Predictions for Table 11

Four parameters (indicated by italics) were estimated from the 18 choice proportions of Table 11 for the decision field model: $\theta = 1.139 \cdot \sigma + .36 \cdot (L - 1) \cdot \sigma$, $L = 1, 2, 3$ (L is the time limit); $z = \tanh(-.10 \cdot x) \cdot \theta(1)$, $x = -3, 0, +3$ (x is the certain value); and $c = -.125 \cdot x$, $x = -3, 0, +3$, (c is the goal gradient parameter). The main difference between these parameters and those obtained from the previous fit to the data in Table 8 is the goal gradient parameter, c, which is a function of the value of the certain alternative. The mean of the uncertain alternative was set to zero, and so the mean input was set as $\delta = (0 - x)$, where x is the value of the certain alternative in Table 11. The standard deviation of the input, σ, was set equal to the standard deviation of the uncertain alternative from Table 11.

For the random walk model, 11 parameters (indicated by italics) were estimated from the 18 choice proportions of Table 11: $d(+3) = -1.67$, $d(0) = .12$, $d(-3) = 2.08$ (d for each certain value); $\sigma(5) = .04$, $\sigma(50) = 50.64$ (σ for each uncertain standard deviation); $\theta(1) = .014 \cdot \sigma$, $\theta(2) = .018 \cdot \sigma$, $\theta(3) = .024 \cdot \sigma$ (θ for each time limit); and $z(+3) = -.47 \cdot \theta(1)$, $z(0) = -.03 \cdot \theta(1)$, $z(-3) = .22 \cdot \theta(1)$ (an initial bias for each certain value). Note that the inhibitory bound, θ, is proportional to the standard deviation of the input for this model (but see Footnote 15). The decision field model has 7 fewer parameters than the random walk model, even though it includes a goal gradient parameter.

These same parameters were then used to predict the mean response times. The time unit and intercept of the time scale are unknowns, and so the predicted response times shown in Table 12 have been adjusted by multiplying by a constant and adding a constant to the original predictions generated from the model. However, this transformation does not change the pattern of the predictions, which is our main concern.

The predictions for both the random walk and the decision field model were generated by using both the discrete-time equations in Busemeyer and Townsend (1992) with $h = .000625$ and continuous-time Equations A3 and A4 from this article. The results shown in Tables 10 and 11 are exactly the same with both versions.

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