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Addendum: Four-loop cusp anomalous dimension in QED

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The cusp anomalous dimension $\Gamma(\varphi, \alpha_s)$ at Euclidean angles $\varphi = \pi - \delta$, $\delta \to 0$ is closely related to the quark-antiquark potential $V(\vec{q}, \alpha_s)$ [1]. Let's define[1]

$$\Delta = [\delta\,\Gamma(\pi - \delta, \alpha_s(|\vec{q}\,|))]_{\delta \to 0} - \frac{\vec{q}^{\,2} V(\vec{q}, \alpha_s(|\vec{q}\,|))}{4\pi}\,. \tag{1}$$

Then $\Delta = 0$ up to 2 loops [1]. The equality $\Delta = 0$ follows from conformal symmetry [3, 4]. E.g., $\mathcal{N} = 4$ SYM is conformally symmetric; the explicit calculation shows that in this theory $\Delta = 0$ up to 3 loops [4]. In QCD (as well as QED and many other gauge theories) conformal symmetry is anomalous, broken by the β function; and [4],

$$\Delta(\alpha_s) = \frac{16}{27}\pi\beta_0 C_F\,(47C_A - 28T_F n_f)\left(\frac{\alpha_s}{4\pi}\right)^3 + \mathcal{O}(\alpha_s^4)\,. \tag{2}$$

Therefore, it seems reasonable to assume [4] that, similarly to the Crewther relation [5, 6], the conformal anomaly has the form

$$\Delta(\alpha_s) = \beta(\alpha_s) C(\alpha_s)\,, \qquad C(\alpha_s) = \frac{16}{27}\pi C_F\,(47C_A - 28T_F n_f)\left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\,. \tag{3}$$

Not much is known about higher orders in (2), (3) (see, e.g., [7]).

[1] At 4 loops, a $C_F C_A^3 \alpha_s^4 \log(\delta)/\delta$ term appears in $\Gamma(\pi - \delta)$ [2], so that the limit $\delta \to 0$ does not exist. It seems probable that after resummation of leading powers of $\log(\delta)$ to all orders this limit will make sense.

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Now we can consider the $C_F^{L-1} T_F n_l \alpha_s^L$ terms in the conformal anomaly Δ. These terms in the quark-antiquark potential in Coulomb gauge are given by a single Coulomb-gluon propagator:

$$
\begin{aligned}
V(\vec{q}) &= -\frac{4\pi\alpha_s}{\vec{q}^{\,2}} \left[C_F + T_F n_l \sum_{L=1}^{\infty} \bar{\Pi}_L \left(C_F \frac{\alpha_s}{4\pi} \right)^L \right] + (\text{other color structures}) \\
&= -\frac{4\pi\alpha_s}{\vec{q}^{\,2}} C_F \left\{ 1 + T_F n_l \frac{\alpha_s}{4\pi} \left[-\frac{20}{9} + \left(16\zeta_3 - \frac{55}{3} \right) C_F \frac{\alpha_s}{4\pi} \right.\right. \\
&\quad - 2 \left(80\zeta_5 - \frac{148}{3}\zeta_3 - \frac{143}{9} \right) \left(C_F \frac{\alpha_s}{4\pi} \right)^2 \\
&\quad \left.\left. + \left(2240\zeta_7 - 1960\zeta_5 - 104\zeta_3 + \frac{31}{3} \right) \left(C_F \frac{\alpha_s}{4\pi} \right)^3 + \mathcal{O}(\alpha_s^4) \right] \right\} \\
&\quad + (\text{other color structures})\,, \qquad (4)
\end{aligned}
$$

where α_s is taken at $\mu = |\vec{q}|$. The terms up to α_s^4 agree with [8]. Comparing (4) with (3.11), we see that the $C_F^{L-1} T_F n_l$ color structures are absent in Δ to all orders in α_s. In particular, this explains the absence of C_F in the bracket in (2). This means that C_F^{L-1} terms are absent in $C(\alpha_s)$ (3) to all orders.

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JHEP01(2019)134