

Research Article

A Supply Chain-Logistics Super-Network Equilibrium Model for Urban Logistics Facility Network Optimization

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The logistics facility decisions may be the most critical and most difficult of the decisions needed to realize an efficient supply chain since these decisions have significant effects on the logistics costs generated in the logistics network. We establish a logistics super network equilibrium integrating urban logistics facilities with members of traditional supply chain network, using the variational inequality theory. This model takes into account the behavior of logistics facilities and the transactions between retailers and logistics facilities are examined in this paper. Furthermore, we obtain the equilibrium condition of the system, and the economic explanation and algorithm are given. Finally, some verification examples are provided to verify the solution and decision-making application.

1. Introduction

The logistics facility plays an important role in the development of urban logistics system, which is provided with storage, processing, transporting, distribution, and other functions. With urbanization the efforts on urban logistics facility have made to optimize urban network structure and improve social benefits.

A considerable number of researches have been done on the urban logistics facility problems during the early time. The facility location problems have been developed extensively and a variety of methods and modelling have been studied on them. Bookbinder and Reece [1] defined a two-tier multicommodity distribution system and established a nonlinear mixed integer programming model for facility location problem. Taniguchi [2] developed a location model of logistics center and heuristic algorithm. In addition, Li and Tang et al. [3] proposed a two-layer logistics network programming model to minimum the total logistics cost. Yan et al. [4] optimized the spatial distribution and scale of intermediate nodes in urban logistics network which consists of nodes and lines, using a location model and genetic algorithm. Also, Sankar Kumar Roy et al. [5] studied

a two-stage transportation problem with the goods collected in warehouses at one stage and then distributed in another stage.

In fact, the optimization of urban logistics facilities is important since it has significantly influence on land use and traffic condition [6]. The approaches mentioned above generally consider the cost from factories to customers through distribution centers and are developed to find one or more than one location with lowest cost. Like the “all or nothing” assignment in traffic planning, these methods are suitable for early logistics market or monopoly market.

Under the condition of the increasing shortage of the land resources in urban area, the logistics facility, which has advantages in location, also has bargaining power in the competitive supply chain networks, influenced by supply and demand factors. On the one hand, suppliers and retailers in the supply chain network compete with other decision-makers and then achieve equilibrium, the derived industry; urban logistics facilities can realize equilibrium in the market on the other hand. We provide the equilibrium theory to explain the urban logistics facility problems, which is proved to be an effective method to deal with a network problem with competition and corporation.

The equilibrium theory of supply chain has been widely applied in many fields such as economy and transportation and studied by many scholars at home and abroad. First Nagurney (2002) proposed a three layers supply chain network equilibrium model including manufacturers, retailers, and consumers. Later, the supply chain equilibrium network model has been developed to consider random demand [7] and electronic commerce [8]. Zhang and Liu et al. [9] analyzed the independent decision-making behavior and interaction of manufacturers, retailers, and decision makers in demand market, respectively, aiming at the supply chain network model with multiple commodity flows; Xu and Zhu [10] assumed that products have differences in origin and brand. They regarded the influence of origin and brand differences as random variables and used stochastic utility theory and polynomial logit model to study the random selection of products in the demand market. Besides, some scholars incorporated the traffic network and supply chain network into a super network model [11–13]. But beyond that, Turan Paksoy et al. [14] studied the trade-offs between various costs using a linear programming model in a closed-loop supply chain and a new mixed integer mathematical model for a CLSC network including forward and reverse flows with multiperiods and multicomponents is presented later [15].

The traditional supply chain network equilibrium model is based on commodity flow (information flow and capital flow). The commodity flow and logistics are essentially unified, interrelated, and restricted with each other. With the refinement of social division of labor and commercialized large production, the variety and quantity of goods used for exchange are increased, and the distance between production and consumption is expanding in time and space. In this case, the commodity flow can be completed by money media and information medium, while logistics need to be carried out by means of warehouse and transportation tools. This difference between the two kinds of flows makes separation of commodity flow and logistics has become a trend.

Within this context, we consider the decision behavior of logistics network and its interaction with supply chain members' decision based on supply chain network model established by Nagurney et al. [16, 17]. We extend the three-echelon supply chain with evolving urban logistics facilities, as shown in Figure 1. The main contributions of the paper are summarized as follows: (1) the relationship characteristics of logistics network are involved and a more practical logistics facility equilibrium model is established; (2) this model describes the behavior of the logistics facilities as decision-makers in the network, and logistics network nodes not only have the characteristics of spatial distribution but also have the characteristics of network based on commodity flow and material demand; (3) logistics activities are carried out well in this supply chain-logistics super network, including storage, transportation, and removal.

The paper is organized as follows. In Section 2, we give the analyzation and hypothesis of urban logistics facilities equilibrium problem. In Section 3, we present the urban logistics facilities equilibrium model as well as the supply chain network model and derive optimality conditions for the decision-makers. The variational inequality formulation

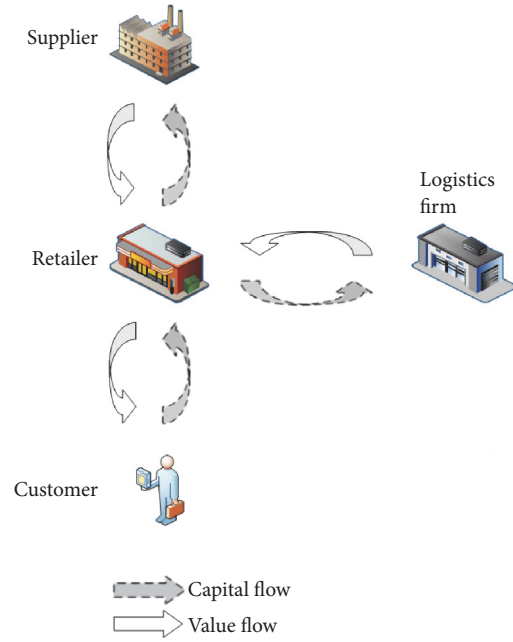


FIGURE 1: The supply chain integrated with urban logistics facility.

of this problem is presented in this section as well. Section 4 promotes an algorithm and then applies this algorithm to several numerical examples and the results of the model are discussed. Finally, we conclude the paper in Section 5.

2. Problem Analysis and Hypothesis

We assume that a large number of logistics facilities in the urban area and the retailers of the supply chain network reach a service transaction and cooperate with each other, and the competing facilities form a new hierarchy, logistics facility layer. The traditional three-layer supply chain network constitutes a business supply and demand layer. The above two layers constitute a multilevel urban supply chain logistics competition super network. Let $G = (N, E)$ express this network in which $N = I \cup J \cup K \cup L$ and E expresses the nodes connection of the network. We denote the set of suppliers by J , the set of retailers by K , the set of demand markets by L , and the set of logistics firms by I . We use i, j, k , and l to denote a typical logistic firm, a typical supplier, a typical retailer, and a typical customer, respectively, such that $i = 1, 2, \dots, I$, $j = 1, 2, \dots, J$, $k = 1, 2, \dots, K$, $l = 1, 2, \dots, L$. There is a direct competition relationship among the members of each node set, which is manifested as the struggle or utilization of limited resources, as shown in Figure 2. We consider that the products supplied by suppliers, serviced by logistics firms and sold to customers by retailers, are homogeneous and denoted by h , $h = 1, 2, \dots, H$.

In this mode, the retailers put the demanding goods in the warehouse of the logistics facilities, and the logistics facilities can get the service instructions of the retailers based on the information sharing. The operation decision of the retailers mainly includes sample display, promotion, and transaction;

TABLE 1: Notation for logistics facilities.

Notation	Definition
q_i^h	The nonnegative production storage of product h by logistics firm i
q^1	ih -dimensional column vector of the production storages with component i denoted by q_i^1
q_{ik}^h	The amount of product h transacted between logistics firm i and retailer k
Q^1	ikh -dimensional column vector of transaction for all products between all logistics firms and all retailers
$c_i^h(Q^1)$	Storing cost of logistics firm i for unit product h
p_{ik}^h	The price charged for product h by logistics firm i to retailer k
$d_{ik}^h(q_{ik}^h)$	Handling costs including stevedoring and managing costs between logistics firm i and retailer k for product h
$\rho_{ik}^h(Q^1)$	Delivery cost for product h associated with each logistics firm i and retailer k

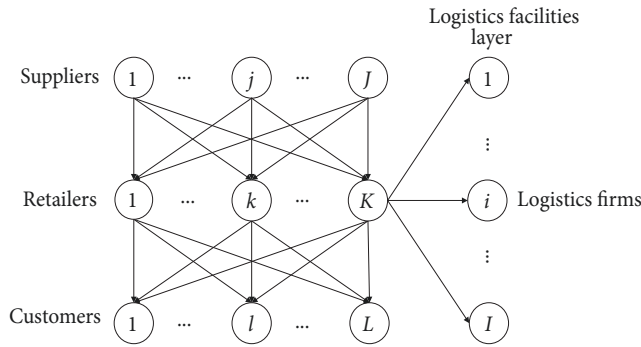


FIGURE 2: The structure of urban logistics supply chain network.

that is to say, the orders are required to be delivered to the designated place by upper level, and the retailers pay the supplier for the purchase cost according to the actual sales.

There are several differences between this model in this paper and SCN model in Nagurney et al. [16, 17], which can be seen in Figure 2 and what mentioned above. As depicted in Figure 2, the model consists of four-tier SCNs and the logistics facilities layer involved implied that the activities of transporting and delivering products were considered. Also, the shipments between layers are various different products which are distinguished from Nagurney et al. [16, 17].

Likely to the actual situation, we assume that there is a noncooperative competition in the same level of urban supply chain logistics competition super network. All commodities in the market need to experience a certain storage cycle; that is, the goods should bear a more fixed inventory cost, and the time spent on distribution between fixed OD pairs has little fluctuation. For convenience, we do not consider the limitation of products supplication and the limitation of service capacity of storage facilities.

3. Urban Logistics Facilities Equilibrium Model

3.1. The Logistics Firms and Their Optimal Conditions. Storage facilities layer is a collection of warehouses, logistics center, and distribution center, whose main task is the integration of input goods and transporting them to destination and storage for goods that may not be delivered in a short time.

Let q_i^h denote the nonnegative production storage of product h by logistics firm i , q_i^1 denote the nonnegative production storage of products by logistics firm i and group the production storages of all logistics firms into the column vector q^1 . The amount of product transacted between logistics firm i and retailer k denoted by q_{ik}^h . We group the amount of transaction for all products between all logistics firms and all retailers into the ikh -dimensional column vector Q^1 . We assume that each logistics firm i is faced with a storing cost function c_i^h for unit product h , which can depend on the entire vector of production storage; that is, $c_i^h = c_i^h(Q^1)$. Let p_{ik}^h denote the price charged for product h by logistics firm i to retailer k . This price means that the retailer should pay the logistics firm for the storage of the transacted products. We let d_{ik}^h denote the handling costs between logistics firm i and retailer k for product h , which mainly contain stevedoring and managing charges of the shipments between retailer k and logistics firm i , and $d_{ik}^h = d_{ik}^h(q_{ik}^h)$. The delivery cost for product h associated with each logistics firm i and retailer k is denoted by ρ_{ik}^h , which depends upon the total volume of production storage; that is, $\rho_{ik}^h = \rho_{ik}^h(Q^1)$. For succinctness, we present the notation for logistics facilities in this model in Table 1.

The quantities stored by logistics firm i are equal to the sum of the quantities transacted between logistics firm i and all retailers; that is,

$$q_i^h = \sum_k^K q_{ik}^h \quad (1)$$

Based on the above-mentioned conditions, we assume that the logistics firms compete in a noncooperative fashion and we can express the criterion of profit maximization for logistics firm i as

$$\begin{aligned} \max \quad & \sum_k^K \sum_h^H p_{ik}^h q_{ik}^h - \sum_h^H c_i^h(Q^1) - \sum_k^K \sum_h^H d_{ik}^h(q_{ik}^h) \\ & - \sum_k^K \sum_h^H \rho_{ik}^h(Q^1) \end{aligned} \quad (2)$$

Formula (2) indicates that each logistics firm seeks to maximize its profit which is equal to the difference between the total storage income and all storage costs, handling costs,

and delivery costs. Storing cost refers to the cost of storage activities to meet the needs of customers. In addition to the quantity of warehouse stock, the storing cost involves the average storage cycle of goods reflecting the processing capacity and efficiency of the storage facilities that can be determined by the statistical analysis of the average inventory period of various goods over the years. The average storage cycle of different types of storage facilities for the same kind of goods is different. The storage cost per unit storage time and quantity is considered to be fixed in a certain period, which can be expressed as follows:

$$c_i^h(Q^1) = \vartheta_i^h(Q^1) T_i^h q_i^h \quad (3)$$

where $\vartheta_i^h(Q^1)$ denotes the storage cost per unit storage time and quantity for product h of logistics firm i ; T_i^h denotes the average storage cycle for product h of logistics firm i .

The handing cost $d_{ik}^h(q_{ik}^h)$ is related to the equipment conditions and management level. We suppose that the delivery cost which refers to the cost vehicles spent on the delivery of goods to customers from facilities to destinations is a function of distributions. The delivery cost depends on the travel time and relates to the different locations and lines (or distribution paths, vehicle type, traffic congestion, unit freight rates, etc.). The average travel speed between OD can be estimated through historical operation data. The delivery cost can be expressed as

$$\rho_{ik}^h(Q^1) = \omega_{ik}^h(Q^1) \frac{L_{ik}^h q_{ik}^h}{\phi \varsigma v_{ik}^h} \quad (4)$$

where $\omega_{ik}^h(Q^1)$ denotes the delivery cost per unit quantity and time; L_{ik}^h denotes the distance from logistics firm i to the destination; ϕ denotes the average utilization coefficient of loading capacity; ς denotes the rated loading capacity (maximum allowable loading capacity of vehicles under safe driving conditions) of distribution vehicles; and v_{ik}^h denotes the average speed of vehicles from warehouse provider i to the destination.

The optimization problem (2) is equal to the following equivalent form:

$$\begin{aligned} \max \sum_k^K \sum_h^H p_{ik}^h q_{ik}^h - \sum_h^H \vartheta_i^h(Q^1) T_i^h q_i^h - \sum_k^K \sum_h^H d_{ik}^h(q_{ik}^h) \\ - \sum_k^K \sum_h^H \omega_{ik}^h(Q^1) \frac{L_{ik}^h q_{ik}^h}{\phi \varsigma v_{ik}^h} \end{aligned} \quad (5)$$

The cost functions $\vartheta_i^h(Q^1)$, $d_{ik}^h(q_{ik}^h)$ and $\omega_{ik}^h(Q^1)$ for each logistics firm are regarded as continuous and convex (the same with $c_i^h(Q^1)$ and $\rho_{ik}^h(Q^1)$). Given that the governing optimization/equilibrium concept underlying noncooperative behavior is that of Cournot (1838) and Nash (1950, 1951), which states that each logistics firm will determine his optimal production quantity and shipments, the given optimal ones of the competitors and the optimality conditions for all logistics firms simultaneously can be expressed as the following variational inequality:

$$\sum_i^I \sum_k^K \sum_h^H \left\{ T_i^h \vartheta_i^h(Q^{1*}) + T_i^h q_i^h \frac{\partial \vartheta_i^h(Q^{1*})}{\partial q_{ik}^h} + \frac{\partial d_{ik}^h(q_{ik}^{h*})}{\partial q_{ik}^h} + \frac{L_{ik}^h}{\phi \varsigma v_{ik}^h} \left[\omega_{ik}^h(Q^{1*}) + \partial q_{ik}^h \frac{\partial \omega_{ik}^h(Q^{1*})}{\partial q_{ik}^h} \right] - p_{ik}^{h*} \right\} \times (q_{ik}^h - q_{ik}^{h*}) \geq 0 \quad (6)$$

$$\forall (Q^1) \in R_+^{IKH}$$

The optimality conditions expressed by (6) have a nice economic interpretation, which is that a logistics firm will accept the logistics business when the marginal storage cost plus the marginal handling cost and delivery cost is equal to the service prices paid to the logistics firm. If the logistics firm's marginal storage, handling, and delivery costs exceed what the retailer is willing to pay for the service, then the flow on the link will be zero.

3.2. The Suppliers and Their Optimality Conditions. Let q_{jk}^h denote the amount of product h shipped (or transacted) between supplier j and retailer k . We group the shipments between the suppliers and the retailers for all products into the jkh -dimensional column vector Q^2 . The supply cost function for each supplier is denoted by f_j^h , which depends on the entire vector of supplies; that is, $f_j^h = f_j^h(Q^2)$. Let p_{jk}^h denote the price charged for product h by supplier j to retailer k . We associate with each supplier and retailer pair (j, k) a

transaction cost denoted by g_{jk}^h , $g_{jk}^h = g_{jk}^h(Q^2)$. Also, we summarize the notation for suppliers in this model in Table 2.

Due to the coordination of supply and marketing for supplier j , we have

$$q_j^h = \sum_k^K q_{jk}^h \quad (7)$$

We can express the criterion of profit maximization for supplier j as

$$\max \sum_k^K \sum_h^H p_{jk}^h q_{jk}^h - \sum_h^H f_j^h(Q^2) - \sum_k^K \sum_h^H g_{jk}^h(Q^2) \quad (8)$$

Assuming that the supply and transaction cost for each supplier is continuous and convex, the optimality conditions for all the suppliers coincide with the solution of the variational inequality:

TABLE 2: Notation for suppliers.

Notation	Definition
q_{jk}^h	The amount of product h shipped (or transacted) between supplier j and retailer k
Q^2	jkh - dimensional column vector of all products between the suppliers and the retailers
$f_j^h(Q^2)$	Supply cost of supplier j for product h
p_{jk}^h	The price charged for product h by supplier j to retailer k
$g_{jk}^h(Q^2)$	Transaction cost between supplier j and retailer k

TABLE 3: Notation for retailers.

Notation	Definition
q_{kl}^h	The shipments for product h between retailer k and customer l
Q^3	klh - dimensional column vector of product shipments between the suppliers and the retailers
p_{kl}^h	The price charged for product h by retailer k to customer l
$v_k^h(Q^3)$	The display and storage cost associated with the product of retailer k

$$\sum_j \sum_k \sum_h \left[\frac{\partial f_j^h(Q^{2*})}{\partial q_{jk}^h} + \frac{\partial g_{jk}^h(Q^{2*})}{\partial q_{jk}^h} - p_{jk}^{h*} \right] \times (q_{jk}^h - q_{jk}^{h*}) \geq 0 \quad \forall Q^2 \in R_+^{IKH} \quad (9)$$

Formula (9) indicates that a supplier will ship a positive amount of the product to a retailer if the price that the retailer is willing to pay for the product is precisely equal to the supplier's supply and transaction costs associated with that retailer. If the supplier's supply and transaction costs exceed what the retailer is willing to pay for the product, then the flow on the link will be zero.

3.3. The Retailers and Their Optimality Conditions. The retailers purchase storing service from the logistics firms and pay the suppliers for shipments and then sell goods to demand markets. Let q_{kl}^h denote the shipments for product h between retailer k and customer l . We group the shipments between the suppliers and the retailers for all products into the klh -dimensional column vector Q^3 . Let p_{kl}^h denote the price charged for product h by retailer k to customer l . We denote an operating cost for a retailer by v_k^h , which may include, for example, the display and storage cost associated with the product. Suppose that this cost is a function of the shipments and depends on the amounts of production held by other retailers, that is, $v_k^h = v_k^h(Q^3)$. All the notations for retailers in this model are presented in Table 3.

We can express the criterion of profit maximization for retailer k as

$$\max \sum_l \sum_h p_{kl}^h q_{kl}^h - \sum_k \sum_h p_{jk}^h q_{jk}^h - \sum_k \sum_h p_{ik}^h q_{ik}^h - \sum_h v_k^h(Q^3)$$

$$\text{s.t.} \quad \sum_j q_{jk}^h \geq \sum_l q_{kl}^h$$

$$\sum_j q_{jk}^h \geq \sum_i q_{ik}^h$$

$$\sum_i q_{ik}^h \geq \sum_l q_{kl}^h$$

$$q_{ik}^h, q_{jk}^h, q_{kl}^h \geq 0 \quad (10)$$

We assume here all cost functions for each retailer are continuous and convex; this assumption is fully backed by experience in articles (see, e.g., Nagurney, 2002, [13, 18]) to guarantee an ideal solution. The optimality conditions for all the retailers coincide with the solution of the variational inequality:

$$\begin{aligned} & \sum_i \sum_k \sum_h (p_{ik}^{h*} - \pi_k^{h*} + \sigma_k^{h*}) \times (q_{ik}^h - q_{ik}^{h*}) \\ & + \sum_j \sum_k \sum_h (p_{jk}^{h*} - u_k^{h*} - \sigma_k^{h*}) \times (q_{jk}^h - q_{jk}^{h*}) \\ & + \sum_k \sum_l \sum_h \left(u_k^{h*} + \frac{\partial v_k^h(Q^{3*})}{\partial q_{kl}^h} + \pi_k^{h*} - p_{kl}^{h*} \right) \\ & \times (q_{kl}^h - q_{kl}^{h*}) + \sum_k \sum_h \left(\sum_j q_{jk}^{h*} - \sum_l q_{kl}^{h*} \right) \\ & \times (u_k^h - u_k^{h*}) + \sum_k \sum_h \left(\sum_i q_{ik}^{h*} - \sum_l q_{kl}^{h*} \right) \end{aligned}$$

TABLE 4: Notation for customers.

Notation	Definition
$c_{kl}^h(Q^3)$	transaction cost for customer l buying product h from retailer k
p	lh -dimensional column vector of demand market prices with component lh denoted by p_l^h
$z_l^h(p)$	demand function of customer l for product h

$$\begin{aligned}
& \times (\pi_k^h - \pi_k^{h*}) + \sum_k \sum_h \left(\sum_j q_{jk}^{h*} - \sum_i q_{ik}^{h*} \right) \\
& \times (\sigma_k^h - \sigma_k^{h*}) \geq 0 \\
& \forall (Q^1, Q^2, Q^3, u, \pi, \sigma) \in R_+^{IKH+JKH+KLH+KH+KH+KH}
\end{aligned} \tag{11}$$

where the terms $u_k^h, \pi_k^h, \sigma_k^h$ are the Lagrange multipliers associated with constraint (11) for retailer k .

Inequality (11) indicates that a retailer will choose storing service when the Lagrange multiplier π_k^{h*} is equal to the sum of the store transaction price of a retailer and the Lagrange multiplier σ_k^{h*} ; a retailer will transact with a supplier if the price p_{jk}^{h*} is precisely equal to the Lagrange multiplier u_k^{h*} plus σ_k^{h*} ; a retailer will sell goods to the customers when the price charged by retailer k is equal to the retailer's Lagrange multiplier u_k^{h*} , Lagrange multiplier π_k^{h*} , and marginal operation cost.

3.4. The Customers and Their Optimality Conditions. In demand market, consumers choose different brands which can replace each other according to their own subjective preferences and consumption abilities. Therefore, consumers' demand is considered as a random variable. We denote the transaction cost for customer l buying product h from retailer k by $c_{kl}^h = c_{kl}^h(Q^3)$. Let p_l^h denote the price of the product at demand market l and p is the lh -dimensional column vector of demand market prices. We assume that z_l^h is the demand of customer l for product h , where z_l^h is influenced by the demand market prices p and $z_l^h = z_l^h(p)$. Then, the notation for customers in this model is shown in Table 4.

The equilibrium conditions of customers at demand market l take the following form: for all retailers $k, k = 1, 2, \dots, K$

$$p_{kl}^{h*} + c_{kl}^h(Q^{3*}) \begin{cases} = p_l^{h*}, & q_{kl}^h > 0 \\ \geq p_l^{h*}, & q_{kl}^h = 0 \end{cases} \tag{12}$$

$$z_l^h(p) \begin{cases} = \sum_k q_{kl}^h, & p_l^h > 0 \\ \leq \sum_k q_{kl}^h, & p_l^h = 0 \end{cases} \tag{13}$$

The equilibrium conditions are equivalent to the following variational inequality problem:

$$\begin{aligned}
& \sum_k \sum_l \sum_h [p_{kl}^{h*} + c_{kl}^h(Q^{3*}) - p_l^{h*}] \times (q_{kl}^h - q_{kl}^{h*}) \\
& + \sum_l \sum_h \left[\sum_k q_{kl}^{h*} - z_l^h(p) \right] \times (p_l^h - p_l^{h*}) \geq 0 \\
& \forall (Q^3, p) \in R_+^{KLH+LH}
\end{aligned} \tag{14}$$

3.5. Urban Logistics Facilities Equilibrium Model. In equilibrium, the outflow between the two adjacent layers is consistent with the inflow. Furthermore, the equilibrium shipment and price pattern in the supply chain must satisfy the sum of inequalities (6), (9), (11), and (14), in order to formalize the agreements between the tiers. We now state this explicitly in the following definition.

The equilibrium conditions governing the urban logistics facilities equilibrium model with competition are equivalent to the solution of the variational inequality problem given by: determine $(Q^1, Q^2, Q^3, u^*, \pi^*, \sigma^*, p^*) \in \Omega$ satisfying

$$\begin{aligned}
& \sum_i \sum_k \sum_h \left\{ T_i^h g_i^h(Q^{1*}) + T_i^h q_i^h \frac{\partial \theta_i^h(Q^{1*})}{\partial q_{ik}^h} + \frac{\partial d_{ik}^h(q_{ik}^{h*})}{\partial q_{ik}^h} + \frac{L_{ik}^h}{\phi \varsigma \nu_{ik}^h} \left[\omega_{ik}^h(Q^{1*}) + \partial q_{ik}^h \frac{\partial \omega_{ik}^h(Q^{1*})}{\partial q_{ik}^h} \right] - \pi_k^{h*} + \sigma_k^{h*} \right\} \times (q_{ik}^h \\
& - q_{ik}^{h*}) + \sum_j \sum_k \sum_h \left[\frac{\partial f_j^h(Q^{2*})}{\partial q_{jk}^h} + \frac{\partial g_{jk}^h(Q^{2*})}{\partial q_{jk}^h} - u_k^{h*} - \sigma_k^{h*} \right] \times (q_{jk}^h - q_{jk}^{h*}) \\
& + \sum_k \sum_l \sum_h \left[u_k^{h*} + \frac{\partial v_k^h(Q^{3*})}{\partial q_{kl}^h} + \pi_k^{h*} + c_{kl}^h(Q^{3*}) - p_l^{h*} \right] \times (q_{kl}^h - q_{kl}^{h*}) + \sum_k \sum_h \left(\sum_j q_{jk}^{h*} - \sum_l q_{kl}^{h*} \right) \times (u_k^h - u_k^{h*}) \\
& + \sum_k \sum_h \left(\sum_i q_{ik}^{h*} - \sum_l q_{kl}^{h*} \right) \times (\pi_k^h - \pi_k^{h*}) + \sum_k \sum_h \left(\sum_j q_{jk}^{h*} - \sum_i q_{ik}^{h*} \right) \times (\sigma_k^h - \sigma_k^{h*}) + \sum_l \sum_h \left[\sum_k q_{kl}^{h*} - z_l^h(p) \right] \times (p_l^h \\
& - p_l^{h*}) \geq 0 \quad \forall (Q^1, Q^2, Q^3, u, \pi, \sigma, p) \in \Omega
\end{aligned} \tag{15}$$

Proof. This proof is similar to that established by Nagurney et al. (2002a). The summation of (6), (9), (11), and (14) yields, after algebraic simplification and elimination, variational inequality (15). Then, we should prove the converse that the solution to (15) satisfies the sum of (6), (9), (11), and (14). To

variational inequality (15) add the term $\sum_i^I \sum_k^K \sum_h^H (-p_{ik}^{h*} + p_{ik}^{h*})(q_{ik}^h - q_{ik}^{h*})$, $\sum_j^J \sum_k^K \sum_h^H (-p_{jk}^{h*} + p_{jk}^{h*})(q_{jk}^h - q_{jk}^{h*})$ and $\sum_k^K \sum_l^L \sum_h^H (-p_{kl}^{h*} + p_{kl}^{h*})(q_{kl}^h - q_{kl}^{h*})$, and the resulting inequality as follows:

$$\begin{aligned}
& \sum_i^I \sum_k^K \sum_h^H \left\{ T_i^h \vartheta_i^h(Q^{1*}) + T_i^h d_i^h \frac{\partial \vartheta_i^h(Q^{1*})}{\partial q_{ik}^h} + \frac{\partial d_{ik}^h(q_{ik}^{h*})}{\partial q_{ik}^h} + \frac{L_{ik}^h}{\phi \zeta v_{ik}^h} \left[\omega_{ik}^h(Q^{1*}) + \partial d_{ik}^h \frac{\partial \omega_{ik}^h(Q^{1*})}{\partial q_{ik}^h} \right] - \pi_k^{h*} + \sigma_k^{h*} \right\} \times (q_{ik}^h - q_{ik}^{h*}) \\
& - d_{ik}^{h*} + \sum_j^J \sum_k^K \sum_h^H \left[\frac{\partial f_j^h(Q^{2*})}{\partial q_{jk}^h} + \frac{\partial g_{jk}^h(Q^{2*})}{\partial q_{jk}^h} - u_k^{h*} - \sigma_k^{h*} \right] \times (q_{jk}^h - q_{jk}^{h*}) \\
& + \sum_k^K \sum_l^L \sum_h^H \left[u_k^{h*} + \frac{\partial v_k^h(Q^{3*})}{\partial q_{kl}^h} + \pi_k^{h*} + c_{kl}^h(Q^{3*}) - p_l^{h*} \right] \times (q_{kl}^h - q_{kl}^{h*}) + \sum_k^K \sum_h^H \left(\sum_j^J q_{jk}^{h*} - \sum_l^L q_{kl}^{h*} \right) \times (u_k^h - u_k^{h*}) \\
& + \sum_k^K \sum_h^H \left(\sum_i^I q_{ik}^{h*} - \sum_l^L q_{kl}^{h*} \right) \times (\pi_k^h - \pi_k^{h*}) + \sum_k^K \sum_h^H \left(\sum_j^J q_{jk}^{h*} - \sum_i^I q_{ik}^{h*} \right) \times (\sigma_k^h - \sigma_k^{h*}) + \sum_l^L \sum_h^H \left[\sum_k^K q_{kl}^{h*} - z_l^h(p) \right] \times (p_l^h - p_l^{h*}) \\
& - p_l^{h*} + \sum_i^I \sum_k^K \sum_h^H (-p_{ik}^{h*} + p_{ik}^{h*})(q_{ik}^h - q_{ik}^{h*}) + \sum_j^J \sum_k^K \sum_h^H (-p_{jk}^{h*} + p_{jk}^{h*})(q_{jk}^h - q_{jk}^{h*}) + \sum_k^K \sum_l^L \sum_h^H (-p_{kl}^{h*} + p_{kl}^{h*})(q_{kl}^h - q_{kl}^{h*}) \\
& - q_{kl}^{h*} \geq 0 \quad \forall (Q^1, Q^2, Q^3, u, \pi, \sigma, p) \in \Omega
\end{aligned} \tag{16}$$

Inequality (16) is equivalent to the price and shipments satisfying the sum of (6), (9), (11), and (14). The proof is completed. \square

4. The Algorithm and Numerical Examples

4.1. Modified Projection Method. In this section, an algorithm, the modified projection method, is presented which can solve any variational inequality problem in standard form. Please refer to literature [19, 20] for details about this method. The algorithm is guaranteed to converge provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). The statement of the modified projection method is as follows, where n denotes an iteration counter.

Step 0 (initialization). Set $X^0 \in \Omega$, let $n = 1$, and let α be a scalar such that $0 < \alpha \leq 1/L$, where L is the Lipschitz continuity constant (cf. [21]).

Step 1 (construction and computation). Compute $\bar{X}^n \in \Omega$ by solving the variational inequality subproblem:

$$(\bar{X}^n + \alpha F(X^{n-1}) - X^{n-1})^T (X - \bar{X}^n) \geq 0$$

Step 2 (adaptation). Compute $X^n \in \Omega$ by solving the variational inequality subproblem:

$$(X^n + \alpha F(\bar{X}^n) - X^{n-1})^T (X - X^n) \geq 0$$

Step 3 (convergence verification). If $\|X^n - X^{n-1}\| \leq \varepsilon$, with $\varepsilon > 0$, a prespecified tolerance, then stop; else, set $n = n + 1$

and go to Step 1. As long as the function F is monotone and Lipschitz is continuous, the algorithm converges.

4.2. Numerical Examples. In order to verify the model, we apply the modified projection method to several examples. The algorithm is implemented in MATLAB and the parameter α was set 0.005 and the Precision parameter was set 0.001 for all examples.

4.2.1. Problem Definition and the Base Case. The first numerical network consists of two logistics firms, two suppliers, two retailers, and two demand markets with two kinds of products. The parameters of logistics firms were set as follows: $T_i^h = 6$, $L_{ik}^h = 50$, $v_{ik}^h = 40$, $\phi = 8$, $\zeta = 0.5$.

The storage cost functions for all logistics firms were given by

$$\begin{aligned}
\vartheta_1^1 &= 0.25q_1^1 + 0.6q_2^1 + 0.4 \\
\vartheta_1^2 &= 0.25q_1^2 + 0.6q_2^2 + 0.4 \\
\vartheta_2^1 &= 0.5q_2^1 + 0.6q_1^1 + 0.4 \\
\vartheta_2^2 &= 0.5q_2^2 + 0.6q_1^2 + 0.4
\end{aligned} \tag{17}$$

The handing cost functions and the delivery cost functions between the logistics firms and retailers, respectively, were given by

$$\begin{aligned}
d_{11}^1 &= 0.4(q_{11}^1)^2 + 0.4q_{11}^1 \\
d_{11}^2 &= 0.4(q_{11}^2)^2 + 0.4q_{11}^2
\end{aligned}$$

$$\begin{aligned}
d_{12}^1 &= 0.4 (q_{12}^1)^2 + 0.4q_{12}^1 \\
d_{12}^2 &= 0.4 (q_{12}^2)^2 + 0.4q_{12}^2 \\
d_{21}^1 &= 0.4 (q_{21}^1)^2 + 0.4q_{21}^1 \\
d_{21}^2 &= 0.4 (q_{21}^2)^2 + 0.4q_{21}^2 \\
d_{22}^1 &= 0.4 (q_{22}^1)^2 + 0.4q_{22}^1 \\
d_{22}^2 &= 0.4 (q_{22}^2)^2 + 0.4q_{22}^2 \\
\omega_{11}^1 &= 2.5q_{11}^1 + 2.5q_{11}^1 \\
\omega_{11}^2 &= 2.5q_{11}^2 + 2.5q_{11}^2 \\
\omega_{12}^1 &= 2.5q_{12}^1 + 2.5q_{12}^1 \\
\omega_{12}^2 &= 2.5q_{12}^2 + 2.5q_{12}^2 \\
\omega_{21}^1 &= 10q_{21}^1 + 4q_{11}^1 \\
\omega_{21}^2 &= 10q_{21}^2 + 4q_{11}^2 \\
\omega_{22}^1 &= 10q_{22}^1 + 4q_{12}^1 \\
\omega_{22}^2 &= 10q_{22}^2 + 4q_{12}^2
\end{aligned}$$

(18)

The supply cost functions for all suppliers were given by

$$\begin{aligned}
f_1^1 &= 2.5 (q_1^1)^2 + 3q_1^1 q_2^1 + 2q_1^1 \\
f_1^2 &= 2.5 (q_1^2)^2 + 3q_1^2 q_2^2 + 2q_1^2 \\
f_2^1 &= 2.5 (q_2^1)^2 + 3q_2^1 q_1^1 + 2q_2^1 \\
f_2^2 &= 5 (q_2^2)^2 + 4q_2^2 q_1^2 + 2q_2^2
\end{aligned}$$

(19)

The transaction cost functions faced by the suppliers and associated with transacting with the retailers were given by

$$\begin{aligned}
g_{11}^1 &= (q_{11}^1)^2 + 2.5q_{11}^1 \\
g_{11}^2 &= (q_{11}^2)^2 + 2.5q_{11}^2 \\
g_{12}^1 &= (q_{12}^1)^2 + 2.5q_{12}^1 \\
g_{12}^2 &= (q_{12}^2)^2 + 2.5q_{12}^2 \\
g_{21}^1 &= (q_{21}^1)^2 + 2.5q_{21}^1 \\
g_{21}^2 &= (q_{21}^2)^2 + 2.5q_{21}^2 \\
g_{22}^1 &= (q_{22}^1)^2 + 2.5q_{22}^1 \\
g_{22}^2 &= (q_{22}^2)^2 + 2.5q_{22}^2
\end{aligned}$$

(20)

The operating cost functions $v_k^h = v_k^h(\sum_l q_{kl}^h)$ for all retailers were given by

$$\begin{aligned}
v_1^1 &= 2.5 (q_{11}^1 + q_{12}^1)^2 \\
v_1^2 &= 2.5 (q_{11}^2 + q_{12}^2)^2 \\
v_2^1 &= 2.5 (q_{21}^1 + q_{22}^1)^2 \\
v_2^2 &= 2.5 (q_{21}^2 + q_{22}^2)^2
\end{aligned}$$

(21)

The transaction cost functions faced by the retailers and associated with transacting with the customers were given by

$$\begin{aligned}
c_{11}^1 &= q_{11}^1 + 1 \\
c_{11}^2 &= q_{11}^2 + 1 \\
c_{12}^1 &= q_{12}^1 + 1 \\
c_{12}^2 &= q_{12}^2 + 1 \\
c_{21}^1 &= q_{21}^1 + 1 \\
c_{21}^2 &= q_{21}^2 + 1 \\
c_{22}^1 &= q_{22}^1 + 1 \\
c_{22}^2 &= q_{22}^2 + 1
\end{aligned}$$

(22)

The demands functions $z_i^h(p)$ of all customers were given by

$$\begin{aligned}
z_1^1 &= -2p_1^1 - 1.5p_2^1 + 1000 \\
z_1^2 &= -2p_1^2 - 1.5p_2^2 + 1000 \\
z_2^1 &= -2p_2^1 - 1.5p_1^1 + 1000 \\
z_2^2 &= -2p_2^2 - 1.5p_1^2 + 1000
\end{aligned}$$

(23)

The modified projection method converged in 1181 iterations and the equilibrium solutions are given in Tables 5–7. The demand prices at demand markets were $p_1^1 = p_2^1 = p_1^2 = p_2^2 = 282.67$.

Table 5 displays the production of transaction between two logistics firms and two retailers. The difference of the storage cost and delivery cost between the two firms, caused by geographic conditions or land factors, led to different results. The storage cost of logistics firm 2 is higher than that of logistics firm 1, and the storage quantities of logistics firm 2 for two products are zero while logistics firm 1 has competitive power which holds the market of two products almost. The main equilibrium results in Tables 6 and 7 have showed that the shipments adjusted by the equilibrium between nodes are the same since the cost and demand functions of a given network are partially symmetrical for the same commodity. The supplies for product 2 of supplier 2 are lower than that of supplier 1 since the supply cost of supplier 2 is higher.

TABLE 5: The production of transaction between logistics firms and retailers.

q_{ik}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
logistics firm 1	9.08	9.06	9.08	9.06
logistics firm 2	0	0	0	0

TABLE 6: The shipments between suppliers and retailers.

q_{jk}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
supplier 1	5.78	11.59	5.78	11.59
supplier 2	5.78	0	5.78	0

TABLE 7: The shipments between retailers and demand markets.

q_{kl}^h	demand market 1		demand market 2	
	product 1	product 2	product 1	product 2
retailer 1	5.31	5.30	5.31	5.30
retailer 2	5.31	5.30	5.31	5.30

4.2.2. *Addition of New Logistics Firm.* The second numerical network consists of three logistics firms, two suppliers, two retailers, and two demand markets with two kinds of products. The rest of the parameters are the same as example 1.

The storage cost functions for all logistics firms were given by

$$\begin{aligned}
 \vartheta_1^1 &= 0.25q_1^1 + 0.6(q_2^1 + q_3^1) + 0.4 \\
 \vartheta_1^2 &= 0.25q_1^2 + 0.6(q_2^2 + q_3^2) + 0.4 \\
 \vartheta_2^1 &= 0.5q_2^1 + 0.6(q_1^1 + q_3^1) + 0.4 \\
 \vartheta_2^2 &= 0.5q_2^2 + 0.6(q_1^2 + q_3^2) + 0.4 \\
 \vartheta_3^1 &= 0.25q_3^1 + 0.6(q_1^1 + q_2^1) + 0.4 \\
 \vartheta_3^2 &= 0.25q_3^2 + 0.6(q_1^2 + q_2^2) + 0.4
 \end{aligned} \tag{24}$$

The handing cost functions between the third logistics firms and the retailers, the same as the former in example 1, were given by

$$\begin{aligned}
 d_{31}^1 &= 0.4(q_{31}^1)^2 + 0.4q_{31}^1 \\
 d_{31}^2 &= 0.4(q_{31}^2)^2 + 0.4q_{31}^2 \\
 d_{32}^1 &= 0.4(q_{32}^1)^2 + 0.4q_{32}^1 \\
 d_{32}^2 &= 0.4(q_{32}^2)^2 + 0.4q_{32}^2
 \end{aligned} \tag{25}$$

The delivery cost functions between the logistics firms and retailers, respectively, were given by

$$\begin{aligned}
 \omega_{11}^1 &= 2.5q_{11}^1 + 2.5(q_{21}^1 + q_{31}^1) \\
 \omega_{11}^2 &= 2.5q_{11}^2 + 2.5(q_{21}^2 + q_{31}^2) \\
 \omega_{12}^1 &= 2.5q_{12}^1 + 2.5(q_{22}^1 + q_{32}^1) \\
 \omega_{12}^2 &= 2.5q_{12}^2 + 2.5(q_{22}^2 + q_{32}^2) \\
 \omega_{21}^1 &= 10q_{21}^1 + 4(q_{11}^1 + q_{31}^1) \\
 \omega_{21}^2 &= 10q_{21}^2 + 4(q_{11}^2 + q_{31}^2) \\
 \omega_{22}^1 &= 10q_{22}^1 + 4(q_{12}^1 + q_{32}^1) \\
 \omega_{22}^2 &= 10q_{22}^2 + 4(q_{12}^2 + q_{32}^2) \\
 \omega_{31}^1 &= 2.5q_{31}^1 + 2.5(q_{11}^1 + q_{21}^1) \\
 \omega_{31}^2 &= 2.5q_{31}^2 + 2.5(q_{11}^2 + q_{21}^2) \\
 \omega_{32}^1 &= 2.5q_{32}^1 + 2.5(q_{12}^1 + q_{22}^1) \\
 \omega_{32}^2 &= 2.5q_{32}^2 + 2.5(q_{12}^2 + q_{22}^2)
 \end{aligned} \tag{26}$$

The modified projection method converged in 1345 iterations and the production of transaction between the three logistics firms and the two retailers are given in Table 8.

Note that there is an additional logistics firm in the network and the competition increased. The results in Table 8 tell that although the storing cost of logistics firm 2 was higher

TABLE 8: The production of transaction between logistics firms and retailers.

q_{ik}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
logistics firm 1	0	0	0	0
logistics firm 2	7.27	7.26	7.27	7.26
logistics firm 3	0	0	0	0

TABLE 9: The production of transaction between logistics firms and retailers.

q_{ik}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
logistics firm 1	10.86	10.82	10.86	10.82
logistics firm 2	0	0	0	0

TABLE 10: The shipments between retailers and demand markets.

q_{kl}^h	demand market 1		demand market 2	
	product 1	product 2	product 1	product 2
retailer 1	6.16	6.13	6.16	6.13
retailer 2	6.16	6.13	6.16	6.13

TABLE 11: The production of transaction between logistics firms and retailers.

q_{ik}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
logistics firm 1	8.76	8.74	8.76	8.74
logistics firm 2	0	0	0	0

TABLE 12: The shipments between suppliers and retailers.

q_{jk}^h	retailer 1		retailer 2	
	product 1	product 2	product 1	product 2
supplier 1	5.62	11.28	5.62	11.28
supplier 2	5.62	0	5.62	0

than the others, it monopolized the market because of the entering of the new member. This means that a company with a high price is more competitive than other companies with lower prices because of the better quality of service.

4.2.3. Decline in Storage Circle of Logistics Facilities. Under the influence of resource sharing and informatization, when a storage facility improves the operate efficiency, the storage circle will decrease. Hence, we have investigated the effect of lowering the value of this parameter and we set the storage circle $T_i^h = 2$, on the basis of Example 1.

The modified projection method converged in 837 iterations and we obtained the changed solutions which are displayed in Tables 9 and 10. Comparing with the base case, the products' amounts this case increased obviously and the demands of customers has increased as well. In the increasingly competitive market environment, the reduce of the storage circle has become a trend in consequence of resource sharing and order processing informatization,

which also has a profound effect on the facilities' market share and social wealth.

4.2.4. Increase in the Delivery Distance. We considered the impact of the changes in distance between the facilities and customers on equilibrium solutions. Therefore, we changed the distance parameter L_{ik}^h to be 80, and the other parameters were the same as the base case.

The modified projection method converged in 1256 iterations and we got the consequences in Tables 11–13. The demand prices at demand markets were $p_1^1 = p_2^1 = 282.75$; $p_1^2 = p_2^2 = 282.76$.

The increase of the distance between the facilities and the customers means that the distribution distance becomes longer. Facility migration leads to changes in the cost of delivery of goods, resulting in a decrease in demand, an increase in the balanced price, and a decrease in the supply of goods. Such consequences would lead to a decrease in both

TABLE 13: The shipments between retailers and demand markets.

q_{kl}^h	demand market 1		demand market 2	
	product 1	product 2	product 1	product 2
retailer 1	5.16	5.15	5.16	5.15
retailer 2	5.16	5.15	5.16	5.15

business and customer satisfaction with the city. Therefore, it is necessary to open up the logistics channel, shorten the delivery time, improve operation efficiency, or decline the unit cost to compensate for the above problems caused by increased distance and urban expansion.

4.3. Discussion. The results of previous examples indicate that the modified projection method could solve the supply chain network problems effectively, and different parameters and network structures have different influences on equilibrium results.

We could find from the equilibrium results that supplies and sales are approximately equal after the adjustment of the equilibrium flow between nodes, because the parameters of the cost and demand function for same products in this network are symmetrical. Besides, storage quantities are lower than supplies as the hypothesis of the model. These results just verify the correctness and rationality of this model.

The results of the storage facilities layer have showed that a firm with better professional service and cost controlling ability will have stronger competitiveness and even monopolize the market. With the development of logistics industry, the number of the facilities has increased, and the efficiency has improved as well. We also noted that the movement of the location of logistics facilities influenced by urbanization, which has become a trend, is called logistics sprawl [22].

5. Conclusion

Logistics facilities' decision-making involves independent decisions of urban commodity supply and demand network, which affects the cost of all decision-makers in a network. In this paper, a hypernetwork competitive equilibrium model of urban logistics facilities supply chain is established by using variational inequalities. Prices connected with logistics facilities, suppliers, retailers, and consumers are endogenous, as well as are the product shipments. After studies we got the following contributions:

- The model is integrated comparing with the previous models; the main feature of this framework is that the interaction of consumers' behavior, suppliers' behavior, retailers' behavior, and logistics operators' behavior has been incorporated. And the equilibrium theory has been applied in the game behavior of logistics operators appropriately.
- We obtained the equilibrium conditions for the system and provided the economic explanation and solution algorithm. Finally, the verification solution and application of decision-making are provided with a specific example.

- Considering that these parameters, such as storage period and distribution distance, are often changed in reality, we carried on the relative example analysis and obtained the corresponding management enlightenment.

This article shows that storage facilities and supply chains interact with and restrict each other; facility efficiency improvement and site relocation may lead to regular changes in market structure, commodity flow and price, and social welfare. It is suggested that a scientific planning scheme for storage facilities should be carried out considering the relation of commodity supply and demand.

This paper still has a large space to expand the research, which needs further precision and improvement. Possible future research may include the following: the dynamic and time-lag of network lack characterization; the impact of different operation mechanisms on network equilibrium behavior is also worth exploring.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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