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Research Article

Finite-Time Nonfragile Dissipative Filter Design for Wireless Networked Systems with Sensor Failures

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In this study, the problem of finite-time nonfragile dissipative-based filter design for wireless sensor networks that is described by discrete-time systems with time-varying delay is investigated. Specifically, to reduce the energy consumption of wireless sensor networks, it is assumed that the signal is not transmitted at each instant and the transmission process is stochastic. By constructing a suitable Lyapunov-Krasovskii functional and employing discrete-time Jensen's inequality, a new set of sufficient conditions is established in terms of linear matrix inequalities such that the augmented filtering system is stochastically finite-time bounded with a prescribed dissipative performance level. Meanwhile, the desired dissipative-based filter gain matrices can be determined by solving an optimization problem. Finally, two numerical examples are provided to illustrate the effectiveness and the less conservatism of the proposed filter design technique.

1. Introduction

In the past few decades, wireless sensor networks (WSNs) have gained considerable attention due to their wide range of applications in various fields, such as mobile communications, target tracking, robotic systems, military, environmental sensing, and monitoring of traffic [1, 2]. WSNs normally consist of a large number of distributed nodes called sensor nodes, where the communication between the nodes is through radio signals. Since the sensors are battery powered, energy consumption is one of the main issues in WSNs. In recent years, different types of protocols have been proposed to reduce the energy consumption of the sensors in WSNs. For instance, in [3], the nonfragile randomly occurring filter gain variation problem is studied for a class of WSNs with energy constraint by using the Lyapunov technique and linear matrix inequality (LMI) approach. The multirate transmission protocols discussed in [4–7] are deterministic, since the transmission instant is pre-set which is not allowed to vary and this may lead to poor performance estimation. The authors in [8] considered not only the transmission rate of signals but also the successive nontransmissions, which leads to much conservatism.

In many practical problems, it is important to focus on the stability and filtering issue of a system over a prescribed time interval, in which the state trajectories remain within a predetermined bound over a given finite-time interval under some given initial conditions [9, 10]. Therefore, much attention has been given to the problem of finite-time filtering for dynamical systems with the use of Lyapunov technique and LMI approach [11–13]. By constructing a probability-dependent Lyapunov-Krasovskii functional, a set of sufficient conditions is established in [14] to obtain an energy-to-peak filter design for networked Markov switched singular systems over a finite-time interval. Wang et al. [15] studied the finite-time filter design problem of switched impulsive linear systems with parameter uncertainties and sensor induced faults by using the mode-dependent Lyapunov-like function approach. Sathishkumar et al. [16] developed a finite-time H_∞ filter design for a class of uncertain nonlinear discrete-time Markovian jump systems represented by Takagi-Sugeno fuzzy model with nonhomogeneous jump process. The asynchronous resilient controller design problem is investigated in [17], for a class of nonlinear switched systems with time delays and uncertainties in a given finite-time interval. The problems of finite-time stability and finite-time stabilisation for T-S fuzzy system with time-varying delay are investigated in [18]. The authors in [19] have designed a finite-time sliding mode controller for a class of conic-type nonlinear systems with time delays and mismatched external disturbances.

On another active research front, dissipativity theory was introduced by Willems in [20], which plays an important role in a wide range of fields, such as systems, circuits, and networks. Compared with passivity and H_∞ performance, dissipative system theory is a more general criterion and it is used for the analysis and the synthesis of dynamical control systems [21, 22]. Specifically, dissipativity means that the increase in energy storage in the system cannot exceed the energy supplied from outside the systems. Recently, few important results have been reported on dissipative-based filtering for various classes of time-varying delay systems. To mention a few, Feng and Lam [23] discussed the robust reliable dissipative filtering problem of uncertain discrete-time singular system with interval time-varying delay and sensor failures, where a set of conditions was derived in terms of LMIs which makes the filtering error singular system regular, causal, asymptotically stable, and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative. In [24], a set of sufficient conditions is developed by using reciprocally convex approach with Lyapunov technique for reliable dissipativity of Takagi-Sugeno fuzzy systems in the presence of time-varying delays and sensor failures. A new criterion of stability analysis for generalized neural networks subject to time-varying delayed signals is investigated in [25]. By employing the LMI approach, a new set of sufficient conditions is obtained in [26] for the existence of reliable dissipative filter which makes the filtering error system stochastically stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative.

On the other hand, perturbations often appear in the filter gain, which may cause instability in dynamic systems and usually lead to unsatisfactory performances. However, in practical problems, the presence of small uncertainties and inaccuracies during the implementation of filters may provide poor performance of the systems [27]. Therefore, it is important and necessary to design a filter that should be reliable and insensitive to some amount of gain fluctuations [28–30]. Xu et al. [31] studied the problem of passive control for fuzzy Markov jump systems with packet dropouts, where a nonfragile asynchronous controller is designed to guarantee that the closed-loop system is mean-square stable with a satisfactory passivity performance index. A novel method to address a proportional integral observer design for the actuator and sensor faults estimation based on Takagi-Sugeno fuzzy model with unmeasurable premise variables is presented in [32]. The nonfragile finite-time filtering problem is studied in [33] for a class of nonlinear Markovian jumping systems with time delays and uncertainties.

It is worth mentioning that, so far in the literature, only few works have been reported on finite-time filter design for wireless sensor networks. However, all the aforementioned works have not unified the external disturbances, time-varying delay, sensor failures, and filter gain variations, despite its practical importance. Motivated by the above, the reliable finite-time dissipative-based nonfragile filtering problem for discrete-time systems with time-varying delays and sensor failures has been investigated in the present study.

The main contributions of this paper are given as follows:

(i) Dissipative-based finite-time filter design problem is formulated for a class of WSNs with energy constraint and filter gain variations, which is represented by discrete-time systems with time-varying delay.

(ii) A reliable nonfragile filter is designed such that the augmented filtering system is stochastically finite-time bounded and dissipative. The proposed filter design includes H_∞ filter and passivity filter designs as special cases.

(iii) A set of sufficient conditions is developed in terms of LMIs to obtain the desired nonfragile filter design.

(iv) A unified filter design is proposed to deal with the external disturbances, time-varying delay, sensor failures, and filter gain variations, which makes the system more practical.

Finally, two numerical examples with simulation results are provided to demonstrate the effectiveness of the obtained results.

The brief outline of this paper is as follows. In Section 2, the problem of WSNs with time-varying delay and sensor faults is formulated, and some essential definitions and lemmas are given. The finite-time boundedness of the filtering error system is analyzed and a nonfragile reliable dissipative-based filter is designed in Section 3. Section 4 provides the simulation results to demonstrate the effectiveness of the obtained results. Some conclusions of this work are given in Section 5.

Notations. The following standard notations will be used throughout this paper. The superscript "T" stands for matrix transportation; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{E}\{\cdot\}$ represents the mathematical expectation; $l_2[0,\infty)$ stands for the space of n-dimensional square integrable functions over $[0,\infty)$; $A > 0$ $(A \geq 0)$ means that A is positive definite (positive-semidefinite); λ_P and λ^P denote the maximum and minimum eigenvalues of the matrix P, respectively; $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. Moreover, the notion $*$ used in matrix expressions represents a term that is induced by symmetry.

2. Problem Formulation

In this study, we consider a class of wireless sensor networks (WSNs), which can be described by the discrete-time system with time-varying delay in the following form:

$$x(k+1) = Ax(k) + A_d x(k - \tau(k)) + Bw(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $w(k) \in \mathbb{R}^q$ is the disturbance signal belonging to $l_2[0,\infty)$; $\tau(k)$ is the time-varying delay satisfying $\tau_1 \leq \tau(k) \leq \tau_2$, where $\tau_1 > 0$ and $\tau_2 > 1 + \tau_1$ are prescribed integers representing the lower and upper bounds of the delay, respectively; A, A_d, and B represent system coefficient matrices with appropriate dimensions. Supposing that there are m distributed sensors for the system (1), the measurement of the p-th sensor is given by

$$y_p(k) = C_p x(k) + D_p w(k), \quad p = 1, 2, \ldots, m, \tag{2}$$

where $y_p(k) \in \mathbb{R}^p$ is the observation collected by the p-th sensor; C_p and D_p are constant matrices with appropriate dimensions. Motivated by the results in [8], in order to save energy in WSNs, in this paper the measurement signal is assumed that it may not be transmitted to the remote filter at each instant. It is to be noted that the measurement signal is transmitted at least once over $N_p(> 0)$ time steps and the transmission can happen at any time in these N_p time steps. Let $y_p(k)$ denote the measurement signal sequence and $\tilde{y}_p(k)$ is the transmitted measurement. The above transmission protocol shows that there is no transmission at some time instants. In such situations, there is no input to the filter and the input to the filter has to be predefined by some rules. Thus, it is reasonable to assume that the filter may use the last transmitted measurement signal as its input [8]. Therefore, the input to the filter must be one member of the transmitted subset $\{y_p(k), y_p(k-1), \ldots, y_p(k - N_p - 1)\}$. Moreover, to reflect the random selection of filter input, a set of stochastic variables $\beta_{p,s}(k) \in \{0, 1\}$, $s = 0, 1, \ldots, N_p - 1$ is introduced such that $\beta_{p,s}(k) = 1$, if $y_p(k - s)$ is selected at time k as the filter input, and $\beta_{p,s}(k) = 0$, otherwise. Furthermore, it is assumed that the expectations of the stochastic variables are known, that is, $\mathbb{E}\{\beta_{p,s}(k) = 1\} = \overline{\beta}_{p,s}$, where $\overline{\beta}_{p,s}$ is the transmission probability and satisfies $\sum_{s=0}^{N_p-1} \overline{\beta}_{p,s} = 1$.

Based on the above transmission protocol, the filter input can further be expressed by

$$\begin{aligned} \overline{y}_p(k) = {} & \beta_{p,0}(k) y_p(k) + \beta_{p,1}(k) y_p(k-1) + \cdots \\ & + \beta_{p,N_p-1}(k) y_p\left(k - N_p + 1\right). \end{aligned} \tag{3}$$

Let $N_0 = \max\{N_1, N_2, \ldots, N_p\}$ and define $\overline{x}(k) = \left[x^T(k) \;\; x^T(k-1) \;\; \cdots \;\; x^T(k - N_0 + 1)\right]^T$, and $\overline{w}(k) = \left[w^T(k) \;\; w^T(k-1) \;\; \cdots \;\; w^T(k - N_0 + 1)\right]^T$. Now, by substituting (2) into (3), we can get

$$\overline{y}_p(k) = \sum_{s=0}^{N_p-1} \beta_{p,s}(k) \left\{C_p F_{p,s} \overline{x}(k) + D_p J_{p,s} \overline{w}(k)\right\}, \tag{4}$$

where $F_{p,s}$ and $J_{p,s}$, respectively, are $n \times nN_0$ and $q \times qN_0$ matrices containing an identity matrix at the $(s+1)$-th block and the rest of elements are zero.

Let $\beta_p(k) = [\beta_{p,0}(k), \beta_{p,1}(k), \ldots, \beta_{p,N_p-1}(k)]$, and $\beta_p(k) \in \{[1, \underbrace{0, 0, \ldots, 0}_{N-1}], \ldots, [\underbrace{0, 0, 0, \ldots, 0}_{N-1}, 1]\}$, then the possible realizations of $\beta_p(k)$ is N_p. Moreover, define $\beta(k) = [\beta_1(k), \beta_2(k), \ldots, \beta_m(k)]$, $\overline{F}_{\beta(k)} = [F^T_{1,\beta_1(k)} \;\; F^T_{2,\beta_2(k)} \;\; \cdots \;\; F^T_{m,\beta_m(k)}]^T$ and $\overline{J}_{\beta(k)} = [J^T_{1,\beta_1(k)} \;\; J^T_{2,\beta_2(k)} \;\; \cdots \;\; J^T_{m,\beta_m(k)}]^T$. Here, it is noted that the total number of possible realizations of $\beta(k)$ is $N = N_1 \times N_2 \times \cdots \times N_m$ and $\beta(k)$ could be viewed as the signal that specifies one particular case of $(\overline{F}_{\beta(k)}, \overline{J}_{\beta(k)})$. Again, introduce a new set of stochastic variables $\sigma_i(k) \in \{0, 1\}$, $i \in \{1, 2, \ldots, N\}$, which could be designed in such a way that if $\beta_p(k) = [1, 0, \ldots, 0]$ for $p = 1, 2, \ldots, m$, then $\sigma_1(k) = 1$, if $\beta_p(k) = [1, 0, \ldots, 0]$ for $p = 1, 2, \ldots, m-1$ and $\beta_m(k) = [0, 1, \ldots, 0]$, then $\sigma_2(k) = 1$, and so on. Therefore, at any time instant, there is only one realization of $\beta(k)$ such that $\sum_{i=1}^{N} \sigma_i(k) = 1$.

By using the probabilities of sensor transmissions $\overline{\beta}_{p,s}$, the probability $\mathbb{E}\{\sigma_i(k) = 1\} = \overline{\sigma}_i$ can be determined. For example, let us consider two sensors and assume that the measurement is transmitted within two time steps stochastically and their probabilities are $\overline{\beta}_{1,0}, \overline{\beta}_{1,1}$ and $\overline{\beta}_{2,0}, \overline{\beta}_{2,2}$, respectively. Now, using probability rules, it can be seen that $\overline{\sigma}_1 = \overline{\beta}_{1,0}\overline{\beta}_{2,0}$, $\overline{\sigma}_2 = \overline{\beta}_{1,0}\overline{\beta}_{2,1}$, $\overline{\sigma}_3 = \overline{\beta}_{1,1}\overline{\beta}_{2,0}$ and $\overline{\sigma}_4 = \overline{\beta}_{1,1}\overline{\beta}_{2,1}$. The main objective of this study is to design an appropriate reliable filter such that the considered WSNs (1) with sensor failures are stochastically finite-time bounded and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative. For this purpose, the sensor failure model in the following form is adopted in this paper: $\overline{y}^G(k) = G\overline{y}(k)$, where G is a diagonal matrix representing sensor fault range defined in the interval $0 \le G \le I$ and $\overline{y}(k)$ is the filter input vector received from sensor and is expressed as $\overline{y}(k) = \sum_{i=1}^{N} \sigma_i(k)\{\overline{C}\,\overline{F}_i \overline{x}(k) + \overline{D}\,\overline{J}_i \overline{w}(k)\}$ with $\overline{C} = \mathrm{diag}\{C_1, C_2, \ldots, C_m\}$, $\overline{D} = \mathrm{diag}\{D_1, D_2, \ldots, D_m\}$, and $\overline{F}_i$ and $\overline{J}_i$ are some appropriate matrices obtained from $\overline{F}_{\beta(k)}$ and $\overline{J}_{\beta(k)}$. On the other hand, define $z(k) = Lx(k)$, where $z(k) \in \mathbb{R}^m$ is the output signal to be estimated and L is a constant matrix with appropriate dimension.

Now, it is the right time to consider the filter equation consisting of gain fluctuations and sensor faults to be designed for the system (1) and that is given by

$$\begin{aligned} x_f(k+1) = {} & \left(A_f + \Delta A_f(k)\right) x_f(k) \\ & + \left(B_f + \Delta B_f(k)\right) \overline{y}^G(k), \\ z_f(k) = {} & C_f x_f(k), \end{aligned} \tag{5}$$

where $x_f(k) \in \mathbb{R}^n$ is the filter's state; $z_f(k) \in \mathbb{R}^m$ is the estimate of $z(k)$; A_f, B_f, and C_f are filter gain parameters to be determined later. Further, the matrices $\Delta A_f(k)$ and $\Delta B_f(k)$ represent the fluctuations in the filter gains and are assumed to satisfy the following structures:

$$\begin{aligned} \Delta A_f(k) &= M_a \Delta(k) N_a, \\ \Delta B_f(k) &= M_b \Delta(k) N_b, \end{aligned} \tag{6}$$

where M_a, M_b, N_a, and N_b are known constant matrices with appropriate dimensions; $\Delta(k)$ is an unknown time-varying matrix function satisfying $\Delta^T(k)\Delta(k) \le I$.

In order to derive the augmented filtering system, we rewrite the discrete-time system (1) and the output signal as follows:

$$\begin{aligned} \overline{x}(k+1) &= \overline{A}\overline{x}(k) + \overline{A}_d \overline{x}(k - \tau(k)) + \overline{B}\overline{w}(k), \\ z(k) &= \overline{L}\overline{x}(k), \end{aligned} \tag{7}$$

where $\overline{A}=\begin{bmatrix} A & 0 \\ I_{(N-1)\times(N-1)} & 0 \end{bmatrix}$, $\overline{B}=\begin{bmatrix} B & 0 \\ 0_{(N-1)\times 1} & 0 \end{bmatrix}$, $\overline{A}_d=\begin{bmatrix} A_d & 0 \\ I_{(N-1)\times(N-1)} & 0 \end{bmatrix}$, and $\overline{L}=\begin{bmatrix} L & 0_{1\times(N-1)} \end{bmatrix}$.

By defining a new augmented state vector as $\eta(k) = \begin{bmatrix} \overline{x}^T(k) & x_f^T(k) \end{bmatrix}^T$ and the estimation error as $e(k) = z(k) - z_f(k)$, the augmented filtering system and the corresponding error system can be formulated as

$$\eta(k+1) = \widetilde{A}\eta(k) + \widetilde{A}_d\eta(k-\tau(k)) + \widetilde{B}\overline{w}(k) + \sum_{i=1}^{N}(\sigma_i(k)-\overline{\sigma}_i)\left[\widehat{A}_i\eta(k) + \widehat{B}_i\overline{w}(k)\right], \tag{8}$$

$$e(k) = \widetilde{L}\eta(k), \tag{9}$$

where

$$\begin{aligned}
\widetilde{A} &= \begin{bmatrix} \overline{A} & 0 \\ \left(B_f+\Delta B_f(k)\right)G\overline{C}\,\overline{F} & A_f+\Delta A_f(k) \end{bmatrix}, \\
\widetilde{A}_d &= \begin{bmatrix} \overline{A}_d & 0 \\ 0 & 0 \end{bmatrix}, \\
\widetilde{B} &= \begin{bmatrix} \overline{B} \\ \left(B_f+\Delta B_f(k)\right)G\overline{D}\,\overline{J} \end{bmatrix}, \\
\widetilde{L} &= \begin{bmatrix} \overline{L} & -C_f \end{bmatrix}, \\
\widehat{A}_i &= \begin{bmatrix} 0 & 0 \\ \left(B_f+\Delta B_f(k)\right)G\overline{C}\,\overline{F}_i & 0 \end{bmatrix}, \\
\widehat{B}_i &= \begin{bmatrix} 0 \\ \left(B_f+\Delta B_f(k)\right)G\overline{D}\,\overline{J}_i \end{bmatrix}, \\
\overline{F} &= \sum_{i=1}^{N}\overline{\sigma}_i\overline{F}_i, \\
\overline{J} &= \sum_{i=1}^{N}\overline{\sigma}_i\overline{J}_i.
\end{aligned} \tag{10}$$

In order to derive the main results in the forthcoming section, we need the following assumption, definitions, and lemmas.

Assumption 1. The disturbance input vector $w(k)$ is time-varying and satisfies $\sum_{k=0}^{\mathcal{N}}\overline{w}^T(k)\overline{w}(k) \le \delta$, where $\delta > 0$.

Definition 2 (see [16]). The augmented filtering system (8) is stochastically finite-time bounded with respect to $(c_1, c_2, \mathcal{M}, \mathcal{N}, \delta)$, where $0 < c_1 < c_2$ and $\mathcal{M}$ is a positive definite matrix, if $\mathbb{E}\{\eta^T(k_1)\mathcal{M}\eta(k_1)\} \le c_1 \Longrightarrow \mathbb{E}\{\eta^T(k_2)\mathcal{M}\eta(k_2)\} < c_2$, $\forall\ k_1 \in \{-\tau_2, -\tau_2+1, \ldots, 0\}$, $k_2 = \{1, 2, \ldots, \mathcal{N}\}$ holds for any non-zero $w(k)$ satisfying Assumption 1.

Definition 3 (see [26]). The augmented filtering system (8) is $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative with respect to $(c_1, c_2, \mathcal{M}, \mathcal{N}, \gamma, \delta)$, where $0 < c_1 < c_2$, $\gamma > 0$ and $\mathcal{M}$ is a positive definite matrix, and if the system is stochastically finite-time bounded with respect to $(c_1, c_2, \mathcal{M}, \mathcal{N}, \delta)$ and under the zero initial condition, the output $z(k)$ satisfies

$$\sum_{k=0}^{\mathcal{N}}\left[e^T(k)\mathcal{Q}e(k) + 2e^T(k)\mathcal{S}\overline{w}(k) + \overline{w}^T(k)\mathcal{R}\overline{w}(k)\right] \ge \gamma\sum_{k=0}^{\mathcal{N}}\overline{w}^T(k)\overline{w}(k), \tag{11}$$

for any non-zero $w(k)$ satisfying Assumption 1, where $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ are real constant matrices in which $\mathcal{Q}$ and $\mathcal{R}$ are symmetric. Also, for convenience, we assume that $\mathcal{Q} \le 0$, then we can have $-\mathcal{Q} = (\overline{\mathcal{Q}}^{1/2})^2$.

Lemma 4 (see [16]). *For given matrices A, $Q = Q^T$ and $P > 0$, the inequality $A^TPA - Q < 0$ holds if and only if there exists a matrix Y such that*

$$\begin{bmatrix} -Q & A^TY^T \\ * & P - Y - Y^T \end{bmatrix} < 0. \tag{12}$$

Lemma 5 (see [9]). *For any two matrices X and Y with appropriate dimensions, $X^TY + Y^TX \le \epsilon X^TX + \epsilon^{-1}Y^TY$ holds for any scalar $\epsilon > 0$.*

Lemma 6 (see [12]). *For any symmetric constant matrix $Z \ge 0 \in \mathbb{R}^{n\times n}$ and two positive integers τ_1 and τ_2 satisfying $\tau_1 \le \tau_2$, the condition $-\sum_{i=k-\tau_2}^{k-\tau_1+1}\eta^T(i)Z\eta(i) \le -(1/(\tau_2 - \tau_1))\sum_{i=k-\tau_2}^{k-\tau_1+1}\eta^T(i)Z\sum_{i=k-\tau_2}^{k-\tau_1+1}\eta(i)$ holds.*

3. Main Results

This section pays attention to solve the problem of robust finite-time nonfragile filter design for the discrete-time system (1) by employing the LMI approach. For this purpose, first, the stochastic finite-time boundedness of the discrete-time system (1) for known filter gains without any fluctuations is discussed. Second, the finite-time $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative performance of the system (1) is analyzed. Third, by taking the filter gain fluctuations into account, the result is extended to obtain the desired finite-time nonfragile reliable filter for the considered system. Precisely, all the aforementioned results are investigated for the system (1) by means of the augmented filtering system (8).

3.1. Stochastic Finite-Time Boundedness Analysis. By constructing a suitable Lyapunov-Krasovskii functional, a new set of sufficient conditions is obtained to ensure the stochastic finite-time boundedness of the augmented filtering system (8).

Theorem 7. *Let Assumption 1 hold, $\mu \ge 1$, $\overline{\sigma}_i$ $(i = 1, 2, \ldots, N)$ and let $c_1 > 0$ be given scalars and $\mathcal{M}$ be a positive definite matrix. Then, the augmented filtering system (8) is stochastically finite-time bounded subject to $(c_1, c_2, \mathcal{M}, \mathcal{N}, \delta)$, if there exist a scalar $c_2 > 0$ and symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$ such that the following matrix inequalities hold:*

$$\left[\Omega_{i,j}\right]_{8\times 8} = \begin{bmatrix} [\Omega]_{7\times 7} & \Omega_1^T P \\ * & -P \end{bmatrix} < 0, \tag{13}$$

$$\psi_1 c_1 + \lambda_W \delta < \lambda_{\overline{P}} c_2 \mu^{-k}, \tag{14}$$

$$\begin{aligned} \lambda^P &\le P \le \lambda_P, \\ 0 &< Q_1 < \lambda_{Q_1}, \\ 0 &< Q_2 < \lambda_{Q_2}, \\ 0 &< Q_3 < \lambda_{Q_3}, \\ 0 &< R_1 < \lambda_{R_1}, \\ 0 &< R_2 < \lambda_{R_2}, \end{aligned} \tag{15}$$

where $\Omega_{1,1} = -\mu P + Q_1 + Q_2 + Q_3 + (\tau_1 - 1)R_1 + (\tau_2 - 1)R_2 + (\tau_2 - \tau_1)Q_3$, $\Omega_{2,2} = -Q_1$, $\Omega_{3,3} = -Q_2$, $\Omega_{4,4} = -Q_3$, $\Omega_{5,5} = -(1/(\tau_1 - 1))R_1$, $\Omega_{6,6} = -(1/(\tau_2 - 1))R_2$, $\Omega_{7,7} = -W$, $\Omega_1 = \left[\widetilde{A} + \sum_{i=1}^N \sqrt{\sigma_i}\widehat{A}_i \;\; \widetilde{A}_d \;\; \underbrace{0 \;\cdots\; 0}_{4} \;\; \widetilde{B} + \sum_{i=1}^N \sqrt{\sigma_i}\widehat{B}_i\right]$, *and* $\psi_1 = \lambda_{\overline{P}} + \lambda_{\overline{Q}_1}\tau_1 + (\lambda_{\overline{Q}_2} + \lambda_{\overline{Q}_3})\tau_2 + \lambda_{\overline{R}_1}((\tau_1)(\tau_1 - 1)/2) + \lambda_{\overline{R}_2}((\tau_2)(\tau_2 - 1)/2) + \lambda_{\overline{Q}_3}((\tau_2 - \tau_1)(\tau_2 + \tau_1 - 1)/2)$.

Proof. Consider the Lyapunov-Krasovskii functional for the augmented filtering system (8) in the following form:

$$V(k) = \sum_{a=1}^{3} V_a(k), \tag{16}$$

where

$$\begin{aligned} V_1(k) &= \eta^T(k) P \eta(k), \\ V_2(k) &= \sum_{s=k-\tau_1}^{k-1} \eta^T(s) Q_1 \eta(s) + \sum_{s=k-\tau_2}^{k-1} \eta^T(s) Q_2 \eta(s) \\ &\quad + \sum_{s=k-\tau(k)}^{k-1} \eta^T(s) Q_3 \eta(s), \\ V_3(k) &= \sum_{s=-\tau_1+1}^{-1} \sum_{j=k+s}^{k-1} \eta^T(j) R_1 \eta(j) \\ &\quad + \sum_{s=-\tau_2+1}^{-1} \sum_{j=k+s}^{k-1} \eta^T(j) R_2 \eta(j) \\ &\quad + \sum_{s=-\tau_2+1}^{-\tau_1} \sum_{j=k+s}^{k-1} \eta^T(j) Q_3 \eta(j). \end{aligned} \tag{17}$$

Computing the forward differences of $V_a(k)$ ($a = 1, 2, 3$) along the solution of augmented filtering system (8) and taking the mathematical expectation, we can get

$$\begin{aligned} \mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}\Bigg\{\Bigg[\widetilde{A}\eta(k) + \widetilde{A}_d \eta(k - \tau(k)) \\ &\quad + \widetilde{B}\overline{w}(k) + \sum_{i=1}^{N} (\sigma_i(k) - \overline{\sigma}_i)\left[\widehat{A}_i \eta(k) + \widehat{B}_i \overline{w}(k)\right]\Bigg]^T \\ &\quad \cdot P \times \Bigg[\widetilde{A}\eta(k) + \widetilde{A}_d \eta(k - \tau(k)) + \widetilde{B}\overline{w}(k) \\ &\quad + \sum_{i=1}^{N} (\sigma_i(k) - \overline{\sigma}_i)\left[\widehat{A}_i \eta(k) + \widehat{B}_i \overline{w}(k)\right]\Bigg] - \eta^T(k) \\ &\quad \cdot P\eta(k)\Bigg\}, \end{aligned} \tag{18}$$

$$\begin{aligned} \mathbb{E}\{\Delta V_2(k)\} &= \mathbb{E}\Bigg\{\eta^T(k)(Q_1 + Q_2 + Q_3)\eta(k) \\ &\quad - \eta^T(k - \tau_1) Q_1 \eta(k - \tau_1) - \eta^T(k - \tau_2) Q_2 \eta(k \\ &\quad - \tau_2) - \eta^T(k - \tau(k)) Q_3 \eta(k - \tau(k)) \\ &\quad + \sum_{s=k+1-\tau_2}^{k-\tau_1} \eta^T(s) Q_3 \eta(s)\Bigg\}, \end{aligned} \tag{19}$$

$$\begin{aligned} \mathbb{E}\{\Delta V_3(k)\} &= \mathbb{E}\Bigg\{(\tau_1 - 1)\eta^T(k) R_1 \eta(k) + (\tau_2 - 1) \\ &\quad \cdot \eta^T(k) R_2 \eta(k) + (\tau_2 - \tau_1)\eta^T(k) Q_3 \eta(k) \\ &\quad - \sum_{j=k-\tau_1+1}^{k-1} \eta^T(j) R_1 \eta(j) - \sum_{j=k-\tau_2+1}^{k-1} \eta^T(j) R_2 \eta(j) \\ &\quad - \sum_{j=k-\tau_2+1}^{k-\tau_1} \eta^T(j) Q_3 \eta(j)\Bigg\}. \end{aligned} \tag{20}$$

Now, applying Lemma 6 to the summation terms in (20), we can have

$$\begin{aligned} &- \sum_{j=k-\tau_1+1}^{k-1} \eta^T(j) R_1 \eta(j) \\ &\quad \le -\frac{1}{\tau_1 - 1} \sum_{j=k-\tau_1+1}^{k-1} \eta^T(j) R_1 \sum_{j=k-\tau_1+1}^{k-1} \eta(j), \end{aligned} \tag{21}$$

$$\begin{aligned} &- \sum_{j=k-\tau_2+1}^{k-1} \eta^T(j) R_2 \eta(j) \\ &\quad \le -\frac{1}{\tau_2 - 1} \sum_{j=k-\tau_2+1}^{k-1} \eta^T(j) R_2 \sum_{j=k-\tau_2+1}^{k-1} \eta(j). \end{aligned} \tag{22}$$

Then, it follows from (18) to (22) that

$$\begin{aligned} &\mathbb{E}\left\{\Delta V(k) - \overline{w}^T(k) W \overline{w}(k)\right\} \\ &\quad \le \mathbb{E}\left\{\xi^T(k)\left[[\Omega]_{7\times 7} + \Omega_1^T P \Omega_1\right]\xi(k)\right\}, \end{aligned} \tag{23}$$

where $\xi^T(k) = [\eta^T(k) \quad \eta^T(k-\tau_1) \quad \eta^T(k-\tau_2) \quad \eta^T(k-\tau(k)) \quad \sum_{j=k-\tau_1+1}^{k-1}\eta^T(j) \quad \sum_{j=k-\tau_2+1}^{k-1}\eta^T(j) \quad \overline{w}^T(k)]$, and the elements of $[\Omega]_{7\times7}$ and Ω_1 are defined in the theorem statement.

By applying Lemma 2.3 in [16] to the matrix terms in the right-hand side of (23), we can obtain the matrix term in (13). If the matrix inequality in (13) holds, it is obvious that

$$\begin{aligned} &\mathbb{E}\left\{\Delta V(k) - (\mu-1)V(k) - \overline{w}^T(k)W\overline{w}(k)\right\} \le 0 \\ &\mathbb{E}\{V(k+1) - V(k)\} \\ &\quad \le (\mu-1)\mathbb{E}\{V(k)\} + \mathbb{E}\left\{\overline{w}^T(k)W\overline{w}(k)\right\} \\ &\quad < (\mu-1)\mathbb{E}\{V(k)\} + \lambda_W\mathbb{E}\left\{\overline{w}^T(k)\overline{w}(k)\right\}. \end{aligned} \tag{24}$$

Thus, we can get $\mathbb{E}\{V(k+1)\} < \mu\mathbb{E}\{V(k)\} + \lambda_W\mathbb{E}\{\overline{w}^T(k)\overline{w}(k)\}$. Further, if $\mu \ge 1$, it follows from Assumption 1 that

$$\begin{aligned} \mathbb{E}\{V(k)\} &\le \mu^k\mathbb{E}\{V(0)\} \\ &\quad + \lambda_W\mathbb{E}\left\{\sum_{n=0}^{k-1}\mu^{k-n-1}\overline{w}^T(k)\overline{w}(k)\right\} \\ &\le \mu^k\mathbb{E}\{V(0)\} + \mu^k\lambda_W\delta. \end{aligned} \tag{25}$$

Moreover, from (16), we can also get

$$\begin{aligned} \mathbb{E}\{V(0)\} &= \eta^T(0)P\eta(0) + \sum_{s=-\tau_1}^{-1}\eta^T(s)Q_1\eta(s) \\ &\quad + \sum_{s=-\tau_2}^{-1}\eta^T(s)Q_2\eta(s) \\ &\quad + \sum_{s=-\tau(0)}^{-1}\eta^T(s)Q_3\eta(s) \\ &\quad + \sum_{s=-\tau_1+1}^{-1}\sum_{j=s}^{-1}\eta^T(j)R_1\eta(j) \\ &\quad + \sum_{s=-\tau_2+1}^{-1}\sum_{j=s}^{-1}\eta^T(j)R_2\eta(j) \\ &\quad + \sum_{s=-\tau_2+1}^{-\tau_1}\sum_{j=s}^{-1}\eta^T(j)Q_3\eta(j). \end{aligned} \tag{26}$$

Let $\overline{P} = \mathscr{M}^{-1/2}P\mathscr{M}^{-1/2}$, $\overline{Q}_1 = \mathscr{M}^{-1/2}Q_1\mathscr{M}^{-1/2}$, $\overline{Q}_2 = \mathscr{M}^{-1/2}Q_2\mathscr{M}^{-1/2}$, $\overline{Q}_3 = \mathscr{M}^{-1/2}Q_3\mathscr{M}^{-1/2}$, $\overline{R}_1 = \mathscr{M}^{-1/2}R_1\mathscr{M}^{-1/2}$, and $\overline{R}_2 = \mathscr{M}^{-1/2}R_2\mathscr{M}^{-1/2}$. Then, we can have

$$\begin{aligned} \mathbb{E}\{V(0)\} &= \mathbb{E}\left\{\eta^T(0)\mathscr{M}^{1/2}\overline{P}\mathscr{M}^{1/2}\eta(0)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau_1}^{-1}\eta^T(s)\mathscr{M}^{1/2}\overline{Q}_1\mathscr{M}^{1/2}\eta(s)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau_2}^{-1}\eta^T(s)\mathscr{M}^{1/2}\overline{Q}_2\mathscr{M}^{1/2}\eta(s)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau(0)}^{-1}\eta^T(s)\mathscr{M}^{1/2}\overline{Q}_3\mathscr{M}^{1/2}\eta(s)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau_1+1}^{-1}\sum_{j=s}^{-1}\eta^T(j)\mathscr{M}^{1/2}\overline{R}_1\mathscr{M}^{1/2}\eta(j)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau_2+1}^{-1}\sum_{j=s}^{-1}\eta^T(j)\mathscr{M}^{1/2}\overline{R}_2\mathscr{M}^{1/2}\eta(j)\right\} \\ &\quad + \mathbb{E}\left\{\sum_{s=-\tau_2+1}^{-\tau_1}\sum_{j=s}^{-1}\eta^T(j)\mathscr{M}^{1/2}\overline{Q}_3\mathscr{M}^{1/2}\eta(j)\right\} \le \Bigg[\lambda_{\overline{P}} \\ &\quad + \lambda_{\overline{Q}_1}\tau_1 + \left(\lambda_{\overline{Q}_2} + \lambda_{\overline{Q}_3}\right)\tau_2 + \lambda_{\overline{R}_1}\frac{(\tau_1)(\tau_1-1)}{2} \\ &\quad + \lambda_{\overline{R}_2}\frac{(\tau_2)(\tau_2-1)}{2} + \lambda_{\overline{Q}_3}\frac{(\tau_2-\tau_1)(\tau_2+\tau_1-1)}{2}\Bigg] \\ &\quad \cdot c_1 \le \psi_1 c_1. \end{aligned} \tag{27}$$

On the other hand, it follows from (16) that $\mathbb{E}\{V(k)\} \ge \mathbb{E}\{\eta^T(k)P\eta(k)\} \ge \mathbb{E}\{\eta^T(k)\mathscr{M}^{1/2}\overline{P}\mathscr{M}^{1/2}\eta(k)\} \ge \lambda^{\overline{P}}\mathbb{E}\{\eta^T(k)\mathscr{M}\eta(k)\}$. Then, it is easy to obtain that $\mathbb{E}\{\eta^T(k)\mathscr{M}\eta(k)\} < (1/\lambda^{\overline{P}})(\psi_1c_1 + \lambda_W\delta)\mu^k$. From inequality (14), it is clear that $\mathbb{E}\{\eta^T(k)\mathscr{M}\eta(k)\} < c_2$ for all $k = \{1, 2, \ldots, \mathscr{N}\}$. Hence, according to Definition 3, the augmented filtering system (8) is finite-time bounded with respect to $(c_1, c_2, \mathscr{M}, \mathscr{N}, \delta)$, which completes the proof. □

3.2. Dissipativity-Based Stochastic Finite-Time Boundedness Analysis. To increase the robustness of the obtained results in Theorem 7, the dissipative performance index is taken into account in the following theorem.

Theorem 8. *Let Assumption 1 hold. For given scalars $\mu \ge 1$, $\overline{\sigma}_i$ $(i = 1, 2, \ldots, N)$, $\gamma > 0$, $c_1 > 0$, and positive definite matrix $\mathscr{M}$, the augmented filtering system (8) is stochastically finite-time bounded and $(\mathscr{Q}, \mathscr{S}, \mathscr{R}) - \gamma$ dissipative with respect to $(c_1, c_2, \mathscr{M}, \mathscr{N}, \delta, \gamma)$ if there exist a constant $c_2 > 0$ and symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$ such that the following matrix inequalities together with (15) are satisfied:*

$$\left[\widehat{\Omega}_{i,j}\right]_{9\times9} = \begin{bmatrix} \left[\widehat{\Omega}\right]_{7\times7} & \widehat{\Omega}_1^T P & \widetilde{L}^T\sqrt{-Q} \\ * & -P & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{28}$$

$$\psi_1 c_1 + \lambda_{\mathscr{R}}\delta < \lambda_{\overline{P}}c_2\mu^{-k}, \tag{29}$$

where $\widehat{\Omega}_{1,7} = -\widetilde{L}^T\mathscr{S}$, $\widehat{\Omega}_{7,7} = -\mathscr{R} + \gamma I$ and the rest of elements of $[\widehat{\Omega}]_{7\times7}$ are the same as of $[\Omega]_{7\times7}$ defined in Theorem 7.

Proof. To discuss the dissipativity of the system (8), we consider the following performance index:

$$J = \mathbb{E}\left\{\sum_{k=0}^{\mathcal{N}}\left[e^T(k)\mathcal{Q}e(k) + 2e^T(k)\mathcal{S}\overline{w}(k)\right.\right. \\ \left.\left. + \overline{w}^T(k)\left[\mathcal{R} - \gamma I\right]\overline{w}(k)\right]\right\}. \tag{30}$$

Following the similar steps carried out in the proof of Theorem 7, it is easy to get that $\mathbb{E}\{\Delta V(k) - (\mu-1)V(k) - J\} \le 0$. Thus, $\mathbb{E}\{V(k+1)\} < \mathbb{E}\{\mu V(k) + e^T(k)\mathcal{Q}e(k) + 2e^T(k)\mathcal{S}\overline{w}(k) + \overline{w}^T(k)[\mathcal{R} - \gamma I]\overline{w}(k)\}$. Further, if $\mu \ge 1$, it follows that

$$\mathbb{E}\{V(k)\} \le \mathbb{E}\left\{\mu^k V(0) + \sum_{n=0}^{k-1}\mu^{k-n-1}e^T(k)\mathcal{Q}e(k) \right. \\ + 2\sum_{n=0}^{k-1}\mu^{k-n-1}e^T(k)\mathcal{S}\overline{w}(k) \\ \left. + \sum_{n=0}^{k-1}\mu^{k-n-1}\overline{w}^T(k)\left[\mathcal{R} - \gamma I\right]\overline{w}(k)\right\}. \tag{31}$$

At the same time, under zero initial condition and $V(k) \ge 0$, $\forall k = 1, 2, \ldots, \mathcal{N}$, we can have

$$\mathbb{E}\left\{\gamma\sum_{n=0}^{k-1}\mu^{k-n-1}\overline{w}^T(k)\overline{w}(k)\right\} \\ \le \mathbb{E}\left\{\sum_{n=0}^{k-1}\mu^{k-n-1}e^T(k)\mathcal{Q}e(k)\right. \\ + 2\sum_{n=0}^{k-1}\mu^{k-n-1}e^T(k)\mathcal{S}\overline{w}(k) \\ \left. + \sum_{n=0}^{k-1}\mu^{k-n-1}\overline{w}^T(k)\mathcal{R}\overline{w}(k)\right\}. \tag{32}$$

This implies that

$$\mathbb{E}\left\{\gamma\sum_{k=0}^{\mathcal{N}}\overline{w}^T(k)\overline{w}(k)\right\} \le \mathbb{E}\left\{\sum_{k=0}^{\mathcal{N}}e^T(k)\mathcal{Q}e(k)\right. \\ \left. + 2\sum_{k=0}^{\mathcal{N}}e^T(k)\mathcal{S}\overline{w}(k) + \sum_{k=0}^{\mathcal{N}}\overline{w}^T(k)\mathcal{R}\overline{w}(k)\right\}. \tag{33}$$

Then, from (33), the inequality in Definition 3 can be easily obtained. Hence, it can be concluded that the augmented filtering system (8) is stochastically finite-time bounded and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative. This completes the proof of the theorem. □

3.3. Dissipativity-Based Finite-Time Nonfragile Reliable Filter Design. In this subsection, we design a finite-time nonfragile reliable filter in the form of (5) for the discrete-time system (1) according to the conditions established in the previous section.

Theorem 9. *Consider the discrete-time system (1). Let Assumption 1 hold, $\mu \ge 1$, $\overline{\sigma}_i$ $(i = 1, 2, \ldots, N)$, $\gamma > 0$, $c_1 > 0$ be given constants, and $\mathcal{Q} \le 0$, $\mathcal{S}$, $\mathcal{R} = \mathcal{R}^T$, $\mathcal{M} > 0$ be known matrices. If there exist positive scalars c_2, ϵ_1, ϵ_2, ϵ_3, symmetric matrices $P_i > 0$ $(i = 1, 2, 3)$, $Q_{1j} > 0$, $Q_{2j} > 0$, $Q_{3j} > 0$, $R_{1j} > 0$, $R_{2j} > 0$ $(j = 1, 2)$, and any matrices Y, A_F, B_F, C_F with appropriate dimensions such that the following LMI together with (29) holds:*

$$\widehat{\Phi} = \begin{bmatrix} [\Phi]_{16\times16} & \epsilon_2\Phi_1 & \Phi_2^T & \Phi_3^T & \epsilon_1\Phi_4 & \epsilon_3\Phi_5 & \Phi_6^T \\ * & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\epsilon_3 I & 0 \\ * & * & * & * & * & * & -\epsilon_3 I \end{bmatrix} < 0, \tag{34}$$

where $\Phi_{1,1} = -P_1 + Q_{11} + Q_{21} + Q_{31} + (\tau_1 - 1)R_{11} + (\tau_2 - 1)R_{21} + (\tau_2 - \tau_1)Q_{31}$, $\Phi_{1,2} = -P_2$, $\Phi_{1,13} = -\overline{L}^T\mathcal{S}$, $\Phi_{1,14} = \overline{A}^T Y_1^T + \overline{F}^T\overline{C}^T G^T B_F^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{F}_i^T\overline{C}^T G^T B_F^T$, $\Phi_{1,15} = \overline{A}^T Y_3^T + \overline{F}^T\overline{C}^T G^T B_F^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{F}_i^T\overline{C}^T G^T B_F^T$, $\Phi_{1,16} = \overline{L}^T\sqrt{-\mathcal{Q}}$, $\Phi_{2,2} = -P_3 + Q_{12} + Q_{22} + Q_{32} + (\tau_1 - 1)R_{12} + (\tau_2 - 1)R_{22} + (\tau_2 - \tau_1)Q_{32}$, $\Phi_{2,13} = C_F^T\mathcal{S}$, $\Phi_{2,14} = -A_F^T$, $\Omega_{2,15} = -A_F^T$, $\Phi_{2,16} = -C_F^T\sqrt{-\mathcal{Q}}$, $\Phi_{3,3} = -Q_{11}$, $\Phi_{4,4} = -Q_{12}$, $\Phi_{5,5} = -Q_{21}$, $\Phi_{6,6} = -Q_{22}$, $\Phi_{7,7} = -Q_{31}$, $\Phi_{7,14} = \overline{A}_d^T Y_1^T$, $\Phi_{7,15} = \overline{A}_d^T Y_3^T$, $\Phi_{8,8} = -Q_{32}$, $\Phi_{9,9} = -(1/(\tau_1 - 1))R_{11}$, $\Phi_{10,10} = -(1/(\tau_1 - 1))R_{12}$, $\Phi_{11,11} = -(1/(\tau_2 - 1))R_{21}$, $\Phi_{12,12} = -(1/(\tau_2 - 1))R_{22}$, $\Phi_{13,13} = -\mathcal{R} + \gamma I$, $\Phi_{13,14} = \overline{B}^T Y_1^T + \overline{J}^T\overline{D}^T G^T B_F^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{J}_i^T\overline{D}^T G^T B_F^T$, $\Phi_{13,15} = \overline{B}^T Y_3^T + \overline{J}^T\overline{D}^T G^T B_F^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{J}_i^T\overline{D}^T G^T B_F^T$, $\Phi_{14,14} = P_1 - Y_1 - Y_1^T$, $\Phi_{14,15} = P_2 - Y_2 - Y_3^T$, $\Phi_{15,15} = P_3 - Y_2 - Y_2^T$, $\Phi_{16,16} = -I$, $\Phi_1 = \left[\overline{F}^T\overline{C}^T G^T N_b^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{F}_i^T\overline{C}^T G^T N_b^T \quad \underbrace{0 \ \ldots \ 0}_{15}\right]^T$, $\Phi_2 = [\underbrace{0 \ \ldots \ 0}_{13} \quad M_b^T Y_2^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}M_b^T Y_2^T \quad M_b^T Y_2^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}M_b^T Y_2^T \quad 0]$, $\Phi_3 = \left[0 \quad M_a^T Y_2^T \quad \underbrace{0 \ \ldots \ 0}_{14}\right]$, $\Phi_4 = \left[\underbrace{0 \ \ldots \ 0}_{13} \quad N_a^T \quad N_a^T \quad 0\right]^T$, $\Phi_5 = [\underbrace{0 \ \ldots \ 0}_{12} \quad \overline{J}^T\overline{D}^T G^T N_b^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}\,\overline{J}_i^T\overline{D}^T G^T N_b^T \quad \underbrace{0 \ \ldots \ 0}_{3}]^T$, $\Phi_6 = [\underbrace{0 \ \ldots \ 0}_{13} \quad M_b^T Y_2^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}M_b^T Y_2^T \quad M_b^T Y_2^T + \sum_{i=1}^N\sqrt{\overline{\sigma}_i}M_b^T Y_2^T \quad 0]$, *then there exists a filter (5) such that the discrete-time system (1) is stochastically finite-time bounded and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative with respect to $(c_1, c_2, \mathcal{M}, \mathcal{N}, \delta, \gamma)$. Furthermore, if the above said matrix inequalities have feasible solutions, the filter gains in (5) can be obtained by $A_f = Y_2^{-1}A_F$, $B_f = Y_2^{-1}B_F$, and $C_f = C_F$.*

Proof. For convenience, define the matrices as follows: $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_2 \end{bmatrix}$, $Q_1 = \mathrm{diag}\{Q_{11}, Q_{12}\}$, $Q_2 = \mathrm{diag}\{Q_{21}, Q_{22}\}$, $Q_3 = \mathrm{diag}\{Q_{31}, Q_{32}\}$, $R_1 = \mathrm{diag}\{R_{11}, R_{12}\}$, and $R_2 = \mathrm{diag}\{R_{11}, R_{12}\}$. Letting $A_F = Y_2 A_f$, $B_F = Y_2 B_f$, and $C_F = C_f$, using Lemma 4 and the partition matrices P_i and Q_i in (28) together with the parameter uncertainties definition in (6), we can get

$$\begin{aligned} \widehat{\Phi} = {} & [\Phi]_{16\times 16} + \Phi_3^T \Delta(k) \Phi_4 + \Phi_4^T \Delta^T(k) \Phi_3 \\ & + \Phi_2^T \Delta(k) \Phi_1 + \Phi_1^T \Delta^T(k) \Phi_2 + \Phi_6^T \Delta(k) \Phi_5 \\ & + \Phi_5^T \Delta^T(k) \Phi_6, \end{aligned} \tag{35}$$

where the elements of $[\Phi]_{16\times 16}$, Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, and Φ_6 are defined in (34). By applying Lemma 5, the matrix expression in (35) can be written as

$$\begin{aligned} \widehat{\Phi} = {} & [\Phi]_{16\times 16} + \epsilon_1^{-1} \Phi_3^T \Phi_3 + \epsilon_1 \Phi_4 \Phi_4^T + \epsilon_2^{-1} \Phi_2^T \Phi_2 \\ & + \epsilon_2 \Phi_1 \Phi_1^T + \epsilon_3^{-1} \Phi_6^T \Phi_6 + \epsilon_3 \Phi_5 \Phi_5^T. \end{aligned} \tag{36}$$

The expression in (36) can be equivalently observed as the matrix term in (34). Therefore, if the LMI in (34) together with (29) holds, the discrete-time system (1) with filter of the form (5) is stochastically finite-time bounded and $(\mathscr{Q}, \mathscr{S}, \mathscr{R})-\gamma$ dissipative. This completes the proof. □

Suppose that there is no time-varying delay term in the discrete-time system (1); then the augmented filtering system (8) and the corresponding error system (9) can be rewritten as

$$\begin{aligned} \eta(k+1) = {} & \widetilde{A}\eta(k) + \widetilde{B}\overline{w}(k) \\ & + \sum_{i=1}^{N} (\sigma_i(k) - \overline{\sigma}_i) \left[\widehat{A}_i \eta(k) + \widehat{B}_i \overline{w}(k) \right], \\ e(k) = {} & \widetilde{L}\eta(k). \end{aligned} \tag{37}$$

Corollary 10. *Consider the discrete-time system (37). Let Assumption 1 hold, $\mu \geq 1$, $\overline{\sigma}_i$ $(i = 1, 2, \ldots, N)$, $\gamma > 0$, $c_1 > 0$ be given constants, and $\mathscr{Q} \leq 0$, $\mathscr{S}$, $\mathscr{R} = \mathscr{R}^T$, $\mathscr{M} > 0$ be known matrices. If there exist positive scalars c_2, ϵ_1, ϵ_2, ϵ_3, symmetric matrices $P_i > 0$ $(i = 1, 2, 3)$, and any matrices Y, A_F, B_F, C_F with appropriate dimensions such that the following LMI together with (29) holds:*

$$\begin{bmatrix} [\overline{\Phi}]_{6\times 6} & \epsilon_2 \Phi_1 & \Phi_2^T & \Phi_3^T & \epsilon_1 \Phi_4 & \epsilon_3 \Phi_5 & \Phi_6^T \\ * & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\epsilon_3 I & 0 \\ * & * & * & * & * & * & -\epsilon_3 I \end{bmatrix} < 0, \tag{38}$$

where $\overline{\Phi}_{1,1} = -P_1$, $\overline{\Phi}_{1,2} = -P_2$, $\overline{\Phi}_{1,3} = -\overline{L}^T \mathscr{S}$, $\overline{\Phi}_{1,4} = \overline{A}^T Y_1^T + \overline{F}^T \overline{C}^T G^T B_F^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i}\, \overline{F}_i^T \overline{C}^T G^T B_F^T$, $\overline{\Phi}_{1,5} = \overline{A}^T Y_3^T + \overline{F}^T \overline{C}^T G^T B_F^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} \overline{F}_i^T \overline{C}^T G^T B_F^T$, $\overline{\Phi}_{1,6} = \overline{L}^T \sqrt{-\mathscr{Q}}$, $\overline{\Phi}_{2,2} = -P_3$, $\overline{\Phi}_{2,3} = C_F^T \mathscr{S}$, $\overline{\Phi}_{2,4} = -A_F^T$, $\Omega_{2,5} = -A_F^T$, $\overline{\Phi}_{2,6} = -C_F^T \sqrt{-\mathscr{Q}}$, $\overline{\Phi}_{3,3} = -\mathscr{R} + \gamma I$, $\overline{\Phi}_{3,4} = \overline{B}^T Y_1^T + \overline{J}^T \overline{D}^T G^T B_F^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i}\, \overline{J}_i^T \overline{D}^T G^T B_F^T$, $\overline{\Phi}_{3,5} = \overline{B}^T Y_3^T + \overline{J}^T \overline{D}^T G^T B_F^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} \overline{J}_i^T \overline{D}^T G^T B_F^T$, $\overline{\Phi}_{4,4} = P_1 - Y_1 - Y_1^T$, $\overline{\Phi}_{4,5} = P_2 - Y_2 - Y_3^T$, $\overline{\Phi}_{5,5} = P_3 - Y_2 - Y_2^T$, $\overline{\Phi}_{6,6} = -I$, $\overline{\Phi}_1 = \Big[\overline{F}^T \overline{C}^T G^T N_b^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i}\, \overline{F}_i^T \overline{C}^T G^T N_b^T \quad \underbrace{0 \ \ldots \ 0}_{5}\Big]^T$, $\overline{\Phi}_2 = [\underbrace{0 \ \ldots \ 0}_{3} \quad M_b^T Y_2^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} M_b^T Y_2^T \quad M_b^T Y_2^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} M_b^T Y_2^T \quad 0]$, $\overline{\Phi}_3 = \Big[0 \ \ M_a^T Y_2^T \quad \underbrace{0 \ \ldots \ 0}_{4}\Big]$, $\overline{\Phi}_4 = \Big[\underbrace{0 \ \ldots \ 0}_{3} \quad N_a^T \quad N_a^T \quad 0\Big]^T$, $\overline{\Phi}_5 = \Big[\,0 \quad 0 \quad \overline{J}^T \overline{D}^T G^T N_b^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i}\, \overline{J}_i^T \overline{D}^T G^T N_b^T \quad \underbrace{0 \ \ldots \ 0}_{3}\,\Big]^T$, $\overline{\Phi}_6 = [\underbrace{0 \ \ldots \ 0}_{3} \quad M_b^T Y_2^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} M_b^T Y_2^T \quad M_b^T Y_2^T + \sum_{i=1}^{N} \sqrt{\overline{\sigma}_i} M_b^T Y_2^T \quad 0]$, *then, there exists a filter (5) such that the discrete-time system (37) is stochastically finite-time bounded and $(\mathscr{Q}, \mathscr{S}, \mathscr{R}) - \gamma$ dissipative with respect to $(c_1, c_2, \mathscr{M}, \mathscr{N}, \delta, \gamma)$. Moreover, the filter gains in (5) can be computed by $A_f = Y_2^{-1} A_F$, $B_f = Y_2^{-1} B_F$, and $C_f = C_F$.*

Proof. The proof of this corollary is similar to that of Theorem 9 and thus it is omitted here. □

Remark 11. It is noted that the construction of Lyapunov-Krasovskii functional plays a constructive role in reducing the conservatism of the developed results. Moreover, the complex Lyapunov-Krasovskii functional with multiple summation terms can bring more number of decision variables to the LMI. Also, when the number of decision variable increases, the computational complexity also increases. So, there should be a trade-off between the summation terms in the construction of Lyapunov-Krasovskii functional and the LMI constraints. However, in this article we have chosen an appropriate LKF of the form (16) without using any free weighting matrix technique, which results in less conservative conditions.

Remark 12. It should be mentioned that the fragility is an important factor in the design of controllers due to the uncertainty and disturbances in control systems. Due to the occurrence of uncertainties, most of the controllers proposed in the existing literature are sensitive to small inaccuracies in their implementation. To overcome these facts, the controller should be designed in such a way that it is insensitive or nonfragile to its own parameter uncertainties. Moreover, dissipativity is an effective concept in designing the feedback controllers for linear and nonlinear systems. Specifically, in Definition 3, if we take $\mathscr{Q} = -I$, $\mathscr{S} = 0$, and $\mathscr{R} = \gamma^2 I$, then it corresponds to the finite-time H_∞ performance index and it minimizes the closed-loop impact of an external perturbation; when $\mathscr{Q} = 0$, $\mathscr{S} = I$, and $\mathscr{R} = \gamma I$, we can obtain the finite-time filter with passivity performance index and use it to measure the excess or shortage of passivity of the system; if $\mathscr{Q} = -\gamma^{-1}\theta I$, $\mathscr{S} = (1-\theta)I$, and $\mathscr{R} = \gamma I$, we can have the finite-time filter with mixed H_∞ and passivity performance index, where $\theta \in [0,1]$ is the weight parameter which deals with the trade-off between the performances of H_∞ and passivity concepts. Also, Theorem 8 provides a filter such that the closed-loop system is not only finite-time bounded, but also $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$ dissipative. Moreover, the proposed filter is more general, which includes H_∞ performance, passivity and mixed H_∞ and passivity as its special cases.

4. Numerical Simulations

In this section, two numerical examples are presented to show the effectiveness of the proposed filter design technique. Specifically, the first example validates the efficiency of finite-time nonfragile reliable filter design proposed in Theorem 9 and the second example presents a comparison result to illustrate the conservativeness of the proposed method with the existing ones.

Example 1. Let us consider the discrete-time system (1) and filter system (5) with parameters as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.2 & 0.05 \\ -0.02 & 0.3 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \\ A_d &= \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0 & 1 \end{bmatrix}, \\ D_1 &= 0.4, \\ D_2 &= 0.3, \\ L &= \begin{bmatrix} 0 & 1 \end{bmatrix}. \end{aligned} \tag{39}$$

The uncertain matrices are taken as $M_a = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$, $M_b = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$, $N_a = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$, and $N_b = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$. Also, the lower bound of the time-varying delay $\tau(k)$ is chosen as $\tau_1 = 2$ and the sensor fault matrix G is assumed as $0.7I$. The other parameters are chosen as $\mu = 1.01$, $\gamma = 0.95$, $c_1 = 0.1$, $\mathcal{M} = I$, $\mathcal{N} = 30$, and $\delta = 0.5$. Now, by solving the LMIs obtained in Theorem 9 with the aid of MATLAB LMI control toolbox, the maximum allowable upper bound of $\tau(k)$ is obtained as $\tau_2 = 6$ and the corresponding filter gain parameters are

$$\begin{aligned} A_f &= \begin{bmatrix} 0.0434 & -0.0066 \\ -0.0519 & 0.1374 \end{bmatrix}, \\ B_f &= \begin{bmatrix} -0.0271 \\ 0.0698 \end{bmatrix}, \\ C_f &= \begin{bmatrix} 0.1368 & -0.6206 \end{bmatrix}. \end{aligned} \tag{40}$$

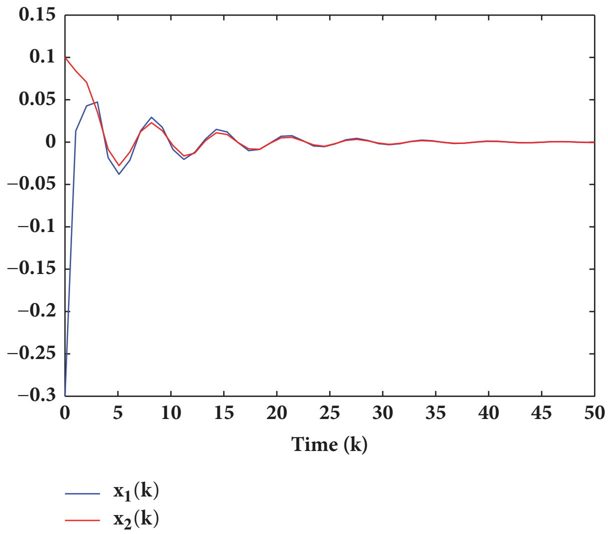
Furthermore, to demonstrate the effectiveness of the filtering performance, the initial conditions for the considered system and the filter are set to be $[-0.3 \quad 0.1]^T$ and $[0 \quad 0]^T$, respectively. Moreover, the disturbance signal is taken as $\overline{w}(k) = 0.05\exp(-0.1k)\sin(k)$. Based on the filter parameters mentioned above, the simulation results are presented in Figure 1. To be more specific, Figures 1(a) and 1(b) show the actual state responses and the designed filter state responses, respectively. The filtering error signal $e(k)$ is shown in Figure 1(c). The trajectories of the system output signal and its estimation are shown in Figure 1(d), wherein the effectiveness of the proposed filter design is clearly exhibited. Moreover, the time history of $\eta^T(k)\mathcal{M}\eta(k)$ is depicted in Figure 1(e). From these simulations, it can be concluded that the considered discrete-time system with time-varying delay (1) is stochastically finite-time bounded with respect to $(0.1, 45.1135, I, 30, 0.5)$ under the proposed dissipative-based filter (5) even in the presence of sensor failures and gain fluctuations.

Example 2. Consider the modified continuous stirred tank reactor (CSTR) system as in [8, 34], where the production of cyclopentanol (B) from cyclopentadiene (A) is considered. The complete reaction is given as follows: Cyclopentadiene (A) $\longrightarrow$ Cyclopentanol (B) $\longrightarrow$ Cyclopentanediol (C), and 2 Cyclopentadiene (A) $\longrightarrow$ Dicyclopentadiene (D). By assuming constant density and an ideal residence time distribution within the reactor, the balance equations can be described in the following form:

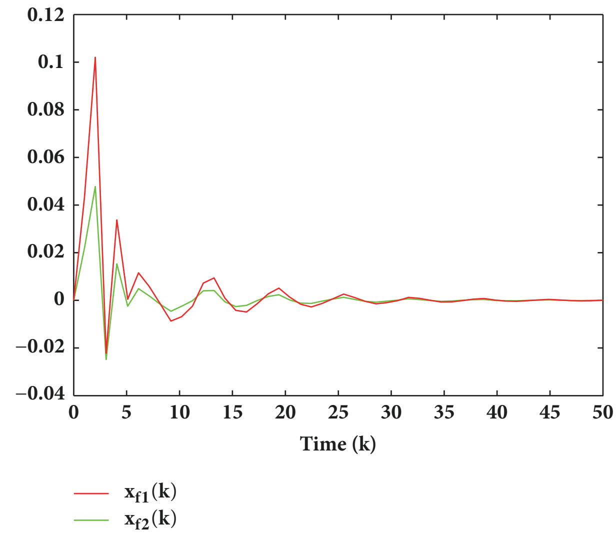
$$\begin{aligned} \frac{dC_A}{dt} &= \frac{\dot{V}}{V_R}\left(C_{A0} - C_A\right) - k_1 C_A - k_3 C_A^2, \\ \frac{dC_B}{dt} &= -\frac{\dot{V}}{V_R} C_B + k_1 C_A - k_2 C_B, \\ \frac{d\vartheta}{dt} &= \frac{\dot{V}}{V_R}\left(\vartheta_0 - \vartheta\right) + \frac{k_w A_R}{\zeta C_p V_R}\left(\vartheta_K - \vartheta\right) \\ &\quad - \frac{k_1 C_A \Delta H_R^{AB} + k_2 C_B \Delta H_R^{BC} + k_3 C_A^2 \Delta H_R^{AD}}{\zeta C_p}, \end{aligned} \tag{41}$$

where C_A denotes the concentration of educt A; C_B represents the concentration of the desired product B within the reactor; ϑ is the reactor temperature. The rate components k_1, k_2, and k_3 depend exponentially on the reactor temperature ϑ via Arrhenius law given by $k_i(\vartheta) = k_{0i}\exp(-E_{Ai}/R\vartheta)$ $(i = 1,2,3)$. Let us assume that the first and second rate components are equal for the reaction system; that is, $k_1 = k_2$. Further, the values for model parameters are defined as $k_{01} = k_{02} = 1.287\times 10^{12} h^{-1}$, $k_{03} = 9.043\times 10^{9} l/molh^{-1}$, $E_{A1}/R = E_{A2}/R = 9758.3K$, $E_{A3}/R = 8560.0K$, $\Delta H_R^{AB} = 4.2kJ/mol$, $\Delta H_R^{BC} = -11kJ/mol$, $\Delta H_R^{AD} = -41.85kJ/mol$, $C_P = 3.01kJ/kgK$, $A_R = 0.215m^2$, $V = 10.01$, $\vartheta_0 = 403.15K$, $k_w = 4032kJ/hm^2K$. Now, linearizing the balance equations (41) at the operating point, we can obtain the state-space model in the following form:

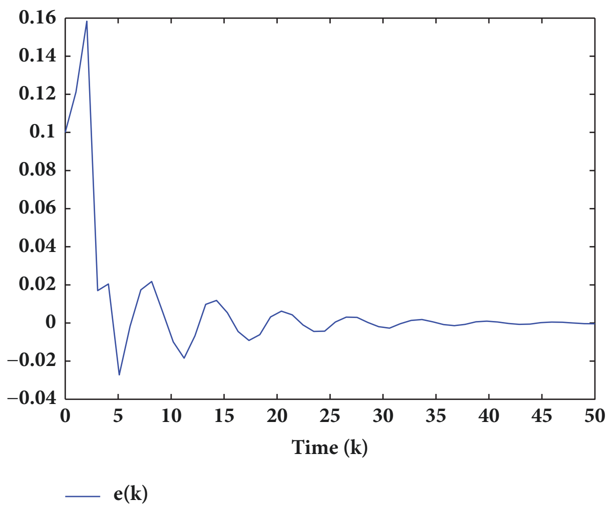
$$\dot{x}(t) = A_p x(t) + B_p u(t), \tag{42}$$



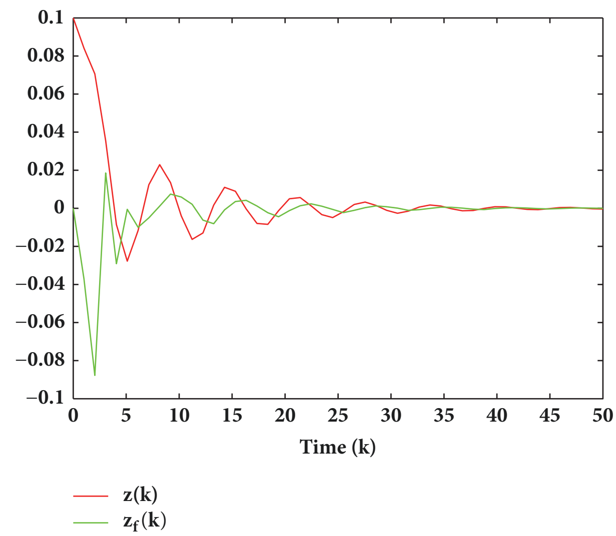
(a) Trajectories of system states



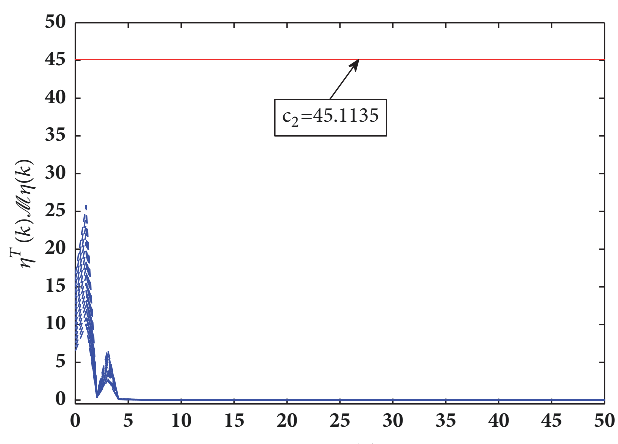
(b) Trajectories of filtering states



(c) Filtering error



(d) Trajectories of output signals



(e) Time history of $\eta^T(k)\mathscr{M}\eta(k)$

FIGURE 1: Simulation results of Example 1.

where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} C_A - C_{As} \\ C_B - C_{Bs} \\ \vartheta - \vartheta_s \end{bmatrix}$, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dot{V} - \dot{V}_s \\ C_{A0} - C_{A0s} \end{bmatrix}$, and the matrices A_p and B_p are given by

$$\begin{gathered} A_p = \begin{bmatrix} -86.0962 & 0 & 4.2077 \\ 50.6146 & -69.4446 & -0.9974 \\ 172.2263 & 197.9985 & -65.5149 \end{bmatrix}, \\ B_p = \begin{bmatrix} 0.3861 & 18.83 \\ -0.0899 & 0 \\ -0.4136 & 0 \end{bmatrix}. \end{gathered} \tag{43}$$

The steady-state values of the main operating point of the reactor are given by $C_{As} = 1.235 mol/l$, $C_{Bs} = 0.9 mol/l$, $\vartheta_s = 407.29K$, $\dot{V}/V_R = 18.83h^{-1}$, $C_{A0s} = 5.1 mol/l$. It should be noted that the control input can be treated as the unknown input signal in the state estimation problem. According to this point, the discrete-time state representation of (42) with the sampling period $T_0 = 1$min can be represented by the following:

$$x(k+1) = Ax(k) + Bw(k), \tag{44}$$

where

$$\begin{gathered} A = \begin{bmatrix} 0.2747 & 0.0345 & 0.0206 \\ 0.2323 & 0.3152 & 0.0033 \\ 1.2566 & 1.1042 & 0.3671 \end{bmatrix}, \\ B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \end{gathered} \tag{45}$$

The product concentration C_B may be needed in practice and the signal processing approaches are used to estimate the concentration. By deploying two sensors for measuring the concentration of educt A and the reactor temperature, the product concentration C_B is estimated. Hence, $C_1 = [1\ 0\ 0]$, $C_2 = [0\ 0\ 1]$, $D_1 = 0.3$, $D_2 = 0.2$, and $L = [0\ 1\ 0]$. Let the unknown input signal $w(k)$ lie in the interval $[-1, 1]$ and assume that the signal is transmitted if one of the following conditions is satisfied: $\| y_p(k) - y_{\text{last},p} \| \geq \delta_{y,p}$, $k - k_{\text{last},p} > \theta_{k,p}$, where $y_{\text{last},p}$ is the last transmitted signal of the pth sensor at time instant $k_{\text{last},p}$ and $\delta_{y,p}$ and $\theta_{k,p}$ are the magnitude and time threshold values, respectively. Moreover, it is also assumed that there is no packet dropouts and set $\delta_{y,1} = 0.1$, $\delta_{y,2} = 0.2$, $\theta_{k,1} = \theta_{k,2} = 1$. From this setting, it is seen that $N_1 = N_2 = 2$. Further, choose the values of the transmission probabilities as $\overline{\beta}_{1,0} = 0.69$, $\overline{\beta}_{1,1} = 0.31$, $\overline{\beta}_{2,0} = 0.44$, and $\overline{\beta}_{2,1} = 0.56$. It should be mentioned that based on the method proposed in [8], the minimum value of γ is 1.9069, and while using the proposed filter design in this paper, the minimum value of γ is 1.6686, which reveals that the proposed filter design technique in this paper is better than that in [8].

Furthermore, for the simulation purposes, we choose $\mu = 1.01$, $\mathcal{N} = 30$, $\mathcal{M} = I$, $c_1 = 0.4$, and $\delta = 0.5$. Then, by Corollary 10, the optimal value of c_2 can be calculated as 838.1240. By solving the LMI-based conditions in Corollary 10, the filter gain parameters are obtained as

$$\begin{gathered} A_f = \begin{bmatrix} -0.1688 & 0.3499 & 0.0052 \\ 0.1143 & 0.2637 & 0.0314 \\ 0.5853 & 0.6015 & 0.8656 \end{bmatrix}, \\ B_f = \begin{bmatrix} -1.5324 & 0.1289 \\ -2.1702 & 0.2194 \\ -6.7899 & -0.5094 \end{bmatrix}, \\ C_f = \begin{bmatrix} -0.0430 & -0.2360 & -0.1018 \end{bmatrix}. \end{gathered} \tag{46}$$

Based on these values, the state responses of the discrete-time system (44) with the proposed nonfragile reliable filter are plotted in Figure 2(a) and the associated filter state responses are presented in Figure 2(b). The system output signal together with its estimation is given in Figure 2(c), respectively. Furthermore, it can be viewed from Figure 2(d) that under the chosen initial condition and the obtained filter parameters, the state responses of the corresponding augmented filtering system satisfy the condition $\eta^T(k)\mathcal{M}\eta(k) < c_2 = 838.1240$. Then, it directly follows that the discrete-time system (44) is stochastically finite-time bounded with respect to $(0.4, 838.1240, I, 30, 0.5)$.

5. Conclusion

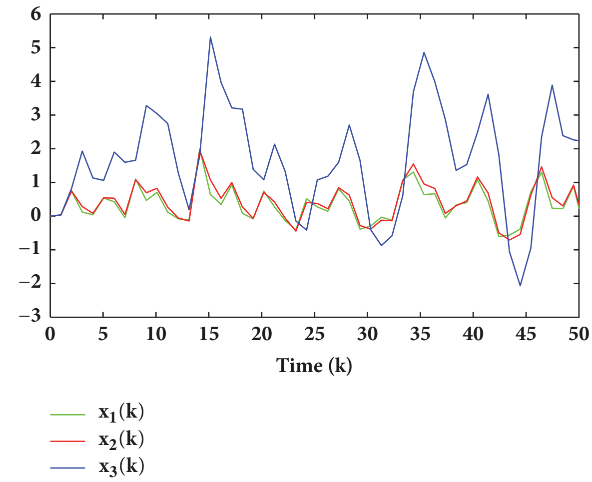
In this paper, the problem of dissipative-based finite-time robust filter design has been discussed for a class of WSNs which is described by discrete-time systems with time-varying delay. More precisely, a reliable nonfragile filter has been designed such that the augmented filtering system is stochastically finite-time bounded and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative. In this connection, a set of sufficient conditions in terms of LMIs has been developed for obtaining the desired nonfragile reliable filter for the system under consideration, wherein the filter gain parameters have been obtained by solving the developed LMIs. Finally, two numerical examples including CSTR model have been presented to demonstrate the effectiveness of the proposed filter design. The problem of finite-time dissipative-based filtering for nonlinear stochastic system with actuator saturation is an untreated topic which will be the future work.

Data Availability

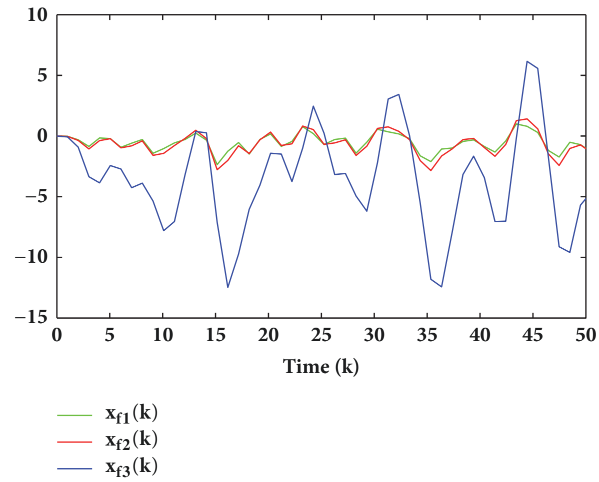
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

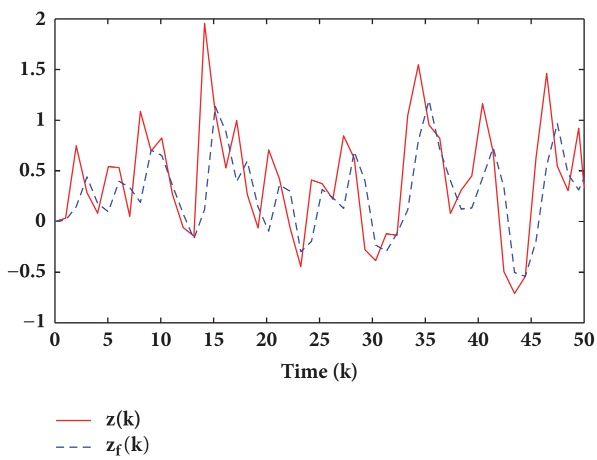
The authors declare that there are no conflicts of interest regarding the publication of this paper.



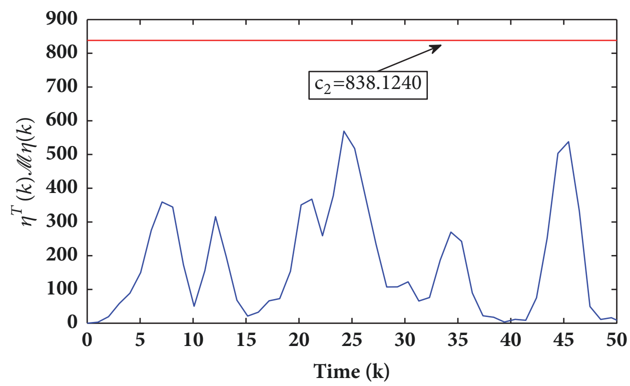
(a) Trajectories of system states



(b) Trajectories of filtering states



(c) Trajectories of output signals



(d) Time history of $\eta^T(k)\mathscr{M}\eta(k)$

Figure 2: Simulation results of Example 2.

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