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A STUDY OF THE LIFE AND WRITINGS OF COLIN MACLAURIN.

BY CHARLES TWEEDIE, M.A., B.Sc., F.R.S.E.

1. COLIN MACLAURIN was one of the most distinguished of the brilliant coterie of Scottish mathematicians that marked the course of the eighteenth century. He was born at Kilmodan in Glendaruel, Argyleshire, in February, 1698, and was the youngest of a family of three sons. His father, John Maclaurin, who was minister of the parish and a man of mark in his day, died when Colin was but six weeks old. His mother, after residing for some time in Argyleshire, removed to Dumbarton, and died in 1707 when Colin was nine years of age. The orphans were taken in charge by their uncle, Daniel Maclaurin, minister of Kilfinnan, who faithfully discharged his duty to them.

2. STUDENT AT THE UNIVERSITY OF GLASGOW. The young Maclaurins shewed a studious temperament, and, in 1709, at the tender age of eleven, Colin, who had already given great promise in the study of the classics, was entered at the University of Glasgow and placed under the charge of Professor Carmichael, with a view to his ultimately entering the ministry of the Scottish Church. So far he had not yet studied Mathematics, but, when twelve years of age, he found in a friend's rooms a copy of Euclid's *Elements*, the perusal of which so fascinated his attention that in a few days he had mastered the contents of the first six books. His extraordinary ability and power of mathematical application, thus early awakened, attracted the attention of Robert Simson, the Professor of Mathematics, then a young man in the twenties. Maclaurin's talents thus developed under the most favourable auspices, for a close friendship sprang up between the professor and his pupil, to be broken only by the death of Maclaurin in 1746.

When fifteen years of age he graduated in the Faculty of Arts, after presenting a thesis on Gravity, which, according to the custom of the times, he publicly defended with great success.* The year after graduating he spent in the Divinity Hall, but the acrimonious disputes that then raged in the Church of Scotland decided him to alter his career.

* It appears that by the time he was sixteen years of age he had already discovered many of the theorems published later in the *Geometria Organica*.

E

Colinus Mc Laurin

He returned to his uncle's manse, where he diligently continued his mathematical and literary studies until an opportunity occurred in 1717, when he presented himself as a candidate for the Chair of Mathematics in the Marischal College of Aberdeen University.

3. Professor at Aberdeen University. There were two candidates for the appointment—Maclaurin, and a young Aberdonian of ability, Walter Bowman,* and the decision was made after a ten days' examination,† the examiners being Professor Gregory of St. Andrews University, and A. Burnet, Regent in King's College, Aberdeen. The following account of the examination and appointment, taken from the *Records of Marischal College* as edited by Mr. Anderson, is of sufficient interest to merit insertion here :

"Sept. 11, 1717.

Mr. Colin M'Laurine, student of Divinity in the College of Glasgow, and Mr. Walter Bowman, son to Walter Bowman of Logie, undergo trial for the vacant chair, before Mr. Charles Gregorie, Professor of Mathematics, St. Andrews, and Mr. Alexander Burnet, Regent in King's College."

The Examiners report :

"They doe think that both the saids Mr. Colin M'Laurine and Mr. Walter Bowman are capable to teach the Mathematicks anywhere.

In most of the tryals in the inferior pairts of the Mathematicks ther was no great odds. Only in Euclid Mr. Bowman was much readier and distincter. And in the last tryall Mr. M'Laurine plainly appeared better acquainted with the speculative and higher pairts of the Mathematicks. And they conclude that they both excell in their own way. Mr. Bowman only hath applyed himself to those things that are commonly taught and Mr. M'Laurine hath made further advances.

So subscryvit Charles Gregorie, Al. Burnet.

M'Laurine is accordingly appointed." ‡

The appointment proved an excellent one, for Maclaurin soon raised to a high standard the study of Mathematics in the northern University.

While faithfully discharging the duties of his chair, he found leisure to draw up two important geometrical memoirs, both published in the *Philosophical Transactions*, viz. :

(i) "On the Construction and Measure of Curves" (No. 356, Anno 1718), which contains the invention and discussion of what is now known as the Pedal Transformation.

(ii) "A New Method of Describing all Kinds of Curves" (No. 359, Anno 1719).

In spite of several inaccuracies in the theory of the double points of the curves investigated, these papers marked a great advance in geometrical research. Their contents he later incorporated in his *Geometria Organica*, the manuscript of which he took with him on his visit to London in 1719. Through his introduction to Dr. Clarke he became acquainted with Sir Isaac Newton, and he was admitted to the Fellowship of the Royal Society. With Newton he formed a warm friendship, which he was wont to describe as "the greatest honour and happiness of his life." His veneration for Newton henceforward took the form of endeavouring to popularise the discoveries of the great mathematician, of whom he was so worthy a disciple. His treatise, the *Geometria Organica*, received the approval of Newton, by whose authority as P.R.S. it was published in 1720.

* This name occurs in the list of subscribers to Maclaurin's *Newton*, published in 1748.

† The method of selecting a professor by examination still applies in certain departments of Aberdeen University.

‡ The salary of the chair amounted to 504 pounds Scots.

On his second visit to London he made the acquaintance of Martin Folkes, who was afterwards to befriend the family of Maclaurin on the untimely death of the latter in 1746, and by whose care Maclaurin's posthumous works on *Algebra* and on *Newton's Philosophical Discoveries* were published.

4. In 1722 he was invited by Lord Polwarth to accompany his son, Mr. Hume, as tutor during a visit of the latter to the Continent, a post he accepted without, however, resigning his chair in Aberdeen. The two visited various parts of France, and settled more definitely in Lorraine. It was during his stay in France that he received from the Académie Royale des Sciences of Paris a prize for his thesis on the *Percussion of Bodies* (1724). In the same year, however, Mr. Hume fell ill during a visit to the South of France, and died at Montpellier, whereupon Maclaurin returned to Aberdeen to resume his duties as professor. The circumstances in which he had left Aberdeen were somewhat peculiar, and appear to have caused some coldness between him and the College authorities.*

It was therefore fortunate for Maclaurin that in the following year (1725) he was, on the recommendation of Newton, invited by the University of Edinburgh to occupy the Chair of Mathematics conjointly with Professor James Gregory, now too old and frail to teach his classes. The conditions of the appointment are of interest, and reveal the pecuniary value of such professorships in Scotland at the time. There was difficulty in finding the means of providing a pension for the aged Gregory, and the agreement made with the patrons (the Town Council of the City) was that Professor Gregory or his family should receive the salary attached to the chair, amounting to £83 6s. 8d. ; while Maclaurin was given a salary of £50 along with the class fees, the arrangement to last for seven years, after which Maclaurin, in the case of the death of Professor Gregory, would enjoy the full salary of £83 6s. 8d.

Newton himself took an active part in the recommendation of Maclaurin to the electors to the Chair in Edinburgh, and even privately offered to them to pay £20 per annum during his own lifetime to augment the salary paid to Maclaurin, but this generous offer was declined. It is little to be wondered at that Maclaurin remained a bachelor until 1733, when he married Anne, daughter of Walter Stewart, the Solicitor-General for Scotland.

5. Professor of Mathematics at Edinburgh University. In November, 1725, Maclaurin was admitted to the Chair. At Edinburgh Maclaurin proved as successful an exponent of Mathematics, more particularly of the Newtonian doctrine, as he had been at Aberdeen, but the demands made upon his time seem nowadays very heavy.

* The following extracts from the *Records of Marischal College* bear upon the matter:

"Dec. 23, 1724.

On consideration that M'Laurine has been abroad and not attended to his charge for near thir three years 'the Council appoint Mr. Daniel Gordon, one of the Regents who had formerly taught mathematicks at the University of St. Andrews,' to teach the class during the current session.

Jan. 20, 1725.

M'Laurine having returned, a committee is appointed to confer with him anent '1st. His going away without liberty from the Council: 2nd. His being so long absent from his charge.'

April 27, 1725.

M'Laurine appears before the Council, expresses regret, and is reponed.

Jan. 12, 1726.

The Council, learning 'by the Public Newsprints,' that M'Laurine has been admitted conjunct professor with Mr. James Gregory in the University of Edinburgh, declare his office vacant.

Jan. 18, 1726.

A Committee is appointed to confer with the Masters of the College anent the vacancy.

Feb. 23, 1726.

M'Laurine intimates that he has sent his demission to the Masters."

According to the *Scots Magazine* for August, 1741,* he "taught three classes during the same session, and sometimes a fourth upon such of the abstruse parts of the science as are not explained in the former three. In the first, he began with demonstrating the grounds of vulgar and decimal arithmetic; then proceeded to Euclid; and after explaining the first six books, with the plain trigonometry, and use of the tables of logarithms, sines, etc., he insisted on surveying, fortification, and other practical parts, and concluded with the elements of algebra. He gave geographical lectures, once in the fortnight, to this class of students.

In the second, he repeated the algebra again from its principles, and advanced farther in it; then proceeded to the theory and mensuration of solids, spherical trigonometry, the doctrine of the sphere, dialling and other practical parts. After this he gave the doctrine of the conic sections, with the theory of gunnery, and concluded with the elements of astronomy and optics.

In the third class, he began with perspective; then treated more fully of astronomy and optics. Afterwards he prelected on Sir Isaac Newton's *Principia*, and explained the direct and inverse method of fluxions. At a separate hour he began a class of experimental philosophy, about the middle of December, which continued thrice every week till the beginning of April; and at proper hours of the night described the constellations, and shewed the planets by telescopes of various kinds." †

Alexander Carlyle, in his delightful *Autobiography*, gives the following description of the Professor:

"Mr. M'Laurin was at this time a favourite professor, and no wonder, as he was the clearest and most agreeable lecturer on that abstract science that ever I heard. He made mathematics a fashionable study, which was felt afterwards in the war that followed in 1743, when nine-tenths of the engineers of the army were Scottish officers. The Academy at Woolwich was not then established."

His popularity in society also made a serious drain upon his leisure, but he still pursued with ardour his favourite studies, though his health, never of the strongest, suffered thereby.

On the death of Newton in 1728, he was invited by Conduitt, the nephew of Newton, to write an account of Newton's *Philosophical Discoveries*. Maclaurin gladly acceded to this request, but owing to the death of Conduitt, this work was not then published, only appearing later, in 1748, through the care of his friend Martin Folkes.

6. In 1732 we find Maclaurin involved in a controversy with Braikenridge regarding the priority of the discovery of certain geometrical theorems, one of which is now known as the Braikenridge-Maclaurin Theorem for the conic section. We return to this later, and in the meantime content ourselves with the remark that both writers seem to have hit independently on the same conclusion in perfect good faith; and, though Braikenridge had the priority of publication, Maclaurin probably discovered the theorem before him.

He was next busied with his great *Treatise of Fluxions*—the development of a pamphlet he had intended to publish in defence of the Newtonian theory of the Calculus against the attack made upon it by the philosopher Bishop Berkeley. Though much of it was in proof in 1737, the *Treatise* was not published until 1742, when it appeared in two quarto volumes.

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* See Bower's *History of the University of Edinburgh*.

† We find him more than once complaining that his duties leave him little time for research.

THE MANSE AT GLENDARUEL, WHERE COLIN MACLAURIN WAS BORN.

Now used as Offices for the modern Manse standing in front of it.

In 1740 he was again awarded a prize by the Parisian *Académie Royale des Sciences*, which he shared with D. Bernoulli and Euler, for his thesis on the *Motion of the Tides*.

The only remaining mathematical work we need here mention is his *Algebra*, written to elucidate the *Arithmetica Universalis* of Newton, and containing as an Appendix an important contribution to Geometry, entitled *De Linearum Geometricarum Proprietatibus Generalibus Tractatus*.*

7. Apart from the immediate duties of his chair, which were a heavy tax upon his energy, Maclaurin gave many proofs of a generous interest in public affairs. His remarkable social gifts made him a welcome member of Edinburgh society.

He took an active part in promoting a scheme for the establishment of an astronomical observatory, preferably in connection with the University. In 1741 he memorialised the Town Council on the question. The interest of influential Scotch noblemen was enlisted in the scheme; various astronomical instruments were collected; and in all probability his efforts would have been crowned with success but for the unsettled condition of the country at the time.

He also instigated and directed a movement to obtain a more accurate survey of the northern coast of Scotland, including the islands of Orkney and Shetland. He was unable to undertake the survey in person, but many of his pupils were engaged in it under his direction.

He took a keen interest in the search of a free passage in the Arctic regions, made himself master of the literature on the subject, and would even, had circumstances permitted, have undertaken the voyage at his own expense. He was consulted regarding the schemes for such an expedition in 1744, but before his memorial was presented to Government, it was enacted that the search should be restricted to the North-west Passage, much to his regret, as he had the idea that a passage, if any, would be found not far from the Pole.

He drew up two elaborate memorials † for the proper gauging of vessels, thus furnishing a system of calculation which was long utilised by the Excise.

On his recommendation a Society, originally intended for medical papers, was extended to include papers on physics and on the antiquities of the country. It was then developed into the Philosophical Society, which acted as the forerunner of what is now known as the Royal Society of Edinburgh.

He drew up an elaborate series of calculations on which to establish on sound principles a fund for the benefit of the widows of ministers of the Church of Scotland and of the professors in the Universities. In this pioneer movement he furnished the city of Edinburgh with a lead which she has never lost in the domain of Insurance with the kindred branches of practical Economic Science, these forming what may be termed a conspicuous industry which has found a congenial home in Edinburgh. The MS. of the relative Memoir is in the Laing Collection of the University of Edinburgh.

8. His interest in public affairs was, in fact, to be the cause of his serious illness and death during the rebellion of the "Forty-Five." When it became known that the rebels had given the Royalist troops the slip and were marching on Edinburgh, Maclaurin endeavoured to rouse the authorities to take active measures for the defence of the capital.

* Perhaps we ought here to add his translation of D. Gregory's *Practical Geometry*, published in 1745.

† One, apparently the first, is now in possession of Sir James Russell, of Edinburgh. A copy of it is in the Advocates' Library, Edinburgh.

The whole responsibility of the construction of trenches and barricades * fell upon Maclaurin. He was not well seconded, for the results of the Porteous Riot in 1737 still caused a coldness to the Government, and his unceasing labours made a strain upon his bodily constitution which it was badly fitted to stand.

When the Highlanders descended upon the city it was found impossible to make any effective resistance. The Volunteers on the side of Government were disbanded. Maclaurin could hope for little mercy at the hands of the rebels, and had to seek refuge by flight to England, where he became the guest of Dr. Herring, Archbishop of York. His health, however, had been undermined, and his condition was aggravated by the stormy weather that prevailed during his return to Scotland.

On reaching Edinburgh he was found to be suffering from dropsy, Attempts to remove the malady proved unavailing, and he gradually sank. His behaviour during his illness was "calm, cheerful, and resigned." He retained his faculties till within a few hours of his death, and he continued to dictate to his amanuensis the closing chapter of his *Account of Sir Isaac Newton's Philosophical Discoveries*. He died on the 14th of June, 1746, at the somewhat early age of forty-eight, in the presence of his friend, Alex. Monro (Primus), the famous Professor of Anatomy, who had been introduced to the University at the same time with Maclaurin.

By his marriage with Anne Stewart he had a family of seven, and was survived by two sons and three daughters. Of the two sons, John and Colin, the elder, John, became a member of the Scottish Bar, and was raised to the Bench as Lord Dreghorn.†

Maclaurin was buried in Greyfriars Churchyard ‡ in Edinburgh, where the following epitaph, composed by his son, Lord Dreghorn, may still be read upon the mural tombstone, high up on the south wall of the church :

INFRA SITUS EST

COLIN MACLAURIN

MATHES. OLIM IN ACAD. EDIN. PROF.
ELECTUS IPSO NEWTONO SUADENTE.

H. L. P. F.§

Non ut nomini paterno consulat.
Nam tali auxilio nil eget
Sed ut in hoc infelici campo
Ubi Luctus regnant et Pavor
Mortalibus prorsus non absit Solatium :
Hujus enim scripta evolve
Mentemque tantarum rerum capacem
Corpori caduco superstitem crede.

---

* "For a few days past M'Laurin the professor had been busy on the walls on the south side of the town, endeavouring to make them more defensible, and had even erected some small cannon near to Potterrow Port which I saw. I visited my old master when he was busy, who seemed to have no doubt that he could make the walls defensible against a sudden attack, but complained of want of service, and at the same time encouraged me and my companions (they were in the University Volunteers) to be diligent in learning the use of arms" (*Autobiography* of A. Carlyle).

† It is interesting to note that the mathematical and legal faculties have been combined at the present day in the person of Richard C. Maclaurin, a descendant of the younger son Colin, born in Edinburgh, and brought up in New Zealand, who took a double first at Cambridge in Mathematics and Law, before becoming a Professor of Mathematics in New Zealand, and later at Columbia University. He was in 1909 called to be President of the Massachusetts Institute. He has published a mathematical treatise on Light—almost the only part of Newton's works which his illustrious ancestor did not attempt to elucidate.
For this information I am indebted to Sir James Russell.

‡ Cantor in his *Geschichte* makes Maclaurin die at York, but this is not the case.

§ H. L. P. F. written in full runs "Hunc Lapidem Posuit Filius."

MACLAURIN'S PUBLISHED WORKS.

Geometria Organica.

9. Maclaurin's first contributions to mathematical literature deal with Geometry.

In 1718 there appeared in the *Philosophical Transactions* his paper entitled "Of the Construction and Measure of Curves," in which he invents what are now termed Pedal Curves, and gives a good account of their general theory. In 1719 his paper on "A New Method of Describing all Kinds of Curves" was printed in the same Transactions.

In these there are several inaccuracies regarding the double points on the curves discussed. The two memoirs are incorporated and amplified in his famous treatise, the *Geometria Organica*, which appeared in 1720, under the imprimatur of Newton. By this important treatise Maclaurin at once took high rank amongst geometers. As it is probably not so well known as it ought to be, some space may here be profitably devoted to it.

The treatise is divided into two parts. In Part I. curves of all orders are described by the sole use of constant given angles and fixed straight lines.*

Chapter I. is devoted to the Newtonian Organic Description of Conics, of which the whole treatise is a development. The conic is obtained by rotating two constant angles POQ and $PO'Q$ round two fixed points, O and O', and restricting P to lie on a straight line, when Q, in general, generates a conic passing through O and O', though it may degenerate into a straight line (taken along with OO'). The generality of this description is established by applying it to trace the conic through five given points. The species and asymptotes of a conic are also determined.

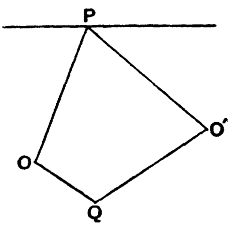

FIG. 1.

10. Chapter II. gives a method of tracing curves of the third order that possess a double point. In this section Maclaurin anticipates what is considered as a modern method of tracing the rational cubic.

Let R be any point on a straight line l; RPQ an angle of constant magnitude, whose vertex lies on a line l', while RP passes through a fixed point O, $RO'Q$ a constant angle rotating round the fixed point O'. Then Q generates a cubic with a double point at O'. If in the quadrilateral $O'QPR$ any two of the three vertices P, Q, R lie in straight lines, the remaining free vertex generates a singular cubic.

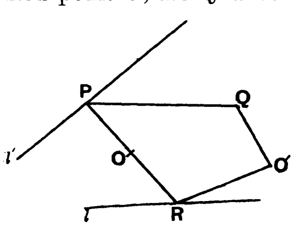

FIG. 2.

He discusses a great variety of particular cases, and gives a general method of tracing the rational circular cubic, treating specially the curve now known as the Strophoid, of which he furnishes the cissoidal generation. In generalising the last result, he comes across part of the Focals of Van Rees. Indeed, many of the rational cubics supposed to have been invented in the nineteenth century may be found here.

* Chasles (*Aperçu Historique*) makes the statement that Maclaurin by his methods obtains all algebraic curves. With this we find difficulty in agreeing. It may be true for cubic curves, but the only quartics obtained in the treatise are either unicursal, possessing three double points, or a unique triple point; or they possess two double points. No quartics are given that possess only one double point, or that are entirely free from singularity.

Chapter III. deals with the description of Quartic Curves with two or three double points, or with a triple point, also of Cubic Curves that have no double point.

The generation of the quartic runs thus: RP_1Q and RP_2Q are two constant angles, whose vertices P_1 and P_2 lie on two fixed straight lines, while QP_1 and QP_2 pass through the fixed points O_1 and O_2. If R lies on a third straight line, then Q generates a quartic with double

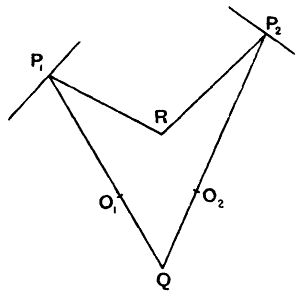

Fig. 3.

points at O_1 and O_2. This quartic may, however, degenerate into the straight line O_1O_2, taken along with a cubic curve devoid of double point.

Also, when any three of the vertices of P_1QP_2R lie on straight lines, the remaining vertex generates in general a quartic curve.

11. Chapter IV. generalises the results of the preceding chapters. Let $OP_1P_2 \ldots P_nQ$ be a broken line which, on deformation, retains unaltered the angles at $P_1, P_2, \ldots P_n$, while the lengths of the segments of the lines may vary. Such a broken line may for convenience be termed a *serrate angle*.

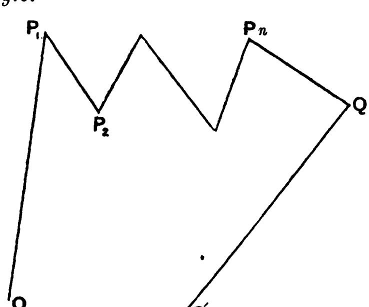

Fig. 4.

Let O be a fixed point, and let $P_1, P_2, \ldots P_n$ lie on fixed straight lines. Then, if O' is another fixed point, and $O'Q$ makes a constant angle with P_nQ, the locus of Q is a curve of degree $n+2$, possessing an $(n+1)$-ple point at O'. It is easy to prove that the curve must be a rational or unicursal curve. The envelopes of P_1P_2, P_2P_3, ... are, in general, curves of class 2 (parabola), 3, ... respectively, while the locus of Q may be considered as an oblique pedal of the envelope of P_nQ.

A further generalisation runs thus:

If $OP_1P_2 \ldots P_mP$ and $O'Q_1Q_2 \ldots Q_nQ$ are two serrate angles, O and O' being fixed points, while $P_1 \ldots P_m$; $Q_1, Q_2, \ldots Q_n$ lie on fixed straight

MEMORIAL SITUATED ON THE OUTSIDE OF THE SOUTH WALL OF GREYFRIARS CHURCH, EDINBURGH.

lines, and if the intersection of P_mP and Q_nQ is restricted to lie on a straight line, then the intersection of $P_{r-1}P_r$ and $Q_{s-1}Q_s$ generates a curve of degree
$$r(n+1)+s(m+1).$$

The fundamental propositions of this chapter Maclaurin proves by the Cartesian Geometry. The demonstrations are not always easy to follow, but other proofs on modern lines may easily be supplied.

12. PART II. In Part II. the linear loci are replaced by curves C_m, C_n, etc., of degrees m, n, etc.

Chapter I. contains the generalisation of Newton's organic description of conics, as summed up in Prop. III. :

In the quadrilateral $POQO'$, O and O' are fixed points and the angles at O and O' constant. If P lies on a curve C_n, Q, in general, generates a curve of degree $2n$, having n-ple points at O and O', and also at a third point O''.

If, however, O, for example, is an r-ple point on C_n, then the locus of Q is a curve of degree $2n-r$.

In Chapter II. we may note the following generalisations of theorems in Part I.

Prop. V. In the quadrilateral $PQO'R$, O' is a fixed vertex, PR passes through a fixed point O, and the angles at P and O' are constant. If P and R lie on curves C_m and C_n, then Q generates a curve C_{3mn}.

Prop. VI. O and O' are fixed points, P and Q lie respectively on C_p and C_q, and the angles OPT, $O'QT$ are invariable. If T lies on a curve C_r, the intersection R of OP and $O'Q$ lies on a curve of degree $4pqr$.

Prop. VIII. Consider two serrate angles,
$$OP_1P_2 \dots P_mP,$$
$$O'Q_1Q_2 \dots Q_nQ,$$
in which O and O' are fixed, while $P_1 \dots P_m$; $Q_1 \dots Q_n$ lie on curves of degree $p_1, \dots p_m$; $q_1, \dots q_n$. If P_mP and Q_nQ intersect on a curve C_r, the intersection of OP_1 and $O'Q_1$ is on a curve of order $r(n+m+2)\,\Pi p \times \Pi q$.

13. Chapter III. contains Maclaurin's Theory of Pedals, and is one of the most interesting chapters in the whole book, although its contents were long relegated to oblivion.

After giving the usual definition of the Pedal * of a given curve, he shows that two sets of curves forming one system may be obtained from a given curve, namely, the Positive and the Negative Pedals.

He gives a very full account of the Pedals of a circle (Limaçon and Cardioid), of a parabola (including the Cissoid of Diocles), and of the central conic, the latter giving rise to the rational Bicircular Quartic.

He shows that a curve may be specified by its $p-r$ equation, which he defines as its *Radial Equation*, and indicates how to deduce the radial equation of its pedals. He points out that the pedal of the parabola is a straight line, and the pedal of a central conic a circle, when the pole is at a focus. He discusses the Podoids, and then proceeds to examine in great detail the curves given by the radial equation
$$p/r=(r/a)^n.$$

The pedals of such curves have a similar radial equation. It is not difficult to show that this group of curves furnishes the *Sine Spirals*. It includes many familiar curves for a suitable pole (*e.g.* the straight line, the circle, the parabola, the equilateral hyperbola, the lemniscate). Regarding the rectification of such curves, he gives a beautiful theorem, which has been strangely overlooked, but which surpasses in elegance

* The nomenclature throughout is modern.

the other theorems known regarding the integration of pedals, in that it admits of the process of differentiation.

The theorem is the following :

Prop. XV. Let B be a point common to a curve and its pedals.

Let $P_1, P_2, \ldots$ be the points on its positive pedals, and $N_1, N_2, \ldots$ the points on its negative pedals corresponding to any point P on the curve.

When the radial equation of the curve is $p/r=(r/a)^n$, we have the relation

$$\text{Arc } BP_1=(n+1)\,(\text{arc } BN_1+\text{the straight line } PN_1).$$

Maclaurin is quite aware of the importance of his theorem, and discusses its application to the pedals of the straight line and the various conic sections. It is not difficult to prove that the theorem applies only to the Sine Spirals ($r^n=A\sin n\theta$), and the interest of the theorem is thereby enhanced.

14. Chapter IV. is concerned with various applications to Mechanics. Chapter V. is of great importance, and furnishes the foundation of the theory of Higher Plane Curves.

He starts with the Lemma, which he cannot yet prove * in its generality that two curves C_m and C_n intersect in mn points.

If, of the points necessary to determine a C_n, $(nr+1)$ lie on a C_r $(r<n)$, then must C_n degenerate into C_r and C_{n-r}.

A curve of degree n is, in general, determined by $\frac{1}{2}(n^2+3n)$ points. Two curves of degree n cut in n^2 points. Hence 9 points may not uniquely determine a cubic, while 10 would be too many.

This is what has come to be termed *Cramer's Paradox*, although Cramer expressly quotes Maclaurin as his authority for the statement.

A curve C_n cannot possess more than $\frac{1}{2}(n-1)(n-2)$ double points without degenerating.

The concluding propositions solve the two following problems :

Prop. XXV. To draw a C_n through $2n+1$ given points, of which one is an $(n-1)$-ple point. The curve is unique.

Prop. XXVI. To draw a curve C_{2n} through as many points as determine a C_n and three other points, each of which is an n-ple point on C_{2n}. There may be several solutions.

The production of this brilliant piece of geometrical research on the part of the youthful professor in Aberdeen would alone be sufficient to render Maclaurin's name immortal.†

The other works of Maclaurin appear to be better known, and do not call for so full a discussion.

THE BRAIKENRIDGE-MACLAURIN THEOREM.

15. In 1733 there appeared in London a small quarto entitled *Exercitatio Geometrica de Descriptione Linearum Curvarum*, written by the Rev. W. Braikenridge. We gather from his preface that several of his theorems were known to him in 1726. In 1727 he met Maclaurin in London, and in conversation spoke of his own geometrical researches. Maclaurin informed him that he had himself already obtained similar

* In the Abstract of a supplement to the *Geometria Organica*, published in the *Phil. Trans.* of 1735, he states that he has obtained a general demonstration.

† Maclaurin's copy of the *Geometria Organica* with his autograph is preserved in the Library of Edinburgh University.

The present writer's copy was bought in May, 1723, by Braikenridge, whom we proceed shortly to discuss in connection with the Braikenridge-Maclaurin Theorem.

E. de Jonquières, in his "Note sur la géométrie organique de Maclaurin" (*Journal de Mathématiques*, 1857) furnishes synthetic demonstrations of Maclaurin's fundamental theorems after the manner of his master Chasles. A fervent admirer of Maclaurin, he speaks of the *Geometria Organica* as "un des titres de gloire de Maclaurin," and adds a French translation of the Latin Preface, which is marked by the fine Latinity and lofty tone of the writer.

theorems, showing him the MS. containing these, while taking care not to place it in his hands. Not until 1733, however, was the work of Braikenridge published, although in 1727 he had entrusted a manuscript containing his theorems to George Gordon, with a view to their being brought before the Royal Society. The manuscript, unfortunately, was lost.

A continuation of the same theorems by Braikenridge was published in the *Philosophical Transactions* for 1735 (No. 436).

In the *Exercitatio Geometrica* occurs the theorem of our heading, viz.: If the sides of a polygon are restricted to pass through fixed points while all the vertices but one lie on fixed straight lines, the free vertex describes a conic section or a straight line.

The rest of the treatise leads up to the theorem that when the sides of a triangle pass through fixed points, while two of the vertices lie on two curves, C_m and C_n, then the maximum degree of the curve traced out by the free vertex is $2mn$, but only mn if the three given points lie on a straight line.

A variety of sub-cases are discussed.

In the contribution to the *Philosophical Transactions* are discussed more especially constructions for cubics and quartics with multiple points.

But near the conclusion he falls into serious error in attempting to give generalisation to his results.

For example, a curve C_n, according to him, is determined by n^2+1 points, which is, of course, true for a conic.

This error is all the more inexcusable, as he had read the *Geometria Organica*, in which Maclaurin gives the correct number $\frac{1}{2}(n^2+3n)$.

His next statement is simply Prop. XXV. Part II. of the *Geometria Organica*, and is correct; but his further statements are doubtful.

Thus far for Braikenridge.

16. Let us now turn to Maclaurin's statement in a letter to Professor Machin in December, 1732, though published in the *Philosophical Transactions* only in 1735 (No. 439).

In this letter he explains that he had intended to issue a supplement to the *Geometria Organica* containing his further geometrical researches on the same lines. Such a supplement was printed in 1721, but not published.

He gives an Abstract of this supplement, and adds a note, dated Nancy, 1722, in which he gives the Braikenridge-Maclaurin Theorem for the triangle, and which he had incorporated in his lectures on Algebra since 1727. The Abstract, as published in the *Philosophical Transactions*, shows a far greater mastery of the subject than Braikenridge could lay claim to.

While Braikenridge could not get beyond the case of a triangle with sides through three fixed points (save in the special case leading to the conic section), Maclaurin generalises to the case of any polygon, and shows that when all the sides pass through fixed points, while all the vertices but one lie on curves C_m, C_n, etc., the free vertex describes a curve whose maximum degree is $2\Pi m$. He sees in his result a generalisation of a porism of Pappus, to which Robert Simson had drawn attention.

Maclaurin's proof and figure for the case of a triangle, two of whose vertices lie on straight lines, lead immediately to the Pascal property of a hexagon inscribed in a conic, which at the time had been lost sight of.

17. But Maclaurin's researches go far beyond this result, and remind one strongly of the *Geometria Organica*.

For example:

Let O_1, O_2 be fixed points and also the point S.

SNP is a constant angle, whose vertex N lies on a curve C_m. The angles QO_2R and RO_1P are constant angles revolving round O_1 and O_2. If Q and R lie on C_n and C_r respectively, then P traces out a curve whose maximum degree is $3mnr$.

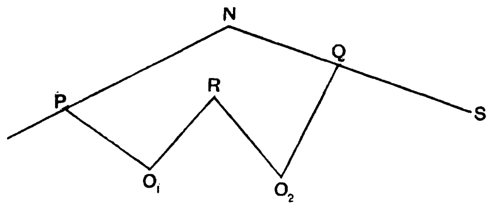

FIG. 5.

If there are more poles, such as O, with constant angles revolving round them while the vertices QR_1R_2 ... lie on curves $C_nC_rC_s$, etc., the maximum degree of the locus of P is $3mnrst$ ... .

Again: Let PMQ and PNR be two invariable angles, PM and PQ passing through two fixed points S_1 and S_2. Let the constant angle QOR revolve round the fixed point O. Then if M, Q, R, N lie on curves C_m, C_q, C_r, C_n, the degree of the locus of P is at most $4mnqr$, there being a similar generalisation to the preceding when there are several points O. Etc., etc.

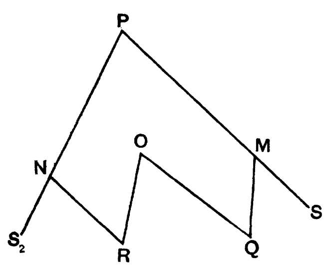

FIG. 6.

Maclaurin adds that he showed his proofs to Braikenridge (who taught Mathematics for some time in Edinburgh), and that he had made public since 1727 an algebraic proof of the Braikenridge-Maclaurin Theorem in his lectures on Algebra. (His well-known *Treatise of Algebra* was not actually published until 1748.)

In the circumstances, the name attached to the theorem is justified by the earlier production of Braikenridge, though Maclaurin's researches seem much more profound.

The dispute furnishes an illustration of the sensitiveness of even great mathematicians in their claims to the priority of invention.

18. We have mentioned that in 1740 Maclaurin shared with Euler and Bernoulli a prize of the *Académie Royale des Sciences* of Paris for his memoir on the Tides, described by Lagrange as a " chef d'œuvre de Géométrie, qu'on peut comparer à tout ce qui Archimède nous a laissé de plus beau et de plus ingénieux." In this paper he solves by geometry the problem of the attraction of an ellipsoid of revolution upon a point taken on its surface or in its interior. A demonstration by analysis was given by Lagrange, in which he generalises the result to the case of an ellipsoid with three unequal principal axes. We return to this problem again when discussing briefly the *Treatise of Fluxions*.

De Linearum Geometricarum Proprietatibus Generalibus Tractatus.

19. Before proceeding to the *Fluxions* and the *Algebra* we may here take up the memoir under the above heading that appeared as an Appendix to the *Algebra* when published, after his death, in 1748. It constitutes a geometrical memoir of the first importance, furnishing many theorems in the foundation of the theory of Higher Plane Curves. Some idea may be gathered of its importance from the use made of it, and of the *Geometria Organica*, by Poncelet in his classical *Traité des Propriétés Projectives des Figures*. The memoir is readily accessible in the *Algebra*, which is easily picked up. In the fifth and later editions there is added an English translation by Lawson, while so late as 1856 a translation into French was made by De Jonquières, and published in his *Mélanges de Géométrie Pure.*

The memoir is based on the following two theorems :

Theorem I. (due to Cotes and sent to Maclaurin by Dr. Smith, the friend of Cotes).

Lines are drawn to a curve C_n through a fixed point O in its plane, cutting C_n in $A_1, A_2, \ldots A_n$.

If P is taken on OA_n, so that OP is the harmonic mean of $OA_1, OA_2, \ldots OA_n$, the locus of P, as the line swings round O, is a straight line.

This is a generalisation of the diameter of Newton for a curve, which is the particular case when O is at infinity in a given direction.

Theorem II. (Maclaurin's Theorem).

Through O is drawn a fixed transversal $OA_1A_2 \ldots A_n$ to a curve C_n. Let $OB_1B_2 \ldots B_n$ be any other transversal, and let the tangents to the curve at the points B cut the fixed transversal in $T_1, T_2, \ldots T_n$. Then

$$\Sigma 1/OB_1 \text{ is constant and equal to } \Sigma 1/OT_1.$$

He then proceeds to discuss the curvature at any point of C_n, a problem again taken up in detail for the conic and cubic in Chapters II. and III.

Chapter II. treats largely of the harmonic properties of the Conic,* most of which were already familiar to mathematicians, though the following is ascribed to Maclaurin by Cantor (*Geschichte der Mathematik*).

Let $ABCD$ be a quadrilateral inscribed in a conic. Let AB and CD cut in P, and AC and BD in Q. Let the tangents at B and C cut in R, and the tangents at A and D in S. Then $PQRS$ are four collinear points.

20. Chapter III. treats of the properties of Cubics, and contains many classical theorems, among which may be noted :

Prop. VI. The tangents at three collinear points on a cubic meet the curve again in three points that are likewise collinear.

Also : If ABC are three collinear points while AP and BQ touch the cubic at P and Q, then, if PQ cuts the cubic again in R, the line CR is tangent at R.

Prop. VII. From A on the cubic are drawn AP and AQ tangent at P and Q. If PQ cuts the curve again in R, the tangents at R and A intersect on the curve. If, however, A is a point of inflexion, the tangent at R passes through A.

Prop. VIII. From A suppose three tangents drawn, AP, AQ, AR. Let PQ cut the curve again in T, and let TR cut it again in S. Then AS is tangent to the curve at S.

From this he concludes that not more than four tangents may be drawn from a point on the cubic to touch the curve elsewhere.

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* "Die Vierecksätze gab im vollstandiger Form zuerst C. Maclaurin" (Schönflies, "Proj. Geom.," *Encykl. d. math. Wiss.*).

We shall, in modern parlance, speak of A as the tangential of P, Q, R, S, which are the pre-tangentials of A.

Prop. IX. If, however, A is a point of inflexion but three tangents, AP, AQ, AR, can be drawn. P, Q, R lie on a straight line which divides harmonically any transversal AMN meeting the cubic in M and N.

Prop. X. The straight line joining two points of inflexion cuts the cubic again in a point which is likewise a point of inflexion (previously discovered by De Gua).

Prop. XIII. If P, Q, R, S have a common tangential A, the lines such as PQ and RS, joining them in pairs, intersect on the cubic.

Prop. XV. Let P and Q have a common tangential A, and let B be any other point on the curve. If BP and BQ cut the cubic again in P' and Q', then P' and Q' have a common tangential C, such that A, C, and the tangential of B are collinear.

Cor. Let P_1, P_2, P_3, P_4 have a common tangential A, and Q_1, Q_2, Q_3, Q_4 a common tangential B. The sixteen lines joining a point P to a point Q cut in sets of four, in four points, R_1, R_2, R_3, R_4, possessing a common tangential C, such that A, B, C are collinear.

Prop. XVI. Let P_1 and P_2 have a common tangential A. Take M on the curve, and let MP_1 and MP_2 cut the cubic again in Q_1 and Q_2. P_1Q_2 and P_2Q_1 cut in a point N of the cubic. The tangents at Q_1 and Q_2 have a common tangential B; and M and N have a common tangential C, such that A, B, C are collinear.*

Prop. XVII. P_1 and P_2 have a common tangential. M_1 on the curve is joined to P_1 and P_2, cutting the cubic again in Q_1, and Q_2 : M_2 on the curve is joined to P_1 and P_2; and M_2P_1 and M_2P_2 cut the cubic again in R_1 and R_2. In that case Q_1R_1 and Q_2R_2 intersect on the cubic, and so do Q_1R_2 and Q_2R_1.

Prop. XVIII. $FLGK$ is a quadrilateral inscribed in a cubic, and such that FK and GL cut on the cubic in P, while LF and GK cut on the cubic in Q. IC, CH, HI are the tangents at Q, P, L, forming the triangle ICH. Let IG cut CH in D, and HF cut CI in E. Then the points DKE lie on a line which is tangent at K.

There are several other theorems dealing with the harmonic properties and the curvature of the cubic, but a full appreciation of the elegance of the structure of the geometrical edifice presented to us by Maclaurin can only be obtained from a perusal of the tractate itself, which has been described by Chasles in the *Aperçu Historique* as " d'une élégance et d'une précision admirables."

The Treatise of Fluxions.

21. In 1734 the Newtonian doctrine of Fluxions was assailed by Bishop Berkeley in an essay entitled *The Analyst: or a Discourse Addressed to an Infidel Mathematician*, which gave rise to a celebrated controversy concerning the foundations of the Calculus.

The defence at first fell into incompetent hands, until in 1735 Benjamin Robins, in his *Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions and of Prime and Ultimate Ratios*, gave a complete justification of Newton.†

It was but natural that the sincere piety of Maclaurin, with his extreme veneration of all that Newton wrote, should take offence at the tone of Berkeley's attack, and he set himself to make a reply. But so many new ideas were developed in his hands during his investigations that his original purpose was changed into the writing of a *Treatise of Fluxions*.

* Prop. XVI. furnishes the simplest example of a closed Steinerian polygon.

† An excellent account of this controversy is given by Professor Gibson in vol. xvii. *Proc. Edinburgh Math. Soc.*

This treatise, which appeared in two volumes in 1742, though the most of the first volume was in print in 1737, is usually considered to be the *magnum opus* of Maclaurin ; and furnishes a splendid monument to the expository powers of the Scotch professor. It immediately became the authority on the subject to the overshadowing of the excellent work of Robins.

In order to establish the logical basis of the theory, a basis that would be incontestable, Maclaurin falls back upon the method of exhaustions of Archimedes. Throughout the course of the treatise synthetic geometric demonstrations play an important part. Like Newton, he was profoundly skilled in the application of Geometry to problems in Pure and Applied Mathematics—a science which since their day has remained undeveloped, the methods of Analysis having completely engrossed the attention of mathematicians.

22. Within the limits at our disposal we can give but a few samples of Maclaurin's handiwork in this treatise. It naturally contains much suitably incorporated from his own geometrical works. There is also an account of his theory of the attractions of confocal ellipsoids, in which he establishes the following theorems.

The attractions of two confocal ellipsoids of rotation upon a point in the axis of rotation or in the equatorial plane are as their masses. The forces of attraction of two confocal ellipsoids upon an external point on their axis produced are as their masses.

He here introduces for the first time the idea of confocal ellipses in a plane and of confocal ellipsoids.

The last of these three theorems had to wait for generalisation until Legendre established its truth for any external point. Maclaurin's treatment of his problems is a triumph of geometric skill. Some idea of the delicacy of his demonstrations may be inferred from the fact that mathematicians like D'Alembert, Lagrange and Legendre thought that he had only stated his theorems without proving them.*

Another example of his geometric skill is furnished by his deduction of the properties of the ellipse by considering the ellipse as an oblique section of a right circular cylinder, and comparing with the corresponding properties of the circular section.

The teacher of elementary geometry may be interested to note on page 141 the "modern" definition of similar and similarly situated figures. Also, on page 262 occurs the locus now known as the *Trisectrix of Maclaurin*, which he defines thus. The base CS of $\triangle CSP$ is fixed. If $\angle TSP = 3\angle PCS$, as in the figure, the locus of P is the trisectrix.

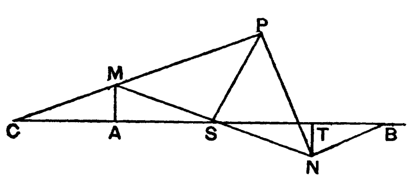


FIG. 7.

Let $CA=AS=ST=TB$. Draw AM and TN perpendicular to AB. Join MS and produce it to N.

Then $NS=SM=SP$. Hence $MPN=\frac{\pi}{2}$ and $PNB=\frac{\pi}{2}$. Thus the locus may be defined as the pedal of the parabola whose focus is B

* Chasles, *Aperçu Historique.*

and vertex T, when the pole is taken at C on AT, such that $CT=3TB$, as in the figure.

His contributions to the rectification of the ellipse are also of interest.

23. In analysis he gives what is universally known as Maclaurin's Theorem for the expansion of $f(x)$, though Maclaurin himself points out that this theorem had already appeared in Taylor's *Methodus Incrementorum.*

He puts the theory of the turning values of $y=f(x)$ upon its proper basis, showing that $f'(x)=0$ is only a first condition, while he adds the test for distinguishing between a maximum and a minimum.

A beautiful combination of geometry and analysis enables him to furnish a test for the convergence or divergence of the series whose general term is of the form

$$y=(Ax^m+Bx^{m-1}+\dots)/(ax^n+bx^{n-1}+\dots).$$

He proceeds thus :

He traces the graph of y (A/a positive), and shows that when $m<n$ the graph descends to the x-axis after passing the last maximum for a finite value of x, the curve being convex to the x-axis, which it approaches asymptotically.

If the ordinate for any value a of x greater than the abscissa of the last maximum be taken, the area, L, between it, the x-axis, and the curve will tend to a finite limit when $n>m+1$, and there is no finite limit when $n \ngtr m+1$.

Next consider the series formed by taking the ordinates for a, $a+1$, $a+2$, etc.

Construct the rectangles of unit breadth upon the corresponding segments of the x-axis, as in the figure.

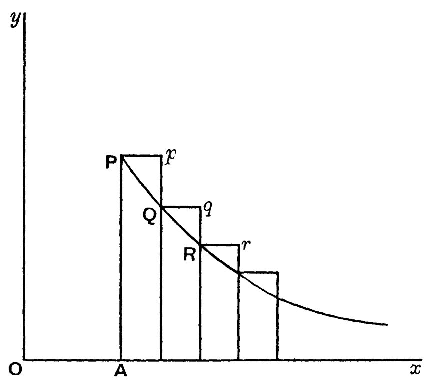


FIG. 8.

Their sum $=y_a+y_{a+1}+y_{a+2}+\dots$ is greater than L. Also, by shifting all the rectangles but the first to the left through unit distance, it is obvious that L is greater than

$$y_{a+1}+y_{a+2}+\dots.$$

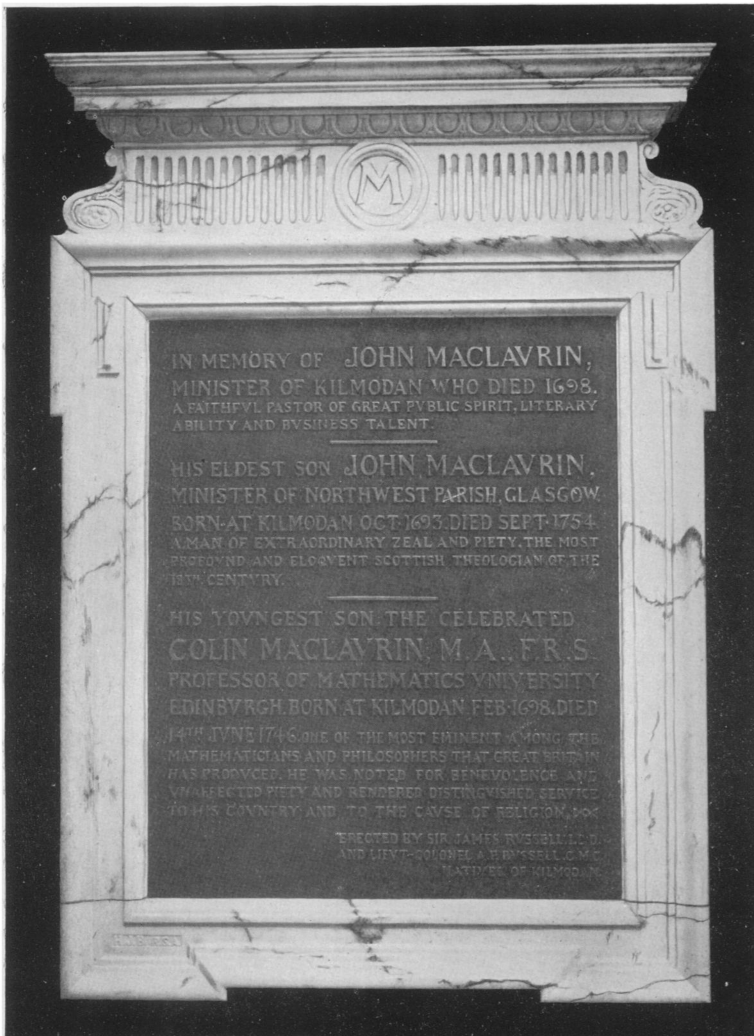
Hence L and the series are both finite or both infinite.

A first measure of the excess of the series over L is furnished by the sum of the triangles

$$PpQ+QqR\dots, \quad i.e. \text{ by } \tfrac{1}{2} \text{ rect. } A_p.$$

Hence, approximately,

$$y_a+y_{a+1}+\dots=L+\tfrac{1}{2}y_a.$$



MURAL MONUMENT, DESIGNED BY HIPPOLYTE BLANC, R.S.A., F.R.I.B.A.

Erected in the Parish Church of Kilmodan.

But Maclaurin's calculations go far beyond this first approximation. Naturally the method extends itself to the series

$$f(a)+f(a+1)+\dots,$$

when, after $x=a$, the graph of $y=f(x)$ continually descends to the x-axis, which it approaches asymptotically.

This leads to *Maclaurin's* (or *Cauchy's*) *Integral Test* for the convergence or divergence of an infinite series. "The test was discovered by Maclaurin, but forgotten, and rediscovered by Cauchy, to whom it is usually attributed" (Hardy, *Pure Mathematics*, p. 305).

POSTHUMOUS WORKS.

24. It being found that Maclaurin's family was but poorly provided for, there being little beyond a small estate in Berwickshire which hardly furnished the bare necessities of life, his friends, conspicuous among whom was Martin Folkes, P.R.S., bestirred themselves to place the widow and orphans in better circumstances. Maclaurin himself had entrusted the care of his manuscripts to Martin Folkes, A. Mitchell, M.P. for Aberdeenshire, and John Hill, Chaplain to the Archbishop of Canterbury.

By their efforts the MSS. of two works were published: *A Treatise of Algebra,* in 1748; and, in the same year, *An Account of Sir Isaac Newton's Philosophical Discoveries.*

Treatise of Algebra.

25. The *Algebra* immediately became a popular text-book, and passed through a large number of editions, maintaining its popularity up to the close of the eighteenth century. As in his other expository works, Maclaurin's intention was to popularise the corresponding work of his friend and master Newton; and the book is to be read as a commentary of the *Arithmetica Universalis.*

His literary executors also incorporated in it his contributions to Algebra published in the *Philosophical Transactions.*

Part I. is devoted to the general Rules of Algebra. Part II. treats of the Genesis and Resolution of Equations of all Degrees, and of the different Affections of the Roots. In Part III. he takes up the Application of Algebra and Geometry to each other, corresponding partly to the modern theory of Graphs. It contains an elementary exposition of Newton's Organic Description of Conics and of the Braikenridge-Maclaurin Theorem, of which mention has already been made. The last chapter discusses the solution of the Biquadratic and Cubic Equations in a single variable x by the intersections of two conics (in particular, by those of a circle and a parabola), a theory already given by Descartes in his *Geometry.*

The valuable *Appendix on the General Properties of Curves* has already been commented upon. A useful analysis of the *Algebra* is given in Moritz Cantor's *Geschichte der Mathematik* (Band II. 588-595).

Account of Sir Isaac Newton's Philosophical Discoveries.

26. This was the last work which received his close attention during his illness, when he had the assistance of an amanuensis, to whom he dictated the concluding chapters.

It was written on broad lines and furnished a popular introduction to the *Principia,* prefaced by a historical account of the study of Natural Philosophy. Some idea may be had of the estimation in which the author was held from the long list of over a thousand subscribers, including many of the aristocracy as well as distinguished contemporary scholars.

Its sale must have greatly helped the widow and family in their somewhat straitened circumstances. The work is prefaced by a very full account of the life and writings of Maclaurin (based mainly on the Panegyric of Dr. Monro, upon which much of the information here recorded depends).

APPENDIX.

(A) Contributions of Maclaurin to

(*a*) *The Philosophical Transactions* :

(i) On the construction and measure of curves. Vol. 30.
(ii) A new method of describing all kinds of curves. Vol. 30.
(iii) On equations with impossible roots. Vol. 34.
(iv) On the roots of equations, etc. Vol. 34.
(v) On the description of curves, etc., and a Paper, dated Nancy, 1722. Vol. 39.
(vi) Observations on a solar eclipse. Vol. 40.
(vii) A rule for finding the meridional parts of a spheroid, etc. Vol. 41.
(viii) An account of the ***Treatise of Fluxions.*** Vol. 42.
(ix) On the basis of the cells wherein the bees deposit their honey. Vol. 42.

The Royal Society is in possession of the MSS. of some of these Papers.

(*b*) *Physical and Literary Essays* (3 vols.) :

(i) On the variation of the obliquity of the ecliptic.
(ii) On the changes observed in the surface of Jupiter's body.

(*c*) *Medical Essays* :

Reference is made by Murdoch to contributions to the *Medical Essays*, but these do not appear in the later editions of these *Essays*.

(*d*) *MSS. in Possession of the University of Aberdeen* (see Library Catalogue) :

(i) On Fluxions. [Detached Papers or Notes, mostly in his own handwriting.]
(ii) An Account of Newton's Philosophical Discoveries. An Essay on the Motion of Fluids. An Essay on the Figure of the Earth.
(iii) Dissertationes Mathematicae Academicae. Letters to him on Mathematical Subjects. Miscellaneous Mathematical Papers.
(iv) Letters to Rob. Simson, etc. Narrative of a quarrel with Braikenridge. A Paper on the Combustion and Vital Powers of Air, with other Papers. An Account of his Tour on the Continent in 1722. Autograph Letters to him from Rob. Simson, Jas. Stirling, etc.

(B) Works Consulted, from which Further Information may be obtained.

Bower, *History of the University of Edinburgh*, 1817.
Carlyle, *Autobiography of the Rev. Dr. Alex. Carlyle*, 1860.
Chambers, *Biographical Dictionary of Eminent Scotsmen.*
Dictionary of National Biography.
Dreghorn, *The Works of John Maclaurin of Dreghorn*, 1798.
Edinburgh Mathematical Society Proceedings. Vol. XVII. (Gibson), Vol. XXX. (Teixeira).
Hutton, *Mathematical Dictionary.*
Murdoch's *Introduction to Maclaurin's Newton*, 1748.
Records of Marischal College, Aberdeen, 1898.
Smith's *Iconographia Scotica.*

M. Chasles, *Aperçu Historique sur l'origine et le développement des méthodes en géométrie*, Paris, 1887.

E. de Jonquières : (*a*) " Sur la géométrie organique de Maclaurin " (*Jour. de Math.* 1857) ; (*b*) *Mélanges de géométrie pure*, Paris, 1856. (Contains French translation of the *Properties of Curve Lines.*)

Poncelet, *Traité des propriétés projectives des figures.*

M. Cantor, *Geschichte der Mathematik*, Leipzig, 1901.

G. Loria, *Spezielle ebene Kurven*, Leipzig, 1902.

Articles on (*a*) " Proj. Geom." (Schönflies) ; (*b*) " Höhere alg. Kurven " (Berzolari) ; (*c*) " Spez. Kurven " (Kohn u. Loria), in the *Encycl. der math. Wissenschaften.*

THE SOLUTION OF NUMERICAL EQUATIONS.

By Eric H. Neville.

About a year ago, to solve a puzzle to which my attention had been drawn, I found it necessary to obtain an approximate numerical solution of extreme accuracy to a set of four trigonometrical equations in four variables. The method by which I succeeded in avoiding some of the labour involved in a direct application of the classical method of approximation I have described elsewhere,* but it has been suggested that an account of it will interest readers of the *Mathematical Gazette.* In this paper the method is introduced by means of a particular pair of equations in two variables, but it is applicable to any set of n independent equations in n variables, whatever integer n may be.

Elementary problems can easily be devised leading to a pair of equations of the form

$$\sin i = \mu \sin r, \quad a \tan i + b \tan r = k,$$

μ, a, b, k being positive constants and i and r being angles which it is required to evaluate. Such a pair of equations has one and only one solution in which i and r are positive acute angles, and it is to this solution that an approximation is wanted ; as a rule, a crude approximation is found by supposing i and r to be equal and to satisfy the second equation, that is, to have the tangent $k/(a+b)$. We will take the pair of equations

$$\sin i = 2 \sin r, \quad \tan i + \tan r = 1$$

to illustrate our method of proceeding with the subsequent calculation.

The first approximation is

$$i_1 = r_1 = 26\tfrac{1}{2}^\circ.$$

For a second approximation, we can take

$$i_2 = i_1 + \iota_2, \quad r_2 = r_1 + \rho_2,$$

where ι_2 and ρ_2 expressed in radians satisfy simultaneously the equations

$$\sin i_1 + \iota_2 \cos i_1 = 2(\sin r_1 + \rho_2 \cos r_1), \quad \tan i_1 + \iota_2 \sec^2 i_1 + \tan r_1 + \rho_2 \sec^2 r_1 = 1,$$

of which the second is equivalent to

$$\iota_2 + \rho_2 = 0,$$

so that

$$\iota_2 = \tfrac{1}{6}, \quad \rho_2 = -\tfrac{1}{6} ;$$

returning to degrees, we may take

$$i_2 = 36\tfrac{1}{2}^\circ, \quad r_2 = 16\tfrac{1}{2}^\circ.$$

* "On the Solution of Numerical Functional Equations", *Proc. Lond. Math. Soc.*, vol. xiv. (1914-5), p. 308.