Optical Orthogonal Code-Division Multiple-Access System—Part I: APD Noise and Thermal Noise

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***Abstract*— In an optical multiple-access system, overall system throughput efficiency and significant implementation cost-reduction would be achieved if many users could access a common optical channel at any time without control among users. Recently one such scheme, an optical orthogonal code division multiple-access system (OOCDMA), was introduced by Salehi *et al.* [1]–[6] for the case of no noise. In this paper, some extensions of the work in [2] are presented, including the effects of avalanche photodiode (APD) noise and thermal noise as well as interference for the OOCDMA direct-detection receiver. Since it has been shown [2], [3] that an optical hard-limiter before the receiver correlator can reduce the interference effect for the OOCDMA system in the absence of noise, the hard-limiter role in the presence of thermal and APD noise is also examined.**

I. Introduction

A COST-EFFECTIVE and throughput-efficient optical orthogonal code division multiple-access system (OOCDMA) was recently introduced by Salehi *et al.* [1]–[6], as shown in Fig. 1. Little or no electronic-optical domain conversion and no synchronization is required in the network. There are N transmitter/receiver pairs (users) (Fig. 1); each user is assigned a unique signature or address sequence. The signature sequence considered in a direct-detection OOCDMA system consists of true (0, 1) sequences that have no negative components, while most documented correlation sequences (e.g., maximal length sequences or Gold sequences) are actually $(+1, -1)$ sequences intended for systems having both positive and negative components. This distinction produces quite different results [1]–[6]. The set of all user signature sequences is called an optical orthogonal code if it satisfies the following two properties: 1) the off-peaks of cyclic autocorrelation of any sequence in the code do not exceed 1, which means the sequence is orthogonal to its shifted version, and 2) the cyclic cross-correlation between any two sequences in the code does not exceed 1, which means the two sequences are mutually orthogonal. In this paper, the optical orthogonal code is considered.

To transmit information from user j to user k, the address sequence for receiver k is imprinted upon the data by user j's optical encoder. The received OOCDMA signal in the common channel is the sum of N users' transmitted signals. At the receiving end, the desired user's receiver is a periodic correlator. The off-peaks of the desired user's correlator output might be higher than the peak at the correlation time (i.e., the proper shifted time), which causes bit errors if $(N-1)$ other users' signals are present and if they interfere the desired signal sequence.

In [1] and [2] the error probability of an ideal OOCDMA was computed including the interference effect on the desired signal, however, any kind of noise at the receiver was not considered. In this paper, some extensions of the work in [2] are presented. In particular, the effects of avalanche photodiode (APD) noise, thermal noise, and interference for the OOCDMA direct-detection receiver are included. The statistical description of the response of APD to incident photons, as given by McIntyre and Conradi [7], [8], is very complicated. Most analyses in the literature have modeled the APD output as a Gaussian process. In this paper the Gaussian approximation of the APD output [9] is employed.

Since it has been shown [2], [3] that an optical hard-limiter before the receiver correlator can reduce the interference effect for the OOCDMA system, in the absence of noise, the hard-limiter role in the presence of thermal and APD noise will also be examined. An exact analysis will be performed for the optical hard-limiter with thermal noise and APD noise present in a chip-synchronous case. In a chip-synchronous OOCDMA system, chips among users are assumed to be synchronized, while in a chip-asynchronous OOCDMA system, all users are allowed to transmit at any time. The chip-synchronous system is considered because 1) it is amenable to analysis while analysis for the chip-asynchronous case is very difficult, and 2) its bit error probability is an upper bound of that of the chip-asynchronous system [2] because the interference effect is more serious if chips are synchronized between users.

Section II describes the OOCDMA system model. Section III presents the analysis of an OOCDMA system without hard-limiter for chip-synchronous, and for chip-asynchronous. Section IV finds the performance of the OOCDMA system with hard-limiter for the chip-synchronous system. Section V provides numerical results, and Section VI gives conclusions.

II. System Model

For N transmitter/receiver pairs, the optical signal (baseband) of the nth user can be written as

$$s_n(t) = Pb_n(t)c_n(t), \qquad 0 \le t \le T = FT_c \tag{1}$$

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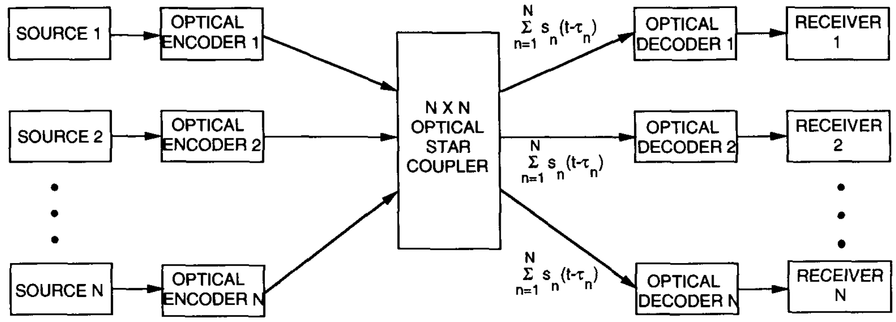

Fig. 1. An optical code division multiple-access system in a star configuration.

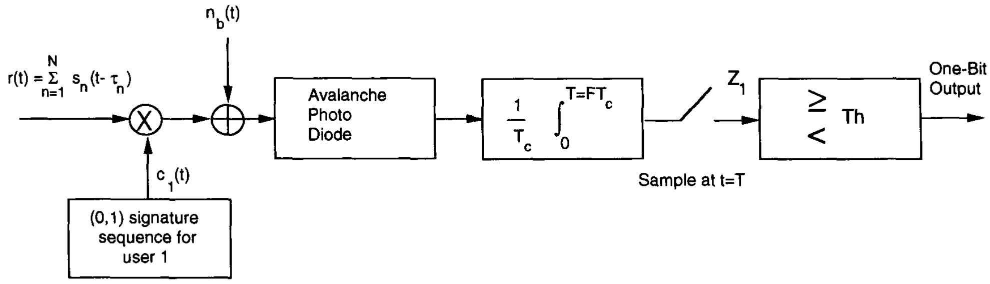

Fig. 2. The configuration of receiver 1 without hard limiter for one-bit/sequence-period transmission.

where P is a user's received laser power, $b_n(t)$ is the nth user's (0, 1) binary data sequence, $c_n(t)$ is the signature code waveform generated by the OOCDMA sequence assigned to the nth user for a single bit transmission per sequence-period, T_c is the chip time interval, F is sequence length, and T is the code symbol time. The signature signal can be written as

$$c_n(t) = \sum_{j=-\infty}^{\infty} c_n(j)P_{T_c}(t - jT_c), \qquad n = 1, 2, \cdots, N \quad (2)$$

where $c_n(j)$ is either 0 or 1, and also $c_n(j) = c_n(j+F)$ for all j and n for sequence length F, and the term P with subscript T_c is a unit rectangular pulse of duration T_c. We assume that sequence-period $T = FT_c$ so that there is one code sequence period $\boldsymbol{c_n} = (c_n(0), c_n(1), \cdots, c_n(F-1))$ per code symbol. The code symbol time interval is a bit time interval. For any sequence $\boldsymbol{c_n}$ and $\boldsymbol{c_m}$ in OOCDMA code, autocorrelation and cross-correlation are written, respectively, as

$$R_{n,n}(l) = \sum_{j=0}^{F-1} c_n(j)c_n(j-l) = \begin{cases} K & \text{for } l = 0 \\ \leq 1 & \text{for } 1 \leq l \leq F-1 \end{cases} \quad (3)$$

and

$$R_{n,m}(l) = \sum_{j=0}^{F-1} c_n(j)c_m(j-l) \leq 1 \qquad \text{for } 0 \leq l \leq F-1 \text{ and } n \neq m. \quad (4)$$

Here, each sequence is an OOCDMA code has an equal number of 1's (or marks), i.e., a uniform weight K. In general, for a given code sequence of length F and weight K, $K(K-1) \leq F-1$ from the autocorrelation property (see [1]). In addition, from the cross-correlation property, the maximum number of users N in OOCDMA is

$$N = \lfloor (F-1)/(K(K-1)) \rfloor \quad (5a)$$

where $\lfloor x \rfloor$ is the largest positive integer less than or equal to x, and given F and N, the number of marks is bounded by

$$1 \leq K \leq \frac{1 + \sqrt{1 + 4(F-1)/N}}{2} \quad (5b)$$

Fig. 2 shows a typical OOCDMA receiver for user 1, using the active optical components.[1] The received signal is modeled as

$$r(t) = \sum_{n=1}^{N} s_n(t - \tau_n) \quad (6)$$

where τ_n is the associated delay for a given receiver. This delay accounts for the lack of synchronization between transmitters, and it is an integer times T_c and a real number times T_c for the chip-synchronous and chip-asynchronous cases, respectively.

We assume that the detection system is synchronous with the first user and all delays are relative to the first user delay (where $\tau_1 = 0$). Furthermore, there is no loss of generality in assuming $0 \leq \tau_n \leq T$ for $2 \leq n \leq N$ since we are concerned only with the time delays modulo T. The received signal $r(t)$ is multiplied by a stored replica of $c_1(t)$ for a

[1] The correlation processing can be implemented by an equivalent passive optical tapped delay line [10]. In this paper, the active optical correlator model is employed for the analysis.

single-bit transmission per sequence-period (see Fig. 2). The actual background light $n_b(t)$ is added before the sequence despreading. However, it is added after the sequence despreading in this analysis because it is assumed that the despread background light has the same characteristics as the unspread background light. The background light (noise) is considered in connection with free-space optical communications; it can be neglected in a fiber network.

The probability that a specified number of photons are absorbed from an incident optical field by an APD detector over a chip interval $(t, t + T_c)$ is given by a Poisson distribution. The average number of absorbed photons is $\lambda_s T_c$ where λ_s represents the photon absorption rate due to a mark[2] transmission in the desired user sequence which can be written as

$$\lambda_s = \frac{\eta P}{hf}. \tag{7}$$

P is the received laser power, η is the APD efficiency with which the APD converts incident photons to photoelectrons, h is Plank's constant equal to 6.624×10^{-34}, and f is the optical frequency. The optical frequency is equal to the speed of light divided by wavelength. A 825 nm wavelength, currently available, is chosen in this paper. In the APD detector, primary photoelectron-hole pairs undergo an avalanche multiplication process which results in the output of m electrons from the APD in response to the absorption of λT_c primary photons, on average. The conditional probability density of m given λT_c is characterized by the Conradi distribution [7], [8]. Here, λ represents the total photon absorption rate due to signal, background light, and APD bulk leakage current

$$\lambda = \begin{Bmatrix} \lambda_s + \lambda_b + I_b/e & \text{for mark} \\ \lambda_s/M_e + \lambda_b + I_b/e & \text{for space} \end{Bmatrix} \tag{8}$$

where λ_s was given in (7), λ_b is the photon absorption rate due to the actual background light $n_b(t)$, e is an electron charge equal to 1.601×10^{-19} Coulomb, I_b/e represents the contribution of the APD bulk leakage current to the APD output, and M_e is the extinction ratio of the laser diode output power in the mark and space states. (See details in Table I for parameters chosen in this paper.) The accumulated output during each chip interval is assumed to be a Gaussian random variable [9]. Suppose that the kth mark of the 1st user sequence $c_1(t)$, $k = 1, 2, \cdots, K$, was hit i_k times (i_k is an integer ≥ 0 for chip-synchronous and a real number ≥ 0 for chip-asynchronous, respectively) by $N - 1$ other users' marks (see Fig. 4). Any undesired user sequence $c_n(t)$ can hit at most once the desired sequence $c_1(t)$ per sequence-period due to the cross-correlation property of optical orthogonal codes. At the kth mark interval of the desired sequence for "1" bit data symbol transmission, $(i_k + 1)$ marks (including the desired signal's mark) are incident upon the APD with λ_s photon arrival rate and $N - 1 - i_k$ spaces are incident upon the APD with λ_s/M_e rate. Hence, at the kth mark interval of $c_1(t)$, the total photon absorption rate due to signal plus interference is $(i_k + 1)\lambda_s + (N - 1 - i_k)\lambda_s/M_e + \lambda_b + I_b/e$, since a sum of

[2]"Mark" and "space" mean "1" and "0," respectively, in a signature sequence, $c_n(t)$.

TABLE I
EXAMPLE NONIDEAL LASER LINK NOMINAL PARAMETERS

Name	Symbol	Value
(Laser wavelength= 825 nm), Optical frequency=light speed/wavelength	f	3.634×10^{14} Hz
APD Quantum Efficiency	η	0.6
APD Gain	G	100
APD Effective Ionization Ratio	k_{eff}	0.02
APD Bulk Leakage Current	I_b	0.1 nA
APD Surface Leakage Current	I_s	10 nA
Background Light Photon Arrival Rate	λ_b	10^9 counts/s
Modulation Extinction Ratio	M_e	100
Data Bit Rate for 1 bit/sequence-period is from $T = FT_c = T_b$ where F =sequence-period	$R_b = 1/T$	30 Mbs
Data Bit Rate for $\log_2 F$ bits/sequence-period is from $T = FT_c = \log_2 FT_b$ (studied in Part II)	$R_b = \log_2 F/T$	$\log_2 F \times 30$ Mbs
Receiver Noise Temperature	T_r	1100°K
Receiver Load Resistor	R_L	1030 Ω

independent Poisson random variables with photon absorption rates λ_a and λ_b is also a Poisson random variable, with photon absorption rate $(\lambda_a + \lambda_b)$.

The receiver integrates the APD output over code symbol interval $T = FT_c$ (see Fig. 2). The integral APD output is

$$Z_1 = \frac{1}{T_c}\int_0^T \text{APD}(r(t)c_1(t) + n_b(t))\, dt \tag{9}$$

at the correlation time. For the analysis, the integral is broken into F number of chip intervals. The random variables (the number of absorbed photons) over each chip interval of $c_1(t)$ are independent. Let the random variable $X_{s,i}$ and $X_{m,k}$, $i = 1, 2 \cdots F - K$, $k = 1, 2 \cdots K$, denote the integrated detector output during the ith space interval and the kth mark interval of $c_1(t)$ in (9), respectively. Then the accumulated output of user 1 in Fig. 2 can be written as

$$Z_1 = \sum_{k=1}^{K} X_{m,k} + \sum_{i=1}^{F-K} X_{s,i} \tag{10}$$

at the correct correlation time for a single-bit transmission per sequence-period. Each $X_{s,i}$ and each $X_{m,k}$, $i = 1, 2 \cdots F - K$, $k - 1, 2 \cdots K$, are independent Gaussian random variables whose distributions are given by [9]. In addition, Z_1 is also a Gaussian random variable (whose distribution is given later).

The receiver for a single-bit transmission per sequence-period chooses "1" if Z_1 is greater than a threshold, Th, and chooses "0" otherwise[3] (Fig. 2). Let I_1 be the total undesired interference contribution to the desired receiver's accumulated output Z_1 during a sequence period at the correlation time (see Figs. 2 and 4):

$$I_1 = \sum_{k=1}^{K} i_k. \tag{11}$$

No photon except the background light is passed through the APD during a space interval of the desired signal because the received laser signal is multiplied by the (0, 1) signature sequence before the APD (see Fig. 2). In other words, photons on K chip intervals of the desired signal are passed and $F-K$ intervals are blocked. Each of N users contribute K pulses (either marks or spaces) during the K chip open intervals of the desired signal. Hence, the total number of either spaces or marks due to N users during the K open chip intervals is KN. Among KN pulses (either marks or spaces), $K+I_1$ marks arrive at λ_s incident photon arrival rate if I_1 marks from the $(N-1)$ undesired signals hit the K open chip intervals of the desired signal and if "1" bit data symbol is transmitted. Vice versa $KN-(K+I_1)$ spaces arrive at λ_s/M_e incident photon arrival rate for the desired receiver's accumulated output, Z_1. In addition, background light, bulk leakage current, surface leakage current, and thermal noise are active for each chip interval. Hence, given I_1 and the first user's bit transmission $b =$"1," the conditional probability density function (pdf) of the accumulated output Z_1 in (10) can be written as

$$p_{Z_1}(z \mid I_1, b=1) = \frac{1}{\sqrt{2\pi\sigma_{b_1}^2}} e^{-(z-\mu_{b_1})^2/2\sigma_{b_1}^2} \tag{12}$$

for a single-bit transmission per sequence-period system where the mean μ_{Z1} and variance σ_{Z1}^2 of Z_1 are

$$\begin{aligned}\mu_{b1} = GT_c[(K+I_1)\lambda_s + (KN-(K+I_1))\lambda_s/M_e \\ + F(\lambda_b + I_b/e)] + FT_cI_s/e,\end{aligned} \tag{13}$$

$$\begin{aligned}\sigma_{b1}^2 = G^2F_eT_c[(K+I_1)\lambda_s + (KN-(K+I_1))\lambda_s/M_e \\ + F(\lambda_b + I_b/e)] + F(T_cI_s/e + \sigma_{th}^2).\end{aligned} \tag{14}$$

Here, G is the average APD gain, I_s the APD surface leakage current, F_e the excess noise factor given by

$$F_e = k_{\text{eff}}G + (2 - 1/G)(1 - k_{\text{eff}}) \tag{15}$$

where k_{eff} is the APD effective ionization ratio, and σ_{th}^2 the variance of thermal noise which can be written as

$$\sigma_{th}^2 = 2k_BT_rT_c/(e^2R_L) \tag{16}$$

where k_B is Boltzmann's constant equal to 1.379×10^{-23}, T_r is the receiver noise temperature, and R_L is the receiver load resistor.

[3]The decision statistic at the desired receiver Z_1 is the number of electrons accumulated for $T = FT_c$ seconds. Hence, the threshold in this paper has no unit. For a practical application, the number of accumulated electrons can be converted into current by multiplying $e = 1.601 \times 10^{-19}$ electron charge and dividing by T seconds.

For a "0" bit data symbol transmission, I_1 marks arrive at λ_s incident photon arrival rate and $KN - I_1$ spaces arrive at λ_s/M_e rate for the desired receiver's accumulated output, Z_1. In addition, background light, bulk leakage current, surface leakage current, and thermal noise are active for each chip interval. Hence, the conditional density of Z_1 is

$$p_{Z_1}(z \mid I_1, b=0) = \frac{1}{\sqrt{2\pi\sigma_{b_0}^2}} e^{-(z-\mu_{b_0})^2/2\sigma_{b_c}^2} \tag{17}$$

for a single-bit transmission per sequence-period system where

$$\begin{aligned}\mu_{b0} = GT_c[I_1\lambda_s + (KN-I_1)\lambda_s/M_e \\ + F(\lambda_b - I_b/e)] + FT_cI_s/e,\end{aligned} \tag{18}$$

$$\begin{aligned}\sigma_{b0}^2 = G^2F_eT_c[I_1\lambda_s + (KN-I_1)\lambda_s/M_e \\ + F(\lambda_b + I_b/e)] + F(T_cI_s/e + \sigma_{th}^2).\end{aligned} \tag{19}$$

III. OOCDMA System Without Hard-Limiter

A. Chip-Synchronous, One-Bit/Sequence-Period, without Hard-Limiter

This code (Fig. 2) assumes that only one (0, 1) binary data symbol is transmitted per sequence-period. In addition, chips are assumed to be synchronous between users (which implies that the number of hits i_k on the kth mark of $c_1(t)$ by $(N-1)$ undesired users' marks is an integer ≥ 0. However, users are not synchronized, i.e., the relative delay τ_n to τ_1 is an arbitrary integer times T_c). Furthermore, the code assumes that the detection system is synchronous with the first user $(\tau_1 = 0)$ and that the hard-limiter is not employed before the optical despreader.

The pdf of I_1 (total undesired interference contribution to the desired receiver's accumulated output at the correlation time) can be written for the chip-synchronous case as [2, eq. (10)]

$$p_{I_1}(i) = \sum_{i=0}^{N-1} \binom{N-1}{i} p^i q^{N-1-i} \delta(I_1 - i) \tag{20}$$

where $p = K^2/2F$, $q = 1-p$, and $\delta(x)$ is the Dirac delta function. Because the receiver chooses a "1" if $Z_1 > Th$, and chooses "0" otherwise, the optimum receiver uses the value of Th which minimizes the overall bit error probability. From (12)–(20), the average bit error probability, P_b(Error), over I_1 and the first user bit b, is

$$\begin{aligned}P_b(\text{Error}) &= \min_{Th} E_{I_1,b}[P_b(\text{Error} \mid I_1, b, Th)] \\ &= \min_{Th} \frac{1}{2}\left[1 + \sum_{i=0}^{N-1} p_{I_1}(i)\left\{Q\left(\frac{Th - \mu_{b0}(i)}{\sigma_{b0(i)}}\right) \right.\right. \\ &\qquad \left.\left. - Q\left(\frac{Th - \mu_{b1}(i)}{\sigma_{b1(i)}}\right)\right\}\right]\end{aligned} \tag{21}$$

where $E_X[Y]$ is the average of Y over X and $Q(\alpha)$ is the integral of the normal density from α to infinity. Here, $\mu_{b1}(i)$, $\sigma_{b1}^2(i)$, $\mu_{b0}(i)$, and $\sigma_{b0}^2(i)$ are given in (13), (14), (18), and (19) with $I_1 = i$, respectively.

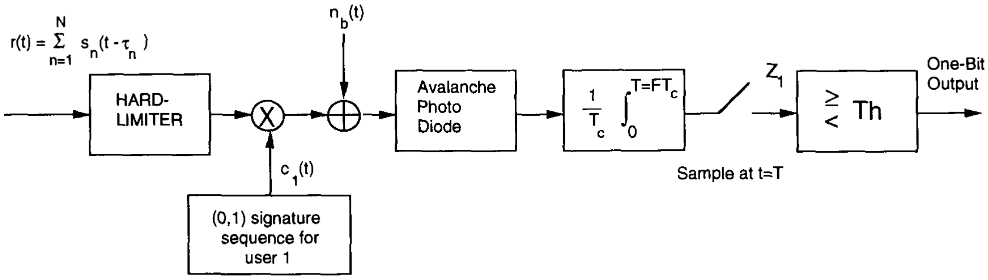

Fig. 3. The configuration of receiver 1 with hard limiter for one-bit/sequence-period transmission.

B. Chip-Asynchronous, One-Bit/Sequence-Period, without Hard-Limiter

This code assumes that only one (0, 1) binary data symbol is transmitted per sequence-period, and that chips are asynchronous between users (which implies that i_k in (11) is a real number ≥ 0). Also, the hard-limiter is not employed before the optical despreader. Then the pdf of total interference I_1 at the correlation time can be approximated (see [2, eq. (11)]) as

$$p_{I_1}(x) = \sum_{i=0}^{N-1} \binom{N-1}{i} p^i q^{N-1-i} f_i(x) \tag{22}$$

where $p = K^2/F$, $q = 1 - p$, $f_0(x) = \delta(x)$,

$$f_1(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases},$$

$$f_2(x) = \begin{cases} 1 - |x-1| & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases},$$

$$f_i(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-(x-m_i)^2/2\sigma_i^2} & \text{for } 0 < x < i \\ 0 & \text{otherwise} \end{cases}$$

for i, $3 \leq i \leq N-1$ where $m_i = i/2$ and $\sigma_i^2 = i/12$.

The average bit error probability over I_1, can be written as

$$P_b(\text{Error}) = \min_{Th}\left\{q^{N-1} P_b(\text{Error} \mid I_1 = 0, Th) + \int_0^{N-1} p_{I_1}(x) P_b(\text{Error} \mid I_1 = x, Th)\, dx\right\} \tag{23}$$

where $P_b(\text{Error} \mid I_1, Th)$ is the average (over the first user bit information) conditional bit error probability given I_1 which can be written as

$$P_b(\text{Error} \mid I_1 = x, Th) = \frac{1}{2}\left[1 + Q\left(\frac{Th - \mu_{b0}(x)}{\sigma_{b0}(x)}\right) - Q\left(\frac{Th - \mu_{b1}(x)}{\sigma_{b1}(x)}\right)\right]. \tag{24}$$

Again, $\mu_{b1}(x)$, $\sigma_{b1}^2(x)$, $\mu_{b0}(x)$, and $\sigma_{b0}^2(x)$ are given in (13), (14), (18), and (19) with $I_1 = x$, respectively.

IV. OOCDMA System with Hard-Limiter (Chip-Synchronous, One-Bit/Sequence-Period)

Fig. 3 shows the receiver block diagram. The receiver has the same configuration as Fig. 2 (for chip-synchronous and one-bit transmission per sequence-period) except a hard-limiter is placed before the optical despreader. If optical light power intensity is larger than or equal to the unit laser chip-pulse power P, the hard-limiter clips the intensity back to the unit laser chip-pulse power; if the optical light power intensity is smaller than the unit power, the response of the optical hard-limiter is zero. As indicated by [2], for the ideal optical link, the hard-limiter would enhance system performance because it would exclude some combinations of interference which cause the decision variable Z_1 to be larger than threshold Th for "0" bit transmission and yield incorrect bit decisions. Fig. 4 shows such an example of an interference pattern on the desired signal over a sequence period. Here i_k denotes the number of hits on the kth mark of the desired user signal $c_1(t)$, $k = 1, 2, \cdots, K$, by $N-1$ undesired interferers' marks. Let $\boldsymbol{i} \equiv (i_1, i_2, \cdots, i_K)$ be the interference state pattern, I_1 the total summation of i_k, and $|\boldsymbol{i}|$ the number of nonzero elements in $\boldsymbol{i}$. Then the performance of the system with hard-limiter depends on $|\boldsymbol{i}|$ as well as I_1, while the system performance without hard-limiter depends on only I_1.

A user is equally likely to incur interference at any one of the K pulse positions independent of all other users. Thus, the interference pattern vector $\boldsymbol{i}$ obeys a multinomial distribution [11]–[12]. Hence, using the results in [11]–[12], the probability that interference pattern $\boldsymbol{i}$ has m nonzero elements, $m = 1, \cdots,$ minimum (K, I_1), can be found as

$$\Pr(|\boldsymbol{i}| = m \mid I_1) = \sum_{\substack{\boldsymbol{i} \in G_{I_1} \\ \text{and } |\boldsymbol{i}| = m}} NDP(\boldsymbol{i}) P(\boldsymbol{i}; F_{I_1}) \tag{25}$$

where F_{I_1} is the set of all interference pattern vectors with total weight equal to I_1, G_{I_1} is the set of representative interference vectors in F_{I_1} with elements in decreasing order, $NDP(\boldsymbol{i})$ is the number of distinct permutations of vector $\boldsymbol{i}$ in G_{I1} which is $(K!)/(\Pi_k R(i_k)!)$ where $R(i_k)!$ is the number of repetition times of an element i_k in vector $\boldsymbol{i}$ and the product is understood to be taken over k for which i_k are distinct, and $P(\boldsymbol{i}; F_{I_1})$ is the multinomial distribution for the interference pattern vector $\boldsymbol{i}$ in F_{I_1} which is $(I_1!)/(K^{I_1}$ times $\Pi_k(i_k!)), m$, $k = 1, \cdots, K)$. For example, consider the case $I_1 = 5$, $K = 3$, and $m = 3$. The set G_5 consists of five distinct vectors: 500, 410, 320, 311, 221. Only two vectors, 311 and 221, have $m = 3$ nonzero elements. The number of distinct permutations of vector 311, $N(311)$, is $3!/(1!2!) = 3$, and $N(221)$ is $3!/(2!1!) = 3$. The multinomial probability distribution for

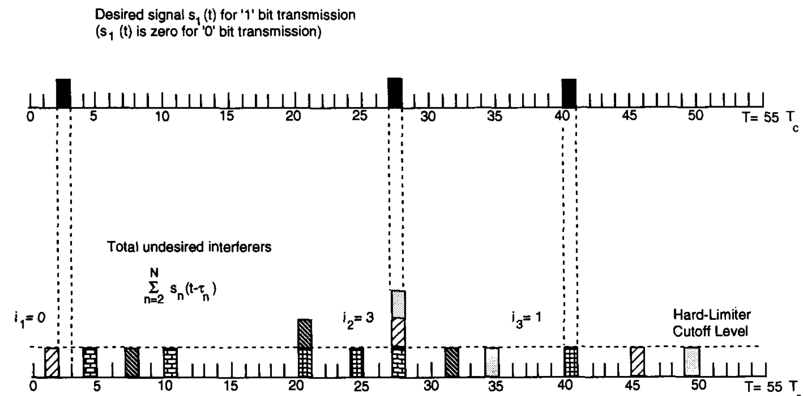

Fig. 4. An example of interference on the desired signal for a chip-synchronous system. Here i_k denotes the number of hits on the kth "mark" of the desired signal $s_1(t)$, $k = 1, 2, \cdots, K$, by $N-1$ undesired interferers' "marks." For illustration, $K = 3$ "marks," $N = 6$ users, and $F = 55$ chips/period were chosen, and signature sequences selected are assumed to satisfy the orthogonality conditions. If the threshold is 2.5, then a bit error happens for "0" bit transmission in the ideal link without hard-limiter, versus no bit error with a hard-limiter in this example.

vector 311, $P(311\colon F_5)$, is $5!/(3^5 3!1!1!)$, and $P(221\colon F_5)$, is $5!/(3^5 2!2!1!)$. Hence, the probability that an interference pattern has $m = 3$ nonzero elements, $\Pr(|\boldsymbol{i}| = m = 3 \mid I_1 = 5)$, is $3 \times 5!/(3!1!1!)/3^5 + 3 \times 5!/(2!2!1!)/3^5 = 0.617$.

For "0" bit transmission and a practical extinction ratio of the laser diode output power in the mark and space states, e.g., $M_e = 100$, the total N users' spaces contribution NP/M_e, to the kth mark of the desired user signal $c_1(t)$, can be larger than the unit laser chip-pulse power P if the number of users N is larger than the extinction ratio M_e. In this case (i.e., $N \ge M_e$) the hard-limiter clips the intensity back to the unit laser chip-pulse power. In the other case (i.e., $N < M_e$), the output of the hard-limiter is zero. Let i_{M_e} denote such an indicator function:

$$i_{M_e} = \left\{\begin{matrix} 1 & \text{if } N \ge M_e \\ 0 & \text{if } N < M_e \end{matrix}\right\}. \tag{26}$$

For "0" bit transmission and $|\boldsymbol{i}| = 0$ case, $K \cdot i_{M_e}$ pulses after the hard-limiter are incident upon the APD with λ_s photon arrival rate. For "0" bit transmission and $|\boldsymbol{i}| = m > 0$, $m + (K-m) \cdot i_{M_e}$ pulses after the hard-limiter are incident upon the APD with λ_s photon arrival rate. For "1" bit transmission and any $\boldsymbol{i}$ case, K pulses after the hard-limiter are incident upon the APD with λ_s photon arrival rate because the optical light power intensity is larger than or equal to the unit power intensity due to the signal presence at any mark interval of $c_1(t)$.

Therefore, the conditional bit error probability for a "0" bit transmission can be expressed as

$$\begin{aligned} &P_b(\text{Error}|\text{“0” bit}, Th) \\ &= \Pr(Z_1 \ge Th \mid I_1 = 0, \text{“0” bit}) p_{I_1}(I_1 = 0) \\ &\quad + \sum_{j=1}^{N-1} \Pr(Z_1 \ge Th \mid I_1 = j, \text{“0” bit}) p_{I_1}(j) \\ &= \Pr(Z_1 \ge Th \mid I_1 = 0, \text{“0” bit}) p_{I_1}(I_1 = 0) \\ &\quad + \sum_{j=1}^{N-1} \sum_{m=1}^{\min(K, I_1 = j)} \Pr(Z_1 \ge Th \mid I_1 = j, |\boldsymbol{i}| = m, \text{“0” bit}) \\ &\quad \cdot p_{I_1}(j) \Pr(|\boldsymbol{i}| = m | I_1 = j) \\ &= Q\left(\frac{Th - \mu_0}{\sigma_0}\right) p_{I_1}(I_1 = 0) + \sum_{j=1}^{N-1} \sum_{m=1}^{\min(K, I_1 = j)} \\ &\quad \cdot Q\left(\frac{Th - \mu_0(m)}{\sigma_0(m)}\right) p_{I_1}(j) \Pr(|\boldsymbol{i}| = m \mid I_1 = j) \end{aligned} \tag{27}$$

where $\mu_0 = \mu_{b0}$ from (18) with I_1 in the first term replaced by $K \cdot i_{Me}$ and $(KN - I_1)\lambda_s/M_e$ in the second term replaced by zero, $\sigma_0^2 = \sigma_{b0}^2$ from (19) with I_1 in the first term replaced by $K \cdot i_{Me}$ and $(KN - I_1)\lambda_s/M_e$ in the second term replaced by zero, $\mu_0(m) = \mu_{b0}$ from (18) with I_1 in the first term replaced by m and $(KN - I_1)\lambda_s/M_e$ in the second term replaced by $(K - m) \cdot i_{Me} \cdot \lambda_s$, and $\sigma_0^2(m) = \sigma_{b0}^2$ from (19) with I_1 in the first term replaced by m and $(KN - I_1)\lambda_s/M_e$ in the second term replaced by $(K - m) \cdot i_{Me} \cdot \lambda_s$, $p_{I_1}(j)$ is in (20), and $\Pr(|\boldsymbol{i}| = m \mid I_1 = j)$ is in (25).

For a "1" bit transmission, the conditional bit error probability can be written as

$$\begin{aligned} &P_b(\text{Error} \mid \text{“1” bit}, Th) \\ &\quad = \sum_{j=0}^{N-1} \Pr(Z_1 < Th \mid I_1 = j, \text{“1” bit}) p_{I_1}(j) \\ &\quad = \sum_{j=0}^{N-1} \left(1 - Q\left(\frac{Th - \mu_1}{\sigma_1}\right)\right) p_{I_1}(j) \end{aligned} \tag{28}$$

where $\mu_1 = \mu_{b1}$ from (13) with $(K + I_1)$ in the first term replaced by K and $(KN - (K + I_1))\lambda_s/M_e$ in the second term replaced by zero, and $\sigma_1^2 = \sigma_{b1}^2$ from (14) with $(K + I_1)$ in the first term replaced by K and $(KN - (K + I_1))\lambda_s/M_e$ in the second term replaced by zero. The overall bit error probability can be computed as

$$P_b = \min_{Th} \frac{1}{2}\{P_b(\text{Error} \mid \text{"0" bit}, Th) + P_b(\text{Error} \mid \text{"1" bit}, Th)\} \quad (29)$$

using (27) and (28).

V. Numerical Results

In this paper, each user's data bit rate is 30 Mb/s, and nonideal laser link parameters are listed in Table I.

A. Chip Synchronous, One-Bit/Sequence-Period, without Hard-Limiter

Fig. 5 shows bit error probability versus decision threshold, Th, with received laser power P as a parameter, for $N = 10$ users and $K = 10$ marks in a code signature sequence of length $F = 1000$. First, observe that the optimum threshold to minimize the bit error probability increases as the received laser power increases. This is simply because more photons arrive as the received laser power increases. Second, observe that (especially for a high received laser power case, like -45 dBW) the bit error probability curve decreases stepwise as the threshold approaches the optimum point from the left-hand side, e.g., for $P = -45$ dBW and $N = 10$ users, there are 10 steps. This is a reasonable result because for large received laser power, the background light, thermal noise, and leakage currents are negligible and hence the performance is sensitive to total interference contribution (which can be $0, 1, 2, \cdots, N-1$ marks because each undesired interferer can contribute at most a mark to the desired receiver accumulated output), while for small received laser power, the background light, noise, and leakage currents are not negligible and hence the performance curve is smooth as the threshold changes. Last, observe that the bit error probability approaches 1/2 as the threshold increases above the optimum point. This is an obvious result because a bit error occurs whenever "1" is transmitted, and there is no bit error whenever "0" is transmitted, due to the decision threshold value being too high.

Fig. 6 shows bit error probability versus threshold normalized by the optimum threshold of $K = 1$, with K marks as a parameter, for $N = 10$ users, a code sequence of length $F = 1000$, and $P = -45$ dBW received laser power. The system performance of the ideal link from [2] (i.e., no APD noise, no thermal noise, no leakage currents, etc.) is also shown on the left side in Fig. 6, for comparison. First, observe that the normalized optimum threshold is quite different from the threshold for the ideal link even for $P = -45$ dBW, and the minimum bit error probability of non-ideal laser link can be two orders larger than that of the ideal link, for given practical conditions in Fig. 6 and Table I. (For the ideal link with $K = 10$, the minimum bit error probability is zero at a normalized threshold = 10, which is not shown in Fig. 6). Second, observe that system performance at the optimum threshold becomes better as K increases. This is because more photons arrive since more marks (with constant power) are open for "1" signal bit transmission and hence total interference positive contribution to the desired receiver accumulated output at correlation time increases as K increases while, for "0" signal bit transmission, the interference negative contribution to the desired receiver accumulated output at correlation time can be suppressed by choosing a threshold larger than the total undesired negative contribution.

Fig. 7 is the same as Fig. 6 except the received laser power P is -20 dBW. The normalized optimum threshold for a nonideal laser link agrees with that for the ideal link, and the minimum bit error probability of a nonideal laser link is somewhat better than that of the ideal link. This is because

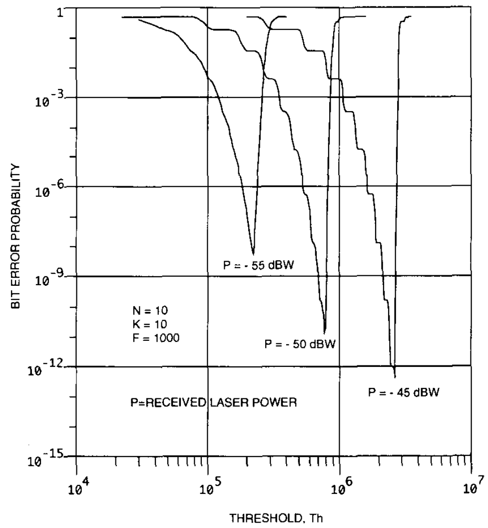

Fig. 5. Bit error probability versus threshold Th with received laser power P as a parameter, for $N = 10$ users, $K = 10$ "marks," $F = 1000$ chips, one bit per sequence-period, and chip-synchronous OOCDMA system without hard-limiter.

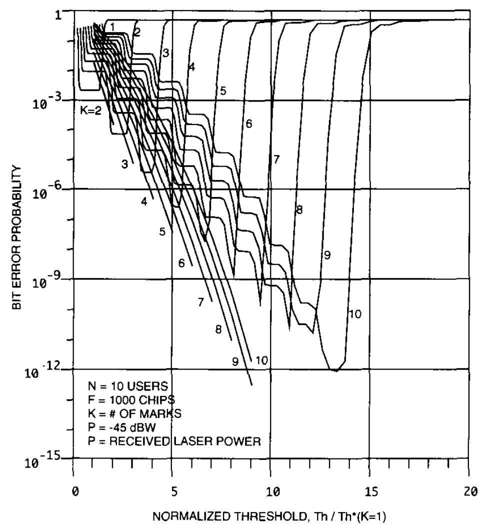

Fig. 6. Bit error probability versus normalized threshold, Th/Th^* ($K = 1$), with K "marks" as a parameter, for $N = 10$ users, $F = 1000$ chips, received laser power, $P = -45$ dBW, one-bit/sequence-period, and chip-synchronous OOCDMA system without hard-limiter.

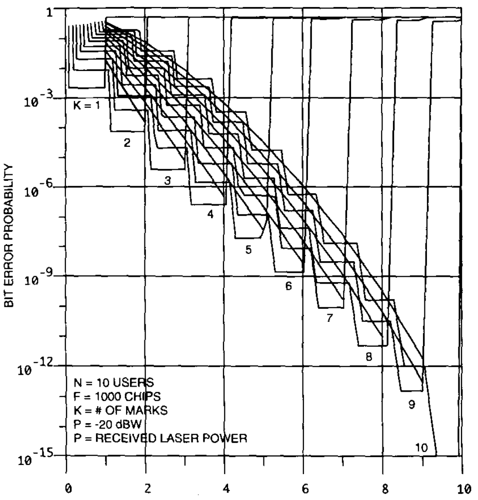

Fig. 7. Bit error probability versus normalized threshold, Th/Th^* ($K = 1$), with K "marks" as a parameter, for $N = 10$ users, $F = 1000$ chips, received laser power, $P = -20$ dBW, one-bit/sequence-period, and chip-synchronous OOCDMA system without hard-limiter.

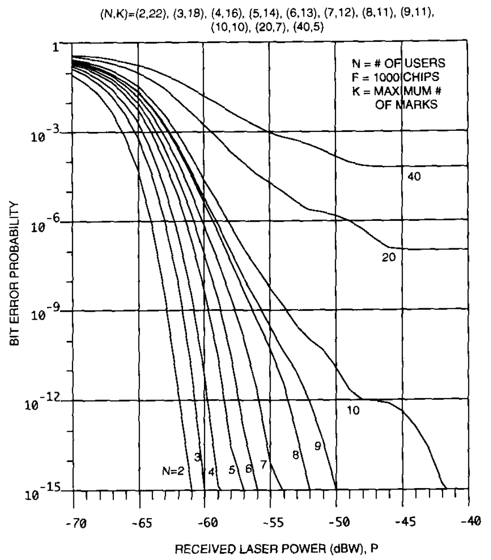

Fig. 8. Bit error probability versus received laser power P using the optimum threshold, with the number of users N as a parameter, for $F = 1000$ chips, one-bit/sequence-period, and chip-synchronous OOCDMA system without hard-limiter. (The maximum number of "marks" K was used for given N and F.)

APD gain G was chosen to be larger than 1 (in this case, $G = 100$ APD gain was chosen), which enhances the system performance.

Fig. 8 shows bit error probability versus received laser power, with the number of users, N as a parameter, for $F = 1000$ length of sequence. The maximum number of marks from (5b) was used for given N and F, because the maximum K achieves the best system performance in general. It is observed that the system becomes worse as the number of users increases. This is mainly because the number of marks K decreases as the number of users N increases, in order to maintain the orthogonality between users' sequences. The received laser power of any mark is assumed to be constant as N changes, hence, total received laser energy (or total number of arrived photons) at the desired receiver decreases, which causes performance to degrade as N increases. A secondary reason is that the total number of marks in the whole system, $K \times N$, increases, which enhances the total interference negative contribution to the decision statistics, as more users share the system. Second, a code with $F = 1000$ chips is better than a code with $F = 2000$ chips when the number of users is less than five, and worse otherwise (though the results for $F = 2000$ chips are omitted). In general, the chip time interval is inversely proportional to F. Hence the energy in a mark decreases as F increases, for constant power. However, more marks can be added to a sequence as F increases. Hence there is a tradeoff in N, K, and F to achieve a specified performance for a given laser power.

B. Chip-Asynchronous, One-Bit/Sequence-Period, without Hard-Limiter

Fig. 9 shows the bit error probability versus received laser power for the chip-asynchronous case with the number of users N as a parameter for $F = 1000$, using the optimum threshold and the maximum number of marks K for given N and F. Conclusions are similar to those of the previous Section V-A (the chip-synchronous case). Also, in general, as expected, system performance of the chip-asynchronous case is superior to that of the chip-synchronous case, because the total interference negative contribution to the desired receiver's final decision statistics, Z_1, for "0" bit transmission in the chip-asynchronous case, is less than that in the chip-synchronous case. The chip-asynchronous case can be about 2 orders of magnitude better, in bit error probability, than the chip-synchronous case under the same conditions. Comparing Figs. 8 with 9 shows the chip-asynchronous case becomes better than the chip-synchronous case as the number of users N increases for example, 0.1 dB better for $N = 4$ and 2 dB better for $N = 10$ in the received laser power to achieve 10^{-9} bit error probability for $F = 1000$ chips. In reality, chip synchronous and chip asynchronous cases are not indicating the exact bit error probability. They simply put an upper and lower bound on the exact bit error probability.

C. Chip-Synchronous, One-Bit/Sequence-Period, with Hard-Limiter

For the ideal link in [2], the performance with hard-limiter can be more than 2 orders in bit error probability better than the performance without hard-limiter. Fig. 10 shows the corresponding results for a chip-synchronous nonideal laser link with hard-limiter and one-bit transmission per sequence-period. The receiver with hard-limiter can be better than the receiver without hard-limiter with high received laser power (compare Fig. 8 with Fig. 10 for $(F, N, K) = (1000, 20, 7)$, and $P \geq -46$ dBW received laser power) because the nonideal link with high received power would behave as the ideal

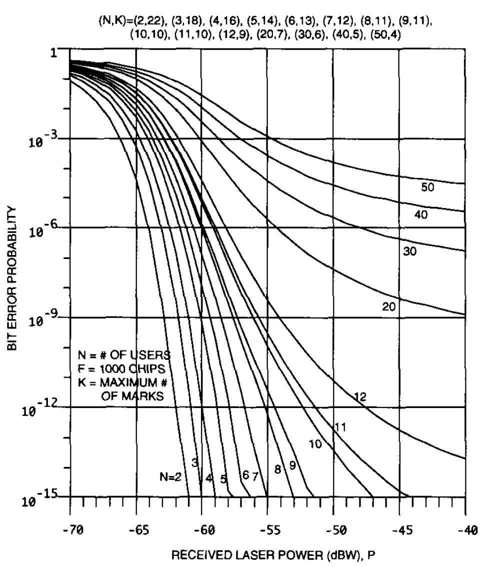

Fig. 9. Bit error probability versus received laser power P using the optimum threshold, with the number of users N as a parameter, for $F = 1000$ chips, one-bit/sequence-period, and chip-asynchronous OOCDMA system without hard-limiter. (The maximum number of "marks" K was used for given N and F.)

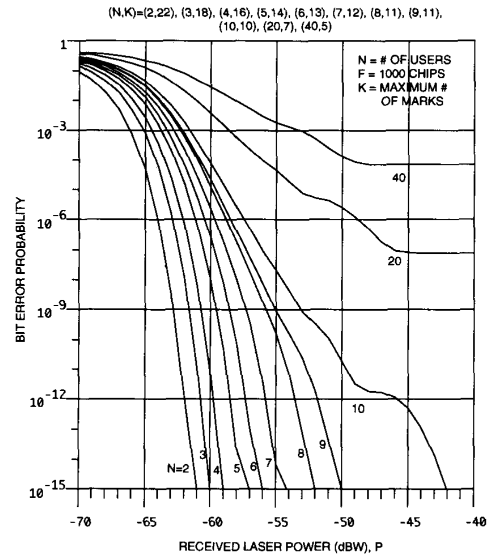

Fig. 10. Bit error probability versus received laser power P using the optimum threshold, with the number of users N as a parameter, for $F = 1000$ chips, one-bit/sequence-period, and chip-synchronous OOCDMA system with hard-limiter. (The maximum number of "marks" K was used for given N and F.)

link. However, in general, for maximum number of marks K given F chips and N users, the improvement from using a hard-limiter, for the nonideal link, is not significant (compare Fig. 8 with Fig. 10). It is stated below why the hard-limiter in a nonideal laser link does not improve the performance significantly while it does for the ideal link. The performance of either the ideal or nonideal laser link is independent of the total positive interference I_1 (for "1" bit transmission, the final decision value Z_1 in the ideal link is a discrete number. Using the decision threshold between $K-1$ and K can completely suppress the total interference negative contribution for "0" bit transmission at correlation time except in the case of all K mark intervals hit by $(N-1)$ users' marks in the ideal link. However, in a nonideal laser link the decision value Z_1 is not discrete and the total interference negative contribution at correlation time cannot be suppressed completely, even though the optimum threshold is employed, because of the presence of APD noise, thermal noise, background light, and leakage currents.

In addition, the bit error probability of the hard-limiter for a nonideal laser link can be 0.5 if the number of users N is larger than or equal to the extinction ratio M_e (= 100 in this paper) of the laser diode output power in the mark and space states. This is because N users' spaces contribution to any kth mark interval of the desired receiver signal is larger than or equal to the unit laser power intensity, for $k = 1, \cdots, K$. Thus, K pulses after the hard-limiter are incident upon the APD with λ_s photon arrival rate for either "0" bit or "1" bit transmission, and there is no difference in the final decision statistics Z_1 between "0" bit and "1" bit transmission.

VI. Conclusion

APD noise and thermal noise were included in the analysis for the performance of a nonideal OOCDMA system with 30 Mbs data bit rate of each user, and the performance of the nonideal link was compared with an ideal OOCDMA system [2]. The bit error probability of a non-ideal link can be 2 orders worse than that of the ideal link for a chip-synchronous one-bit/sequence-period transmission for moderate received laser power, such as $P = -55$ dBW. A careful analysis is required in an application of the OOCDMA code to a practical environment because the nonlinearity of the nonideal link depends on power, number of users, number of chips, number of marks, APD noise, thermal noise, and so on.

In addition, exact analysis was performed for the system with hard-limiter placed at the front of the receiver, in the presence of APD and thermal noise, and its performance was compared to that of the system without hard-limiter, for the chip-synchronous and one-bit transmission per sequence-period system. The improvement from using a hard-limiter is not significant because the total interference negative contribution (for "0" bit transmission) at correlation time cannot be negligible because of the presence of APD and thermal noise, while it can be completely suppressed in the ideal-link.

Furthermore, a chip-asynchronous system can be more than 2 orders of magnitude better in bit error probability or, equivalently, 2 dB better in required received power, than a chip-synchronous system for the one-bit/sequence-period case.

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Hyuck M. Kwon (S'82–M'84) for a photograph and biography, please see the May 1994 issue of this TRANSACTIONS, p. 2126.