Syllabus for 'Philosophy of Mathematics'
Thomas Donaldson; Winter Quarter, 2015

Basic Information

Course Numbers: PHIL 162, MATH 162, PHIL 262.
Instructor: Thomas Donaldson
Email: tmedonaldson@gmail.com (or: tmed@stanford.edu)
Office hours: Tuesday 1400-1700, or by arrangement
Office location: Building 90, Room 92A
Class schedule: Mon, Wed 1315 - 1445
Class location: 120-314

Course Description

This course is a general survey of the philosophy of mathematics; we'll focus on epistemological issues. We begin with a survey of some basic concepts (*proof, axiom, definition, number, set,* …). Some of this discussion will be historical in character. We then discuss some mind-bending theorems about the limits of our current mathematical knowledge: Gödel's Incompleteness Theorems, and the independence of the continuum hypothesis from the current axioms of set theory. We then discuss some of the major philosophical accounts of mathematics, proceeding more-or-less in chronological order:

- Logicism
- Intuitionism
- Hilbert's program
- Quine's empiricism
- Field's program
- Structuralism

We finish with a discussion of Eugene Wigner's 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences'.

This is a course on philosophy, not mathematics – students won't be expected to prove theorems or complete mathematical exercises. However, students will have to read some material of a technical nature.

Disability Accommodation

Students who have a disability which may necessitate an academic accommodation or the use of auxiliary aids and services in a class must initiate the request with the Office of Accessible Education (OAE). The OAE will evaluate the request with required documentation, recommend appropriate accommodations, and prepare a verification letter dated in the current academic term in which the request is being made. Please contact the OAE as soon as possible: timely notice is needed to arrange for appropriate accommodations. The OAE's contact details are as follows.

Address: 563 Salvatierra Walk, Stanford, CA 94305
Phone: (650) 723-1066
Web address: http://studentaffais.stanford.edu/oae

Recommended Preparatory Reading

Timothy Gower's *Mathematics: A Very Short Introduction* is magnificent. An afternoon wandering through the first two parts of the *Princeton Companion to Mathematics* would be fun. Mark Colyvan and Stewart Shapiro have each written excellent introductions to the philosophy of mathematics, called respectively *An Introduction to the Philosophy of Mathematics,* and *Thinking about Mathematics: The Philosophy of Mathematics*.

Prerequisites

A familiarity with basic mathematical logic is essential. I expect all participants to have taken PHIL 151, or at least 150, or at least PHIL 50.

Office Hours

My office hours are noted at the top of the first page of this syllabus. If you cannot make these times because of a scheduling conflict please email me to request an appointment at some other time. However, please make sure you know when office hours are normally held before requesting a meeting outside of office hours.

Assessment

From week 2 until week 10, each student must answer a series of short questions on the previous week's topic. These questions are intended to test basic comprehension, and shouldn't take long. Answers are due at the end of each week.

In addition, each student should choose EITHER

> (1) to write two short papers (approx. 3,500 words) to be handed in at the end of week 5 and week 10; OR
> (2) to write one short paper (approx. 3,500 words) to be handed in at the end of week 5, and to complete an exam at the end of the quarter (the exact time and location of the exam have yet to be arranged); OR
> (3) to write one long paper (7,000 words or longer) to be handed in at the end of week 10.

A 'week', in my sense, ends at 2359 on a Friday evening.

Attendance in class is compulsory. Any student who takes more than two unexcused absences will be penalized.

Absences and Late Arrivals

Please do not arrive late to class. As noted above, absences will adversely affect a student's final grade. Do not take this class if you expect to be absent several times.

Honor Code & Plagiarism

Students are not merely bound by Stanford University's Honor Code. It is also their responsibility to know what the Honor Code states. Students unfamiliar with the Honor Code should consult the following site:

http://www.stanford.edu/dept/vpsa/judicialaffairs/guiding/honorcode.ht

Read this page and download the PDF. Students are also responsible for knowing what constitutes plagiarism. If you are unsure about what counts as plagiarism, ask the instructor.

Course Readings

Readings will be made available at coursework.stanford.edu at least one week before they are discussed in class. Students are not required to purchase books. Students will be expected to have the readings with them in class, either on a computer or in hard copy.

Schedule

<u>Week One</u>   Some basic concepts: proof, axiom, definition. The foundational role of set theory.

Reading:

- Excerpts from Part II of the *Princeton Companion to Mathematics:*
    - ◦ 'From Numbers to Number Systems', by Fernando Q. Gouvêa
    - ◦ 'Geometry', by Jeremy Gray
    - ◦ 'The Development of Rigor in Mathematical Analysis', by Tom Archibald
    - ◦ 'The Development of the Idea of Proof', by Leo Corry
    - ◦ 'The Crisis in the Foundations of Mathematics', by José Ferreirós

(I realize that this is a lot of text for one week! But don't worry – you don't need to understand every detail. This is preparatory, background reading. Make a note of your confusions: we'll sort them out as the quarter goes on.)

<u>Week Two</u>   Limitative Results: Gödel's Incompleteness Theorems and The Continuum Hypothesis.

Reading:
- Colyvan, *An Introduction to the Philosophy of Mathematics,* ch. 2

<u>Week Three:</u>   The Benacerraf/Field Problem

Reading:
- Benacerraf, 'Mathematical Truth'
- Field, Excerpts from *Realism, Mathematics and Modality*

<u>Week Four</u>   Logicism

Reading:

- Frege, excerpt from *The Foundations of Arithmetic*
- Russell, excerpt from *Introduction to Mathematical Philosophy*
- Shapiro, *Thinking about Mathematics,* ch. 5

<u>Week Five</u>   Intuitionism

Reading:

- Shapiro, *Thinking about Mathematics,* ch. 7
- Brouwer, "The Intuitionist Foundations of Mathematics" and "Disputation," in Benacerraf and Putnam (eds.) *The Philosophy of Mathematics: Selected Readings*.

<u>Week six</u>   Hilbert's Program

Reading:

- Shapiro, *Thinking about mathematics,* ch. 7
- Hilbert, 'On the Infinite'

<u>Week Seven</u>   Quine's Empiricism

Reading:

- Quine, *Pursuit of Truth,* Section 40
- Quine, *From Stimulus to Science*, ch. 5

<u>Week Eight</u>   Field's Program

Reading:

- Field, "Realism and Anti-Realism about Mathematics," in his *Realism, Mathematics, and Modality*

<u>Week Nine</u>   Structuralism

Reading:

- Benacerraf, 'What Number Could not Be'
- Paseau, 'Reducing Arithmetic to Set Theory', in Bueno and Linnebo *New Waves in the Philosophy of Mathematics*
- Hellman, 'Structuralism, Mathematical', forthcoming in *The Encyclopedia of Philosophy* (2nd ed.), MacMillan

<u>Week Ten</u>   Wigner and the 'unreasonable effectiveness' of mathematics.

Reading:

- Wigner, 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences'
- Colyvan, *An Introduction to the Philosophy of Mathematics,* ch. 6