# ANALOGY AND CONFIRMATION THEORY*

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The argument from analogy is examined from the point of view of Carnap's confirmation theory. It is argued that if inductive arguments are to be applicable to the real world, they must contain elementary analogical inferences. Carnap's system as originally developed (the $\lambda$-system) is not strong enough to take account of analogical arguments, but it is shown that the new system, which he has announced but not published in detail (the $\eta$-system), is capable of satisfying the conditions of inductive analogy. Finally it is shown that an elementary analysis of analogical inference yields postulates of the $\eta$-system with a minimum of arbitrary assumptions.

Recent work in the philosophy of science has laid stress on the function of *models* in the structure and development of theories, and has interpreted the relation between a theoretical model and the world as that of *analogy*.[1] If the use of models is regarded as a mere crutch to aid the imagination in construction of theories, it may not raise any deep philosophical problems, and may be more appropriately regarded as a study for the psychology or sociology of scientific discovery rather than for philosophy and logic. But if stronger claims are made for models, namely that an analogy with a more familiar domain of phenomena can provide *rational predictions* in a new domain, then the logic of such claims must involve what has traditionally been called the "argument from analogy".

These considerations provide the motive for a fresh examination of the argument from analogy in the context of modern formulations of inductive method. Writers on probabilistic induction including Keynes, Broad, and Von Wright[2] have explicitly considered this argument to be a species of induction, but have dealt with it within their respective inductive theories by means of somewhat cumbersome and *ad hoc* postulates (for example Keynes' "generators") which afford little scope for simplification or generalization. Writers on induction whose main inspiration comes from statistical theory, on the other hand, practically ignore the argument from analogy, presumably because, as will appear below, an argument which depends upon *similarities* between individuals as well as upon *identities* requires considerably stronger assumptions than those of most applications of statistical method. Only Carnap's confirmation theory appears to be sufficiently general and sufficiently detailed to give some hope of dealing with this problem. In this paper, therefore, I shall use the techniques of Carnap's theory to elucidate the conditions of the argument from

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[1] See especially L. Apostel: "Towards the Formal Study of Models in the Non-Formal Sciences," in *The Concept and Role of the Model in Mathematics and Natural and Social Sciences* (ed. H. Freudenthal), Dordrecht, 1961; R. B. Braithwaite: "Models in the Empirical Sciences" in *Proc. Congress of the International Union for the Logic, Methodology and Philosophy of Science* (ed. P. Suppes et al.), Stanford, 1960; E. Nagel: *The Structure of Science*, Ch. VI; and the present author's *Models and Analogies in Science*, London: Sheed & Ward, 1963.

[2] C. D. Broad: "Problematic Induction", *Proc. Aris. Soc.*, 28, 1927-8, 1; J. M. Keynes: *A Treatise on Probability*, London, 1921, Ch. XVIII, XIX; G. H. von Wright: *The Logical Problem of Induction*, Helsingfors, 1941, p. 134.

analogy, and I shall show in particular that his new axiom-system for c-functions[3] is capable of development as a satisfactory explication of the argument, and also that his own choice of c-function within that system can be introduced with a minimum of arbitrary and *ad hoc* assumptions if the characteristics of analogy are kept in mind.

**1. Analogy in Carnap's λ-system.** In Appendix D to *LFP* Carnap defines the inference by analogy as follows: "The evidence known to us is the fact that individuals $b$ and $c$ agree in certain properties and, in addition, that $b$ has a further property; thereupon we consider the hypothesis that $c$ too has this property" (p. 569). In terms of this definition he shows that the evidence increases the value of the c-function $c^*$ for the hypothesis above its initial value on tautological evidence, and this result can easily be extended to the $\lambda$-continuum of c-functions defined in *CIM*. If we take, for example, a language containing two primitive predicates '$P_1$' and '$P_2$', and two individuals $a$ and $b$, we have (by *CIM* 5-6 and 10-7):

$$c(P_2b, P_1P_2a \cdot P_1b) = \frac{m(P_1P_2a \cdot P_1P_2b)}{m(P_1P_2a \cdot P_1b)}$$

$$= \frac{\lambda/4 + 1}{\lambda/2 + 1} \qquad (0 \leqslant \lambda < \infty)$$

which is greater than the initial value, $\frac{1}{2}$, of $c(P_2b)$ on tautological evidence. We also have

$$c(P_2b, P_2a) = \frac{\lambda/2 + 1}{\lambda + 1} \leqslant c(P_2b, P_1P_2a \cdot P_1b).$$

Hence the evidence of similarity between $a$ and $b$ in respect of '$P_1$' increases the confirmation of '$P_2b$' except in the case of the 'straight rule', $\lambda = 0$.

It has, however, been pointed out[4] that this type of argument is not what has been traditionally understood by argument from analogy, since analogical inference has always supposed *differences* as well as similarities between the two analogues. Furthermore, it can be argued that if any theory of confirmation is to have application to the real world, it must provide a justification in terms of degree of confirmation for analogy arguments of this type as well as for those described by Carnap. For consider the concept of the 'next instance' or of 'all instances of a given kind' in an inductive inference. In general, what we recognize to be such instances in applications of induction, are already known *not* to be identical in all respects. That is to say the assumption, made in Carnap's type of inference, that the evidence ascribes to the individuals only the same property $P_1$ in both cases, and that there are not initially known to be any differences between them, is at best an idealization of the real situation. It will generally be the case that, if the total evidence is taken into account, superficially similar instances will be found to be different in some respects. Now two kinds of reasons might be advanced for ignoring such differences and treating the instances

[3] The new system is outlined in R. Carnap & W. Stegmüller: *Induktive Logik und Wahrscheinlichkeit*, Vienna, 1959, Appendix B (abbreviated *ILW*). References to Carnap's previous system will be to *The Logical Foundations of Probability*, Chicago, 1950 (abbreviated *LFP*), and *The Continuum of Inductive Methods*, Chicago, 1952 (abbreviated *CIM*).

[4] By the present author in *Models and Analogies in Science*, p. 121, and in more detail by Peter Achinstein: "Variety and Analogy in Confirmation Theory", *Phil. Sci.* 30, 1963, p. 216.

for the purposes of the induction as so far identical. First we may have reason to believe that the differences are of a kind which is causally irrelevant to the induction. But this presupposes that we have already accepted a causal law determining what is and is not relevant, and acceptance of such a law must itself rest, at least in part, upon previous decisions to ignore differences among instances which were otherwise identical, either in the case of instances of the law itself, or of instances of other laws which are taken to justify acceptance of this law in the context of a hypothetico-deductive theory. Secondly, we might decide to ignore differences between instances, either when they appear small compared with similarities, or when there are many instances of correlation of the similar properties with the problematic property in instances which are otherwise different. That is to say, we might feel justified in ignoring differences in these two cases:

(a) $a$ has properties $P_1P_2 \ldots P_rP_s$; $b$ has properties $P_1P_2 \ldots P_{r-1}\bar{P}_s$ (where '$\bar{P}_s$' denotes the property not—$P_s$); and the hypothesis is '$P_rb$'; or

(b) $a_1$ has properties $P_1P_{r-1}P_r$; $a_2$ has properties $P_2P_{r-1}P_r$; ... $a_s$ has properties $P_sP_{r-1}P_r$; $b$ has properties $\bar{P}_1 \ldots \bar{P}_sP_{r-1}$; and the hypothesis is '$P_rb$'.

But what is the justification for such procedures? It might be suggested that the justification is a higher-level induction concerning a large number of cases of types (a) and (b), where we have found that ignoring the property $P_s$ in type (a), and the properties $P_1 \ldots P_s$ in type (b), has resulted in successful predictions. This suggestion, however, begs the question, because different cases of types (a) and (b) respectively are themselves not identical in all respects, and so the higher-level induction makes just the assumption we set out to justify.

It seems therefore that we must seek to provide a justification for arguments of types (a) and (b) within a theory of confirmation if we wish the theory to apply to practical forms of inductive argument. Now it is clear that a necessary condition for such a justification is that the hypothesis '$P_rb$' should have a confirmation in both cases greater than the initial confirmation ($\frac{1}{2}$ on tautological evidence). It can, however, be shown that none of the inductive methods of Carnap's $\lambda$-system satisfy this condition

In the paper referred to above, Achinstein has proved in general that the confirmation of the hypothesis 'If an individual $b$ has some properties in common with another individual $a$ and some properties different from $a$, then it has property $P_r$' is the same whichever of the following three types of evidence is adduced:

(i) $a$ has property $P_r$ and also many properties in common with the known properties of $b$, and a few different;

(ii) $a$ has property $P_r$ and few properties in common with the known properties of $b$, and many different;

(iii) $a$ has property $\bar{P}_r$ and few properties in common with the known properties of $b$, and many different.

Achinstein does not point out, however, that the confirmation of the more relevant hypothesis '$P_rb$', given any one of the same three types of evidence, is equal to the *initial* confirmation of the hypothesis '$P_rb$'. Achinstein's proof concerns the confirmation of the hypothesis '$Rb \supset P_rb$', where '$R$' denotes the known properties of $b$. This hypothesis, however, includes the confirmation of the hypothesis '$\bar{R}b$', and we require rather the confirmation of '$Rb \supset P_r b \cdot Rb$', that is, of $P_r b \cdot Rb$. This can be obtained from the following theorem which is a minor modification of Achinstein's:[5]

*Theorem:* Let $M_1$, $M_2$, $M_3$ be three distinct molecular predicates such that $\bar{M}_1 M_2 M_3 y$ is neither $L$-true nor $L$-false.

Then, if $x$ and $y$ are distinct individual constants and $c$ any confirmation function satisfying Carnap's conditions of adequacy $C1 - C9$ (*CIM* pp. 12-14),

$$c(M_3 y, M_1 M_2 M_3 x \cdot \bar{M}_1 M_2 y) = c(M_3 y, t) = \tfrac{1}{2},$$

where $c(M_3 y, t)$ is the confirmation of $M_3 y$ on tautological evidence.

**2. Conditions for analogical inference.** It follows from this theorem that neither of the analogical arguments (a) and (b) are justifiable within Carnap's $\lambda$-system. This means that a large number of similarities between instances as in (a), and a large number of occurrences of the same correlation in otherwise different instances as in (b), both fail to give increasing confirmation as these numbers respectively increase, although such arguments are usually regarded as justified and are frequently resorted to. Worse still, the result means that, far from approaching the confirmation of the inductive arguments obtained by ignoring the properties $P_s$ in (a) and $P_1 \ldots P_s$ in (b), the confirmation in these cases has a value just equal to the initial confirmation; in other words no process of learning from analogous instances is justifiable. This is a very serious limitation on the treatment of induction in the $\lambda$-system, since if our previous remarks about the nature of practical applications of induction are accepted, it means that any application of the theory rests on an arbitrary decision to ignore part of the evidence. Such a decision not only conflicts with Carnap's requirement of total evidence (*LFP* p. 211), but also has the consequence that the confirmation values obtained cannot be regarded as approximations, corrigible by a deeper level of analysis which *does* take into account differences between instances, because if such analysis were carried out, all confirmations would be reduced to their initial values irrespective of the evidence.

We must therefore require of any confirmation theory satisfaction of the following condition:

I. If two individuals $a$ and $b$ are known to agree in certain properties and differ in others, and if in addition $a$ has a further property, then the confirmation of the hypothesis that $b$ also has this property is greater than its initial confirmation; at least if the weight of the similarities is sufficiently great compared with the weight of the differences.

This condition is still not sufficiently strong to deal with Achinstein's examples, however, so let us consider what other demands might reasonably be made of analogical arguments.

It might be demanded that, if $M_1$, $M_2$ are two logically independent molecular predicates each independent of the primitive predicate $P_3$, then

$$c(P_3 b, \bar{M}_1 M_2 P_3 a \cdot M_1 M_2 b) = c(P_3 b, M_2 P_3 a \cdot M_2 b),$$

that is, that differences between instances should be irrelevant to the confirmation.

[5] Achinstein's theorem is proved in Appendix I to his paper.

Such a demand, would, however, be quite implausible, since it is reasonable to assume that the occurrence of differences between $a$ and $b$ *reduces* the confirmation of '$P_3b$' compared with $c(P_3b, M_2P_3a \cdot M_2b)$. What may reasonably be demanded, however, are the two following conditions:

II. $c(P_3b, \bar{M}_1M_2P_3a \cdot M_1M_2b)$ increases or decreases with increase or decrease of the weight of $M_2$ compared with the weight of $M_1$;

III. $c(P_3b, \bar{M}_1M_2P_3a \cdot M_1M_2b) > c(P_3b, \bar{M}_1M_2\bar{P}_3a \cdot M_1M_2b)$.

Satisfaction of conditions I-III would ensure reasonable confirmation values for the cases (a) and (b) above, and also for the examples of analogy arguments adduced by Achinstein, namely prediction of a new property $H$ of the metal rhodium on the basis of the following kinds of evidence:

(i) A very similar metal platinum has property $H$;

(ii) A quite dissimilar substance, say oxygen, has $H$;

(iii) A quite dissimilar substance which has not $H$.

A confirmation function satisfying conditions I-III would give greater confirmation with evidence (*i*) than with (*ii*), and greater confirmation with (*ii*) than with (*iii*). A function satisfying I alone would not be sufficient to ensure this, although it would, unlike Carnap's $\lambda$-system, show that evidence of (*i*), and perhaps (*ii*), is better than no evidence at all. In the previous discussion referred to above[6] I suggested general reasons why it might be necessary to forego satisfaction of conditions II and III, remaining content with a justification of analogy arguments on the basis of I only. But it now appears that Carnap's own modification of his $\lambda$-system, introduced partly for other reasons, is sufficient to satisfy all three conditions[7]—a much more satisfactory state of affairs; for a confirmation theory could hardly be regarded as an adequate explication of induction if it did not deal with examples of Achinstein's types (*i*)-(*iii*).

**3. Carnap's $\eta$-system.** In Appendix B of Carnap and Stegmüller's *Induktive Logik und Wahrscheinlichkeit*, a new axiom system for c-functions is presented, whose main object is to extend the system of *CIM* to deal with languages whose primitive predicates belong to families. Thus, a primitive predicate '$P_1$' is not alternative to just one primitive predicate '$\bar{P}_1$', as in the $\lambda$-system, but is one of a set of predicates of equal initial weight, say '$P_{11}$' ... '$P_{1k}$', comprising the family '$F_1$'. Axioms *NA* 1-15 repeat the axioms of *CIM* for regular symmetric *c*-functions (*C* 1-7), introduce the notion of predicate-families, and modify and extend the other C-axioms to account for languages containing such families. In particular the previous axiom *C* 8, which stated that c-functions are symmetrical with respect to *Q*-predicates in the *CIM* system (which are the conjunctions of every primitive predicate of the language, either negated or unnegated), is replaced by *NA* 8 and 9, which state symmetry only with respect to primitive predicates, and with respect to families having equal numbers of primitive predicates. Since *C* 8 is sufficient to ensure that any c-function of the $\lambda$-system violates the analogy conditions I-III, and *NA* 8 and 9 are not strong enough to violate them, it is possible to look for a c-function or c-functions satisfying both *NA* 1-15 and conditions I-III.

[6] Note 1.

[7] That this is the case is surmised but not proved by Carnap in his reply to Achinstein's paper *Phil. Sci.* 30, 1963, p. 225).

The form of c-function is not uniquely determined by *NA* 1-15, and Carnap and Stegmüller proceed (*ILW* p. 251) to suggest two definitions of $c$ in the special case of a language containing two predicate families '$F_1$' and '$F_2$', and a further c-function consisting of a combination of these. With the usual definition of a measure-function $m$ as the confirmation function on tautological evidence (*CIM* 5-1), we have (*CIM* 5-6)

$$c(h, e) = \frac{m(e \cdot h)}{m(e)} .$$

Then $m$-values for each family separately are determined from the axioms *NA* 1-15, and are equivalent to the $m$-values in the $\lambda$-system for a language containing as many $Q$-predicates as there are predicates of the family. Denote the $m$-value for the first family by '$m_\lambda{}^1$', and for the second family by '$m_\lambda{}^2$'. Then the second suggested definition of $c$ is derived by taking as $m$-value

$$m_\lambda{}^{1/2} = m_\lambda{}^1 \times m_\lambda{}^2.$$

For the first suggested definition, the whole set of $Q$-predicates of $F_1$ and $F_2$ (the set of conjunctions of one predicate from $F_1$ and one predicate from $F_2$) is regarded as a "pseudo-family". The $m$-values are then equivalent to those in the $\lambda$-system for a language whose set of $Q$-predicates is the set of $Q$-predicates of $F_1$ and $F_2$. Thus, denoting $m$-values according to the definition by '$m_\lambda{}^{1,2}$', in the case of two families containing $k_1$ and $k_2$ predicates respectively and hence yielding $k_1 k_2$ $Q$-predicates, the values of $m_\lambda{}^{1,2}$ are the same as those in the $\lambda$-system for a language of $k_1 k_2$ $Q$-predicates.

Carnap next proposes to test these two suggested definitions by means of three simple examples for a language with $k_1 = k_2 = 2$. Simplifying still further by considering only four individuals $w$, $x$, $y$, $z$, the examples are essentially as follows. Consider the state descriptions:

(A) $P_1 P_2 w \cdot P_1 \bar{P}_2 x \cdot \bar{P}_1 P_2 y \cdot \bar{P}_1 \bar{P}_2 z$
(B) $P_1 P_2 w \cdot P_1 P_2 x \cdot \bar{P}_1 \bar{P}_2 y \cdot \bar{P}_1 \bar{P}_2 z$
(C) $P_1 P_2 w \cdot P_1 P_2 x \cdot P_1 \bar{P}_2 y \cdot P_1 \bar{P}_2 z$.

We should expect (a) $m(\mathrm{A}) < m(\mathrm{B})$, since, for example, we want

$$c(P_1 \bar{P}_2 x \cdot \bar{P}_1 P_2 y,\ P_1 P_2 w \cdot \bar{P}_1 \bar{P}_2 z) < c(P_1 P_2 x \cdot \bar{P}_1 \bar{P}_2 y,\ P_1 P_2 w \cdot \bar{P}_1 \bar{P}_2 z)$$

and we should expect (b) $m(\mathrm{B}) < m(\mathrm{C})$, since we want

$$c(\bar{P}_1 \bar{P}_2 y \cdot \bar{P}_1 \bar{P}_2 z,\ P_1 P_2 w \cdot P_1 P_2 x) \qquad c(P_1 \bar{P}_2 y \cdot P_1 \bar{P}_2 z,\ P_1 P_2 w \cdot P_1 P_2 x).$$

Now $m_\lambda{}^{1/2}$ satisfies (*b*) but not (*a*), and $m_\lambda{}^{1,2}$ satisfies (a) but not (b). Therefore Carnap suggests a third solution which is a weighted mean of these, and satisfies both conditions, namely

$$m_{\lambda,\eta}(e) =_{df} \eta\, m_\lambda{}^{1/2}(e) + (1 - \eta)\, m_\lambda{}^{1,2}(e) \tag{3.1}$$

where $0 < \eta < 1$. Thus if the parameter $\eta$ is nearer to 1, weight is given to the *similarities* of $Q$-predicates appealed to in requirement (b), and if $\eta$ is near to zero, weight is given to the *identities* of $Q$-predicates appealed to in (a), but no weight to their similarities.

This introduction of the function $m_{\lambda,\eta}(e)$ appears somewhat arbitrary and *ad hoc*, and is not further developed by Carnap here. Neither is any generalization given for more than two families although Carnap states that he and Kemeny are working on one. If, however, we take the simplest generalization of the same form consistent with *NA* 1-15, we obtain for $n$ families, with an obvious notation,

$$m_{\lambda,\eta}(e) =_{df} \eta m_\lambda^{1|2|\dots|n}(e) + (1-\eta)\, m_\lambda^{1,2,\dots n}(e). \tag{3.2}$$

It will now be shown, in the case of three families, that $m_{\lambda,\eta}(e)$ thus defined satisfies conditions I-III.

Let us take a language of three families containing respectively the predicates '$P_1$', '$\bar{P}_1$'; '$P_2$', '$\bar{P}_2$'; '$P_3$', '$\bar{P}_3$'; and two individuals $a$ and $b$. Thus $k_1 = k_2 = k_3 = 2$. Dropping the $\lambda,\eta$ suffixes, let us put

$$\begin{aligned} c_3 &= c(P_3b,\, P_1P_2P_3a \cdot P_1P_2b) \\ c_2 &= c(P_3b,\, \bar{P}_1P_2P_3a \cdot P_1P_2b) \\ c_1 &= c(P_3b,\, \bar{P}_1\bar{P}_2P_3a \cdot P_1P_2b) \\ c_0 &= c(P_3b,\, \bar{P}_1\bar{P}_2\bar{P}_3a \cdot P_1P_2b) \end{aligned}$$

$$\begin{aligned} m_3 &= m(P_1P_2P_3a \cdot P_1P_2P_3b) = \eta m_3^{1/2/3} + (1-\eta)m_3^{1,2,3} \\ m_2 &= m(\bar{P}_1P_2P_3a \cdot P_1P_2P_3b) = \eta m_2^{1/2/3} + (1-\eta)m_2^{1,2,3} \\ m_1 &= m(\bar{P}_1\bar{P}_2P_3a \cdot P_1P_2P_3b) = \eta m_1^{1/2/3} + (1-\eta)m_1^{1,2,3} \\ m_0 &= m(\bar{P}_1\bar{P}_2\bar{P}_3a \cdot P_1P_2P_3b) = \eta m_0^{1/2/3} + (1-\eta)m_0^{1,2,3}. \end{aligned}$$

Then conditions I-III entail $c_3 > c_2 > c_1 > \frac{1}{2}$ and $c_1 > c_0$. But by *ILW NA* 3, 7, 8, 9

$$c_r = \frac{m_r}{m_r + m_{r-1}} \qquad (r = 1, 2, 3)$$

$$c_0 = \frac{m_0}{m_0 + m_1}.$$

Hence the necessary and sufficient conditions for I-III in this case are

$$\frac{m_3}{m_2} > \frac{m_2}{m_1} > \frac{m_1}{m_0} > 1$$

By *ILW* p. 251, D 4, we have

$$m_2^{1,2,3} = m_1^{1,2,3} = m_0^{1,2,3}$$

and

$$m_3^{1/2/3} = m \times m \times m, \quad m_2^{1,2,3} = m \times m \times m',$$

$$m_1^{1/2/3} = m \times m' \times m', \quad m_0^{1,2,3} = m' \times m' \times m',$$

where

$$m = m(P_ra \cdot P_rb) = \frac{\lambda/2 + 1}{2(\lambda+1)},$$

and

$$m' = m(\bar{P}_ra \cdot P_rb) = \frac{\lambda/2}{2(\lambda+1)}, \qquad (r = 1, 2, 3).$$

Hence

$$m + m' = \tfrac{1}{2}, \qquad m > m' \quad \text{and} \quad \frac{m_1}{m_0} > 1.$$

It also follows from D 4 that $m_2 m_0 - m_1 m_1 > 0$ for $\eta < 1$.
Again, by D 4, $m_3^{1,2,3} > m_0^{1,2,3}$
Putting $m_3 = \eta m_3^{1/2/3} = (1-\eta)\ (m_0^{1,2,3} + \alpha)$, where $\alpha > 0$, and by an argument similar to the above we obtain

$$\frac{\eta m_3^{1/2/3} + (1-\eta)\, m_0^{1,2,3}}{m_2} > \frac{m_2}{m_1} \qquad \text{for} \quad \eta \leqslant 1.$$

Hence

$$\frac{m_3}{m_2} > \frac{m_2}{m_1} \qquad \text{for} \quad \eta \leqslant 1.$$

Hence conditions I-III are satisfied by the $m$-function (3.2) for a language of three families containing two predicates each. The proof can be extended immediately to show that the same functions $c_3$, $c_2$, $c_1$, $c_0$ satisfy conditions I-III in a language containing $n$ families, by using *NA* 11, which states that $c(h, e)$ is independent of families other than those occurring in $h$ and $e$. The proof is also independent of the number of individuals in the language, by *NA* 10.

It should be noticed that for $\eta = 1$, that is $m_{\lambda,\eta}(e) = m_\lambda^{1/2/3}(e)$, the m-values do *not* satisfy the condition $m_2/m_1 > m_1/m_0$, that is, the simple generalization (3.2) of Carnap's suggested definition does not conform to condition II for analogy arguments, although it does satisfy conditions I and III, and in fact gives c-values equivalent to those obtained by restricting the evidence to '$P_3 a$', and ignoring other similarities and differences between $a$ and $b$. Carnap's new "Axiom of Analogy" *NA* 16 is satisfied by $m^{1/2/\ldots/n}(e)$; it therefore follows that the axiom-system *NA* 1-16 is not sufficient for conditions I-III, nor, incidentally, as Carnap implicitly recognizes, is it sufficient to deal with Carnap's own example (a) described above. A further axiom is required to eliminate the possibility $\eta = 1$.

**4. A derivation of $m_{\lambda,\eta}(e)$.** We have seen that at least one generalization of Carnap's $\eta$-system satisfies conditions I-III for arguments involving three two-predicate families. Let us now try to make the form of $m_{\lambda,\eta}(e)$ for two families a little more luminous by relating it to general considerations regarding induction and analogy. The fundamental inductive argument has always been conceived on the model of the drawings of balls out of bags; an artificial situation where it is natural to suppose that for purposes of generalization and prediction some of the balls are identical with each other. This, I have already argued, is not a situation found in nature, and it is not surprising that, with this starting-point, the more realistic situation of similar but not identical instances cannot be adequately represented in confirmation-theory without the special introduction of *ad hoc* assumptions. Let us therefore suppose that the fundamental inductive argument that needs explication is not the inference of '$P_1 b$' from '$P_1 a$,' but the inference of '$P_2 b$' from '$\bar{P}_1 P_2 a \cdot P_1 b$'. The demand that this inference shall have confirmation greater than $\frac{1}{2}$ is stronger than condition I, since the evidence here contains *no* known similarity between $a$ and $b$, but it turns out that no weaker demand can be satisfied without violating other requirements for the c-functions. Suppose we now analyse this elementary analogical inference into two idealized situations in which the evidence is denoted by, respectively, '$P_2 a$' and '$\bar{P}_1 a \cdot P_1 b$', and treat these situations according to Carnap's $\lambda$-system.

As before we assume only two individuals, and initially we consider a language with two families of predicates '$P_1$', '$\bar{P}_1$' and '$P_2$', '$\bar{P}_2$'. To be an adequate representation of the elementary analogical inference the confirmation function $c(P_2 b, \bar{P}_1 P_2 a \cdot P_1 b)$

must have a value greater than $c(P_2b, t) = \frac{1}{2}$, but less than $c(P_2b, P_2a)$, since the evidence '$P_2a$' does not decide between the favorable case '$P_1a \cdot P_1b$' and the unfavorable case '$\bar{P}_1a \cdot P_1b$' specified in the analogical inference. Thus we should expect

$$\tfrac{1}{2} < c(P_2b, \bar{P}_1P_2a \cdot P_1b) < c_\lambda(P_2b, P_2a),$$

where $c_\lambda$ is calculated as in the $\lambda$-system, for a language of four $Q$-predicates.

In other words we expect the evidence '$\bar{P}_1a \cdot P_1b$' to diminish the favorability of the evidence '$P_2a$'. The required confirmation function can therefore be regarded as lying between the extreme values $c_\lambda(P_2b, \bar{P}_1a \cdot P_1b)$ (which itself has value $\frac{1}{2}$), and $c_\lambda(P_2b, P_2a)$. Let us therefore put $c_\eta(P_2b, \bar{P}_1P_2a \cdot P_1b) = c_\lambda(P_2b, P_2a) + \frac{1}{2}(1 - \eta)$, where

$$0 < \eta < 1. \tag{4.1}$$

Let us further assume that the new c-function has the same value as $c_\lambda(h, e)$ for $h = P_rb$, $e = P_ra$, and for $h = P_rb$, $e = \bar{P}_ra$, that is

$$c_\eta(P_rb, P_ra) = c_\lambda(P_rb, P_ra), \quad c_\eta(P_rb, \bar{P}_ra) = c_\lambda(P_rb, \bar{P}_ra). \tag{4.2}$$

D4 and (4.2) give

$$\begin{aligned} m_\eta(P_ra \cdot P_rb) &= m_\lambda^{1,2}(P_ra \cdot P_rb) = 2[m_\lambda^{1,2}(P_rP_sa \cdot P_rP_sb) - m_\lambda^{1,2}(P_r\bar{P}_sa \cdot P_rP_sb)] \\ &= 2[m_\eta(P_rP_sa \cdot P_rP_sb) + m_\eta(P_r\bar{P}_sa \cdot P_rP_sb)] \\ m_\eta(\bar{P}_ra \cdot P_rb) &= m_\lambda^{1,2}(P_ra \cdot P_rb) = 4\, m_\lambda^{1,2}(\bar{P}_r\bar{P}_sa \cdot P_rP_sb) \\ &= 2[m_\eta(\bar{P}_rP_sa \cdot P_rP_sb) + m_\eta(\bar{P}_r\bar{P}_sa \cdot P_rP_sb)] \\ &\qquad (r, s = 1, 2; \quad r \neq s). \end{aligned}$$

Hence from (4.1)

$$\begin{aligned} m_\eta(\bar{P}_r\bar{P}_sa \cdot P_rP_sb) &= \eta[m_\lambda^{1,2}(\bar{P}_ra \cdot P_rb)]^2 + (1 - \eta)\, m_\lambda^{1,2}(\bar{P}_r\bar{P}_sa \cdot P_rP_sb), \\ m_\eta(\bar{P}_rP_sa \cdot P_rP_sb) &= \eta m_\lambda^{1,2}(\bar{P}_ra \cdot P_rb)\, m_\lambda^{1,2}(P_sa \cdot P_sb) + (1 - \eta)\, m_\lambda^{1,2}(\bar{P}_rP_sa \cdot P_rP_sb), \\ m_\eta(P_rP_sa \cdot P_rP_sb) &= \eta[m_\lambda^{1,2}(P_ra \cdot P_rb)]^2 + (1 - \eta)\, m_\lambda^{1,2}(P_rP_sa \cdot P_rP_sb). \end{aligned} \tag{4.3}$$

Thus the function $m_\eta$ corresponds to Carnap's $m_{\lambda,\eta}$ for two families.

The derivation of (4.3) from (4.1) and (4.2) can be extended at once to a language of $N$ individuals by using *NA* 10, and to a language of $n$ two-predicate families by using *NA* 11, irrespective of any particular generalization of (3.1) to $n$ families. But the generalization to $m(M_ra \cdot M_sb)$, where $M_r$, $M_s$ are molecular predicates made up of the primitive predicates of more than two families is not entailed by (4.1) and (4.2), although the $m$-function (3.2) is the simplest generalization consistent with *NA* 1-15 and I-III.

**5. Summary.** We have investigated the conditions under which a theory of confirmation of Carnap's type can provide an adequate explication of analogy arguments, and shown that a natural generalization of Carnap's $\eta$-system for c-functions satisfies these conditions in the simplest non-trivial case. It has been suggested that the primary explicatum of a confirmation theory which is to be applicable to the real world is an elementary analogical inference which takes account of differences between instances as well as similarities; and finally it has been shown that a certain analysis of this inference yields Carnap's $\eta$-system with a minimum of arbitrary assumptions.

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