The Structure of the Models of Structural Change and Kaldor's Facts: A Critical Survey[*]

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Abstract

Although structural analysis was one of the central subjects in economics, its importance fell by the wayside, especially after aggregate macroeconomic growth models became popular in the 20th century. However, structural analysis has been revived recently and a new research agenda has emerged: to examine whether structural change can be reconciled with Kaldor's facts. This is an interesting agenda from both the theoretical and empirical point of view. Since Kaldor's facts are thought of as a sort of balanced growth path, the concept of balanced growth is extended so as to reconcile structural change with Kaldor's facts. In this study, we review the multi-sectoral models in which structural change can be reconciled with Kaldor's facts. We demonstrate that the common feature of all reviewed multi-sectoral models of structural change is that they are regarded as *natural* extensions of the one-sector model of growth and then somehow transformed into the one-sector model. However, we assert that it is not an adequate treatment of multi-sectoral models when structural change is focused. The transformation of multi-sectoral models into the one-sector model assumes a homogeneous capital but capital consists of heterogeneous commodities in modern capitalist economies. It reminds us of the lessen of the Cambridge capital controversies that the properties obtained by the one-sector model do not necessarily hold in multi-sectoral models when capital consists of heterogeneous commodities and the choice of techniques is allowed. From the empirical point of view, it is one of the important characteristics that the change in the composition of physical capital is systematically related to income growth. However, the models in which only homogeneous capital is included cannot focus on the characteristic. Whether or not structural change can be reconciled with Kaldor's facts in the models with heterogeneous capital is still an open question.

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1 Introduction

Since the advent of classical economics, the analysis of economic structures, which refers to the structures of prices, quantities, expenditure, and employment from the multi-industrial or multi-sectoral perspective, has been one of the central subjects in the principles of political economy. Smith (1979) argued for the *natural* process of economic development from a multi-industrial perspective. Ricardo (1951) constructed the growth model in which the corn and gold industries are included. Marx (1967) constructed the schema of reproduction with two sectors. As is well known, Walras (1984) constructed the model of general equilibrium.

After the aggregate models of economic growth, such as Solow (1956), became popular in the 20th century, the attention given to structural analyses faded away in macroeconomics, although the input–output table was used frequently in microeconomics. Only Goodwin (1949, 1974) and Pasinetti (1965, 1981, 1993) continued to focus on structural analysis.[1] As Silva and Teixeira (2008) showed, however, the attention to structural change revived in the 1990s.[2]

Structural change occurs for demand-side or supply-side reasons, or a mixture of both. The demand-side reason implies that non-hommothetic preferences are assumed and the supply-side reason implies that the industrial or sectoral differences in the growth rates of productivity or in factor proportions are assumed. There is ample literature on structural change caused by the demand-side reason: Falkinger (1994), Echevarria (1997, 2000), Laitner (2000), Kongsamut et al. (2001), Foellmi (2005), Bonatti and Felice (2008), and Foellmi and Zweimüller (2008). Since Herrendorf et al. (2013) argued that demand-side effects are the dominant force behind changes in final consumption expenditure share, models of structural change caused by the demand-side reason have assumed great significance.[3] On the contrary, there is relatively scarce literature of structural change caused by the supply-side reason: Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Bonatti and Felice (2008). Even scarcer is literature emphasising that structural change is caused by *both* reasons: Pasinetti (1965, 1981, 1993) and Boppart (2014a). In addition, by using a pure labour model with two commodities, Baumol (1967) emphasised the supply-side reason, which changes the relative price. He demonstrated that the change in relative price disproportionally affects consumption expenditure if the elasticity of substitution is not assumed to be unity.

It is noteworthy that a new research subject related to structural change has emerged: to examine whether structural change can be reconciled with Kaldor's (1961) facts. Kaldor's facts can be summarised as follows:

1. continued growth of aggregate production and labour productivity at steady trend rates;
2. a continued increase in the amount of capital per worker;
3. a steady rate of profit on capital that is substantially higher than the rate of interest;
4. steady capital–output ratios over long periods;
5. high correlation between profit share and investment share; and
6. appreciable differences in the rate of growth of labour productivity and total output in different societies, the rate of variation being of the order of 2–5%.

In the research subject, Kaldor's facts are interpreted as a sort of balanced growth path. Thus, the new research agenda involves investigating whether the model of structural change is consistent with balanced growth at the *aggregate* levels. However, structural change is the phenomenon of an economic system changing the *sectoral* level. In principle, therefore, it cannot be reconciled with the balanced growth path in the strict sense. The concept of 'balanced growth' must be extended for it to be reconciled with structural change. Two extended

[1] Kerr and Scazzieri (2013) demonstrated that Goodwin and Pasinetti were exceptional figures in Cambridge in that they continued to have an interest in structural analysis.

[2] In addition, the growing attention to structural change is verified by the fact that the term 'structural change' has been added to the 2008 version of *The New Palgrave Dictionary of Economics*. See Matsuyama (2008).

[3] On the contrary, Herrendorf et al. (2013) asserted that the change in income is much less important and that relative prices are much more important if sectors are categorised by the consumption value-added component, not final consumption expenditure.

concepts of the balanced growth path are presented: the *generalised balanced growth path* and the *aggregate balanced growth path*.

It is demonstrated that the common feature of the models which reconcile structural change with Kaldor's facts is to consider multi-sectoral models as a *natural* extension of the one-sector model of growth (i.e. Ramsey model); then, multi-sectoral models of structural change are reduced to a sort of one-sector model.

However, we assert that the multi-sectoral models of structural change cannot be *natural* extensions of the one-sector model of growth. This is because, first, all the models reviewed in this study have only a homogeneous capital, which contradicts the 'stylized' fact that capital generally consists of heterogeneous and reproducible commodities in capitalist economies. If capital consists of heterogeneous and reproducible commodities and the choice of techniques is allowed, it is the lessen of the Cambridge capital controversies that multi-sectoral models cannot be *natural* extensions of the one-sector model (Harcourt, 1972). The *neo-classical parable* of the one-sector model does not necessarily hold in multi-sectoral models. More importantly, we cannot pay attention to the change in physical capital composition by using the models in which only a single and homogeneous capital is included. Not only the change in capital-labour ratio but also that in physical capital composition are accompanying by economic growth. The former change has already been sufficiently analysed but the latter change has seldom been given attention yet. The change in physical capital composition would be one of important features in capitalist economies. In order to focus on the change in the composition, we must construct the models in which heterogeneous capital is included.

The rest of this paper is organised as follows. Section 2 summarises two extended concepts of the balanced growth path. Section 3 reviews a representative model that reconciles structural change caused by the demand-side reason with Kaldor's facts. Section 4 reviews a model that reconciles structural change caused by the supply-side reason with Kaldor's facts. Section 5 reviews the model which reconciles structural change caused by both demand-side and supply-side reasons with Kaldor's facts. Section 6 discusses the characteristics of the models reviewed in this study and shows that the reconciliation is based on the supposition that multi-sectoral models are *natural* extensions of the one-sector model. However, we assert that it is not an adequate treatment of multi-sectoral models, given the 'stylized' fact that capital generally consists of heterogeneous commodities and the change in physical capital composition is one of the important characteristics of economic growth. Section 7 presents concluding remarks.

2 Extension of the Balanced Growth Path Concept

As stated in the previous section, Kaldor's facts have similar properties to a sort of balanced growth path; for example, Kaldor's facts require the rate of profit and the capital–output ratio to be constant despite growth in aggregate output and labour productivity. These are the results obtained by the standard neo-classical growth models if the Harrod neutral technical progress is assumed. On the contrary, structural change is the phenomenon in which the structures of prices, quantities, and employment change over time. In principle, therefore, it cannot be reconciled with the balanced growth path in the strict sense.

It is thought that the definition of balanced growth needs to be extended so as to be able to reconcile structural change with Kaldor's facts. Two extended definitions of balanced growth have been presented so far.

Definition 1 *The generalised balanced growth path (GBGP) is the path along which the real rate of profit is constant.*

Definition 2 *The aggregate balanced growth path (ABGP) is the path along which aggregate output, consumption or expenditure, and capital grow at the same rate.*

The former originates from Kongsamut et al. (2001) and the latter from Ngai and Pissarides (2007). The former definition was adopted in Echevarria (1997, 2000), Kongsamut et al. (2001), and Boppart (2014a) as well. In addition, Herrendorf et al. (2014) focused on the former concept. Although Acemoglu and Guerrieri (2008) used the term *constant growth path*, it is substantially equivalent to the GBGP. The latter definition was adopted in Foellmi (2005), Ngai and Pissarides (2007), and Foellmi and Zweimüller (2008).

It is obvious that the former definition is weaker than the latter; it requires only the constancy of the rate of profit. The reasons why the ABGP does not exist is dependent on the structure of each model; non-existence of the ABGP results from the assumption of the utility function in some models and that of the production function in other models.

3 Reconciliation of Structural Change Caused by the Demand-side Reason with Kaldor's Facts

In this section, we examine the characteristic of models which attempt to reconcile structural change caused by the demand-side reason with Kaldor's facts. As the representative example, we closely review Kongsamut et al. (2001). See Kurose (2015) concerning other models which reconcile structural change caused by the demand-side reason with Kaldor's facts.

There are three sectors: Agriculture $\left(A(t) \in \left[\overline{A}, \infty\right)\right)$, Manufacturing $(M(t) \in \mathbb{R}_+)$, and Services $(S(t) \in \mathbb{R}_+)$. All sectors share the standard neo-classical production function, F, which is identical up to the constant of proportionality. It is assumed that only manufacturing goods can be consumed and invested and the rest of goods are just consumed. Since structural change is caused by the demand-side reason, the assumptions of production are quite normal:

$$A(t) = B_A F\left(\phi^A(t) K(t), N^A(t) X(t)\right), \tag{1}$$

$$M(t) + \dot{K}(t) + \delta K(t) = B_M F\left(\phi^M(t) K(t), N^M(t) X(t)\right), \tag{2}$$

$$S(t) = B_s F\left(\phi^S(t) K(t), N^S(t) X(t)\right), \tag{3}$$

$$\phi^A(t) + \phi^M(t) + \phi^S(t) = 1, \tag{4}$$

$$N^A(t) + N^M(t) + N^S(t) = 1, \tag{5}$$

$$\dot{X}(t) = gX(t), \tag{6}$$

where $N^i(t), \phi^i(t)$ denote labour and the share of capital employed in sector i at period t $(i = A, M, S)$, respectively. The total amount of labour is normalised to unity, which is shown by (5). $X(t)$ denotes the labour augmenting technical progress, the rate of which is $g > 0$, as shown by (6). δ is the depreciation rate.

Since capital and labour are assumed to be freely mobile, the condition for efficient allocation is that the marginal rates of transformation are equal across the three sectors. Therefore, we obtain:

$$\frac{\phi^A(t)}{N^A(t)} = \frac{\phi^M(t)}{N^M(t)} = \frac{\phi^S(t)}{N^S(t)}. \tag{7}$$

Since the proportionality of production functions is assumed, the relative prices of agriculture and services to manufacturing are given as follows:

$$p_A = \frac{B_M}{B_A}, p_S = \frac{B_M}{B_S}. \tag{8}$$

Using (1)–(8), the resource constraint for the whole economy is as follows:

$$M(t) + \dot{K}(t) + \delta K(t) + p_A A(t) + p_S S(t) = B_M F(K(t), X(t)). \tag{9}$$

The demand-side factor is characterised by non-homothetic preferences as follows:

$$U = \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} \mathrm{d}t, \text{ where } c(t) \equiv \left(A(t) - \overline{A}\right)^\beta M(t)^\gamma \left(S(t) + \overline{S}\right)^\theta, \tag{10}$$

where $\sigma, \beta, \gamma, \theta, \rho$ (rate of time preference), $\bar{A}, \bar{S}$ are assumed to be strictly positive and $\beta + \gamma + \theta = 1$. The income elasticity of demand is less than 1 for agricultural goods, equal to 1 for manufacturing goods, and greater than 1 for services, and according to Kongsamut et al. (2001), $\overline{A}$ and $\overline{S}$ can be interpreted as the level of subsistence consumption and home production of services, respectively. $c(t)$ in (10) is called the Stone–Geary preferences.

The problem to solve here is to maximise (10) subject to (9). Thus, the equilibrium real rate of profit r is given by:

$$r(t) = B_M f'(k(t)) - \delta, \tag{11}$$

where $k(t) \equiv K(t)/X(t), f(k(t)) \equiv F(k(t), 1)$. Moreover, the optimal allocation of consumption across sectors must satisfy:

$$\frac{p_A\left(A(t) - \overline{A}\right)}{\beta} = \frac{M(t)}{\gamma} \text{ and } \frac{p_S\left(S(t) + \overline{S}\right)}{\theta} = \frac{M(t)}{\gamma}. \tag{12}$$

(8) and (12) imply that both $A(t) - \overline{A}$ and $S(t) + \overline{S}$ are proportional to $M(t)$. By using (11) and (12), the optimal path for the consumption of manufacturing goods is given as:

$$\frac{\dot{M}(t)}{M(t)} = \frac{r(t) - \rho}{\sigma}. \tag{13}$$

Since $\overline{A}, \overline{S}$ are positive, there is no balanced growth path in this model; even when the real rate of profit is constant, (12) and (13) imply that consumption of A and S does not grow at a constant rate. Then, Kongsamut et al. (2001) adopted the GBGP.

As seen from (11), the constancy of the real rate of profit requires the constancy of $k(t)$. Let k^* be the value at which the real rate of profit is kept constant. Rewriting (9), the resource constraint is given as follows:

$$M(t) + \dot{K}(t) + \delta K(t) + p_A A(t) + p_S S(t) = B_M f(k^*) X(t).$$

As is clear from (6), the right-hand side grows at rate g. In the left-hand side, $A(t)$ and $S(t)$ do not grow at rate g. However, the following proposition shows the existence of the GBGP in this model:

Proposition 3 *The GBGP exists for some initial value of $k > 0$ if $\overline{A}B_S = \overline{S}B_A$ is satisfied. The GBGP for this model features constant relative prices as shown by (8), a constant growth rate of capital and aggregate output, a constant capital–output ratio, a constant share of capital income, time-varying sectoral growth rates, and employment share. As time goes by, the employment share of agriculture declines, that of manufacturing remains constant, and that of services rises.*

Proof. See Kurose (2015). ■

Proposition 3 demonstrates that households tend to spend a greater fraction of their income on services and a smaller fraction on agriculture as their incomes grow. This tendency makes equilibrium with fully balanced growth impossible. Instead, different sectors grow at different rates, and capital and labour are reallocated across sectors. However, the proposition demonstrates that the GBGP exists under such a knife-edge condition as $\overline{A}B_S = \overline{S}B_A$, and structural change occurs even though the real rate of profit and the share of capital income in national income are constant.

However, note that $\lim_{t\to\infty} \dot{A}(t)/A(t) = \lim_{t\to\infty} \dot{S}(t)/S(t) = g$ and $\lim_{t\to\infty} \dot{N}^A(t) = \lim_{t\to\infty} \dot{N}^S(t) = 0$. These results are crucially dependent on utility function (10), which combines the Stone–Geary preferences with the constant relative risk averse (CRRA) utility function. Therefore, when $A(t)$ and $S(t)$ are sufficiently large, the utility function has no substantial difference compared with a homothetic utility function. This implies that the Engel curves are almost linear, given the relative prices (8), when $A(t)$ and $S(t)$ are sufficiently large. In the limit, therefore, demand for both agriculture and services grows at the same rate. This means that structural change ceases to occur in the limit. Note that the characteristic of the reconciliation of structural change with Kaldor's facts in Kongsamut et al. (2001) model is that the three-sector model is transformed into the one-sector model, as is shown above.

According to Kongsamut et al. (2001), the knife-edge condition should be interpreted such that each agent has a positive endowment of services and a negative endowment of agricultural goods. The endowments in terms of the relative prices are such that $p_S\overline{S} = p_A\overline{A}$. The knife-edge condition implies a specific equality between technology and preference parameters, and it is obviously restrictive. In fact, Herrendorf et al. (2013) argued that the condition is not trivially consistent with the final consumption expenditure data of the US economy since the relative price of services to goods has been increasing steadily after the Second World War whereas $\overline{A}$ and $\overline{S}$ are constants. Furthermore, Kongsamut et al. (2001) has such a deficiency that the process of structural change does not fit with Kuznets' facts.[4] In other words, in the manufacturing sector, there is no change in the share of employment and the growth rate of output is kept constant at rate g. However, in reality, those shares increase at the early stage of structural change. Other models, such as Laitner (2000), add land as an additional factor of production so that the increase in manufacturing production can be explained. In addition, the assumption that all three sectors have the same production function is restrictive. Owing to this assumption, the shares of employment coincide with the output shares in this model.

In addition to Kaldor's facts, Herrendorf et al. (2014) pointed out the quantitative differences in structural patterns, depending on whether variables are measured in real or nominal terms. However, relative prices remain constant in this model, which implies that the model cannot account for the quantitative differences between real and nominal measures. Moreover, according to the model, the consumption and employment of services are zero in a very poor economy. However, Herrendorf et al. (2014) asserted that value-added and employment of services are far from zero even in the poorest economy.

4 Reconciliation of Structural Change Caused by the Supply-side Reason with Kaldor's Facts

In this section, we take Ngai and Pissarides (2007) as a representative example of the models which reconcile structural change caused by the supply-side reason with Kaldor's facts. See Kurose (2015) concerning other models which reconcile structural change caused by the supply-side reason with Kaldor's facts.

There are m sectors, among which $m-1$ sectors $(i = 1, \cdots, m-1)$ produce pure consumption goods and the last sector $(i = m)$ produces a special good which can be consumed and invested. Moreover, it is assumed that the labour force grows at an exogenous rate of g.

The household's preferences are represented by the following utility function:

$$U = \int_0^\infty e^{-\rho t} \upsilon \left[c_1(t), \cdots, c_m(t)\right] \mathrm{d}t, \text{ where} \tag{14}$$

$$\upsilon \left[c_1(t), \cdots, c_m(t)\right] \equiv \frac{\phi(\cdot)^{1-\theta} - 1}{1-\theta}; \phi(\cdot) \equiv \left(\sum_{i=1}^m \omega_i c_i(t)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)},$$

and $\rho > 0, c_i(t) \geqq 0$ denote the rate of time preference and per capita consumption level of good i at period t, respectively. Moreover, $\theta, \varepsilon, \omega_i > 0$, and $\sum_{i=1}^m \omega_i = 1$ are satisfied. If $\theta = 1$, then $\upsilon\left[c_1(t), \cdots, c_m(t)\right] = \ln \phi(\cdot)$, and if $\varepsilon = 1$, then $\ln \phi(\cdot) = \sum_{i=1}^m \omega_i \ln c_i(t)$. These are standard assumptions on preferences; demand functions have constant price elasticity $-\varepsilon$ and unit income elasticity.

On the contrary, the production function of each sector is formulated as follows:

$$c_i(t) = A_i(t) F\left(n_i(t) k_i(t), n_i(t)\right), \text{ for } i = 1, \cdots, m-1, \tag{15}$$

$$\dot{k}(t) = A_m(t) F\left(n_m(t) k_m(t), n_m(t)\right) - c_m(t) - (\delta + g) k(t), \tag{16}$$

where $n_i(t), k_i(t), k(t) \geqq 0$ denote the employment share and the capital–labour ratio in sector i, and the aggregate capital–labour ratio at period t, respectively. F is the standard neo-classical production function and $A_i(t)$ $(i = 1, \cdots, m)$ denote Hicks neutral technical progress such that $\dot{A}_i(t)/A_i(t) \equiv \gamma_i$ $\left(\gamma_i \neq \gamma_j \text{ if } i \neq j\right)$ is

[4] Kuznets' facts are the tendency, as pointed out by Kuznets (1957), implying a shift of allocation of production factors from agriculture and manufacturing to services as an economy grows.

satisfied: $A_i(t)$ is total factor productivity (TFP). Free mobility of both factors is assumed. Moreover, the following constraints are satisfied:

$$\sum_{i=1}^{m} n_i(t) = 1, \quad \sum_{i=1}^{m} k_i(t) = k(t). \tag{17}$$

As in Section 2, an optimal allocation condition requires that the marginal rates of substitution are equal to the marginal rates of transformation, which implies the following:

$$\frac{\upsilon_i(t)}{\upsilon_m(t)} = \frac{A_m(t)}{A_i(t)}, \text{ for } i = 1, \cdots, m-1, \tag{18}$$

where $\upsilon_i(t) \equiv \partial\upsilon/\partial c_i$. Conditions (17) and (18) immediately imply

$$k_i(t) = k(t) \text{ for } \forall i, \text{ and } \frac{p_i(t)}{p_m(t)} = \frac{\upsilon_i(t)}{\upsilon_m(t)} = \frac{A_m(t)}{A_i(t)} \text{ for } i = 1, \cdots, m-1. \tag{19}$$

The dynamic problem to solve is to maximise (14) subject to (15) and (16). The optimal conditions are given as follows:

$$-\frac{\dot{\upsilon}_m(t)}{\upsilon_m(t)} = A_m(t) F_k - (\delta + g + \rho), \tag{20}$$

where $F_k \equiv \frac{\partial F}{\partial k}$.

Given utility function (14), (19) yields:

$$\frac{p_i(t)\, c_i(t)}{p_m(t)\, c_m(t)} = \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m(t)}{A_i(t)}\right)^{1-\varepsilon} \equiv x_i(t), \text{ for } i = 1, \cdots, m-1. \tag{21}$$

$x_i(t)$ is a variable denoting the ratio of consumption expenditure on good i to that on manufacturing good at period t. Let us define the aggregate consumption expenditure and output per capita in terms of manufacturing as follows: $c(t) \equiv \sum_{i=1}^{m} \frac{p_i(t)}{p_m(t)} c_i(t)$, $y(t) \equiv \sum_{i=1}^{m} \frac{p_i(t)}{p_m(t)} A_i(t) F(n_i(t) k_i(t), n_i(t))$, which can be rewritten by using (15), (16), and (19):

$$c(t) = c_m(t) X(t), \quad y(t) = A_m(t) F(k(t), 1),$$

where $X(t) \equiv \sum_{i=1}^{m} x_i(t)$.

Structural change is defined in this model as the state in which at least some of the labour share changes over time: $\dot{n}_i(t) \neq 0$ for at least some sectors. The employment share can be obtained by (15) and (21):

$$n_i(t) = \frac{x_i(t)}{X(t)} \left(\frac{c(t)}{y(t)}\right), \quad n_m(t) = \frac{x_m(t)}{X(t)} \left(\frac{c(t)}{y(t)}\right) + \left(1 - \frac{c(t)}{y(t)}\right),$$

which immediately yields:

$$\frac{\dot{n}_i(t)}{n_i(t)} = \frac{\mathrm{d}(c/y)/\mathrm{d}t}{c/y} + (1-\varepsilon)(\overline{\gamma}(t) - \gamma_i), \text{ for } i = 1, \cdots, m-1, \tag{22}$$

$$\frac{\dot{n}_m(t)}{n_m(t)} = \left[\frac{\mathrm{d}(c/y)/\mathrm{d}t}{c/y} + (1-\varepsilon)(\overline{\gamma}(t) - \gamma_m)\right] \times \frac{(c/y)(x_m/X)}{n_m(t)} + \left(\frac{-\mathrm{d}(c/y)/\mathrm{d}t}{1 - c/y}\right)\left(\frac{1 - c/y}{n_m(t)}\right), \tag{23}$$

where $\overline{\gamma}(t) \equiv \sum_{i=1}^{m} \left(\frac{x_i(t)}{X(t)}\right) \gamma_i$, which is a weighted average of sectoral TFP growth rates, with the weight given by each good's consumption share. Therefore, we obtain the following proposition:

Proposition 4 *Structural change occurs in this model*

i) $\gamma_i = \gamma_m$ *for* $\forall i = 1, \cdots, m-1$*: structural change occurs between the aggregate of consumption sectors and the capital good if and only if* c/y *changes over time.*

ii) $\gamma_i \neq \gamma_m$ *for* $\forall i = 1, \cdots, m-1$ *and* $\varepsilon \neq 1$.

Proof. The validity of the proposition is guaranteed by (22), (23), and the definition of structural change. ■

Since we are interested in the relationship between structural change and Kaldor's facts, it must be assumed that F takes a Cobb–Douglas form: $F(n_i(t)k_i(t), n_i(t)) \equiv (n_i(t)k_i(t))^{\alpha} n_i(t)^{1-\alpha} = k_i(t)^{\alpha} n_i(t)$ for $\alpha \in (0,1)$.[5] Note that α is a common parameter to all sectors; this implies that factor intensities are equal in all sectors.

Given the above production function, the following proposition is obtained:

Proposition 5 *Given any initial* $k(0) > 0$*, the necessary and sufficient condition for the existence of the ABGP is given by:*

$$\theta = 1, \varepsilon \neq 1, \text{ and } \exists i \in \{i = 1, \cdots, m-1 \,|\gamma_i \neq \gamma_m\}.$$

Proof. Although case i) of Proposition 4 indicates the condition for structural change, it is inconsistent with the ABGP. This is because by definition, it requires c/y to be constant. Therefore, we examine the only case of $\varepsilon \neq 1$ and $\exists i \in \{i = 1, \cdots, m-1 \,|\gamma_i \neq \gamma_m\}$.

The equilibrium path for $\{k, c\}$ must satisfy the following differential equations:[6]

$$\begin{aligned} \dot{k}(t) &= A_m(t) k(t)^{\alpha} - c(t) - (\delta + g) k(t), \\ \theta \frac{\dot{c}(t)}{c(t)} &= (\theta - 1)(\gamma_m - \overline{\gamma}) + \alpha A_m(t) k(t)^{\alpha - 1} - (\delta + g + \rho). \end{aligned}$$

The transversality condition is given as follows:

$$\lim_{t \to \infty} k(t) \exp\left[-\int_0^t \left(\alpha A_m(\tau) k(\tau)^{\alpha-1} - \delta - g\right) d\tau\right] = 0.$$

Let us measure the aggregate consumption and the capital–labour ratio in the above system of differential equations in terms of *efficiency* units, meaning that both sides of the equations are divided by $A_m(t)^{\frac{1}{1-\alpha}}$ and the control and state variables are denoted as $\widetilde{c}(t) \equiv c(t) A_m(t)^{\frac{-1}{1-\alpha}}$ and $\widetilde{k}(t) \equiv k(t) A_m(t)^{\frac{-1}{1-\alpha}}$, respectively. In what follows, we prove sufficiency and necessity.

Sufficiency: If $\theta = 1$, then $(\theta - 1)(\gamma_m - \overline{\gamma}) = 0$ holds. In this case, this model expressed in terms of $\widetilde{c}(t)$ and $\widetilde{k}(t)$ is equivalent to the one-sector Ramsey model; it has a saddle-path equilibrium and steady state $\left(\widetilde{k}^*, \widetilde{c}^*\right)$, implying the balanced growth of $k(t), c(t)$. Their growth rate is $\frac{1}{1-\alpha}\gamma_m$. Given the definition of aggregate output $y(t) = A_m(t) F(k(t), 1) = A_m(t) k(t)^{\alpha}$, it also grows at rate of $\frac{1}{1-\alpha}\gamma_m$ on the ABGP.

Necessity: In order for the model to have an ABGP, $(\theta - 1)(\gamma_m - \overline{\gamma}(t))$ must be constant. As shown by (21), $x_i(t)$ depends on the TFP growth rates, which implies that $\overline{\gamma}(t)$ cannot be constant when structural change occurs.[7] Therefore, it is only when $\theta = 1$ that $(\theta - 1)(\gamma_m - \overline{\gamma}(t))$ is constant even though $\overline{\gamma}(t)$ is changing. The growth rate of $k(t), c(t)$ and $y(t)$ is $\frac{1}{1-\alpha}\gamma_m$. ■

Furthermore, Ngai and Pissarides (2007) derived the following proposition:

[5] Otherwise, as Uzawa (1961) showed, no steady state exists given the assumption of Hicks neutral technical progress.
[6] See Kurose (2015).
[7] See Lemma shown in the Appendix of Kurose (2015) concerning this point.

Proposition 6 *Let sector γ_h be the smallest TPF growth rate when $\varepsilon < 1$ or be the highest TFP growth rate when $\varepsilon > 1$. Then, n_h increases monotonically on the ABGP. Employment in the other sectors is either hump-shaped or declines monotonically. Asymptotically, the economy converges to an economy with*

$$\begin{aligned} n_m^* &= \widehat{\sigma} = \alpha \left(\frac{\delta + \eta + g_m}{\delta + \eta + \rho + g_m} \right), \\ n_h^* &= 1 - \widehat{\sigma}, \end{aligned}$$

where $\widehat{\sigma}$ is the saving rate (i.e. the ratio of investment to output) along the ABGP.

Proof. See Kurose (2015). ■

Proposition 6 implies that h and m are asymptotic dominant sectors and thus structural change ceases to occur in the limit (recall that structural change is defined in terms of sectoral employment share). However, this does not necessarily imply that the other sectors disappear. The growth rates of consumption and output in each sector is positive, and then sectors never vanish even though their employment shares in the limit converge to zero if $\varepsilon \leqq 1$. On the contrary, the growth rates of output may be negative in some low growth sectors if $\varepsilon > 1$, and due to Lemma in Section 8.2 of Kurose (2015) $\overline{\gamma}(t)$ is rising over time in this case, their growth rate remains indefinitely negative until they vanish.

The characteristic of the model is that the existence of an ABGP is ensured by transforming the multi-sectoral model into the one-sector model. Note that a stronger assumption about the utility function is required. Given function (14), Proposition 5 requires it to be logarithmic in the consumption composite ϕ, which implies that intertemporal elasticity of substitution is equal to unity. The ABGP requires aggregate consumption to be a constant fraction of aggregate output, since aggregate income and consumption grow at the same rate. Given homothetic utility function (14), this can hold either when consumption is independent of the rate of profit or when the rate of profit is constant. Since the rate of profit is determined by the marginal productivity of capital in this model, the constancy of the rate of profit and structural change are obviously inconsistent. Therefore, consumption must be independent of the rate of profit, which implies the logarithmic utility function. Moreover, the existence of the ABGP is dependent on the forms of production functions given by (15) and (16), in which function F is identical for all sectors while the TFP is different across sectors. Due to the identical F, the growth of aggregate consumption expenditure and output at the same rate is possible. Since factor intensities are the same across sectors in the identical F, the consumption expenditure and output can be aggregated easily.

Moreover, Proposition 6 demonstrates that the model can generate sectors with increasing employment, sectors with employment declining monotonically, and sectors with hump-shaped employment. This is an advantageous property of the model, since it can account for a 'shallow bell-shape' for manufacturing that is observed in most advanced economies.

The limitation, as Herrendorf et al. (2014) pointed out, is that the assumptions of relative TFPs and an inelastic CES utility function (i.e. $\varepsilon \in [0, 1)$) cannot generate the decrease in the real quantities of agriculture and manufacturing relative to services, which is widely observed in the growth process in most advanced economies. As Ngai and Pissarides (2004) showed, the ABGP can account for the empirical evidence for the share of employment and nominal value-added. It implies that the nominal shares of agriculture and manufacturing decline relatively. However, if a CES utility function is assumed, nominal and real shares necessarily move in opposite directions. In other words, the assumption of relative TFPs and the CES utility function cannot account for both nominal and real declines in the shares.

5 Reconciliation of Structural Change Caused by Both Reasons with Kaldor's Facts

Most models of structural change can be categorised into one of the two types reviewed so far. However, there are some models in which structural change occurs as a result of both demand-side and supply-side reasons. A recent example is Boppart (2014a).

His motivation is to present a model which is consistent with the following empirical regularities with respect to the relationship among goods, services, and the level of expenditure:

1. the share of goods in total personal consumption expenditure declines at a constant rate over time;
2. the price of goods relative to services declines at a constant rate over time; and
3. poor households spend a larger proportion of their budgets on goods than do rich households.

The model has two sectors: goods (G) and services (S). It is assumed that each household consists of $N(t)$ identical members, where $N(t) = \exp[nt]$, where $n \geqq 0$, and each member of household i is endowed with $l_i \in (\bar{l}, \infty), \bar{l} > 0$ units of labour and labour is supplied inelastically at every period of time. Therefore, the aggregate labour supply $L(t) \equiv N(t) \int_0^1 l_i di$ grows at rate n. Household i, which is indexed by $i \in [0,1]$, has the following intertemporal preferences:

$$U_i(0) = \int_0^\infty \exp[-(\rho - n)t]\, v(p_G(t), p_S(t), e_i(t))\, \mathrm{d}t, \text{ where} \tag{24}$$

$$v(p_G(t), p_S(t), e_i(t)) = \frac{1}{\varepsilon}\left(\frac{e_i(t)}{p_S(t)}\right)^{\varepsilon} - \frac{\eta}{\gamma}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma} - \frac{1}{\varepsilon} + \frac{\eta}{\gamma}, \tag{25}$$

and ρ, n are the rates of time preference and population growth, respectively, and $\rho > n > 0$ is assumed. Function v is the indirect instantaneous utility function, where $0 \leqq \varepsilon \leqq \gamma < 1$ and $\eta > 0$ are assumed. Moreover, $p_G(t), p_S(t), e_i(t)$ are the price of goods, services, and nominal per capita expenditure of household i, respectively. Function (25) shows a preference with a property such that the aggregate expenditure share coincides with that of a *representative* household whose expenditure level is the same expenditure share as that of the aggregate economy.[8] Moreover, the preferences ensure that the representative expenditure level is independent of prices within a given period.[9]

The static problem of a household is to maximise (25) subject to the budget constraint $e_i(t) = p_G(t)x_G^i(t) + p_S(t)x_S^i(t)$, where $x_G^i(t), x_S^i(t)$ denote the per capita consumption of goods and services at period t, respectively, and the dynamic problem of a household is to maximise (24) subject to the following constraints:

$$\dot{a}_i(t) = (r(t) - n)a_i(t) + w(t)l_i - e_i(t), \tag{26}$$
$$\lim_{t\to\infty} e_i(t)^{\varepsilon-1} p_S(t)^{-\varepsilon} a_i(t) \exp[-(\rho - n)t] = 0,$$

where a_i, r, w, l_i denote the per capita wealth of household i, nominal rate of profit, nominal wage rate, and labour input of household i, respectively. (26) is a usual intertemporal budget constraint and the latter is the transversality condition. Utility function (25) must represent a locally non-satiated preference, which implies:

$$e_i(t)^{\varepsilon} \geq \left(\frac{1-\varepsilon}{1-\gamma}\right)\eta p_G(t)^{\gamma} p_S(t)^{\varepsilon-\gamma}. \tag{27}$$

The production of goods and services requires an investment good, which is transformed one-to-one into capital:

$$\begin{aligned} Y_j(t) &= \exp[g_j t] L_j(t)^{\alpha} K_j(t)^{1-\alpha}, \text{ for } j = G, S, \\ Y_I(t) &= AK_I(t), \end{aligned} \tag{28}$$

[8] Instantaneous utility function (25) includes broad classes of homothetic preferences as special cases. If $\varepsilon = 0$, we obtain the limit case: $v(\cdot) = \ln\left(\frac{e_i(t)}{p_S(t)}\right) - \frac{\eta}{\gamma}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma} + \frac{\eta}{\gamma}$; if $\varepsilon = \eta = 0$, we obtain: $v(\cdot) = \ln\left(\frac{e_i(t)}{p_G(t)^{\eta}p_S(t)^{1-\eta}}\right)$; if $\eta = 0$, the model is reduced to a one-sector model and the utility function is transformed into CRRA preferences. Moreover, the case of $\varepsilon = 0$ under (25) reflects the result obtained by Ngai and Pisarrides (2007) in that if preferences are homothetic, the intertemporal substitution elasticity of expenditure must be unity in order to reconcile structural change with Kaldor's facts.

[9] Therefore, the property of the preferences is termed the 'price independent generalised linearity'. See the Appendix of Kurose (2015) concerning (25).

where $\alpha \in (0,1)$, $A > \delta$ and $Y_j(t)$ for $j = G, S, I$ denote the output of goods, services, and investment good at period t, respectively. $L_j(t)$ and $K_j(t)$ denote the input of labour and capital employed at sector j at period t, respectively. A special form of function (28) is assumed in order to prevent transitional dynamics and to focus on the co-existence of structural change and aggregate balanced growth. Both factors of production are freely mobile, and thus, wage rate $w(t)$ and the rate of profit $R(t)$ equalise across sectors. The TFPs grow at constant, exogenous, and sector-specific rates $g_j \geqq 0$ for $j = G, S$. The law of motion of capital is given as follows:

$$\dot{K}(t) = X_I(t) - \delta K(t), \tag{29}$$

where $X_I(t)$ denotes the aggregate gross investment at period t. Moreover, $A > \delta$ is assumed. The investment good is competitively produced. The price of the investment good is adopted as the numéraire at each period: $p_I(t) \equiv 1$ for $\forall t$.

The conditions for factor-market clearing are given as follows:

$$L(t) = L_G(t) + L_S(t) \text{ and } K(t) = K_G(t) + K_S(t) + K_I(t). \tag{30}$$

Let us denote the aggregate demand as $X_j(t) = N(t)\int_0^1 x_j^i(t)\,di$, for $j = G, S$. Therefore, the market-clearing condition for goods, services, and investment good is given as:

$$Y_j(t) = X_j(t), \text{ for } j = G, S, I. \tag{31}$$

Since the price of the investment good is adopted as the numéraire, the asset market clearing condition implies:

$$N(t)\int_0^1 a_i(t)\,\mathrm{d}i = K(t).$$

The market rate of return of capital is given as: $r(t) = R(t) - \delta$.

By using Roy's identity, indirect utility function (25) gives household i's expenditure functions for goods $\left(x_G^i\right)$and services $\left(x_S^i\right)$ as follows:

$$x_G^i(t) = \eta\frac{e_i(t)}{p_G(t)}\left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma} \text{ and } x_S^i(t) = \frac{e_i(t)}{p_S(t)}\left[1 - \eta\left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}\right].$$

Therefore, the expenditure shares of household i, $\varphi_j^i(t) \equiv \frac{p_j(t)x_j^i(t)}{e^i(t)}$ for $j = G, S$ can be given as follows:

$$\varphi_G^i(t) = \eta\left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma} \text{ and } \varphi_S^i(t) = 1 - \eta\left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}. \tag{32}$$

Furthermore, the elasticity of substitution between goods and services is less than or equal to 1 for all households at any period under the assumption of $0 \leqq \varepsilon \leqq \gamma < 1$.[10]

Note that $\lim_{e_i(t)\to\infty} \varphi_G^i(t) = 0$ and $\lim_{e_i(t)\to\infty} \varphi_S^i(t) = 1$ for $\varepsilon > 0$. This implies that rich households spend a larger proportion of their expenditure on services than do poor households. This is consistent with the abovementioned Empirical Regularity 3. Moreover, (32) implies that the composition of the expenditure of household i changes even in absence of the change in relative prices.

By solving the household's intertemporal optimisation problem, we obtain:[11]

$$(1-\varepsilon)\,g_{e_i}(t) + \varepsilon g_{p_s}(t) = r(t) - \rho, \tag{33}$$

[10] An elasticity of substitution below 1 implies that the sector whose relative price increases grows in terms of expenditure shares.

[11] The current-value Hamiltonian for the problem is given as follows:

$$\hat{H} = v\left(p_G(t), p_S(t), e_i(t)\right) + \lambda_i(t)\left[a_i(t)\left(r(t) - n\right) + w(t)\,l_i - e_i\right].$$

The first-order conditions are $\dot{\lambda}_i(t) = \lambda_i(t)\left(\rho - r(t)\right), e_i(t)^{\varepsilon-1}p_S(t)^{-\varepsilon} = \lambda_i(t)$.

where $g_{e_i}(t) \equiv \dot{e}_i(t)/e_i(t)$ and $g_{p_s}(t) \equiv \dot{p}_S(t)/p_S(t)$. The right-hand side and the second term of the left-hand side are common to all households, which implies that the growth rate of per capita expenditure levels must be the same for all households at a given period:

$$g_{e_i}(t) = g_e(t). \tag{34}$$

Because of the preferences that have the property of the price-independent generalised linearity, we obtain:

$$\begin{aligned} X_G(t) &= N(t)\int_0^1 x_G^i(t)\,\mathrm{d}i = N(t)\int_0^1 \eta\frac{e_i(t)}{p_G(t)}\left(\frac{p_S(t)}{p_G(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}\mathrm{d}i \\ &= \eta\frac{p_S(t)^{\varepsilon}}{p_G(t)}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}\left(\frac{E(t)}{N(t)}\right)^{-\varepsilon}E(t)\,\phi(t), \end{aligned}$$

where $\phi(t) \equiv \int_0^1 \left(\frac{e_i(t)N(t)}{E(t)}\right)^{1-\varepsilon}\mathrm{d}i$. Similarly, we can obtain $X_S(t)$. Then, the aggregate expenditure $E(t) \equiv N(t)\int_0^1 e^i(t)\mathrm{d}i$ is obtained as follows:

$$E(t) = p_G(t)\,X_G(t) + p_S(t)\,X_S(t). \tag{35}$$

In fact, $\phi(t)$ is a constant over time because it is scale invariant in all $e_i(t)$ and (34) holds.

Moreover, the aggregate expenditure share of goods $\varphi_G(t) \equiv \frac{p_G(t)X_G(t)}{E(t)}$ can be obtained:

$$\varphi_G(t) = \eta\left(\frac{p_S(t)\,N(t)}{E(t)}\right)^{\varepsilon}\left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}\phi(0). \tag{36}$$

The comparison of (36) with (32) reveals that a household with $e_i(t) = \frac{E(t)}{N(t)}\phi(0)^{-1/\varepsilon}$ is the representative agent whose expenditure level is equal to the aggregate economy.

From (33) and (34), the condition for the aggregate intertemporal optimisation is obtained:

$$(1-\varepsilon)(g_E(t) - n) + \varepsilon g_{p_S}(t) = r(t) - \rho, \tag{37}$$

where $g_E(t) \equiv \dot{E}(t)/E(t)$. In addition, the aggregate constraints are rewritten:

$$\dot{a}_i(t) = (r(t) - n)\,a_i(t) + w(t)\,l_i - e_i(0)\exp\left[\int_0^t (g_E(\tau) - n)\,\mathrm{d}\tau\right] \text{ for } \forall i, \tag{38}$$

$$\lim_{t\to\infty} a_i(t)\exp\left[\int_0^t (r(\tau) - n)\,\mathrm{d}\tau\right] = 0 \text{ for } \forall i. \tag{39}$$

where $a_i(0) > 0$ is given exogenously.

Then, the proposition concerning approximate consistency between structural change and Kaldor's facts is obtained:

Proposition 7 *Suppose that the exogenous parameters satisfy the conditions shown below:*

$$A - \delta - \rho + \varepsilon g_S > 0, \tag{40}$$

$$\rho > (1-\alpha)\,\varepsilon\,(A - \delta - n) + n + \varepsilon g_S, \tag{41}$$

$$\alpha^{\varepsilon}\bar{l}^{\varepsilon} \geqq \eta\left(\frac{1-\varepsilon}{1-\gamma}\right)\left(\frac{L(0)}{K(0)}\frac{A(1-(1-\alpha)\varepsilon)}{\rho - n - \varepsilon g_S - \varepsilon(1-\alpha)(A-\delta-n)}\right)^{\varepsilon(1-\alpha)} \tag{42}$$

$$\gamma(g_S - g_G) - \varepsilon\left(\frac{g_S + (1-\alpha)(A-\delta-\rho)}{1-\varepsilon(1-\alpha)}\right) \leqq 0. \tag{43}$$

Then, the GBGP exists. On the path, the following is obtained:

$$g_E^* - n = g_w^* = \frac{A - \delta - \rho + \varepsilon g_s}{1 - \varepsilon (1 - \alpha)}, \tag{44}$$

$$g_K^* = g_{K_G + K_S}^* = g_E^*, \tag{45}$$

$$r^* = A - \delta, \tag{46}$$

$$g_{p_j}^* = -g_j + \alpha \left(g_E^* - n \right), \text{ for } j = G, S, \tag{47}$$

$$g_{\varphi_G}^* = -\gamma \left(g_G - g_S \right) - \varepsilon \left[g_S + (1 - \alpha) \left(g_E^* - n \right) \right] \leqq 0, \tag{48}$$

$$g_{K_G}^* = g_K^* + g_{\varphi_G}^* \leqq g_K^* \leqq g_{K_S}^* = g_K^* + g_{\varphi_S}^*, \tag{49}$$

$$g_{L_G}^* = n + g_{\varphi_G}^* \leqq n \leqq g_{L_S}^* = n + g_{\varphi_S}^*, \tag{50}$$

$$g_{p_G}^* - g_{p_S}^* = g_S - g_G, \tag{51}$$

where g_w^* denotes the growth rate of the wage rate on the path.

Proof. See Kurose (2015). ■

Proposition 7 demonstrates that the asymptotic equilibrium, defined as a dynamic competitive equilibrium toward which the economy converges over time, reconciles structural change with the GBGP. (44)–(47) are results consistent with the balanced growth path; the per capita consumption expenditure, wage rate, profit rate, aggregate capital, and capital allocated to the consumption sectors grow at constant rates. The constant rate of profit, which is a central feature of the GBGP, is obtained trivially by special production function (28). The constant growth of per capita consumption expenditure implies a constant saving rate. (47) implies that the prices of goods and services change at constant rates. In addition, the capital income share is constant over time.[12]

Moreover, (48)–(51) show the sectoral unbalanced features in equilibrium. Although Kaldor's facts aggregately hold, the expenditure shares and relative prices change over time at constant rates (see (48) and (51)). This is consistent with the abovementioned Empirical Regularity 1.[13] (49) and (50) show that changing aggregate demand structure of consumption is reflected in changing sectoral resource allocation; $g_{\varphi_G}^* \leq 0$ means that capital allocated to the goods sector grows at a lower rate than capital allocated to the services sector, and the same applies to the allocation of labour.

In asymptotic equilibrium, $\lim_{t \to \infty} \varphi_G(t) = 0$ holds: the expenditure share of goods becomes zero. The existence of an asymptotic dominant sector is a characteristic also found in Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007). However, note that the asymptotic dominance of the services sector does not imply the disappearance of the goods sectors; the consumption of goods grows infinitely in absolute terms. The elasticity of substitution between goods and services is equal to $1 - \gamma$ for all households and the expenditure elasticity of demand is $1 - \varepsilon$ for goods and unity for services in the asymptotic equilibrium. Furthermore, note

[12] Let us define the aggregate income as $Y(t) \equiv p_G(t) Y_G(t) + p_S(t) Y_S(t) + Y_I(t)$. It can be rewritten as follows:

$$\begin{aligned} Y(t) &= E(t) + AK_I(t) = \left(\frac{A}{1-\alpha} \right) \left(K_G(t) + K_S(t) \right) + AK_I(t) \\ &= AK(t) + \left(\frac{\alpha A}{1-\alpha} \right) \left(K_G(t) + K_S(t) \right). \end{aligned}$$

Therefore, the capital income share on the path is given as follows:

$$\frac{r^* K(t)}{Y(t)} = \frac{r^*}{A + \left(\frac{\alpha A}{1-\alpha} \right) \left(\frac{K_G(t) + K_S(t)}{K(t)} \right)}.$$

Since, as Proposition 6 shows, $g_{K_G+K_S}^* = g_K^*$ is satisfied, the capital income share remains constant on the GBGP.

[13] (51) certainly claims that the relative price changes at a constant rate. However, this is not necessarily consistent with Empirical Regularity 2. Although $g_G > g_S$ is required for Empirical Regularity 2 to hold, it is not explicitly assumed in Boppart (2014a).

that the multi-sectoral models is transformed into the one-sector model in Boppart's (2014a) model as well as in other models reviewed above.

As already mentioned, the characteristic of this model is to introduce the 'price independent generalised linearity' preferences, shown by (25). The advantage of the introduction of such a function is that when we analyse aggregate consumption/expenditure, we only have to investigate the level of consumption/expenditure of the *representative* household. The second advantage of using function (25) is that it enables us to analyse the difference in the levels of consumption expenditure between richer and poorer households within a given period. Thanks to this advantage, the model can address the abovementioned Empirical Regularity 3 and overcome the deficiency of other non-homothetic preferences. The difference in the expenditure levels between richer and poorer households, which indicates the effects of inequality in a society, is not a major subject in existing models of structural change.[14] Although Boppart (2014a) stated that parametric conditions (40)–(43) are innocuous, it is difficult to interpret them intuitively. The restrictive assumption that both goods and services sectors have an identical production function continues to hold in this model. Furthermore, the crucial condition for existence of the GBGP is

$$\lim_{t\to\infty} a_i(t)\exp\left[-(A-\delta-n)t\right] = 0 \text{ for } \forall i.$$

This is the condition that rewrites (39). Although Boppart (2014a) does not emphasise, it is without doubt very restrictive. This shows how difficult the reconciliation of structural change is with Kaldor's facts, even though the concept of the balanced growth path is extended.

5.1 Comments on Boppart (2014a)

Although Boppart (2014a) stated that no one has so far constructed a model in which structural change is caused by both demand-side and supply-side reasons, this might be incorrect.

Echevarria (1997,2000) presented a three-sector model of structural change caused by both demand-side and supply-side reasons. She used special non-homothetic preferences which have similar properties to the Stone–Geary preferences:

$$U_i = \sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{3}\left(\alpha_j\ln C_j(t) - \eta C_j(t)^{-\rho_j}\right), \text{ where}$$
$$\sum_{j=1}^{3}\alpha_j = 1, \alpha_j > 0, \beta\in(0,1), \rho_j, \eta > 0,$$

and i is the index denoting an individual. The advantage of the utility function is that an interior solution to the static problem exists for any positive level of income. This is the demand-side reason for structural change. Moreover, she assumed that sectorally different TFP growth rates and different factor intensities, which implies that the three sectors have different production functions. This is the supply-side reason for structural change. Although any kind of balanced growth is impossible under the assumptions, the property of the utility function that she assumed is closer to that of the CRRA utility function as $C_j(t)$ becomes larger. If $\eta = 0$, the GBGP exists; labour in the three sectors remains constant while capital in the three sectors, total capital, investment, and consumption of manufacturing all grow at the same rate (manufacturing goods are consumable and invested), and consumption of primary goods and services grows at different rates. However, $\eta = 0$ means that the preferences take the homothetic log form. In other words, although structural change occurs by both the demand-side and supply-side reasons in Echevarria's (1997) model, the existence of the GBGP is ensured by eliminating demand-side reason for structural change. Boppart's (2014a) contribution is to show the existence of the GBGP, not the emergence of structural change, when both reasons are included.

Furthermore, Pasinetti (1965, 1981, 1993) has definitely constructed the models of structural change caused by both demand-side and supply-side reasons, although his model of structural change lacks micro-foundations.

[14] In addition, Foellmi (2005) deals with the effect of inequality on economic growth.

Pasinetti persistently emphasised the importance of *structural*, not aggregate, analysis of economic growth and continued to pay attention to the effects of both the demand-side reason (non-linear Engel curves) and the supply-side reason (dispersion of sectoral growth rates of labour productivity) on economic growth accompanying structural change. He took into account not only technical progress and human learning but also the hierarchy of needs and wants (Pasinetti, 1981, 1993). However, Pasinetti's model cannot reconcile structural change with Kaldor's facts. It is explicitly asserted that Pasinetti (1962), which is a particular aggregate model that exhibits balanced growth, is incompatible with his model of structural change. This is because his model of structural change has a particular property termed a *natural economic system*. It is the pre-institutional level of economic analysis.[15] The steady state is never an analytical point of reference in Pasinetti's model of structural change; the structures of prices, quantities, and employment continue to evolve in his model, and the rate of profit and wage rate continue to change, even in the long run, although they become relatively stable.[16] Perhaps, the reconciliation of structural change with Kaldor's facts is the research area belonging to the *institutional* level of investigation.

6 Discussion: The Reconciliation and Theory of Capital

Apart from Fact 6, which is related to international comparison of the performance of each economies' growth, Burmeister (1980) has already proposed the neo-classical one-sector model which can account for Kaldor's facts although Kaldor (1961) himself had asserted that none of the facts can be 'explained' plausibly by the theoretical constructions of neo-classical economic theory. As already pointed out, structural change is not the phenomenon that is perfectly consistent with Kaldor's facts. Therefore, we should closely examine how well the reviewed models reconcile structural change with Kaldor's facts.

Fact 1 (persistent growth of aggregate output and labour productivity) holds on the GBGP and the ABGP. However, a constant growth is not always obtained on the GBGP; for example, in Kongsamut et al. (2001), the constant growth of aggregate output is obtained only in the limit. Fact 2 (persistent increase in capital–labour ratio) is satisfied on the GBGP and ABGP in all the models. Fact 3 (steady rate of profit) is satisfied on the GBGP and ABGP in all the models, due to their definitions. Moreover, Fact 4 (steady capital–output ratio) holds on the ABGP due to its definition but does not necessary hold on the GBGP. This is because the growth rate of aggregate output is not necessary kept constant along the GBGP, as already pointed out. Fact 5 (high correlation between the profit share and investment share) is satisfied on the GBGP and ABGP. For the models in which the Cobb–Douglas production function is assumed, however, Fact 5 is obviously irrelevant. This is because the share of factor income is given exogenously in models in which the Cobb–Douglas production function is assumed, irrespective of the share of investment in national income.

Boppart (2014a) is the distinctive model of the reconciliation of structural change with Kaldor's facts. This is because it exhibits structural change both along the extended balanced growth path and in the limits while structural change ceases to occur asymptotically in other models reviewed in this paper. The co-existence of balanced growth at *aggregate* level and structural change at *sectoral* level in the limit is particularly interesting. Whether or not the model of structural change generates hump-shaped growth is one of the important points. Ngai and Pissarides (2007) generates the hump-shaped growth of manufacturing employment and Boppart (2014a) generates that of relative quantity of services. Moreover, Boppart (2014b) showed alternative indirect instantaneous utility function to (25) necessary to generate the hump-shaped growth of manufacturing expenditure.

Would Kaldor be satisfied with the reconciliation of structural change with the facts if he was still alive? Absolutely, his answer would be no. In the discussion with Champernowne, Hicks, Samuelson, Solow, and others at the Round Table Conference on the Theory of Capital held on the Island of Corfu in 1958, Kaldor persistently criticised the neo-classical production function (Lutz and Hague, 1961, pp. 289–403). First, he said that there are inherent logical difficulties of defining capital used by the neo-classical production function. Second, he criticised the smooth substitutability between capital and labour as an unrealistic assumption. Instead, Kaldor

[15] See Pasinetti (2007) in detail.
[16] See Kurose (2013) concerning Pasinetti's model of structural change.

(1961) assumed strict complementarity between capital and labour, according to him, which has more affinities with the classical economics of Ricardo and Marx as well as the von Neumann model. Third, he asserted that the marginal productivity has no relevance in determining the share of factor income.[17] As Pasinetti (1959) did, furthermore, Kaldor (1957) criticised that Solow's (1957) distinction between the 'movement' along the production function and the 'shift' of the production function is arbitrary and artificial.

The common feature to all the reviewed models is to consider multi-sectoral models of structural change as *natural* extensions of the one-sector model of economic growth (i.e. Ramsey model), and thus, the neo-classical economists somehow attempt to transform multi-sectoral models into a type of Ramsey model that encompasses the existing theories of structural change. If this is an adequate strategy to study the reconciliation of structural change with Kaldor's facts, the problem is only to find the combination of utility and production functions, such as homothetic, Cobb–Douglas, CES functions, for a sort of balanced growth path to exist, like in the Ramsey model.

All the models reviewed neglect such a 'stylized' fact in modern economic systems that capital consists of a bundle of heterogeneous and reproducible commodities. Although capital is reproducible in the models, it is assumed that only a homogeneous capital is treated. This is without doubt a restrictive assumption. If we take into account the stylized fact, the very lesson of the Cambridge capital controversies is that we cannot in general consider a multi-sectoral model as a natural extension of a one-sector model. The neo-classical production function works perfectly only in a one-sector model; each technique has a one-to-one correspondence to the specific rate of profit and the capital–output/capital–labour ratios are a monotonically decreasing function of the rate of profit. On the contrary, phenomena that never occur in a one-sector model can be observed in a multi-sectoral model in which capital is heterogeneous and reproducible commodities and the choice of techniques is allowed. One phenomenon is called the *reswitching of techniques*. Suppose that α is the cost-minimising technique at a rate of profit $r \in [0, r_1]$ and β is the cost-minimising technique at a rate of profit $r \in [r_1, r_2]$. Then, if α is the cost-minimising technique again at a rate of profit $r \in [r_2, r_3]$ where $r_2 < r_3$, the reswitching of techniques occurs. The other phenomenon is called *reverse capital deepening*. This means that the capital–output ratio at rate r_1 is higher than that at rate r_2 $(r_1 < r_2)$ when the cost-minimising technique is chosen.[18] *Reverse capital deepening* implies that the rate of profit cannot be always a measure of the scarcity of capital.

The consideration that multi-sectoral models can be regarded as *natural* extensions of the one-sector model is crucially dependent on assuming the neo-classical production function. It is not a *natural* extension but an artificial device from the viewpoint of capital theory, since the phenomena that are not observed in the one-sector model can occur in multi-sectoral models. The parable of a one-sector model is the essence of neo-classical economics. Moreover, neo-classical economists consider the parable as conveying desirable properties, and regard the phenomena of *reswitching of techniques* and *reverse capital deepening* as 'paradoxical behaviour' (Burmeister, 1980, p. 124). Therefore, the neo-classical economists attempt to search for the condition under which the paradoxes are excluded from the parable of the one-sector model.[19] Burmeister (1980, p. 131) characterised the condition as the concept of a *regular economy*. The natural extension of the one-sector model to multi-sectoral models extremely simplifies the complexity of real economies that we observe.

Why are the properties of the one-sector model desirable? While capital deepening is defined in terms of *physical* capital-labour ratio in the neo-classical one-sector model, capital deepening in the neo-Ricardian arguments of the *reswitching of techniques* and *reverse capital deepening* is defined in terms of the *value* of the per capita capital stock. According to Burmeister and Turnovsky (1972), therefore, the latter definition cannot generalise the results of the one-sector model. Although this seems to be a deficiency from the neo-classical point of view, why is the inability to generalise the result of the one-sector model the deficiency? Rather, regarding the inability as the deficiency would be ideological. Regarding the results of the one-sector

[17] With respect to this point, see also Kaldor (1956, 1966).

[18] See Harcourt (1972) and Pasinetti (1977) for details.

[19] The reverse capital deepening is a more embarrasing phenomenon for the neo-classical economists than the reswtiching of techniques. This is because the former may contradict such a neo-classical property that per capita consumption is a monotonically decreasing function of the rate of profit. In other words, the paradoxical behaviour of consumption can occur without the reswitching of techniques. See Burmeister (1980) in detail.

model as desirable may overlook the complex effects of price changes inherent in capitalist economies in which capital consists of heterogeneous commodities, as we have already pointed out. Without doubt, the complex effects are not negligible in the analysis of structural change. In fact, even Burmeister (2000, p. 312) conceded that methodology that relies on one-capital models may, for some questions at least, lead to serious mistakes. Similarly, Herrendorf et al. (2014) honestly confessed that the neo-classical multi-sectoral models that have the GBGP or ABGP are overly restrictive.

The most serious deficiency of the attempt to transform multi-sectoral models into the one-sector model is that it cannot focus on the change in capital composition caused by economic growth. According to Nomura (2004, p. 155), the proportion of 'Construction' to total capital stock declined by about 13 % in real term in Japan from 1960 to 2000 and the average growth rate of 'Construction' (5.8%) is lower than the average growth rate of total capital stock (6.8%) during the period. On the contrary, the proportions of 'General Instrumentation' and 'Electric Machine' tend to increase and the average growth rates of them are much higher than the average growth rate of total capital stock.[20] These results imply that the composition of physical capital changes as income grows.

Mutreja (2014) asserted that the relation between composition of physical capital and income differences has not been sufficiently paid attention while the relation between capital-output ratio and income differences has closely analysed. Moreover, she demonstrated that the composition of physical capital is systematically related with the income level, according to her, which is one of the important factors to explain the differences in income levels across countries.[21] According to her, the cross-country differences in equipment capital are much larger than the differences in structure capital; the equipment capital-output ratio a factor of approximately 7 between rich and poor while the structures capital-output ratio is a factor of only 3. The results should be carefully taken into account when we pay attention to Fact 6. Moreover, the cross-country dispersion in the equipment capital-output ratio has also increased over time while the dispersion of in the structures capital-output ratio has declined. It was also demonstrated that the standard growth accounting has attributed a larger fraction of the income differences to the TFP differences in the models in which heterogeneous capital is excluded. Mutreja's (2014) results imply that in growth process of a country's income the composition of physical capital changes in such a way that the proportion of equipment capital to aggregate capital increases. The importance of the change in the composition of physical capital in the growth process is not focused in some comprehensive surveys of structural change, such as Herrendorf et al. (2014). As is shown by the property of non-homothetic preferences, the demand structure is affected by the income level, which is systematically related to the composition of physical capital. The effect caused by the change in the composition of physical capital should not be overlooked when we analyse structural change and economic growth. Mutreja's results strongly support the importance of the existence of heterogeneous capital in the analysis of structural change.

In summary, the transformation of multi-sectoral models into the one-sector model is not an satisfactory approach to the reconciliation of structural change with Kaldor's facts. At first, some restrictive assumptions on the form of functions are required in order for multi-sectoral models to transform into the one-sector model successfully. Second, more importantly, the change in the composition of physical capital accompanying economic growth cannot be analysed since the transformation of multi-sectoral models into the one-sector model requires capital to be homogeneous. The change in the composition of physical capital is one of the essential features of economic growth, and then the analyses of the change in the composition should not be disregarded. The change in the composition can be analysed only in the models in which heterogeneous capital is included. The multi-sectoral models which reconcile structural change with Kaldor's facts shall account for not only the change in the allocation of capital to each sector, like (49), but also the change in the composition of allocated capital.

In order to examine the condition for reconciling structural change with Kaldor's facts, however, we make the neo-Ricardian models dynamic. In fact, it is related to Burmeister's critiques of the neo-Ricardian models. Burmeister (1980, p. 5) asserted that the neo-Ricardian analysis of the comparison of alternative steady state equilibria is irrelevant, even though the paradoxes occur between the two steady states. According to him, the

[20] As Herrendorf et al. (2014) pointed out, the change in capital structure considerably differs, depending on whether they are measured in real or nominal terms. In general, the proportions are more stable when they are measured in nominal term.

[21] Caselli (2005) also indicated the effect of capital composition on the income differences.

only relevant facts from growth theory concern the properties of the set of technologically feasible dynamic paths emanating from, for example, steady state A as the initial condition. In fact, there may not exist a feasible path along which it is possible to move from A to alternative steady state B. The neo-Ricardian economists, with a few exceptions, have focused on only steady states, and generally refused to consider the states in which economic system is changing (e.g. Eatwell, 1977). In order to increase our understanding of the relationship between the reconciliation of structural change with Kaldor's facts and economic growth, we shall attempt to construct the multi-sectoral model which can account for the reconciliation without neglecting the importance of the change in the composition of physical capital.

7 Concluding Remarks

In this study, we review the representative models in which structural change is reconciled with Kaldor's facts. Here, Kaldor's facts are reduced to the extended concepts of the balanced growth path: the GBGP and the ABGP. All the models show that multi-sectoral models exhibit a sort of balanced growth on either the GBGP or the ABGP. The common feature to all the models is to consider multi-sectoral models as the *natural* extensions of of the one-sector model. Therefore, all the models attempt to somehow transform the multi-sectoral models into the one-sector model by imposing a set of assumptions.

We argue that the transformation is not an adequate treatment of the models of structural change. This is because, at first, very restrictive assumptions are required to transform. Second, all the models assume that capital is reproducible but is homogeneous. It implies that all the models cannot focus on the change in the composition of physical capital, which is one of important aspects of structural change and economic growth. No one has so far confirmed whether or not structural change can be reconciled with Kaldor's facts in the multi-sectoral models in which heterogeneous capital is included.

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