

# *Welcher Weg* Experiments, Duality and the Orthodox Bohr's Complementarity Principle

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## Abstract

In its most orthodox form, Bohr's Complementarity Principle states that a quanton (a quantum system consisting of a Boson or Fermion) can either behave as a particle or as wave, but never simultaneously as both. A less orthodox interpretation of this Principle is the "duality condition" embodied in a mathematical inequality due to Englert [B-G Englert, Phys. Rev. Lett., 77, 2154 (1996)] which allows wave and particle attributes to co-exist, but postulates that a stronger manifestation of the particle nature leads to a weaker manifestation of the wave nature and vice versa. In this Letter, we show that some recent *welcher weg* ("which path") experiments in interferometers and similar set-ups, that claim to have validated, or invalidated, the Complementarity Principle, actually shed no light on the orthodox interpretation. They may have instead validated the weaker duality condition, but even that is not completely obvious. We propose simple modifications to these experiments which we believe can test the orthodox Complementarity Principle and also shed light on the nature of wavefunction collapse and quantum erasure.

**Keywords:** Complementarity Principle, Wave-Particle Duality, *Welcher Weg*, Wavefunction Collapse

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The orthodox Bohr’s complementarity principle [1] states that a quanton can behave *either* as a particle *or* as a wave, but never as *both at the same time*. *Welcher weg* experiments conducted with two-path interferometers (or analogous set-ups) are a suitable vehicle to test this strict complementarity between the wave- and particle-nature. If one can determine - even in principle - which of the two paths in the interferometer was traversed by the quanton, then the entity behaves as a “particle” since a “wave” would have sampled both paths simultaneously. In this case, there should be no interference (or any other wave-like behavior) if the orthodox Complementarity Principle holds. On the other hand, if there is interference, then the wave property is intact in which case it should have been impossible to discern the particle attribute, i.e. to tell which path was traversed. The orthodox Complementarity principle therefore allows only sharp wave or sharp particle attribute, but not both. A somewhat tempered version of the Complementarity Principle is the *duality principle* due to Englert [2] which states that a quantum system can simultaneously exhibit wave and particle behavior, but sharpening of the wave character blurs the particle character and vice versa. In fact, Englert derives an inequality

$$P^2 + V^2 \leq 1 \quad (1)$$

where  $P$  is a measure of the “which-path” infor-

mation (particle attribute) and  $V$  is a measure of the “(interference) fringe-visibility” (wave attribute). Equation (1) immediately shows that stronger wave or stronger particle behavior can be manifested only at the expense of each other.

### **Welcher Weg experiments that question the orthodox Complementarity Principle**

Experiments purported to demonstrate violation of the *orthodox* version of the Complementarity Principle were proposed and carried out in the past. Ghose, Home and Agarwal [3] had proposed a biprism experiment schematically depicted in Fig. 1(a). A single photon source emits a single photon which is split into orthogonal states  $\psi_r$  and  $\psi_t$  by a 50:50 beam splitter. They are detected by two photon detectors  $D_r$  and  $D_t$ . If the photon behaves truly as a particle, then it should be detected at either  $D_r$  or  $D_t$  (but never at both) since a particle cannot traverse two paths simultaneously. That is, there should be perfect anti-coincidence between  $D_r$  and  $D_t$ , or, in other words, *either*  $D_r$  *or*  $D_t$  will click but *both* will never click in between the arrival of two successive photons. The clever twist in this experiment, motivated by an experiment performed in the 19th century by Jagdish Chandra Bose [4], is the placement of the biprism with a small tunneling gap in the path of the transmitted photon. If  $D_t$  clicks *and*  $D_r$  *does not*, then we have made a “which path” determina-

tion (the particle took the path of transmission as opposed to reflection) and a *sharp* particle nature is demonstrated [5]. Yet, to arrive at  $D_t$ , the particle must have tunneled through the biprism and tunneling is a *sharp* wave attribute. In this experiment, later conducted by Mizobuchi and Ohtake [6], perfect anti-coincidence was found between  $D_t$  and  $D_r$  demonstrating the particle nature. Yet, the very fact that  $D_t$  ever clicked required tunneling and hence the existence of a wave nature. It was claimed that in this experiment, a photon was behaving both sharply as a particle and as a wave in violation of Bohr's Complementarity Principle. A slight modification of this experiment has been proposed by Rangwala and Roy [7] where interference is used instead of tunneling to showcase the wave-like behavior. They claimed that quantum mechanics does not prohibit the demonstration of simultaneous wave and particle behavior; rather, it prohibits their simultaneity only when the wave and particle attributes are "complementary" [8] in the sense that projection operators associated with them do not *commute*. This is actually consistent with Englert's work [2] in that Englert takes pain to point out that Equation (1) does not rely on Heisneberg type uncertainty, i.e. the following relation need *not* hold:

$$\Delta P \Delta V \geq \frac{1}{2} | \langle [P, V] \rangle | . \quad (2)$$

In the experiments of refs. [3, 6, 7], the wave

and particle behavior supposedly are not truly complementary and hence not subject to the restrictions of the orthodox Complementarity Principle.

A mathematical framework to determine true "complementarity" between wave and particle attributes was first addressed by Kar. et. al. [9]. Complementary observables as those whose projection operators *do not commute*, i.e. have no common eigenvectors. In the experiments of refs. [3, 6, 7], the Hilbert spaces  $H_r$  and  $H_t$  associated with the reflected state  $\psi_r$  and the transmitted state  $\psi_t$  are orthogonal (since there is always anti-coincidence between reflection and transmission). Hence the projection operators  $P_r$  and  $P_t$ , corresponding to reflection and transmission respectively, always commute. If we assume that the wave property (tunneling) is represented by some projection operator  $P_{wave}$ , its Hilbert space is contained within the Hilbert space of  $P_t$ . i.e.  $\langle \psi | P_{wave} | \psi \rangle \leq \langle \psi | P_t | \psi \rangle$  and is hence orthogonal to  $H_r$ . Thus,  $P_{wave}$  commutes with  $P_r$ . But  $P_{wave}$  must also commute with  $P_t$  since every state  $\psi_t$  that tunnels through the biprism and reaches  $D_t$  is a common eigenvector of these two operators. Thus,  $P_{wave}$ ,  $P_t$  and  $P_r$  all commute with each other. Hence the sharp wave property (tunneling) and the sharp particle property (anti-coincidence between detectors  $D_t$  and  $D_r$ ) are *not* complementary and their simultaneous observation is *not*

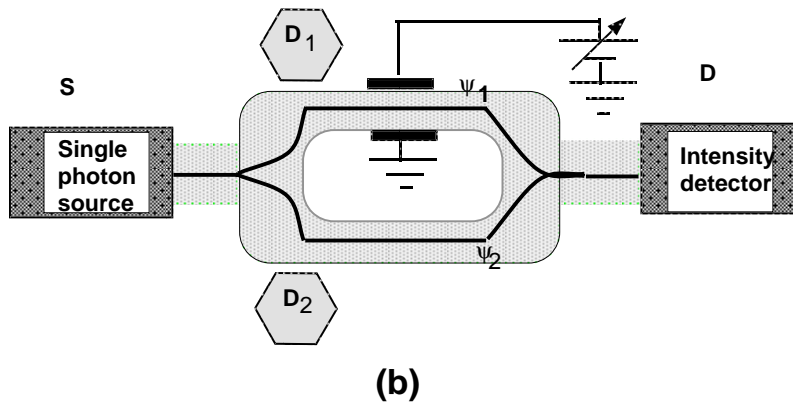
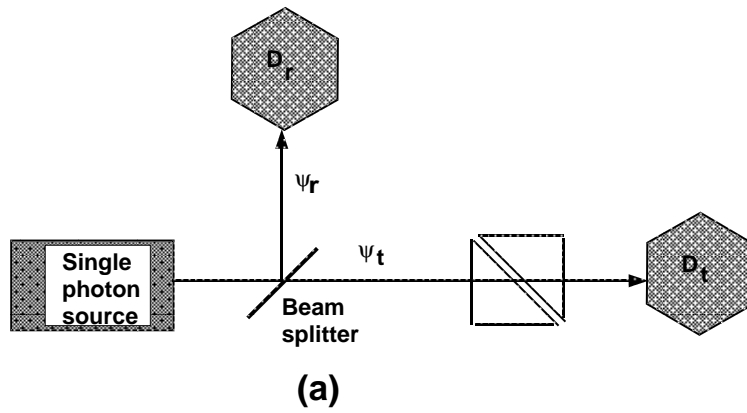


Figure 1: (a) A welcher-weg biprism experiment to demonstrate violation of the Complementarity Principle; (b) a welcher-weg Mach-Zender interferometry experiment that can test the Complementarity Principle more rigorously.

prohibited by the Complementarity Principle (equation (1) however, must still be obeyed, but the experiments did not test this inequality). In concluding their paper, Kar et. al. [9] point out that the experiments of refs. [3, 6, 7] do not test wave and particle properties that are complementary and hence can draw no conclusion about the validity or invalidity of the “orthodox” Complementarity Principle.

**An alternate *welcher weg* experiment to test the orthodox Complementarity Principle**

We propose an alternate *welcher weg* experiment where sharp particle behavior and sharp wave behavior would be complementary in the sense defined by Kar and co-workers. Consequently, this is an *unambiguous* experiment where simultaneous exhibition of sharp particle- and wave-character will give lie to the orthodox version of the Complementarity Principle. We point out that these experiments are worth conducting since their outcome is by no means a foregone conclusion. The orthodox Complementarity Principle is not sacrosanct (even though it is viewed by some as a cornerstone of the Copenhagen interpretation of quantum mechanics); viewpoints due to Einstein and DeBroglie do not subscribe to the Complementarity Principle [10, 11].

Consider a Mach-Zender type interferometer as shown in Fig. 1(b). Proximity photon detectors  $D_1$  and  $D_2$  are placed near each limb as

terminal  $S$  which reaches a screen  $D$  after traveling along the two possible paths comprising the arms of the ring (actually there are a denumerably infinite number of paths possible if we take into account multiple reflections, but they are not important in this context).

To demonstrate an invalidation of the orthodox Complementarity Principle, we need to demonstrate *two* effects simultaneously:

1. Perfect anti-coincidence between  $D_1$  and  $D_2$  (sharp particle nature).
2. Existence of an interference pattern at  $D$  (sharp wave nature)

In this example, the projection operators  $P_1$  and  $P_2$  corresponding to the traversal of the two paths of the interferometer are orthogonal, but  $P_{wave}$  is manifestly not orthogonal to either one of them. Note that the Hilbert space  $H_{wave}$  is not contained within either  $H_1$  or  $H_2$  since neither of the two paths alone is sufficient to cause interference (both paths are needed). Also note that  $\psi_1$  and  $\psi_2$  (while eigenfunctions of  $P_1$  and  $P_2$  respectively), are not eigenfunctions of  $P_{wave}$ . Hence, unlike in the experiments in ref. [3, 7],  $P_{wave}$  does not commute with  $P_1$  and  $P_2$  and therefore the wave and particle properties are indeed complementary. Thus, their simultaneous manifestation will definitely give lie to the Complementarity Principle.

## *Welcher weg* experiments that claim to have validated Complementarity or Duality

The duality principle embodied in Englert’s inequality [Equation (1)] was verified in atom interference experiments [12] and perhaps even in recent experiments conducted with electrons traversing an Aharonov-Bohm (A-B) quantum interferometer whose one arm contained a quantum dot (QD) with a nearby quantum point contact (QPC) [13, 14]. The QD has a non-critical, peripheral role; it merely serves to trap an electron traveling that path long enough for the QPC to detect it. The trapping changes the transmission probability through the QPC (and hence its conductance) thus allowing “which path” detection.

The experiment [13] showed that the A-B interference was diluted if one could even *in principle* detect which path was traversed, irrespective of whether the detection actually took place. When the QPC detector was turned on, the interference peaks were diluted regardless of whether one monitored or not any change in the QPC conductance caused by a fleeting electron in the nearby path. This result, remarkable as it is, does *not* shed any light on the orthodox version of Bohr’s complementarity principle. Validation of the orthodox version would require demonstrating *complete* vanishing of interference along with the demonstration of perfect anti-coincidence between *two* QPC detectors placed

near the two paths of the interferometer. Unfortunately, this was not attempted in the experiment. Second, the experimental result may be consistent with the Duality Principle [Equation (1)] (like ref. [12]), but does not quite validate it either (validation is a stronger condition than consistence). We say this because turning the QPC on introduces an *asymmetry* into the interferometer (e.g. decrease the transmittivity of one path relative to another) and this alone can cause a dilution of the interference as we show in the appendix.

We suggest some modifications to this experiment to test the orthodox version of Bohr’s complementarity principle. The configuration of the interferometer used in ref. [13] is shown in Fig. 2(a). It was defined on a high mobility two-dimensional electron gas using standard split-gate technology. The modifications are the following:

1. Introduce electrons one at a time into the interferometer using single charge tunneling (single electron pump or turnstile) [15]. This can be done by delineating a small island at the mouth of the emitter and isolating it from the emitter with a tunnel barrier whose resistance is much higher than  $h/e^2$  ( $h$  = Planck’s constant and  $e$  = single electron charge). Modulating the barrier with an external potential can cause elec-

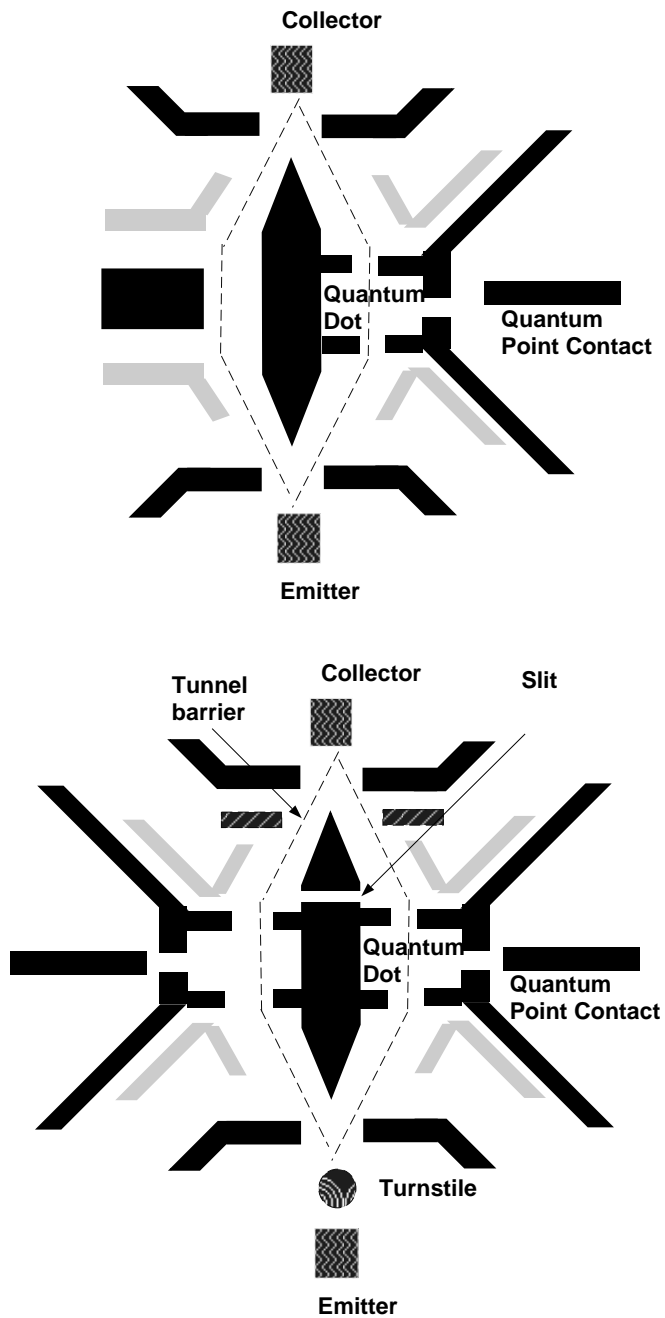


Figure 2: (a) A welcher-weg Aharonov-Bohm interferometry experiment conducted with electrons to test the Complementarity Principle; (b) a modified welcher-weg Aharonov-Bohm interferometry experiment that can test the Complementarity Principle more rigorously as well as test “quantum erasure”.

trons to be injected one at a time. There may be other, more complicated, schemes for pumping single electrons [16, 17] that may be more appropriate depending on the measurement approach.

2. Place two tunnel barriers in the two arms of the A-B interferometer near the detector  $C$ . They can also be defined by split gates and are shown as striped metal gates in Fig. 2(b). If an electron is detected and it collapses to a “particle”, it *cannot* tunnel through the barrier and reach the collector  $C$  since only waves tunnel and particles do not tunnel. Therefore, it must be reflected back into the emitter. Thereafter it cannot take the other path to the collector either because there is a tunnel barrier in that path as well. Consequently there should be no current. The evolution of the particle nature should not only destroy the interference, but actually reduce the current to zero.
3. Place a narrow slit defined by split gates in between the QPC detectors and the detector  $C$ . This connects the two paths. Normally this slit is pinched off by the split gates to isolate the two paths, but it can be opened (made conducting) by the split gate voltages, if necessary. This arrangement is relevant to the study of “quantum

erasure” as shown later.

4. Place *two* QD-QPC detectors alongside both arms. They serve two purposes. They can make the interferometer *symmetric* and they could also be used to demonstrate a perfect anti-coincidence between the detection events in the two paths (detection events correspond to a change in the QPC current when an electron passes by).

With the above modifications, the A-B interferometer can now be used to remove all the objections that we raised in relation to the experiment of ref. [13]. The interferometer is nominally symmetric as long as both detectors are turned on. Perfect anti-coincidence can be demonstrated if it exists, and the tunnel barriers in the interferometer arms will conclusively demonstrate if the wave nature is present or absent. Thus *sharp* particle- and wave-behavior can be exhibited with no ambiguity.

### **Which path determination and quantum erasure**

In addition to other differences, a major point of departure between the experiment of ref. [3] and the above experiment is that the photon experiment purports to demonstrate wave behavior *before* the particle behavior is evidenced in the transmission path, whereas here the opposite is proposed. The tunneling gap in Fig. 1(a) *precedes* the detector  $D_t$ . In our experiment (Fig.



2(b)), the reverse is true; the detector precedes the tunnel barrier. Thus, simultaneous exhibition of wave- and particle- nature in our proposed experiment would require an electron to first “become” a particle and then be reincarnated as a wave. “Which path” determination does not prohibit this since entanglement of the electron’s wavefunction with that of the detector does not cause the entangled pure state to evolve into a mixed state irreversibly. We show this below.

Let  $\psi_1$  and  $\psi_2$  be the wavefunctions in the two arms of the interferometer and  $\theta$  is the A-B phase difference between the two arms. Then, neglecting multiple reflection effects [19], the wavefunction  $\psi$  at the detector D is

$$\psi = \psi_1 + e^{i\theta}\psi_2 \quad (3)$$

The current density at the detector is

$$J = \frac{ie\hbar}{2m^*} [\psi(\nabla\psi)^* - \psi^*\nabla\psi] \quad (4)$$

Thus, for propagating states (plane waves along the direction of propagation), Equations (1) and (2) yield

$$J \propto |\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + e^{i\theta}\psi_1^*\psi_2 + e^{-i\theta}\psi_2^*\psi_1 \quad (5)$$

The last two terms account for the A-B interference.

A fundamental result of quantum measurement theory is that if a detector tries to detect which arm was traversed by the interferometer,

the wavefunction of the detector becomes entangled with that of the electron. The entangled wavefunction can be written as

$$\Phi = \psi_1|1\rangle + e^{i\theta}\psi_2|2\rangle \quad (6)$$

where the wavefunctions  $|1\rangle$  and  $|2\rangle$  span the Hilbert space of the detector. The state  $|1\rangle$  corresponds to detecting the particle in path 1 and  $|2\rangle$  corresponds to detecting the particle in path 2.

The current density associated with this wave function is

$$J_{entangled} \propto |\Phi|^2 = |\psi_1|^2 \langle 1|1\rangle + |\psi_2|^2 \langle 2|2\rangle + \langle 1|2\rangle e^{i\theta}\psi_1^*\psi_2 + \langle 2|1\rangle e^{-i\theta}\psi_2^*\psi_1 \quad (7)$$

If the detector is an “unambiguous” detector which unambiguously determines which path is traversed by the particle, then  $|1\rangle$  and  $|2\rangle$  are orthonormal and hence the interference terms vanish in Equation (5) and the wave behavior is lost. This is interpreted as dephasing or collapse.

$$J_{collapsed} \propto [|\psi_1|^2 + |\psi_2|^2] \quad (8)$$

In truth however, the collapse is not quite irreversible since if we design an experiment whose result is the probability of a particular outcome of the *welcher weg* determination *and* finding the detector in the symmetric state  $[|1\rangle + |2\rangle]$ , we find

$$J_{erase} \propto [|\langle 1| + \langle 2| \rangle \Phi \rangle|^2$$

$$\begin{aligned}
&= |[\langle 1| + \langle 2|][\psi_1|1\rangle + e^{i\theta}\psi_2|2\rangle]|^2 \\
&= |\psi_1|^2 + |\psi_2|^2 + e^{i\theta}\psi_1^*\psi_2 + e^{-i\theta}\psi_2^*\psi_1 \\
&= |\psi_1 + e^{i\theta}\psi_2|^2 = |\Phi|^2
\end{aligned} \tag{9}$$

Thus, the original wavefunction of Equation (4) (along with the interference terms) is regenerated from the entangled wavefunction once we choose to erase the "which path" information. This is termed "quantum erasure" [20, 21, 22] and is possible because the entangled state of Equation (4) is still a pure state and not a mixed state. Evolution of a pure state into a mixed state (termed orthodox collapse) [18] is irreversible, but that is not the case here.

We can test the quantum erasure by opening the slit which, in principle, allows conduction between the two paths and hence effectively erases the which path information. This can then regenerate the wave nature and allow tunneling through the tunnel barriers in the two paths. Consequently, the current (which is ideally zero) with the slit closed and the QPC detectors on, will rise to a non-zero value when the slit is opened. This will be a demonstration of quantum erasure.

In conclusion, we have proposed modifications to some welcher weg experiments that we believe can rigorously test the Complementarity Principle.

## Appendix

We show that introducing asymmetry between the two paths of an interferometer degrades the visibility of the interferograms independent of any other effect.

Consider a two-path interferometer. The current between the source and the detector can be written as (neglecting multiple reflection of the electron wave between the emitter and collector (due to geometric discontinuities)

$$I = |i_1 + i_2 e^{i\theta}|^2 \tag{10}$$

where  $i_1$  ( $i_2$ ) is the complex amplitude of the current in path 1 (2) and we assume that  $|i_1| = \alpha|i_2|$  ( $0 \leq \alpha \leq 1$ ), i.e. the transmissivity of path 2 is equal to or larger than that of path 1. Thus,  $\alpha$  is a measure of the asymmetry; the farther  $\alpha$  is from unity, the more is the asymmetry. The angle  $\theta$  is the phase difference between the two paths. We now find

$$I = |i|^2 [1 + \alpha^2 + 2\alpha \cos\theta] \tag{11}$$

We can adopt a suitable metric for the visibility of the oscillation. This could be the relative modulation  $M$  of the current

$$M = \frac{I_{max} - I_{min}}{I_{max}} = \frac{4\alpha}{(1 + \alpha)^2} \tag{12}$$

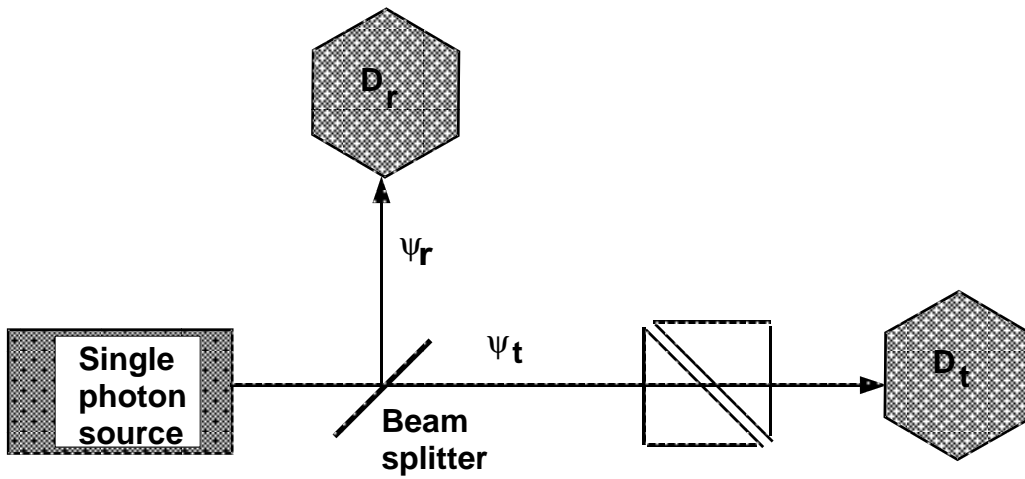
From the above equation, we can see that  $M$  is 100% if  $\alpha = 1$ ; otherwise,  $M$  decreases as  $\alpha$  decreases. Thus, increasing the asymmetry in a two-path interferometer degrades the visibility of

the interference oscillations independent of any other effect.

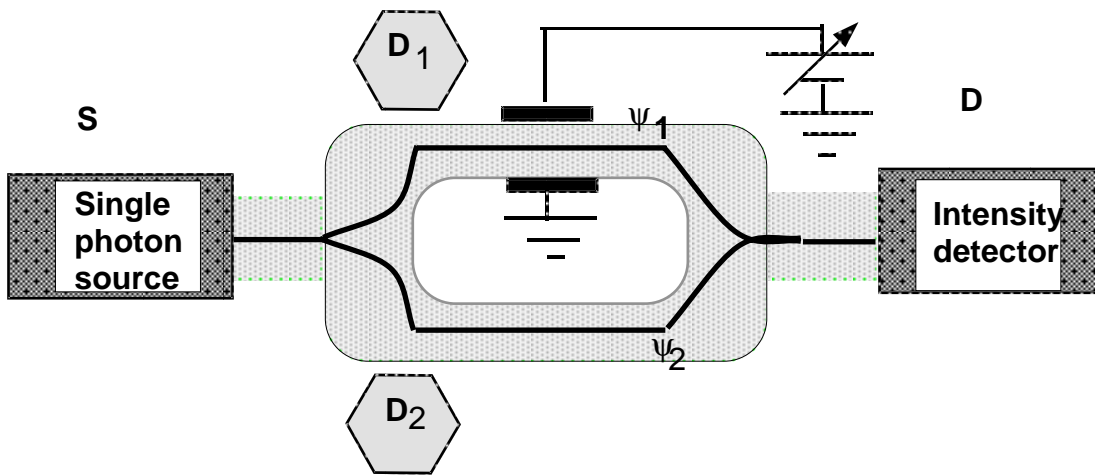
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(a)



(b)

