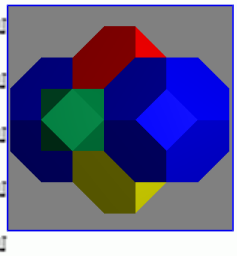
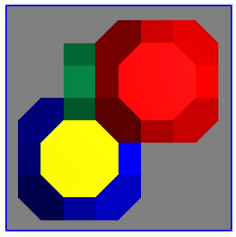
Uniform Honeycombs in 3-Space



- Here is a collection of uniform regular-polyhedral honeycombs. George Olshevsky defines a uniform honeycomb in three-space as "one all of whose cells are uniform polyhedra and whose vertices are transitive on its symmetry group, and which covers three-space exactly once". Banko Grünbaum lists all 28 of these in his 1994 paper: "Uniform tilings of 3-space," Geombinatorics 4, 49-56. I have not read seen the paper yet, so any errors with the figures are my own. Gurnbaum's paper does not represent the original discovery of these figures, as they appear to have been known for some time (dating back at least to the 1950's). But in any case, here is a simple presentation of these figures for those who are interested.



Note: Each figure shows all of the polyhedra surrounding a single vertex, with the exceptions of figures 24, 25, 27 and 28. See the explanation below the table for those figures.

Model	.Stel	Long Name and Alternate Names	Components per vertex
01-Chon	.Stel	Cubic honeycomb	8 Cubes
02-Rich	.Stel	Rectified cubic honeycomb	4 cuboctahedra, 2 octahedra
03-Tich	.Stel	Truncated cubic honeycomb	4 truncated cubes, 1 octahedron
04-Srich	.Stel	Cantellated cubic honeycomb (small rhombated)	2 rhombicuboctahedra, 1 cuboctahedron, 2 cubes
05-Batch	.Stel	Bitruncated cubic honeycomb; truncated octahedral honeycomb	4 truncated octahedra
06-Grich	.Stel	Cantitruncated cubic honeycomb (great rhombated)	2 truncated cuboctahedra, 1 truncated octahedron, 1 cube
07-Prich	.Stel	Runcitruncated cubic honeycomb (prismatorhombated)	1 truncated cube, 1 rhombicuboctahedron, 2 octagonal prisms, 1 cube
08-Otch	.Stel	Omnitruncated cubic honeycomb	2 truncated cuboctahedra, 2 octahedral prisms
09-Tassiph	.	Truncated square prismatic	4 octagonal prisms, 2 cubes

	Stel	honeycomb	
10-Sassiph	Stel	Snub square prismatic honeycomb	6 triangular prisms, 4 cubes
11-Octet	Stel	Octahedral-Tetrahedral honeycomb (oct-tet struss)	8 tetrahedra, 6 octahedra (see 26-Gytoh)
12-Ratoh	Stel	Runcinated alternated cubic honeycomb; runcinated tetrahedral-octahedral honeycomb	3 rhombicuboctahedra, 1 tetrahedron, 1 cube
13-Tatoh	Stel	Truncated alternated cubic honeycomb; truncated tetrahedral-octahedral honeycomb	2 truncated octahedra, 2 truncated tetrahedra, 1 cuboctahedron
14-Gratoh	Stel	Cantitruncated alternated cubic honeycomb; cantitruncated tetrahedral-octahedral honeycomb	2 truncated cuboctahedra, 1 truncated cube, 1 truncated tetrahedron
15-Batatoh	Stel	Bitruncated alternated cubic honeycomb; bitruncated tetrahedral-octahedral honeycomb	6 truncated tetrahedra, 2 tetrahedra
16-Hiph	Stel	Hexagonal prismatic honeycomb	6 hexagonal prisms
17-Thiph	Stel	Triangular-hexagonal prismatic honeycomb	4 hexagonal prisms, 4 triangular prisms
18-Tiph	Stel	Triangular prismatic honeycomb	12 triangular prisms
19-Thaph	Stel	Truncated hexagonal prismatic honeycomb	4 dodecagonal prisms, 2 triangular prisms
20-Rothaph	Stel	Rhombitriangular-hexagonal prismatic honeycomb	2 hexagonal prisms, 2 triangular prisms, 4 cubes
21-Otathaph	Stel	Omnitruncated triangular-hexagonal prismatic honeycomb	2 dodecagonal prisms, 2 hexagonal prisms, 2 cubes
22-Snathaph	Stel	Snub triangular-hexagonal prismatic honeycomb	2 hexagonal prisms, 8 triangular prisms
23-Gytoph	Stel	Gyrated triangular prismatic honeycomb	12 triangular prisms
24-Etoph=x	Stel	Elongated triangular prismatic honeycomb	6 triangular prisms, 4 cubes (see note below)
25-		Gyroelongated triangular prismatic	6 triangular prisms, 4 cubes (see

Gyetaph=x	Stel	honeycomb	note below)
26-Gytoh	Stel	Gyrated Tetrahedral-Octahedral honeycomb	8 tetrahedra, 6 octahedra (see 11-Octet)
27-Etoh=x	Stel	Elongated cubic honeycomb; elongated tetrahedral-octahedral honeycomb	3 octahedra, 4 tetrahedra, 6 triangular prisms (see note below)
28-Gyetoh=x	Stel	Gyroelongated cubic honeycomb; gyroelongated tetrahedral-octahedral honeycomb	3 octahedra, 4 tetrahedra, 6 triangular prisms (see note below)

Note: Figures labeled with "=x", (numbers 24, 25, 27 and 28), include the polyhedra around 2 vertices instead of 1, so as to differentiate between related pairs.

Thanks to George Olshevsky and Jonathan Bowers for introducing me to this collection, and the names associated with them.

Thanks also to Robert Webb for his very cool "Stella" program. I have used it extensively to generate VRML files for my site. It is a great tool for rapid exploration of augmentations and excavations of polyhedra (among other things).

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