[1] G. T. Trammell, *Chemical Effects of Nuclear Transformations* (International Atomic Energy Agency, Vienna, 1961), p. 75; C. Tzara, in *Proceedings of Second International Conference on the Mössbauer Effect* (John Wiley & Sons, Inc., New York, 1962), p. 14.

[2] S. Bernstein and E. C. Campbell, "Nuclear Anomalous Dispersion in $Fe^{57}$ by the Method of Total Reflection," Phys. Rev. (to be published); see also S. Bernstein and E. C. Campbell, Rev. Mod. Phys. **36**, 462 (1964) [this issue].

### Mössbauer Scattering of 14.4-keV Gamma Rays from $Fe^{57}$ as a Function of Temperature

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The scattering of $Fe^{57}$ gamma rays from a stainless steel foil has been studied for temperatures of the scatterer from 20 to 300°C. The source, $Co^{57}$ in stainless steel, and the stainless steel scatterer exhibited single lines. The total scattering and the transmission of the scatterer have been determined as a function of the source velocity. For the same temperatures the line shape of the scattered radiation has been measured by standard transmission techniques. The energy distribution of the source was centered on the scatterer resonance. This required that the source be Doppler shifted by a constant velocity to compensate for the temperature shift. From the line-shape data the ratio of the recoilless to the nonrecoilless scattering has been determined and used to evaluate $f$. The recoilless fraction $f$ is found to vary linearly from $f = 0.69 \pm 0.01$ at room temperature to $f = 0.51 \pm 0.02$ at 300°C. These values are consistent with the Debye model using $\theta_D = (331 \pm 12)$°K.

# *Session* VIII

## RELATIVITY, NUCLEAR PHYSICS, AND POSTDEADLINE PAPERS

CHAIRMAN: *R. V. Pound*

THE PHYSICAL STRUCTURE OF GENERAL RELATIVITY: *D. W. Sciama*

CONTRIBUTED PAPERS

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# The Physical Structure of General Relativity

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### 1. INTRODUCTION

The formal structure of general relativity is fairly well understood, but its physical structure is not. This is illustrated by the following three quotations.

"Mach conjectures that in a truly rational theory inertia would have to depend upon the interaction of the masses, precisely as was true for Newton's other forces, a conception which for a long time I considered as in principle the correct one. It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics: masses and their interaction as the original concepts. The attempt at such a solution does not fit into a consistent field theory, as will be immediately recognized."

"So, if one regards as possible, gravitational fields of arbitrary extension which are not initially restricted by spatial limitations, the concept of the 'inertial system' becomes completely empty. The concept, 'acceleration relative to space,' then loses every meaning and with it the principle of inertia together with the entire paradox of Mach."[1]

"One should note that Einstein did not fully appreciate the inadequacy of a local description and the importance of boundary conditions. This is why it is necessary to change substantially Einstein's statement of the basic problems of gravitational theory; . . ."

"Enough has been said to make clear that the use of the terms 'general relativity,' 'general theory of relativity' or 'general principle of relativity' should not be admitted. This usage not only leads to misunderstanding, but also reflects an incorrect understanding of the theory itself. However paradoxical this may seem, Einstein, himself the author of the theory, showed such a lack of understanding when he named his theory and his publications and when in his discussions he stressed the word 'general relativity,' not seeing that the new theory he had created, when considered as a generalization of the old, generalizes not the notion of relativity but other, geometrical, concepts."[2]

"When, in a relativistic discussion, I try to make things clearer by a space–time diagram, the other participants look at it with polite detachment and, after a pause of embarrassment as if some childish indecency had been exhibited, resume the debate in their own terms. Perhaps they speak of the Principle of Equivalence. If so, it is my turn to have a blank mind, for I have never been able to understand this Principle."

"The Principle of Equivalence performed the essential office of midwife at the birth of general relativity, but, as Einstein remarked, the infant would never have got beyond its long-clothes had it not been for Minkowski's concept. I suggest that the midwife be now buried with appropriate honours and the facts of absolute space–time faced."[3]

While the disagreement over the physical meaning of general relativity stems in part from the mathematical difficulty of finding general solutions of Einstein's nonlinear field equations, the physical path leading to these equations ought to be discernible. In this paper I have attempted to find such a path. In order to stress the physical ideas no mathematical results are derived, but references are given where necessary. In some cases these references carry the physical considerations somewhat farther. I have deviated occasionally from the standard theory, where the physical argument seemed to demand it. The most glaring example of this *lèse-majesté* is the introduction of a torsion tensor (skew part of the affine connection) to describe the spin angular momentum of matter (Sec. 2.2). I hope the reader will agree that this innovation is actually in the spirit of Einstein's great theory.

[1] A. Einstein, in *Albert Einstein: Philosopher-Scientist*, edited by P. A. Schilpp (Harper & Brothers, New York, 1949), pages 29, 67.

[2] V. Fock, *The Theory of Space Time and Gravitation* (Pergamon Press Ltd., London, 1959), pages xv, xviii.

[3] J. L. Synge, *Relativity: the General Theory* (North-Holland Publishing Company, Amsterdam, 1960), page ix.

## 2. MACH'S PRINCIPLE

There are many ways of stating Mach's principle: we shall adopt the form "inertial forces are exerted by matter, not by absolute space." In this form the principle contains two ideas:

(i) inertial forces have a dynamical rather than a kinematical origin, and so must be derived from a *field theory*,[4]

(ii) the *whole* of the inertial field must be due to sources, so that in solving the inertial field equations the boundary conditions must be chosen appropriately.

We consider these two ideas in turn.

### 2.1 A Field Theory of Inertial Forces

We know from special relativity that a free particle moves along a timelike geodesic of Minkowski space–time. Its world-line in an arbitrary noninertial coordinate system is given by

$$\frac{d^2x^i}{ds^2} + \begin{Bmatrix} i \\ j\,k \end{Bmatrix} \frac{dx^j}{ds}\,\frac{dx^k}{ds} = 0\,,$$

where the Christoffel symbols $\{{}_j{}^i{}_k\}$ are given in terms of the metric tensor $g_{ij}$ by

$$\begin{Bmatrix} i \\ j\,k \end{Bmatrix} = \tfrac{1}{2}\, g^{il} \left( \frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l} \right), \qquad (1)$$

and $ds$ is given by

$$ds^2 = g_{ij}dx^i dx^j\,.$$

The necessary and sufficient condition that we can find a coordinate system which is everywhere inertial, that is in which the inertial field $\{{}_j{}^i{}_k\}$ is everywhere zero, is that the curvature tensor of $\{{}_j{}^i{}_k\}$ is zero, that is,[5]

$$R^i_{jkl} \equiv \frac{\partial\{{}_j{}^i{}_k\}}{\partial x^l} - \frac{\partial\{{}_j{}^i{}_l\}}{\partial x^k} + \begin{Bmatrix} m \\ j\,k \end{Bmatrix}\begin{Bmatrix} i \\ m\,l \end{Bmatrix} - \begin{Bmatrix} m \\ j\,l \end{Bmatrix}\begin{Bmatrix} i \\ m\,k \end{Bmatrix} = 0\,.$$

[4] —or possibly an action at a distance theory in the sense of J. A. Wheeler and R. P. Feynman [Rev. Mod. Phys. 21, 425 (1949)].

[5] J. A. Schouten, *Der Ricci-Kalkül* (Springer–Verlag, Berlin, 1924).

At this stage of the argument the inertial force field has a purely kinematical origin, arising out of a nonlinear coordinate transformation from an original inertial frame into a noninertial one. We now make the physical hypothesis, with Mach, that the inertial force field is part of a dynamical force field $\Gamma^i_{jk}$, which is coupled to matter in the same way as $\{{}_j{}^i{}_k\}$ is, and which transforms in the same way under a coordinate transformation. This step will be recognized as an example (of course the first) of the now-fashionable "gauge trick."[6] The crucial assumption which gives physical content to the trick is that the new field is nontrivial, that is, it cannot be everywhere annihilated by a coordinate transformation. This implies that the curvature tensor of $\Gamma^i_{jk}$ is not in general zero, that is,

$$R^i_{jkl} \equiv \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \frac{\partial \Gamma^i_{jl}}{\sigma x^k} + \Gamma^m_{jk}\Gamma^i_{ml} - \Gamma^m_{jl}\Gamma^i_{mk} = 0\,.$$

This condition on the dynamical inertial force field does not prevent us from deriving $\Gamma^i_{jk}$ from a potential function. In fact we can tentatively write by analogy with (1),

$$\Gamma^i_{jk} = \tfrac{1}{2} g^{il}\left(\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right). \qquad (2)$$

This implies, inter alia, that $\Gamma^i_{jk}$ is symmetrical in $j$, $k$, a restriction which we will later relax. It also implies that the potential of the inertial force field is a second-rank tensor, rather than a scalar or a vector.[7] Since this potential must be coupled to the mass or equivalently the energy of a particle, it is clearly the material energy–momentum tensor $T_{ij}$ which is the source of $g_{ij}$. This is confirmed by the fact that if $L$ is the Lagrangian density of the material system, then $\delta\mathfrak{L}/\delta g^{ij}$ is equal to the (symmetrized) energy–momentum tensor.[8]

By virtue of our assumption about the coupling of $\Gamma^i_{jk}$ to matter, the equation of motion of a particle in the $\Gamma^i_{jk}$ field is

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = X^i \qquad (3)$$

where $ds$ is now defined in terms of the new potential $g_{ij}$ by

$$ds^2 = g_{ij}dx^i dx^j\,,$$

and $x^i$ represents a possible force exerted by the curvature tensor and vanishing with it. This new force will be determined later. In view of (3) we will take $g_{ij}$ as the metric of space–time. This metric is covariantly constant relative to $\Gamma^i_{jk}$ as defined by (2), that is,

$$\frac{\partial g_{ij}}{dx^k} - \Gamma^l_{ik}g_{lj} - \Gamma^l_{jk}g_{il} = 0\,.$$

Conversely, the constancy of the metric implies (2) if and only if $\Gamma^i_{jk}$ is symmetric in $j,k$.[9] It is important to note that our development of Mach's principle up to this point has made no mention of gravitation. This aspect of the theory we can defer till Sec. 3 on the principle of equivalence. As far as the present argument is concerned we have simply used the gauge trick to introduce a new field into physical theory.

We now consider the problem of setting up field equations for the new $g_{ij}$ field. We can immediately take over Einstein's classic arguments and introduce the equations

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = -\kappa T_{ij} \qquad (4)$$

where the Ricci tensor $R_{ij}$ is $R^a_{ija}$, the curvature scalar $R$ is $g^{ij}R_{ij}$, and $\kappa$ is a coupling constant. A more physical way of arriving at the same equations is to work in close analogy with Maxwell's equations, which can be written in the form

$$\Box^2 A^i \equiv \frac{\partial^2 A^i}{\partial x^2} + \frac{\partial^2 A^i}{\partial y^2} + \frac{\partial^2 A^i}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A^i}{\partial t^2} = J^i\,,$$

$$\frac{\partial A^i}{\partial x^i} = 0\,,$$

which together imply the conservation of charge

$$\partial J^i/\partial x^i = 0\,.$$

Analogous equations for a tensor potential $g^{ij}$, or more conveniently, for the density $\mathfrak{g}^{ij} = (-g)^{\frac{1}{2}}g^{ij}$ would be

$$\Box^2\mathfrak{g}^{ij} = -\kappa\mathfrak{T}^{ij}$$

$$\partial\mathfrak{g}^{ij}/\partial x^j = 0\,,$$

where

$$\mathfrak{T}^{ij} = (-g)^{\frac{1}{2}}T^{ij}\,.$$

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[6] H. Weyl, Z. Physik **56**, 330 (1929). C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954). R. Utiyama, Phys. Rev. **101**, 1597 (1956). J. J. Sakurai, Ann. Phys. (N.Y.) **11**,1 (1960). A. Salam and J. C. Ward, Nuovo Cimento **19**, 165 (1961). T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961). D. W. Sciama, *Recent Developments in General Relativity* (Pergamon Press, Ltd., London, 1962), p. 415, etc.

[7] D. W. Sciama, Monthly Notices Roy. Astron. Soc. **113**, 34 (1953); *The Unity of the Universe* (Doubleday & Company, Inc., Garden City, New York, 1959).

[8] F. J. Belinfante, Physica **7**, 449 (1940). L. Rosenfeld. Acad. Roy. Belg. Classe Sci. Mem. **18**, 6 (1940).

[9] E. Schrödinger, *Space-Time Structure* (Cambridge University Press, Cambridge, England, 1950).

However, these equations are not satisfactory since they imply too strong a conservation law, namely,

$$\partial \mathfrak{T}^{ij}/\partial x^j = 0 \, ,$$

which does not permit the required interaction between the inertial field and the matter field. What is required is rather the covariant equation

$$\partial \mathfrak{T}^{ij}/\partial x^j = -\Gamma^i_{jk}\mathfrak{T}^{jk} \, .$$

If we can write this equation in the form

$$\partial \mathfrak{T}^{ij}/\partial x^j = -\partial \theta^{ij}/\partial x^j \, ,$$

then a satisfactory set of field equations would be

$$\Box^2 \mathfrak{g}^{ij} = -\kappa(\mathfrak{T}^{ij} + \theta^{ij}) \, , \qquad (5)$$

$$\partial \mathfrak{g}^{ij}/\partial x^j = 0 \, . \qquad (6)$$

We could then interpret $\theta^{ij}$ as representing the energetic properties of the inertial field itself, which by Mach's principle we would expect to be a source of the inertial field just like material forms of energy.[10] Papapetrou[11] has in fact shown that $\theta^{ij}$ is the canonical energy pseudotensor of the gravitational field, symmetrized by the usual Belinfante–Rosenfeld prescription.[8] He has also shown that the resulting field equations (5), (6) are equivalent to Einstein's field equations (4).

As is well known, these field equations can be derived from an action principle, the Lagrangian density being $(-g)^{\frac{1}{2}}R + \kappa\mathfrak{L}$. The most general method of variation is that now known as the Palatini method, in which $g_{ij}$ and $\Gamma^i_{jk}$ (assumed symmetrical) are varied independently.[9] The $g_{ij}$ variation leads to the usual field equations (4), while the $\Gamma^i_{jk}$ variation leads to (2) *if L is independent of* $\Gamma^i_{jk}$. In general this will not be so, and the material variables will appear in the relation between $g_{ij}$ and $\Gamma^i_{jk}$. This would mean that the covariant derivative of the metric tensor would not be zero, that is, that the metric would not be covariantly constant. Such a possibility was studied by Weyl[12] many years ago, in an attempt to geometrize the electromagnetic field (indeed this theory provided the origin of the phrase "gauge transformation"). However, it is difficult to introduce spinorial variables into a space of this type, since a local coordinate system (vierbein) does not undergo a Lorentz transformation under a parallel displacement. As a result one cannot define the covariant derivative of a spinor field in a natural way. Since we need to be able to introduce spinor fields in order to describe fermions this is an undesirable result. Rather than give up the Palatini method of variation, which itself may be necessary for the quantization of the theory,[13] we prefer to use the vierbein formalism, which is explained in the next section. We again arrive at a non-Riemannian geometry in the general case, and again it is a geometry which has been discussed in the context of unified field theory, namely one in which $\Gamma^i_{jk}$ is not symmetric in $j,k$. So far as I am aware, the introduction of a nonsymmetrical $\Gamma^i_{jk}$ does not lead to any difficulties, and indeed it has a simple physical interpretation.

### 2.2 The Gauge Trick in Vierbein Form

A vierbein field consists of four linearly independent vectors $e_i(\alpha)$, defined at each point. Here the Greek label $\alpha$ numbers the vectors. The vierbein affine connexion $0_j(\alpha\beta)$ is then defined by[14]

$$\partial e^i(\alpha)/\partial x^j + \Gamma^i_{jk}e^k(\alpha) + 0_j(\alpha\beta)e^i(\beta) = 0 \, .$$

This definition holds in a general affinely connected space, where neither $\Gamma^i_{jk}$ nor $0_j(\alpha\beta)$ have any special symmetry properties. The skew part of $\Gamma^i_{jk}$ is known as the torsion tensor,[5] and vanishes in a Riemannian space by definition. If in addition lengths are preserved by parallel transfer relative to $\Gamma^i_{jk}$, then $0_j(\alpha\beta)$ is skew-symmetric in $\alpha,\beta$ (relative to the Minkowski metric tensor). In this case the transferred vierbein suffers at most a Lorentz transformation, so that the vierbein at each point can be taken to be orthonormal. Moreover, we can now introduce differentiable spinor fields into the space, defined in terms of Lorentz transformations of the vierbeins. The affine connection for a four spinor field $\psi$, for example, will then be $\frac{1}{2}\,0_j(\alpha\beta)\gamma(\alpha)\gamma(\beta)$, where $\gamma(\alpha)$ is a Dirac matrix, and its covariant derivative will be $\partial\psi/\partial x^j + \frac{1}{2}\,0_j(\alpha\beta)\gamma(\alpha)\gamma(\beta)\psi$.

The vierbein formulation is thus particularly convenient if one wants to study a material system which has spin. Consider first Minkowski space with an arbitrary vierbein field (the analog of a noninertial frame). The affine connection $0_j(\alpha\beta)$ then has the form

$$0_j(\alpha\beta) = \tfrac{1}{2}\, e_j(\gamma)\{[e(\gamma),e(\alpha)](\beta) + [e(\beta),e(\gamma)](\alpha) - [e(\alpha),e(\beta)]\gamma\}$$

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[10] This point was already made by Einstein in his classic paper [Ann. Phys. **49**, 769 (1916)].

[11] A. Papapetrou, Proc. Roy. Irish Acad. **A52**, 11 (1948).

[12] H. Weyl, *Space, Time, Matter* (Dover Publications, Inc., New York, 1951).

[13] R. L. Arnowitt, S. Deser, and C. W. Misner, *Gravitation*, edited by L. Witten (John Wiley & Sons, Inc., New York, 1962).

[14] H. Weyl, Z. Physik **56**, 330 (1929).

where

$$[e(\gamma),e(\alpha)](\beta) = e_k(\beta)\left(e^j(\alpha)\frac{\partial e^k(\gamma)}{\partial x^j} - e^j(\gamma)\frac{\partial e^k(\alpha)}{\partial x^j}\right),$$

and the corresponding curvature tensor is given by

$$R_{jk}(\alpha\beta) \equiv R^i_{jkl}e_i(\alpha)e^l(\beta) \equiv \frac{\partial 0_k(\alpha\beta)}{\partial x^j} - \frac{\partial 0_j(\alpha\beta)}{\partial x^k} + 0_j(\alpha\gamma)0_k(\gamma\beta) - 0_k(\alpha\gamma)0_j(\gamma\beta) = 0\,.$$

If the material system is described in a parallel vierbein field by a Lagrangian density $\mathfrak{L}$ which depends on the material field function $\psi$ and its first derivatives $\partial\psi/\partial x^i$ only, then in an arbitrary vierbein field the Lagrangian density becomes[15]

$$\mathfrak{L} + s^j(\alpha\beta)0_j(\alpha\beta)\,,$$

where $s^j(\alpha\beta)$ is the flux of the spin density of the material system. We thus obtain a term in the Lagrangian density coupling the "inertial" $0_j(\alpha\beta)$ field with the material spin. We now play the gauge trick and assume the existence of a dynamical (skew) $0_j(\alpha\beta)$ field which cannot be transformed away everywhere by suitable choice of the vierbein field, that is, whose curvature tensor $R_{ij}(\alpha\beta)$ is nonzero. This dynamical field is then coupled to the material spin just as the vierbein field is coupled to the material energy. This is shown by the variational formulas

$$\frac{\delta\mathfrak{L}}{\delta e^i(\alpha)} = T_i(\alpha)\,, \qquad \frac{\delta\mathfrak{L}}{\delta 0^i(\alpha\beta)} = S_i(\alpha\beta)\,.$$

The second of these formulas has an important implication for the theory when expressed in terms of the variables $g_{ij},\Gamma^i_{jk}$. For if we take as the total Lagrangian density $\epsilon R + L$ where $\epsilon = \det e_i(\alpha)$ and $R = e^i(\alpha)e^j(\beta)R_{ij}(\alpha\beta)$, then the vanishing of the variation with respect to $0_i(\alpha\beta)$ leads to the equation[15,16]

$$\Gamma^i_{jk} = S^i_{jk} - \tfrac{1}{2}\,\delta^i_j S^l_{lk} - \tfrac{1}{2}\,\delta^i_k S^l_{jl}\,, \qquad (7)$$

where

$$S^i_{jk} = S^i(\alpha\beta)e_j(\alpha)e_k(\beta)\,.$$

Thus the torsion vanishes only if the material system has no spin. We may note in passing that the result (7) suggests that unified field theories based on a nonsymmetric connection have nothing to do with electromagnetism.[17]

The theory of the inertial field in this vierbein form has one direct physical consequence typical of a theory based on the gauge trick, namely that it implies the existence of a physical force involving the curvature tensor, which of course was identically zero before the trick was performed. The most familiar example of this extra force is provided by the electromagnetic field which can be introduced by a gauge trick involving phase transformations of the matter variables. In this case we obtain an "affine connection" $A_i$ and a coupling term in the Lagrangian density $J^iA_i$ where $J^i$ is the current-density. This coupling term then leads to the Lorentz force $J^iF_{ij}$, where $F^i_j$ is the "curvature" associated with $A_i$, whose nonvanishing assures that the potential $A_i$ leads to a nontrivial electromagnetic field. In the same way the coupling term $s^j(\alpha\beta)0_j(\alpha\beta)$ leads to a force of the form $s^j(\alpha\beta)R_{jk}(\alpha\beta)$, as can be seen either from the form of the contracted Bianchi identities when there is torsion present or by a variation of the Lagrangian in which the field variables are kept constant, and the world lines of matter are varied.[16,17] This spin-curvature force is then the force $X_i$ mentioned earlier which deviates a (spinning) particle from a geodesic. The existence of this force has been known for a long time,[18] but the present derivation appears to be the most natural. In addition there is a further purely geometrical identity involving the torsion which corresponds physically to the conservation of total angular momentum (spin plus orbital).[15]

A further point in connection with (7) is that it would be interesting to quantize the theory with torsion, since spin has such characteristic quantum properties. The quantization of the conventional theory in vierbein form has recently been studied by Schwinger,[19] and that of the theory with torsion is under investigation by Lemmer. It will also be interesting to see whether in the quantized theory the inertial waves have zero rest mass.[20]

### 2.3 The Minimal Coupling Principle

A final feature of the gauge trick we have been performing is worth stressing. In introducing the new dynamical field we assumed that it is coupled to matter in *exactly* the same way as the inertial force field. We could, however, have added to the Lagrangian density a further coupling between the curvature tensor and the matter field without inconsistency with any of our other principles. For simplicity we assume that this coupling is absent.[15] This is the *minimal coupling principle* familiar to field

[15] D. W. Sciama, *Recent Developments in General Relativity* (Pergamon Press, Ltd., London, 1962), p. 415.
[16] T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961).
[17] D. W. Sciama, J. Math. Phys. **2**, 472 (1961).
[18] M. Mathisson, Acta. Phys. Polon. **6**, 163 (1937). A. Papapetron. Proc. Roy. Soc. (London) **A209**, 248 (1951).
[19] J. Schwinger, Phys. Rev. **130**, 1253 (1963).
[20] P. W. Anderson, Phys. Rev. **130**, 439 (1963).

theorists in connection with the electromagnetic field and phase (gauge) transformations.[21] Its physical significance will be mentioned later in connection with the principle of equivalence (Sec. 3).

### 2.4 Boundary Conditions for the Inertial Field Theory

Before studying the boundary conditions for the somewhat complicated inertial field theory, we consider the analogous problem in electrodynamics.[22] We want to find a precise expression for the somewhat vague condition that the electromagnetic field must be entirely due to sources. To do this it is convenient to express Maxwell's equations in the Kirchhoff form

$$\phi(p,t) = \frac{1}{4\pi}\int_v \frac{[\rho]}{\nu}\,dv + \frac{1}{4\pi}\int_s \left\{[\phi]\frac{\partial}{\partial n}\left(\frac{1}{\nu}\right) - \frac{1}{\nu}\frac{\partial \nu}{\partial n}\left[\frac{\partial\phi}{\partial t}\right] - \frac{1}{\nu}\left[\frac{\partial\phi}{\partial n}\right]\right\} ds\,,$$

where $\phi(p,t)$ is any component of the electromagnetic field at the event $(p,t)$, $\rho$ is the corresponding component of the source, $S$ is the closed 2-surface of a 3-volume $V$ surrounding $p$, $\nu$ is the distance from $p$ to a volume or surface element, $\partial/\partial n$ represents differentiation along the inward normal to $S$, and [ ] represents *either* retarded or advanced values (but the same in both integrals). The surface integral represents the contributions to $\phi$ from the sources outside $V$ and from any source-free radiation that may be coming in from or going out to infinity. It is this last, source-free contribution that we require to vanish. We therefore take as our boundary condition

$$\lim_{s\to\infty}\int_s \left\{[\phi]\frac{\partial}{\partial n}\left(\frac{1}{\nu}\right) - \frac{1}{\nu}\frac{\partial\nu}{\partial n}\left[\frac{\partial\phi}{\partial t}\right] - \frac{1}{\nu}\left[\frac{\partial\phi}{\partial n}\right]\right\} ds = 0\,, \tag{8}$$

in *both* the past and the future light cones. It can be shown that in some models of the universe this condition is violated (e.g., the Einstein–de Sitter model with slowly cooling galaxies) while in others the condition is automatically satisfied (e.g., the steady state model).

We now propose a similar boundary condition for the inertial field theory. If we take the inertial field equations in the form (5), (6), we can again use the Kirchhoff representation, with $\kappa(\mathfrak{T}^{ij} + \theta^{ij})$ acting as the source. Thus we again impose (8), with $\phi$ representing any component of $\mathfrak{g}^{ij}$ in a coordinate system satisfying (6), and $\rho$ representing the corresponding component of the source. A detailed investigation of (8) is now under way. The main problem is to give a careful discussion of what is meant by $S \to \infty$, that is, of the topology of the light cone. It would appear that Penrose's[23] methods of studying conditions at infinity are ideally suited to this problem. In the meantime it is clear that (8) rules out the obviously anti-Mach solutions of Minkowski space and the exterior Schwarzschild solution.

## 3. THE PRINCIPLE OF EQUIVALENCE

We shall take the principle of equivalence in the following form: the inertial field introduced by the gauge trick is the gravitational field. The most obvious evidence for this principle is that gravitational fields, like inertial fields, are coupled to the inertial mass of a particle. But the most striking evidence works in the reverse sense, from inertial field to gravity. For as we shall see, the inertial field theory implies that the earth produces a static inverse square $\Gamma^i_{jk}$ force field which has about the same strength as gravity—so if it is not gravity *what is it?*

It is clear from the form (5), (6) of the field equations that the earth of mass $M$ produces an "inertial" field of the form $\kappa M/\gamma^2$ (neglecting to first order the contribution of $\kappa\theta^{ij}$ which is permissible if $\kappa M/c^2\gamma \ll 1$). To determine the strength of this field, and to confirm that $\kappa M/c^2\gamma \ll 1$ we need to know the value of the coupling constant. We can estimate the value of $\kappa$ by using the condition that in a noninertial frame of reference which has acceleration $\alpha$, the sources in the universe must exert an inertial field of the observed magnitude, that is, $\alpha$. For our present purpose a rough approximation will suffice, so we shall again tentatively ignore the nonlinear source term $\kappa\theta^{ij}$. We recognize from the similarity of the inertial field equations with those of electrodynamics that the decisive term in $\Gamma^i_{jk}$ will be the acceleration-dependent radiative field of the sources $i$, namely $\sum_i \kappa m_i\alpha/c^2\gamma$ (ignoring factors like $1/4\pi$ and the angular dependence of this field). Cutting this sum off at the radius of the universe $c\tau$, we obtain for the total inertial acceleration $\kappa\rho\tau^2\alpha$, where $\rho$ is the mean density of matter in the universe. Hence we require that

$$\kappa \backsim 1/\rho\tau^2\,,$$

which gives us an estimate for the value of $\kappa$. Using

[21] M. Gell-Mann, Nuovo Cimento Suppl. **4**, 848 (1956).
[22] D. W. Sciama, Proc. Roy. Soc. (London) **A273**, 484 (1963).
[23] R. Penrose, Reports to Warsaw Conference on Gravitation, 1962, and Cornell Conference on the Nature of Time, 1963 (unpublished).

this estimate, the acceleration produced by the static earth $\sim M/\rho\tau^2\gamma^2$. Now $\tau \sim 10^{10}$ light years and $\rho$ probably lies between $10^{-31}$ gm/cc (a lower limit for the density due to galaxies) and $10^{-27}$ gm/cc (an upper limit for the mean density within a typical cluster of galaxies). Hence $10^{-8} < \kappa < 10^{-4}$, so that indeed $\kappa M/c^2\gamma \ll 1$. With this estimate of $\kappa$, the acceleration produced by the earth lies between $10^2$ cm/sec$^2$ and $10^6$ cm/sec$^2$. It is therefore natural to identify this acceleration with the acceleration due to gravity of 981 cm/sec$^2$, and thus to assert the principle of equivalence. In this case $\kappa \sim G$, and we arrive at the well-known relation

$$G\rho\tau^2 \backsim 1 ,$$

leading to a mean density $\rho \sim 10^{-28}$ gm/sec$^3$.

Of course the principle of equivalence is further confirmed by the three crucial tests. The advance of perihelion is of particular interest since it involves a nonlinear contribution from $\kappa\theta^{ij}$ of order $\kappa^2$. The bending of light is linear in $\kappa$ and is therefore less interesting, although it does confirm that the gravitational potential is a tensor. Finally the gravitational red shift is of interest because it might be used to test the minimal coupling principle. For in the laboratory test using the Mössbauer effect,[24] the $\gamma$ rays move along a path short compared with the length scale of the earth's gravitational field. Hence any nonminimal coupling that might exist between the electromagnetic field and the curvature tensor is unlikely to be important. On the other hand, in the astronomical tests, the light path is long compared with the length scale of the gravitational fields involved so that the effect of a nonminimal coupling would be more likely to be measurable. While we do not anticipate that such an effect will be observed, it appears to be worthwhile to continue the astronomical attempts, which also of course may have direct astrophysical interest.

[24] R. V. Pound and G. A. Rebka, Phys. Rev. Letters **3**, 439 (1959).

## CONTRIBUTED PAPERS FOR SESSION VIII

### Relativity Experiments Using a Rotor

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Experiments to test relativistic effects with a Mössbauer source and absorber mounted on a spinning rotor are described. With the source mounted at the center and an absorber at the tip, a relative change in frequency of $\Delta\nu/\nu = +K(u_a^2/2c^2)$ is found with $K = 1.03 \pm 0.03$, where $\Delta\nu$ is the apparent excess of source frequency caused by rotation, $u_a$ is the absorber velocity, and $c$ is the velocity of light. The use of the apparatus as a sensitive variant of the Michelson Morley experiment is described, an ether drift of velocity $\bar{v}$ past the apparatus giving rise to a linear shift of $\Delta\nu/\nu = +\bar{v}\cdot(\bar{u}_a - \bar{u}_s)/c^2$, where $\bar{u}_s$ and $\bar{u}_a$ are the source and absorber velocities in the laboratory reference frame. Using the fact that with source and absorber at opposite tips no quadratic Doppler effect is expected leaving only the linear term, experiments are described which indicate that the diurnal amplitude of $\bar{v}$ in the east-west direction at Birmingham (England) was less than some 5 m/sec in the second half of August 1963.

### Internal Conversion Coefficient of 14.4-keV $Fe^{57}$ Transition*

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Using conventional nuclear spectroscopic techniques the total internal conversion coefficient of the 14.4-keV transition in $Fe^{57}$ has been measured in an attempt to clarify the discrepancy between a recent value $\alpha_T = 9.94 \pm 0.60$ reported by Thomas *et al.*[1] and the generally accepted previous value of Lemmer *et al.*,[2] $\alpha_T = 15 \pm 1$. In the present determination both argon–methane proportional counters and a NaI(Tl) scintillation crystal spectrometer were used to measure the intensity of the $K$ x rays in the $Co^{57}$ decay relative to the 14-keV $\gamma$-ray intensity. From this result, $I_K/I_\gamma = 5.58 \pm 0.3$, we obtain the following $K$-shell and total conversion coefficients for the 14-keV transition: $\alpha_K = 8.44 \pm 0.5$, $\alpha_T = 9.51 \pm 0.5$. This $\alpha_T$, which is in agreement with the result of Thomas *et al.*, is compared to other measurements and to theory. The $\alpha$'s estimated from Mössbauer experiments are all less than 10. There is some evidence[3] that $\alpha$ (Mössbauer) is significantly smaller than $\alpha$ (nuclear). This possibility is not understood.

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[1] H. C. Thomas, C. F. Griffin, W. E. Phillips, and E. C. Davis, Bull. Am. Phys. Soc. 7, 120 (1962); Nucl. Phys. **44**, 268 (1963).

[2] H. R. Lemmer, O. J. A. Segaert, and M. A. Grace, Proc. Phys. Soc. (London) **A68**, 701 (1955).

[3] S. S. Hanna, R. S. Preston, and W. S. Denno, Rev. Mod. Phys. **36**, 469 (1964) [preceeding abstract].

### Mössbauer Cross Section in Metallic Iron*

S. S. Hanna, R. S. Preston, and W. S. Denno, *Argonne National Laboratory*

A method has been evolved for measuring the Mössbauer cross section $f\sigma_0$. The total area in the absorption dip (or dips) is measured for a thin absorber and for a very thick one, the thickness ratio being as high as 40. As is well known in resonant absorption processes, the area ratio for a given thickness ratio determines the resonant cross section. In order to minimize the effect of nonresonant background, the absorbers are prepared so as to have very nearly the same electronic absorption. This can often be achieved by using samples of different isotopic content. The entire problem, from fitting the data to a set of overlapping