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Stabilization of Motion of the Segway

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ABSTRACT

This article is devoted to the stabilization of the segway model in the form of mechanical model. Model consists of a wheel with two-link inverted pendulum which is attached to the mechanism by a hinge. Connections between the links are tied by elastic coiling spring. The mechanism is driven by electromotor which rotates the wheels of the segway. Segway can be moved or rolled over the surface with the help of the same driver. The control parameters are the voltage of bounded source and the elastic coefficient of the coiling spring. Stabilization of region of attraction of the pendulum vertical position is controlled by manipulating the voltage and the elastic coefficient of the coiling spring.The aim of the research is to investigate effect of flexible body of the monocycle model to the region of attraction of pendulum vertical position. The considered work has researched and analyzed the planar motion of an inverted flexible pendulum mounted on a pair of wheel. The maximal area of attraction of upright position was obtained.

Keywords: *Stabilization, planar motion, controllability domain, model of segway, inverted flexible pendulum.*

1. INTRODUCTION

The monocycle is a interested object of research both from view point of theoretical and applied research. A new means of transportation, which is called as Segway Human Transforter, is designed as a monocycle seating one person. Lacking control, the motion of such a monocycle is unstable; it has to be stabilized by a system controlling the motion of the device. In any practical system, control actions are limited in some way or another, so that an unsatable object can not be put into the necessary mode of operation from any state. In other words, the controllability domain, namely, the set of states from which, with the available control resources, the object may be brought to the desired mode of operation, occupies only part of the phase space. The domain of attraction of the desired mode of operation, resulting from the construction of specific law of feedback control, is a subdomain of the controllability domain; most frequently, it occupies only part of the latter. In that situation it is an extremely important problem to maximize the domain of attraction for bounded control action.

Models of vehicle wheeled inverted pendulum have attracted the attention of many researchers in recent years. A monocycle which consisting of a wheel and inverted pendulum is controlled and stabilized in the upper unstable equilbrium position, while the monocycle is maintained in position or moved (see 1). The plane motion of one-link pendulum with fixed suspension point has also been investigated theoretically and experimentally (see 2, 3, and 4).

In the considered work, the dynamic modeling is done directly in terms of variables which are of interest with respect to the planning and control of vehicle's position and orientation. A Lagrangian approach is used to derive the equations. The equations are then simplified, reduced in order, and then checked for the strong-accessibility condition.

Purpose of the considered work is to investigate effect of flexible body of the monocycle model to the region of attraction of pendulum vertical position.

2. EQUATIONS OF MOTION OF THE SYSTEM

It is considered the planar motion of two-link pendulum whose suspension point is at the center O, mass of wheel M and radius R, as depicted in Figure 1. Center of mass of the wheel is in its center. Wheel rolls without slipping on a horizontal line, adopted as the axis X. Centers of mass of the pendulum units, connected by a hinge at point D, located at the points C_1, C_2. The center of mass define the length of $OC_1=r_1$, $DC_2=r_2$, length of the first link of the pendulum .OD=l Hinge axis at the points O, D perpendicular to the plane of motion.

We denote φ_o angle of rotation counterclockwise which is fixed radius (labeled at the wheel), which in the beginning of the movement is oriented along the horizontal axis X, let x denote the displacement of center of mass of the wheel O along the horizontal line so that $x= -\varphi_o R$. Deflection angles of the links of the pendulum from the vertical, reckoned counterclockwise, denoted by φ_1, φ_2.

It is described a mechanical system consisting of three rigid bodies, has three degrees of freedom, its position is uniquely determined by the vector of generalized

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coordinates $\boldsymbol{\varphi} = [\varphi_0 \quad \varphi_1 \quad \varphi_2]^T$. If the controlled moment L does not depend on the angle of rotation of the wheelφ_0, then the variable φ_0 is cyclic.

The kinetic energy T of the system is a homogeneous quadratic form of generalized velocities

$$T = \frac{1}{2}(a_{00}\dot{\varphi}_0^2 + a_{11}\dot{\varphi}_1^2 + a_{22}\dot{\varphi}_2^2 2a_{01}\cos\varphi_1\,\dot{\varphi}_1\dot{\varphi}_0 + 2a_{02}\cos\varphi_2\,\dot{\varphi}_2\dot{\varphi}_0 + 2a_{12}\cos(\varphi_1-\varphi_2)\,\dot{\varphi}_1\dot{\varphi}_2) \tag{1}$$

Here

$a_{00} = M(R^2+\rho^2) + (m_1+m_2)R^2$, $a_{11} = I_1 + m_2 l^2$, $a_{22} = I_2$, $a_{12} = m_2 r_2 l$, $a_{01} = (m_1 r_1 + m_2 l)R$, $a_{02} = m_2 r_2 R$

, I_1, I_2 – moments of inertia of the first and second link with respect to pivot O and D, respectively; m_1 and m_2 – the mass of the first and second link of the pendulum, r_1 and r_2 – distance from the hinges O and D to the centers of mass of the first and second link respectively, l – length of second link OD. The length of the second link in the expression (1) in the equations of motion is not included. The equations of motion contain only the distance r_2 from joint D to the center of mass of the second link. We assume that r_1 r_2> 0, i.e. center of mass of the first link does not coincide with the hinge O, and the center of mass of the second link with the hinge D.

The potential energy of the system Π has the form

$$\Pi = m_1 g y_{C1} + m_2 g y_{C2} + \frac{1}{2}c_0(\varphi_1-\varphi_2)^2 = \frac{a_{01}g}{R}\cos\varphi_1 + \frac{a_{02}g}{R}\cos\varphi_2 + \frac{1}{2}c_0(\varphi_1-\varphi_2)^2 \tag{2}$$

Here, g - acceleration of gravity, c_0 - the stiffness of the spring hinge inter-link D.

To describe the dissipative effects in the relative motion of links of a pendulum it is use a linear viscous friction model and introduced the function of Rayleigh

$$\Phi = \frac{1}{2}b_0(\dot{\varphi}_1-\dot{\varphi}_2)^2 \tag{3}$$

Here b_0 - coefficient of viscous friction in the inter-link joint.
Virtual work done by momentum L is $\delta W = L(\delta\varphi_1 - \delta\varphi_0)$

Using the Lagrangian formalism, we construct using expressions (1) (2) (3) the equations of motion of the system in the matrix form

$$\mathbf{A}\ddot{\boldsymbol{\varphi}} + \mathbf{B}\,\dot{\boldsymbol{\varphi}}^2 + \left(\frac{\partial\Pi}{\partial\boldsymbol{\varphi}}\right)^T + \left(\frac{\partial\Phi}{\partial\dot{\boldsymbol{\varphi}}}\right)^T = \mathbf{Q} \tag{4}$$

Here A - positive definite symmetric matrix of kinetic energy, Q - a generalized vector of controlled forces, T - stands for transposition,

In this case, the angular momentum L in the equation of motion (4) creates a DC motor; we assume that it depends linearly on the applied voltage u [5]

$$L = c_u u + c_v(\dot{\varphi}_1 - \dot{\varphi}_0) \tag{5}$$

Error! Bookmark not defined. Constant positive coefficients c_u and c_v (coefficient of back e.m.f) can be calculated from nameplate values of starting and rated torque, the nominal angular velocity and the nominal motor voltage (see 5). Applied to the motor voltage u is bounded in absolute value

$$|u| \le u_0, \qquad (u_0 = \text{const}) \tag{6}$$

Voltage u will assume as a control to the considered problem. Thus, the problem of stabilizing steady motions of system (4) under the bounded control u is the problem of motion control system with a deficit value of control actions (system with three degrees of freedom has only one control u).

Multiplying equation (4) with the inverse matrix A^{-1}, we obtain a system of three nonlinear differential equations which is solved for the highest derivatives

$$\ddot{\boldsymbol{\varphi}} = \mathbf{A}^{-1}\left(\mathbf{Q} - \mathbf{B}\,\dot{\boldsymbol{\varphi}}^2 - \left(\frac{\partial\Pi}{\partial\boldsymbol{\varphi}}\right)^T - \left(\frac{\partial\Phi}{\partial\dot{\boldsymbol{\varphi}}}\right)^T\right) \tag{7}$$

When $c_v = 0$ by the presence of the zero column of matrices B the angular velocity of the wheel $\dot{\varphi}_0$ not included in the last two equations of system (7), which are separated from the system (7)

$$\ddot{\varphi}_1 = f_1(\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_0, u), \qquad \ddot{\varphi}_2 = f_2(\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_0, u)$$

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The equations allow us to consider the problem of stabilization of the pendulum motion, regardless of the wheel motion.

3. SIMULTANEOUS STABILIZATION OF THE WHEEL AND THE PENDULUM IN THE UPPER UNSTABLE POSITION

The statement of the above approach is fully applicable to the problem of simultaneous stabilization of the wheel in a position $\varphi_0 = \varphi_0^0$ and the two-link pendulum in the upper unstable position $\varphi_1 = \varphi_2 = 0$ (without loss of generality, the angle φ_0^0 can be set equal to zero). In this case, we can omit that has being worked in the previous section the assumption that the coefficient of opposite electromotive force $c_v = 0$, allow to consider the equations of motion of the pendulum away from the equations of motion of the wheel.

3.1 Linearized Equation

We assume that the process of stabilization of the upper equilibrium position of the pendulum angle φ_1, φ_2 and their derivatives are close to zero. Then, linearizing equation (4), we obtain a system of differential equations with constant coefficients

$$\ddot{\boldsymbol{\varphi}} = \mathbf{A}_3^{-1}(\mathbf{b}_3 u - \mathbf{B}_3\dot{\boldsymbol{\varphi}} - \mathbf{C}_3\boldsymbol{\varphi}) \qquad (8)$$

Introducing the angular velocity $\boldsymbol{\omega} = (\omega_1 \quad \omega_2 \quad \omega_3)^T$, we write the system (8) in the Cauchy form

$$\begin{pmatrix}\dot{\boldsymbol{\varphi}}\\ \dot{\boldsymbol{\omega}}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \mathbf{E}_3\\ -\mathbf{A}_3^{-1}\mathbf{C}_3 & -\mathbf{A}_3^{-1}\mathbf{B}_3\end{pmatrix}\begin{pmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\omega}\end{pmatrix} + \begin{pmatrix}\mathbf{0}\\ \mathbf{A}_3^{-1}\mathbf{b}_3\end{pmatrix}u \qquad (9)$$

Or

$$\dot{\mathbf{q}}_6 = \mathbf{A}_6\mathbf{q}_6 + \mathbf{B}_6 u \qquad (10)$$

Where

$$\mathbf{A}_6 = \begin{pmatrix}\mathbf{0} & \mathbf{E}_3\\ -\mathbf{A}_3^{-1}\mathbf{C}_3 & -\mathbf{A}_3^{-1}\mathbf{B}_3\end{pmatrix} \mathbf{B}_6 = \begin{pmatrix}\mathbf{0}\\ \mathbf{A}_3^{-1}\mathbf{b}_3\end{pmatrix}$$

Linear system (10) can also be reduced to the Frobenius form with a non degenerate change of variables

$$\mathbf{q}_6 = G_6\mathbf{x}_6, \quad \mathbf{x}_6 = (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6)^T \qquad (11)$$

Substituting (11) into (10) we obtain the equation

$$\dot{\mathbf{x}}_6 = \mathbf{D}_6\mathbf{x}_6 + \mathbf{B}_6 \boldsymbol{u} \qquad (12)$$

The scalar equation that equivalent to the system (12) can be represented as

$$(D-\lambda_1)(D^5+\delta_5 D^4+\delta_4 D^3+\delta_3 D^2+\delta_2 D+\delta_1)x_1 = u \qquad (13)$$

Here λ_1 – positive eigenvalues of matrix $\mathbf{A}_6$.

4. NUMERICAL ANALYSIS AND DISCUSSION OF RESULTS

In organizing numerical experiments, we chose the following parameters for the system

$$M = 10\text{ kg}, R = 0.2\text{ m}, \rho = 0.1\text{ m}, m_1 = 25\,kg, \rho_1 = 0.1\text{ m}, r_1 = 0.25\,m, l = 0.5\,m, \quad m_2 = 50\,kg, \rho_2 = 0.3\,m, r_2 = 0.25\,m, g = 9.8\,{}^m/_{s^2}. \qquad (14)$$

Graph of dependence of the limit angle $\varphi_1^0 = \frac{u_0}{\lambda_1\sigma_1}$ as shown in Figure 2 versus the stiffness of the hinge of the inter-link when c_0 from $100\,N.m$ to $500\,N.m$ at different values of the coefficient of viscous friction, is constructed in Figure 3.

As can be seen from Figure 3, with increasing stiffness of inter-link joint limit the deflection angle of the first link of the pendulum of the Figure 2 increases, while the curves in Figure 3 approaching the horizontal asymptote corresponding to the limiting value of the angle of deflection of a rigid pendulum. The growth of the viscous friction coefficient on the contrary leads to a decrease in this limit.

Figure 4 shows the solution of the non linear system (4), where $c_0 = 100\,N.m$, $b_0 = 5\,N.m.s$,

$$\lambda_0 = 3\tfrac{1}{s}\, c_v = 4\,N.m.s.$$
$$\varphi_0(0) = 0, \varphi_1(0) = 12.7^o, \varphi_2(0) = -13.5^o, \varphi0'(0) = 6\tfrac{1}{s},\ \varphi1'(0) = 0\,\tfrac{1}{s},\ \varphi2'(0) = 0\,\tfrac{1}{s}.$$

5. CONCLUSION

In this article, it is considered model of the segway in the form of mechanical model. The control parameters are the voltage of bounded source and the elastic coefficient of the coiling spring. Stabilization of region of attraction of the pendulum vertical position is controlled by manipulating the voltage and the elastic coefficient of the coiling spring. It is researched the properties of the domains of attraction and controllability of the two-link pendulum at the wheel. It is shown that with increasing stiffness, controllability domain of the two-link elastic pendulum increases.

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7. LIST OF CAPTIONS

1. Figure 1 Two-link pendulum with a suspension point at the wheel.
2. Figure 2 Controllability domain P in the plane of the variables .
3. Figure 3 Dependence of the limiting values (in degrees) the deflection angle of the first link of the pendulum from the inter-link joint stiffness for different values of the coefficient of viscous friction in the hinge of inter-link: 1), $b_0 = 5$[2]), 2) $b_0 = 15$[3]), 3) $b_0 = 25$.
4. Figure 4 Transient processes in nonlinear system without calculating the restrictions on the control.

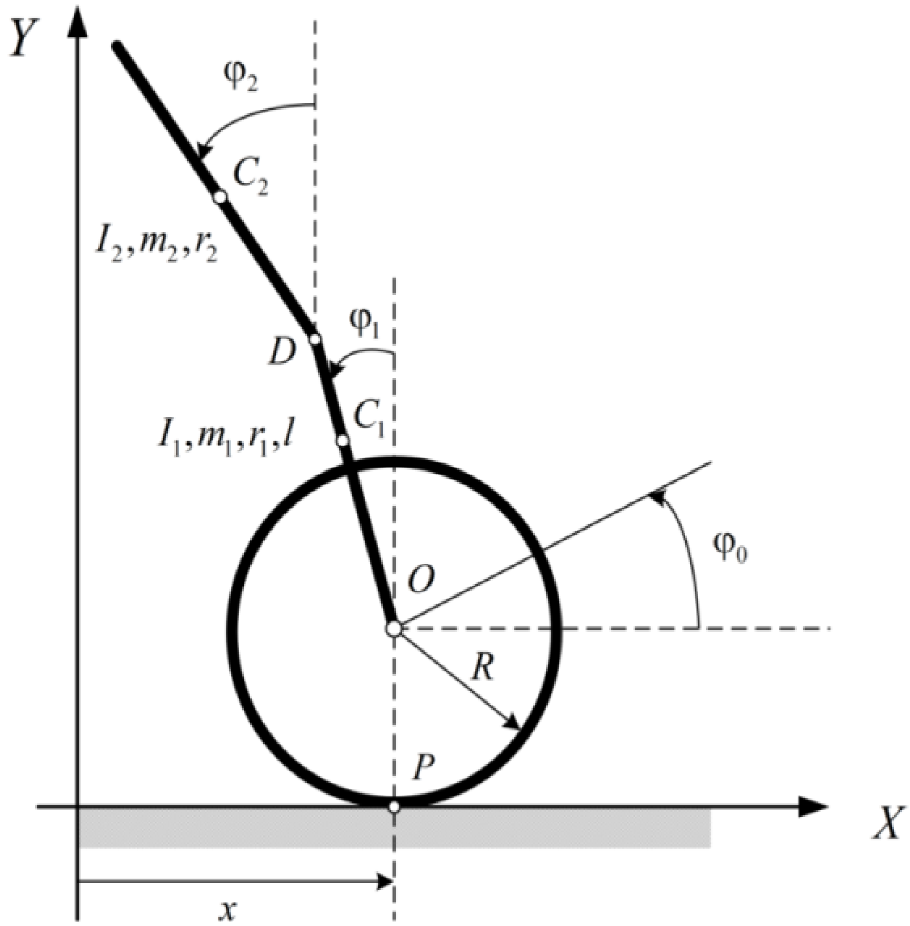

Fig 1: Two-link pendulum with a suspension point at the wheel.

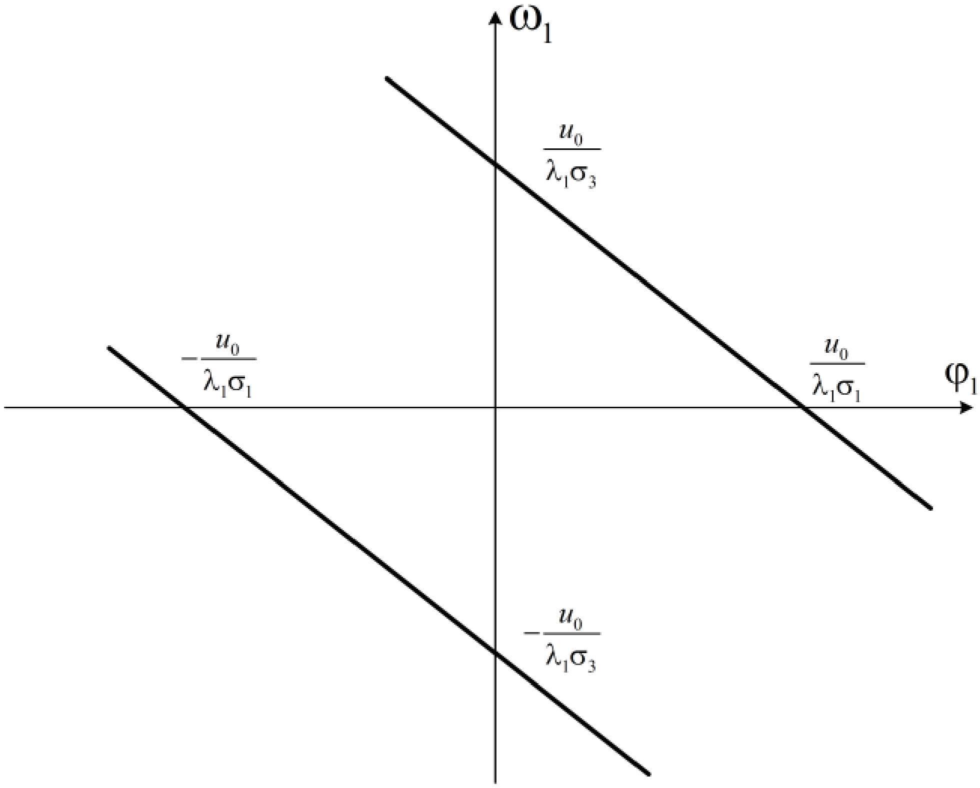

Fig 2: Controllability domain P in the plane of the variables φ_1, ω_1.

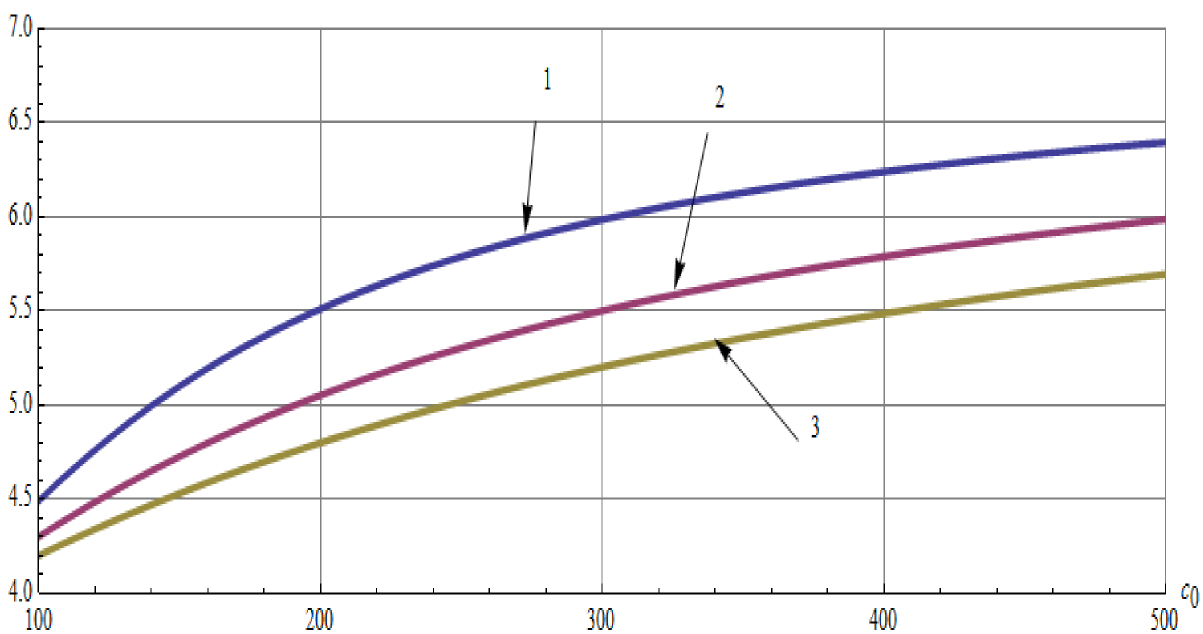

Fig 3: Dependence of the limiting values (in degrees) the deflection angle of the first link of the pendulum from the inter-link joint stiffness for different values of the coefficient of viscous friction in the hinge of inter-link: 1) $b_0 = 5$), 2) $b_0 = 15$), 3) $b_0 = 25$.

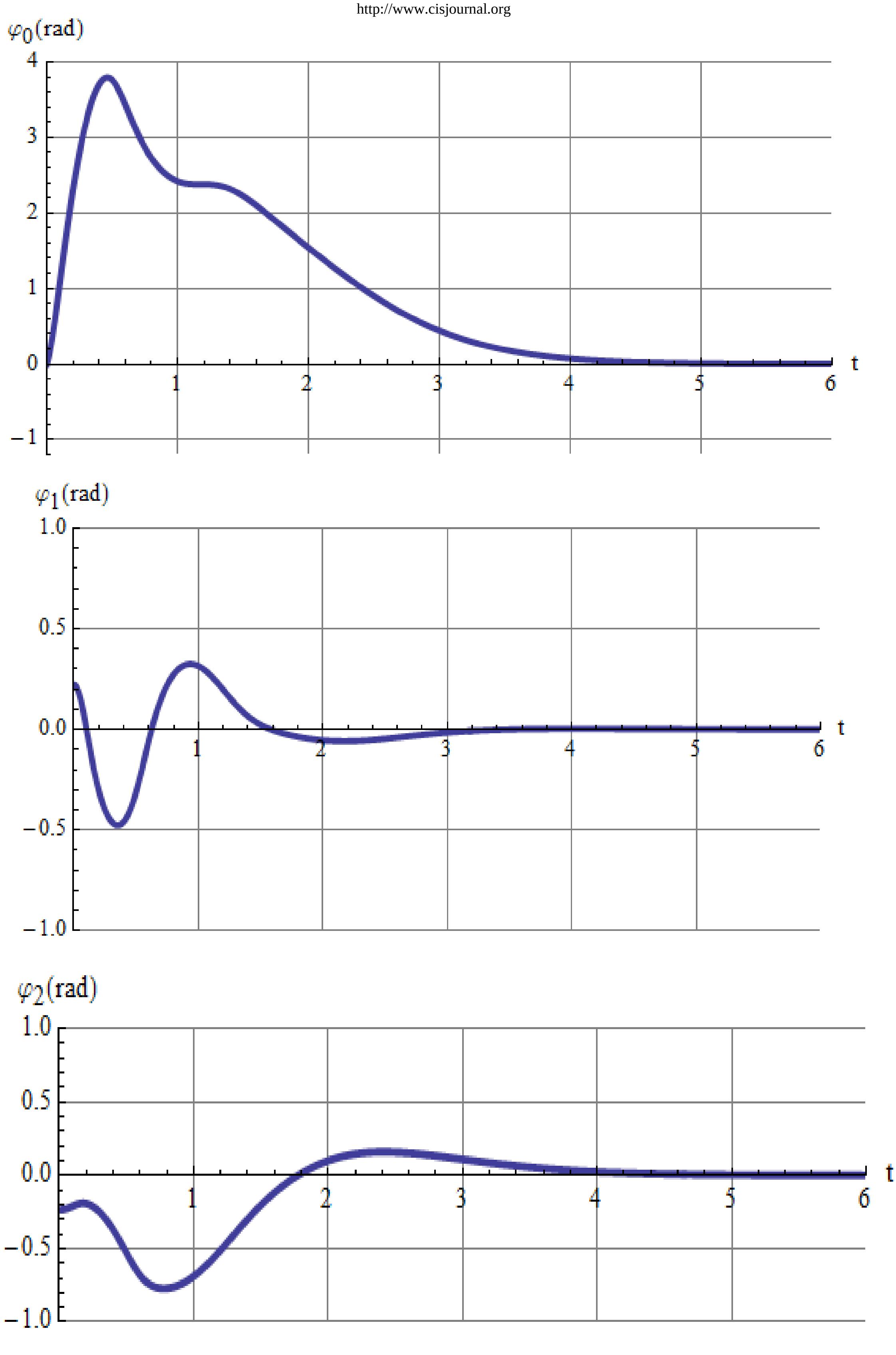

Fig 4: Transient processes in nonlinear system without calculating the restrictions on the control.

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