

Quantum Interference and the Quantum Potential*

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Abstract

We re-examine the notion of the quantum potential introduced by de Broglie and Bohm and calculate its explicit form in the case of the two-slit interference experiment. We also calculate the ensemble of particle trajectories through the two slits. The results show clearly how the quantum potential produces the bunching of trajectories that is required to obtain the usual fringe intensity pattern. Hence we are able to account for the interference fringes while retaining the notion of a well-defined particle trajectory. The wider implications of the quantum potential particularly in regard to the quantum interconnectedness are discussed.

1 Introduction.

In spite of the undoubted success of the quantum formalism, its interpretation continues to present difficulties [1]. In fact, Feynman [2] writes “I think I can safely say that nobody understands quantum mechanics . . .” and goes on to suggest that questions like “how can nature be like that?” should be avoided. We would like to propose that such questions can be meaningfully raised and we will show how detailed considerations of the quantum potential [3],[4] can be used to give a different insight into quantum interference.

Let us start by contrasting the nature of classical and quantum ensembles. In classical physics each individual particle has well-defined properties

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and all movement is described in terms of trajectories on a space-time manifold. Any statistical properties of an ensemble arise as a result of a frequency distribution over the individual properties themselves. In quantum theory, however, one usually denies the possibility of specifying completely all the properties of the individual even in principle, so that the meaning of an ensemble becomes unclear. Of course individual properties can be calculated from the wave function, but the relation between the individual and the wave function is essentially ambiguous and it is because of this ambiguity that the statistical ensembles used in quantum theory do not have the same epistemological status as those used in classical physics.

To Bohr [5],[6] this ambiguity was essential to the development of the Copenhagen interpretation. It was not a question of disturbance, but a consequence of the indivisibility of the quantum of action. As a result, it is not meaningful to ask what goes on between measurements even though the wave function and the entire formalism can be used to predict the probabilities of the outcome of a given measurement. Thus in the Copenhagen interpretation one is forced to renounce all possibility of conceiving how a particle moves between measurements. As Bohr [5] puts it “there is no question of reverting to a mode of description that fulfills to a higher degree the accustomed demands regarding pictorial representation of the relation between cause and effect”. Yet the terms particle, position, momentum, etc. are essential to the theory and their retention has carried implications that lead, almost inevitably, to the type of difficulties raised by Schrödinger [7] and Renniger [8]. It will be recalled that Schrödinger’s difficulty arose from the identification of the wave function with the state of a macroscopic object, namely a cat, while Renniger showed that under certain conditions the wave function of a particle changes even when a measuring instrument gives no response. Similar interpretative difficulties also occur in the two-slit interference experiment when the pattern is analysed in terms of individual events.

Of course these particular features of the Copenhagen interpretation have received many criticisms and have led to the development of a number of alternative attitudes to the relation between the wave function and the individual. In this paper we reconsider the de Broglie-Bohm approach to this problem and report some new results that show much more clearly its implications. In particular, we calculate the quantum potential for the two-slit situation and show how it gives rise to “interference” without the need to abandon the notion of a well-defined particle trajectory, thus supporting the qualitative claims made both by de Broglie [9] and Bohm [4].

Futhermore, we are able to show that even though the theory uses par-

ticles with well-defined properties it does not imply a return to the classical paradigm. The quantum potential suggests a radical change in our conceptual outlook and provides two new interesting possibilities that could have a direct bearing on the subsequent development of the theory. Firstly it provides a fresh perspective on the microworld by giving clear intuitive representations of physical processes without the need for the ambiguous relation between the individual and the wave function. We believe that clear intuitive structures are a necessary requirement for suggesting new concepts and new experiments.

Secondly the quantum potential offers a clearer insight into the quantum interconnectedness or “quantum wholeness” that Bohr saw as the essential new feature of quantum phenomena¹. Although Bohm did recognise this aspect of the quantum potential, he did not immediately pursue its implications. In fact, it was only much later that these considerations led to the realization that a different kind of causality was implicit in quantum mechanics. This has been discussed in a somewhat different context by Baracca, Bohm, Hiley and Stuart [10] and Philippidis [11] and its relation to this work will be discussed elsewhere. Here we will show in what way the quantum potential implies interconnectedness by examining the exact form of the potential in the case of the two-slit system.

2 The quantum potential formalism.

Let us follow Bohm [4] and write the wave function in the form $\psi = R \exp[iS/\hbar]$, where R and S are real. Then Schrödinger’s equation reduces to the following two equations:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \quad (1)$$

where V is the classical potential and Q is the quantum potential,

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \quad (2)$$

and

$$\frac{\partial P}{\partial t} + \nabla \left(P \frac{\nabla S}{m} \right) = 0 \quad (3)$$

¹It has been shown that the quantum potential associated with two particles gives rise to a force between them which does not necessarily decrease as the distance between them increases. This may be considered as an instance of quantum interconnectedness. For further details see D. Bohm and B. J. Hiley: *Fonds of Phys.*, **5**, 93 (1975).

with

$$P = R^2 = \psi^* \psi. \quad (4)$$

Equation (1) is recognized immediately to be the classical one-particle Hamilton-Jacobi equation with an additional term which vanishes when $\hbar = 0$. Thus we see that a new quantity Q , the quantum potential, appears alongside classical quantities. It is this feature that allows us to retain the localized particle with well-defined positions and momenta, while the novel aspects of the quantum phenomena can be accounted for in terms of the quantum potential.

Although this new potential formally appears in an equation that suggests a dynamical origin, a closer examination reveals a conceptual structure that is radically different from that used in classical physics. For example, it carries nonlocal features which seem to be essential for a proper description of some quantum effects [12] [13] and it appears to have no well-defined source, so that its interpretation as a dynamical field is inappropriate. Bohm originally considered this to be a weakness of the model and thought it was a temporary feature of the unrefined theory. No doubt it is the nondynamical nature of the quantum potential that has generated an attitude best summarized in a quotation by Bopp [14] “we say that Bohm’s theory cannot be refuted . . . however we don’t believe it”. But it is not a matter of faith, and novelty is not a sufficient reason for rejection. We share Bohm’s later views and take the nondynamical features of the quantum potential to reflect the essential novelty of the quantum domain.

Equation (3) together with eq.(4) is taken to be an expression for the conservation of probability. In the quantum potential approach the probability density arises as a result of a distribution of initial conditions of the particles in the ensemble together with the density of trajectories in that region, the trajectories themselves being determined from the modified Hamilton-Jacobi equation. Thus, in this case, the probability is interpreted as a frequency distribution over individuals with well-defined properties so that the ensembles are of the type used in classical physics. However, we must emphasize that there is an essential difference in the sense that in classical physics the initial distribution can be arbitrarily reduced by careful preparation of the initial conditions. But in the model we are considering, the role played by the quantum potential itself is such as to make it impossible to reduce the uncertainty in the initial conditions below that given by quantum mechanics in any given measurement. Whether there exist situations in which this is not true will be left open. However, once the initial

distribution satisfies the quantum mechanical condition, the presence of the quantum potential in eq.(1) ensures that the probability P is equal to $\psi^*\psi$ for all subsequent times.

3 The quantum potential for the two-slit experiment.

We shall now indicate how the quantum potential was calculated for the usual two-slit set-up comprising an electron source S_1 , two slits A and B and a screen S_2 . In the co-ordinate system with origin at O shown in fig. 1, the centres of the slits have co-ordinates $(0, Y)$ and $(0, -Y)$.

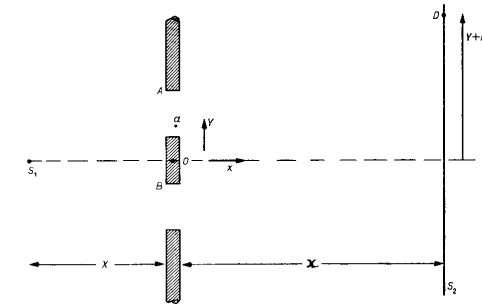


Figure 1: Two-Slit Arrangement

We begin by computing the wave function using the path integral method from which the quantum potential can be obtained by using equation (2). This method of obtaining the quantum potential does not imply that the wave function has any physical significance, but rather may be considered as a mathematical aid from which the physically significant quantum potential is calculated. First we calculate the free-particle kernel for a path starting at S_1 , passing through the point a inside slit A at a distance δY from its centre, and ending at a point D on the screen. If D has co-ordinates $(x, Y + \eta)$, where η is measured from the centre of A , the kernel can be written in the form

$$K_{\delta Y}^A(-X, 0, 0; x, Y + \eta, t_D) \exp \left[\frac{im}{2\hbar} \left[\frac{X^2 + (Y + \delta Y)^2}{T} \right] \right] \cdot \exp \left[\frac{im}{2\hbar} \left[\frac{x^2 + (Y + \eta - Y - \delta Y)^2}{\tau} \right] \right] \quad (5)$$

where $T = X/V_x$ and $\tau = x/V_x$, V_x being the velocity along the x -axis. $F(T, \tau)$ is a normalizing factor.

The probability amplitude ψ_A is obtained by integrating over all positions of a within the slit. For convenience we assume the slit to be Gaussian [15] so that the probability amplitude is then given by the following integral:

$$\psi_A = F(T, \tau) \int_{-\infty}^{\infty} K_{\delta Y}^A \exp \left[-\frac{\delta Y^2}{2\beta^2} \right] d(\delta Y) \quad (6)$$

where β is the half-width of the slit. We obtain finally

$$\begin{aligned} \psi_A = F(T, \tau) \exp \left[\frac{im}{2\hbar} \left(\frac{X^2}{T} + \frac{x^2}{\tau} \right) \right] \exp \left[\frac{im}{2\hbar} \left(\frac{Y^2}{T} + \frac{\eta^2}{\tau} \right) \right] \\ \cdot \exp \left[\frac{(m^2/2\hbar^2\tau^2)(V_y\tau - \eta)^2}{im/\hbar T + im/\hbar\tau - 1/\beta^2} \right] \end{aligned} \quad (7)$$

similarly for the slit B we find

$$\begin{aligned} \psi_B = F(T, \tau) \exp \left[\frac{im}{2\hbar} \left(\frac{X^2}{T} + \frac{x^2}{\tau} \right) \right] \exp \left[\frac{im}{2\hbar} \left(\frac{Y^2}{T} + \frac{(2Y + \eta)^2}{\tau} \right) \right] \\ \cdot \exp \left[\frac{(m^2/2\hbar^2\tau^2)(V_y\tau + 2Y + \eta)^2}{im/\hbar T + im/\hbar\tau - 1/\beta^2} \right] \end{aligned} \quad (8)$$

Here V_y is the packet velocity in the y -direction.

These solutions give two wave packets immediately behind the slits, each moving with velocity V_x , in the x -direction and separating from each other with a relative velocity $2V_y$. The half-widths of these two packets in the y -direction are given by

$$\Delta y = \left[\beta^2 \left(\frac{x}{V_x T} + 1 \right)^2 + \frac{\hbar^2 x^2}{m^2 \beta^2 V_x^2} \right]^{1/2}. \quad (9)$$

In order to obtain a clear visualization of the shape of the quantum potential under ‘‘steady state’’ conditions, we have carried out numerical computations using data based on the experiments performed by Jonsson [16]. The energy of the electrons is taken to be 45 keV and we have used the velocities $V_x = 1.3 \cdot 10^8 \text{ms}^{-1}$ and $V_y = \pm 1.5 \cdot 10^2 \text{ms}^{-1}$. The separation between the centres of the two slits, A and B , is $1.0 \cdot 10^{-4} \text{cm}$ and their half-width is assumed to be $0.1 \cdot 10^{-4} \text{cm}$. The quantum potential was calculated in a region between the slits and the screen bounded by $0 < x \leq 35 \text{cm}$ and $-1.9 \cdot 10^{-4} \text{cm} \leq y \leq 1.9 \cdot 10^{-4} \text{cm}$.

Figure 2 shows the quantum potential when viewed from the screen S_2 looking towards the slits. The position of the slits coincides with the two parabolic peaks in the background. In fig.3 we plot the corresponding particle trajectories for various initial positions within each of the slits². The trajectories are calculated by integrating the equation

$$\nabla S = mv, \quad (10)$$

which relates the S -function to the particle velocity in the usual way. Initially the trajectories from each slit fan out in a manner that is consistent with diffraction at a single Gaussian slit. The subsequent “kinks” in the trajectories coincide with the troughs in the quantum potential. They arise because, when a particle enters the region of a trough, it experiences a strong force in the y -direction which accelerates the particle rapidly through the trough into a plateau region where the forces are again weak. In consequence most of the trajectories run along the plateau regions giving rise to the bright fringes, while the troughs coincide with the dark fringes.

At about 35 cm, which corresponds to the foreground of fig.2, one has already reached the Fraunhofer limit in which the separation between the fringes is given by $\delta = \lambda x/2Y$, λ being the de Broglie wavelength. In fact, this result is used to provide a check on the numerical data.

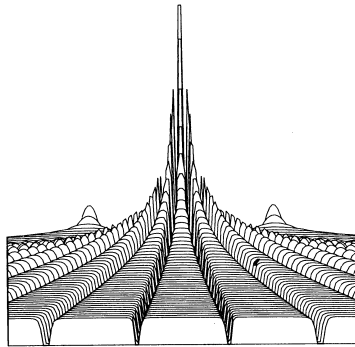


Figure 2: The quantum potential for two Gaussian slits viewed from S_2 ,

The density distribution of trajectories alone does not provide the actual intensity distribution, but must be supplemented by the particle distribution

²A preliminary attempt to calculate the particle trajectories has been made by J. P. Wesley, *Phys. Rev.*, **122**, 1932 (1961). However, his premises and assumptions are radically different from those used here and, furthermore, his results do not correspond with those derived from quantum mechanics.

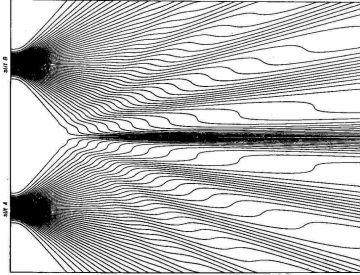


Figure 3: Ensemble of trajectories through two Gaussian slits

function. If we assume a Gaussian distribution at the slits, then the intensity distribution in the Fraunhofer limit agrees with that expected from the usual considerations.

In fig. 2 the finer details of the quantum potential are not evident, so we have plotted a cross-section in fig. 4 which shows the depth and gradient of each trough at about 18 cm from the slits. The diminishing depth of the troughs for larger values of $|y|$ is responsible for the intensity envelope of the fringes.

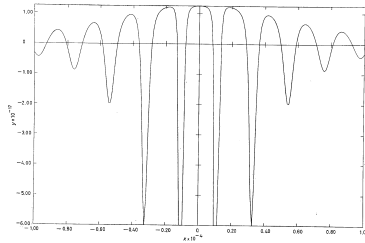


Figure 4: Cross-section of quantum potential at 18cm from slits

4 Discussion.

In the usual quantum-mechanical interpretation of the two-slit interference pattern, it is argued that the question as to which slit the electron passes through should not be raised. Naturally such a conclusion would follow from the assertion that it is not meaningful to consider what happens between measurements, but in this particular case a further reason is often given. If an actual experiment is performed to try to answer such a question, say by

placing a small counter behind one of the slits, we find that the outcome of the original experiment is changed and the fringes are no longer produced. It is then argued that, since we cannot design an experiment to answer such questions, we must not raise them because they are, in fact, meaningless. Thus we are left with point electrons producing interferencelike phenomena with no intuitive structure to comprehend such a behaviour and, by following Bohr, we must combine incompatible concepts like wave and particle through complementarity.

Our calculations show very clearly that this position is not necessary to account for interference. The approach through the quantum potential retains a pointlike particle and each particle in the original ensemble follows a well-defined trajectory which passes through one or other of the slits. This ensemble produces the required interference pattern and, at the same time, shows that the final position of the particle on the screen allows us to deduce through which slit it actually passes. Thus it is possible to retain the trajectory concept and, at the same time, account for the interference. There is no longer a mystery as to how a single particle passing through one slit “knows” the other slit is open. This information is carried by the quantum potential so that we no longer have a conceptual difficulty in understanding the results obtained in very low intensity interference experiments.

Notice that the trajectory of a single particle is not obtained by direct observation at the slits. In fact, if a counter is placed at one of the slits, the resulting quantum potential can be shown to be different from the one we have calculated and this new potential will show no interference properties. Thus altering the experimental arrangement can radically change the outcome of the experiment. This feature of quantum mechanics was continually emphasized by Bohr [5] when he talked about “the impossibility of subdividing quantum phenomena” . The quantum potential actually provides a clear expression of this inseparability. For example, the quantum potential calculated from eq.(7) shows that the properties of the particle (such as mass and velocity) cannot be isolated from those of the apparatus (such as width and separation of the slits). In other words, the observed system and the observing apparatus are linked in an essential and irreducible way. Our disagreement with Bohr’s position concerns his assertion that it is a logical consequence of this irreducibility that we are presented with a choice of *either* tracing the path of a particle *or* observing interference effects. Our results show that this is not the case since the essential features of the wholeness of quantum phenomena can be retained without the need to give up the idea of point particles following well-defined trajectories. The wholeness is now expressed through the quantum potential which depends

irreducibly on the properties of both the particle and the apparatus.

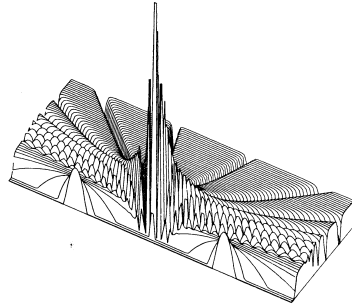


Figure 5: 150° azimuthal view of quantum potential.

From the remarks above and those in sect.2, it is clear that the quantum potential is unlike any other field used in classical physics. Indeed we can bring out yet another interesting feature of this potential by examining the 150° azimuthal view shown in fig.5. Immediately behind the slits the cross-section of the initial parabolic peaks first increase slowly, causing the trajectories to spread out radially. This feature corresponds to the spread of the wave packet in the usual approach.

As one moves away from the slits, high, rapidly varying spikes appear at about 1.5 cm from the slit plane and finally decay into the background at about 6 cm. Very few electrons actually reach this region, so it can be regarded as lying in the geometric shadow of the two slits. However, this does not imply that its role is negligible. On the contrary fig.2 shows that the high peaks are the source of the overall pattern because all the troughs and ridges emerge from here and radiate outwards.

Furthermore, the magnitude of the quantum potential contributes negligibly to the total energy of the electrons. Its absolute maximum value is only about 10^{-4} eV, whereas the kinetic energy of the electrons is about 45 keV. Its effect is almost entirely confined to the relatively narrow troughlike regions, where it gives rise to large positive and negative accelerations which produce the short kinks found in the trajectories; its role may, therefore, be thought of as one of ordering and structuring the particle trajectories in a way that reflects the peculiarities of the quantum domain. Thus the geometric shadow can be thought of as an organizing centre for the whole ensemble of trajectories which unfold as the particle moves towards the screen. This suggests a natural link with the recent ideas proposed by Thom [17] in a different context. We have already begun to explore this relationship and

a more detailed study of the structural characteristics and function of this region will be discussed in a later paper.

5 Conclusion.

Our results using the quantum potential show that one can, in fact, remove the ambiguity of whether quantum objects are waves or particles and provide, instead, a clear intuitive understanding of quantum interference in terms of well-defined particle trajectories. More important than this, however, is the new perspective it gives to quantum interconnectedness. We have shown that the quantum potential combines properties of all the participating elements—masses, velocities of particles, widths and separation of slits—in an irreducible way and suggests that, as far as the quantum domain is concerned, space cannot be thought of simply as a neutral back cloth. It appears to be structured in a way that exerts constraints on whatever processes are embedded within it. More surprisingly still, this structure arises out of the very objects on which it acts and the minutest change in any of the properties of the contributing objects may result in dramatic changes in the quantum potential.

This gives a new appreciation of Bohr's insistence that quantum phenomena and the experimental situation are inseparable. Moreover, it recalls the relativistic relationship between space and inertial mass, and seems to extend this relationship to include the geometrical and possibly the topological configurations of matter.

It is clear, therefore, that the quantum potential is unlike any other field employed in physics. Its globalness and homogeneity in the sense of not being separable into well-defined source and field points indicates that it calls for a different conceptual framework for its assimilation. Notions of structure, structural relationships and stabilities seem to be more appropriate than those of dynamics (even though here we have started with what appeared to be dynamical equations). However, a more detailed discussion of these points will be presented in a further paper.

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