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Editor

# Humanizing Mathematics and its Philosophy

Essays Celebrating the 90th Birthday  
of Reuben Hersh

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## **Preface – Reuben Hersh: *Humanizing Mathematics and Its Philosophy***

It is difficult to find words to succinctly capture a mathematician who has systematically deconstructed the cold edifice of the institution of mathematics. Reuben Hersh turns 90 on December 9, 2017, and his nine-decade journey to the present moment offers glimpses into the world of mathematics as well as the changing nature of the American landscape. Born in the Bronx to working-class immigrant parents, Reuben embraced the ideals of the “working class” in spite of graduating from Harvard at the age of 19. After a stint at *Scientific American*, Reuben spent the 1950s working as a machinist. When I asked Reuben about this, he said “Being so young and naive, I just felt frustrated at his (Svirsky’s) constant dissatisfaction with my work. Then there were political reasons too. I was deluded into thinking that only the working class could save the world, so I ought to be part of the working class. Learning to run a lathe was interesting and in a way gratifying work” (see interview in “An Interview with Reuben Hersh”).

One could say that becoming a machinist created a dual identity, namely, that of a “working man” as well as a “thinking man” and one that seems *different* with the way professions are structured today, particularly if one looks at the ivory tower of academia. However, being able to do many things was the hallmark of learned people for centuries. Gauss’s “day job” was that of a surveyor; Euler worked as an engineer; von Helmholtz started out as a physician but went on to make astonishing contributions to mathematical physics.

Reuben went into mathematics after obtaining a degree in literature and then working as a machinist. He chose mathematics because he always enjoyed it. It should not be surprising then to know that he went on to complete a PhD under Peter Lax and had a fruitful career as a mathematician for many decades with research in partial differential equations, random evolutions, and operator equations. If one imagines these decades of his life as that of a “working mathematician” with literary leanings analogous to the “working machinist” with literary leanings, then these leanings came into full force when he started to write about the nature of mathematics, what it means to be a mathematician, the social nature of mathematics, the burden of proof, and what it means to question the status of mathematics. His first expository book *The Mathematical Experience*, cowritten with Phil Davis,

won a National Book Award in Science in 1983. His subsequent book *What Is Mathematics, Really?* picked up where the Courant-Robbins classic ended, with a rhetorical question, but provided a more detailed exposition on what it means to be a mathematician, how mathematicians think about their work (as opposed to “science” or “craft” or “art”), and philosophical problems that arise when doing mathematics. Many of us can “do” things, i.e., do mathematics, do physics, do biology or do garden work, do woodwork, do cooking, etc., but very few of us are able to articulate what it really means to do something in a way that would appeal and interest a layperson. Reuben’s expository books have been very impactful to many of us, and his position of mathematics as a human endeavor or humanistic allows the foot soldiers and the lay mathematicians a doorway through which they can examine their own mathematical endeavors. A well-known mathematician once told me that there are very few that can leap from one area of research mathematics to another and then be able to clearly articulate the connections that led them to make these “creative leaps.” Reuben’s journey as writer, machinist, mathematician, and philosopher over the nine decades of his life contains many such leaps, not simply within mathematics but between disciplines and not simply between disciplines but between completely different “working lives.” His corpus of writings that range from technical mathematics to reviews to expository and philosophical writings offer clear glimpses of Reuben’s many leaps.

I conferred with Reuben about putting together a Festschrift for his 90th birthday, and he agreed to it based on some conditions of course, namely, being able to shape this in order to break convention and be *different!* So he sent me a list of colleagues/friends/scholars whose work he has been influenced by, and I invited them to contribute to this book. When I asked Reuben what he would like contributors to address, he said the following:

Forty years ago, Paul Cohen enraged me by predicting that (at some unspecified future time) mathematicians would be replaced by computers. So I now ask you, 1. Can practicing mathematicians, as such, contribute anything to philosophy of math? Can or should philosophers of math, as such, say anything to practicing mathematicians? 2. 20 or 50 years from now, what will be similar, and what will, or could, or should be altogether different: About the philosophy of math? About math education? About math research institutions? About data processing and scientific computing?

The colorful and eclectic essays in this Festschrift from numerous well-known mathematicians, philosophers, logicians, and linguists offer in part his colleagues’ attempts at answering Reuben’s questions and also in part glimpses into Reuben’s fertile mind and his influence on many generations and decades of mathematical life. In his 90th year, he continues to produce mathematics and writings about it that is accessible to us all. Reuben Hersh epitomizes the phrase “humanist mathematician and philosopher,” and I hope this Festschrift celebrates his many accomplishments and contributions to the field. I am deeply honored to be able to edit this collection and join the authors in this book to wish him a happy birthday, and I hope for another decade of contributions from Reuben.

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