ON BLACKBODY RADIATION IN ELECTRODYNAMICS

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In a paper appearing in this journal[1] a short time ago entitled "Zur Stralungstheorie" (On Radiation Theory), LORENTZ came to the conclusion, that the theory developed by himself[2] and JEANS[3], on black body radiation, is not compatible with certain observed facts; but now as such a deficiency must imply something for the derivation of the formula for radiation, if significant modifications are to be made to the fundamentals of electrodynamics, then such modification are necessary, and must, in the sense of PLANCK's theory, have the nature of injecting a notion with a time-energy character into the theory.

Considering the great importance of this issue, and the difficulties, which, as LORENTZ has shown, persist in spite of PLANCK's introduction of atoms [quanta], it must be taken, that an error in the JEANS-LORENTZ theory is indicated, which points to the core of the deficiency.

The assumption regarding electric and magnetic forces, that is made in this matter, is too general, it stands in conflict with the requirement that any physics theory must satisfy, namely that only retarded potentials are acceptable. This condition severely restricts the number of coordinates for the free aether, i.e. those that LORENTZ (*loc. cit.*) denoted with q_3, q_3'. These are the coordinates that in the end determine the form of the radiation formula; and the conflict between theory and experiment is caused by the fact that these coordinates, that are quite numerous, on a basis analogue to BOLTZMANN's spectral distribution law over the degrees of freedom, such coordinates tend to pull the whole energy of the system to themselves.

As is well known, the fundamental equations for the LORENTZ theory of electromagnetism can be unified in the single equation

$$\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} - \triangle f = \varphi(\vec{x},t) \tag{1}$$

where c is the speed of light, φ a given function of $\vec{x},t$, and f vanishes at infinity. The general solution of this equation, say by the method of POISSON, introduces two arbitrary functions of $\vec{x}$, namely those giving values of f and $\partial f/\partial t$ at the initial time t_0. Particular solutions are

$$f_1(\vec{x},t) = \frac{1}{4\pi}\int \frac{\varphi(\vec{x}', t-r/c)}{r} d\vec{x}',$$

[1] *Physikal. Zeitschrift.* **IX**, 562 (1908).

[2] LORENTZ, H.-A., "Le partage de l'énergie entre la matière et l'éther," Confénce tenud au congrès de Rome (Roma, Tipografia dell R. Accad. dei Lincei) April 1908.

[3] *Proc. Roy. Soc.,* **LXXXVI** 296, 545 (1906).

and

$$f_2(\vec{x},t) = \frac{1}{4\pi}\int \frac{\varphi(\vec{x}',t+r/c)}{r}d\vec{x}';$$

where

$$r^2 = (\vec{x}-\vec{x}')\cdot(\vec{x}-\vec{x}'),$$

and, moreover, solutions can be any linear combination of f_1 and f_2, say $f_3 = a_1 f_1 + a_2 f_2$, where $a_1 + a_2 = 1$; and, finally, also

$$f_4(\vec{x},t) = \frac{-1}{4\pi^2}\int \frac{\varphi(\vec{x}',t)d\vec{x}'dt'}{r^2-(t-t')^2/c^2}.$$

The function f_1 represents divergent waves, f_2 convergent waves originating at infinity, f_3 their sum, all other solutions would represent waves which at points in pure aether, i.e., where $\varphi = 0$, converge or diverge; f_4 represents waves, that according to the POISSON or KIRCHHOFF laws in infinitely many ways can take place continuously, but are never seen. In the end, only solutions of the form of f_1 actually are ever observed, and MAXWELL's theory imposes this fact as a precondition. Just how necessary this precondition is, is already evident, in that, whereas for f_1, a body whose electrons are accelerated, energy is emitted such that at a distance the POYNTING vector points away from the source, for f_2 (where $c \to -c$) this vector also changes its sign which corresponds to energy flowing from infinity to the charge *without the involvement of other charges losing energy*. Such a situation would allow a body to extract energy from the aether at infinity and would constitute a *perpetuum mobile*, a simple physical impossibility.

Now in order to satisfy all such conditions—also those at infinity—for solutions that are still not physically admissible, one must some how limit the initial conditions. The necessary and sufficient condition that $f_1$is valid for all time, is that it be valid at $t = t_0$ and at $t = t_0 + dt$. That exactly this condition has no reasonable representation in MAXWELL theory, is enlightening, and one had tried to replace it with other preconditions. If one takes it, as is usually done, that at $t = t_0$ at large distance from the source the field vanishes, then it follows that f_1 is valid at later times, but *is not valid for* f_2 *at earlier times.* Moreover, now the validity of f_1 is subject to entirely different restrictions (field $= 0$ at $t = t_0$), which, for example, a uniform translation does not satisfy. Actually, it is a characteristic of hyperbolic differential equations, such as Eq. (1), that when the initial conditions are just closely satisfied, it does not follow that the solutions will behave in the same way; for example, weak waves that at $t = t_0$, can grow to be arbitrarily large at particular points.

Likewise, the other conditions mentioned above withstand careful scrutiny in this regard no better[4]; the transition from time reversible differential equations to retarded potentials, *through which irreversibility is uniquely introduced into electrodynamics*, can not be established within the MAXWELL-LORENTZ theory. Thus, it is important to emphasize, the the complete expression of the laws of radiation, and of MAXWELL-LORENTZ theory in general, can not be the given differential equations, rather, elementary interactions, which result from the introduction of retardation in LORENTZ's expression for the ponderomotive force. In this format, both the electric and magnetic vectors are also precluded, they are entities, which, in any case, are unobservable, but rather play only a role only as mathematical crutches[5], while the physical results of the theory are expressed only in terms of space, time and electric charge.

[4]For analysis of these and other weak points in the MAXWELL-LORENTZ theory, see: RITZ, W., *Recherches critiques sur lélectrodynamique général. Œuvers,* **XVIII**, p.317.

[5]*Loc. cit.* p.318.

For derivation of the radiation formula, JEANS and LORENTZ consider a rectangular cavity with reflecting walls in which there is a body, K; the electric and magnetic forces within the cavity are expanded in FOURIER series as functions of $\vec{x}$, with time dependant coefficients, and play the role of LAGRANGE coordinates, with which, using HAMILTON's principle, their differential equations can be derived. With these preconditions, the important stipulation restricting consideration to retarded potentials is not taken into account, this condition as otherwise admitted and specified by two arbitrary functions of $\vec{x}$, however, precludes, as described above, an infinity of physically inadmissible initial conditions for the aether. In particular, it requires that forces remain static whenever the source charges do not move. Strictly from the differential equations, this is not necessary; a solution of the homogeneous equation $\partial^2/\partial t^2 - c^2\triangle = 0$ can always be appended, which for the cavity must satisfy initial conditions on the reflecting walls, and thereby contribute to the total electric field within the cavity (excluding body K). Such solutions appear in the JEANS-LORENTZ derivation, as LORENTZ[6] emphasized, as acceptable, which should not be the case. Much more likely, the multiple infinity of coefficients, which are involved (that is, the expansions coefficients of the general solution to$\partial^2/\partial t^2 - c^2\triangle = 0$ in terms of the eigenfrequencies of the cavity, i.e., the FOURIER series expansion coefficients), must be zero for all times. This infinity of parameters of the "pure aether" are exactly the ones, which according to the uniform distribution of energy over the degrees of freedom, that tend to be distributed over the short wave lengths and in effect absorb the whole energy. In other words, *The* JEANS-LORENTZ *theory is unacceptable*.

One could object, that the just mentioned solution expanded in terms of "retarded" forces, results from the electrons in the reflecting walls of the cavity. In so far, however, as a perfectly reflecting wall would require *infinitely* many electrons, and is therefore an inadmissible abstraction, the actual number of degrees of freedom of the of the body K (or the number of electrons contained it) and in the reflecting walls of the cavity can not be considered infinite, and this is the crucial point. When the number of electron is very large, then the JEANS-LORENTZ assumptions remain valid, but then only for eigen frequencies of the cavity, for which, in fact, the discontinuity in the structure of the reflecting walls and their electron structure with its conduction peculiarities do not intervene, that is *for long wavelength oscillations or low temperatures. This is the reason their law pertains in this region for such wave, and only for such waves. For wave shorter than these, the conditions can not be brought in accord with the retarded potentials, as there are simply too many solutions taken into account.*

To take the condition of the necessity of using retarded potentials into account, appears to be difficult, and it is unclear if it is sufficient to determine the spectral distribution from its experimentally observed character. To do so, it is first of all necessary to determine how many and which arbitrary constants of the general solution of the equation of motion of electrons in the system are involved when the forces' retarded potentials are used. Only on these arbitrary elements may the static considerations be extended. For mechanical problems the question can be greatly simplified, in that by specifying the coordinates q and momenta p, the subsequent development is determined. In the electron theory the situation is different, however, and in this cast there may a sore point. Even the equations of the force free motion of a inflexible electron, as shown by HERGLOTZ[7], in addition to uniform translation, there are an infinity of other solutions; for very small velocity the general solution is representable as a sum of an infinity of oscillations with arbitrary amplitudes

[6] loc. cit. p.13.

[7] HERGLOTZ, G., *Gött. Nacht.* **6** (1903); *Math. Ann.* **LXV,** 87 (1908).

for which the wavelengths are quite far beyond the ultraviolet, that is at most of the order of the diameter of an electron but with no lower bound. The method use by HERGLOTZ, which remains applicable and leads to similar integral equations, can be expected to yield solutions with a similar behavior. In the end this behavior is determined by the fact that in the theory of electrons, accelerations are determined by *earlier* positions, velocities and accelerations of other electrons, or charged bodies. To begin, if we restrict ourselves on the case where all functions of the form $\psi(t-r/c)$ are expanded according to

$$\psi\left(t-\frac{r}{c}\right)=\psi(t)-\frac{r}{c}\psi'(t)+\frac{r^2}{1.2\cdot c^2}\psi''(t)-\cdots,$$

then the differential equations that result have endlessly high order for which the solutions depend on endlessly many constants[8], which in this special case must satisfy convergence conditions. Studies by SOMMERFELD[9] and HERTZ[10] on a solid spherical electron show, that for a given external force one can specify an arbitrary motion in a time interval T, which equals the diameter of the electron divided by the speed of light; in particular, for uniform surface distribution of charge, every function with period T satisfies the equation of motion for a force e free electron, and also any function $P(t)$ can be added to each solution of the problem for given exterior forces. If the solution is to be analytic, then th values of $P(t)$ within a period can not be arbitrary, but is can be such that $P=(Q+Q*)/2$, where

$$Q=Q\left(e^{\frac{i2\pi t}{T}}\right)=Q(x)=a_0+a_1x+a_2x^2+\ldots,$$

and the a_i (up to convergence conditions for the series) are arbitrary.

In general the solution for arbitrary motion of systems of electrons requires the determination of infinitely many constants; such that they allow oscillations of unlimitedly small wavelength. These in turn are determined by the infinity of “degrees of freedom of the aether;” and, it is to be anticipated, that they, on the principle of uniform energy distribution, finally also tend to concentrate the radiation totally on the shortest wave lengths, even if the formula should deviate slightly from the form given by JEANS. But even aside from these considerations, the existence of force free eigenoscillations of the electron, for example, which may be superposed to give any solution and with which any solution can be constructed, and which must appear everywhere, can be considered experimentally improbable. Should a beam of extremely short wave length be unobservable with our observation methods, it still should reveal itself through a corresponding energy defect, that has, however, not been detected.

From this one might conclude, that *just as we were forced to constrain the multiplicity of available solutions to* MAXWELL*’s equations by restricting consideration to those resulting from retarded potentials, a new restricting principle is needed to similarly restrict the constants for solutions to a finite number.*

That among all the possible, infinitely many solutions, one is always distinguished, as those for partial differential equations are distinguished as being from retarded potentials, is easily rationalized. Suppose that one considers gravitational interaction not to be instantaneous, but delayed as is electric interaction. Thus, for given initial conditions, i.e., the positions and momenta, one would calculate subsequent motion using classical laws as a first approximation; which would then be inserted in the (very small) additional terms from the new law; thereby giving new differential equations of second order that are to be

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[8]See: LALESCO, T. *Sur l’équation de Volterra*, (Thése, Paris, 1908).

[9]*Gött. Nachr.*, , 363 (1904).

[10]*Math. Ann.* **LXV**, 1 (1908)

integrated using the same initial conditions, etc. Finally one would obtain each coordinate x as an analytic function of time t and the initial conditions, x_0, x'_0, which, however, would be valid only in a restricted domain. This solution can be continued, however, both as an analytic function of t or of x_0, x'_0 and then gives for arbitrary initial values of the coordinates and velocities a particular solution, dependant on only these initial values, beside which, however, an infinity of other solutions exist, for which this procedure is *never* valid, and would correspond to extremely outlandish planetary systems. For the case of a force free electron, one gets the solution of uniform motion. This is the only one permitted, *in which case the variety of constants, that is, the variety of solutions, is no larger than in mechanics, namely twice the number of degrees of freedom of the electron.*

This can be realized by introducing an additional principle in the form of a *maximalization principle,* where the variations vanishes for all solutions, but gives only a real *minimum* for the correct specific solution. Similar results obtain in the theory of vibrating strings, membranes, etc.[11], where, while for all oscillations, infinite in number, the variation vanishes, only for the principle frequency is a minimum. Likewise, one could, aside from conditions at infinity, also introduce conditions at very large times t, which again could be deduced through a variational calculation.

The difficulties in the theory of black body radiation brought to attention by LORENTZ, *lead us not so much to. along with* PLANCK *to introduce an energy-time unit, rather to the stipulation, that the principle, violated by the current theory of electrodynamics of unity in the sense of classical mechanics, be restored by principle of minimization,* so that a finite number of determining factors specify the motion of electron for all time.

With that we see the last of what was to be the aether disappear from the laws of nature. Stepwise it was seen necessary to deny it motion and ever more other properties of matter; from a more or less complicated mechanism it became an unchangeable carrier of electromagnetic effects. In this diminished competence it could have shown its existence through the equations from which matter, i.e., electrons are dependant (i.e., satisfy the equation $d^2/dt^2 - c^2\triangle = 0$). Experiments however, forced us to abandon this solution. Then they compelled us to eliminate completely equations among space and time expressed for field strengths, or “the state of the aether.” Aether sank to being an abstraction, it is now only an absolute coordinate system and a mathematical construction which insinuates into the equations infinitely many constants. Experiments seem to deny aether any of these characteristics, they simply banish it from physics.

Through this development a substantial fundamental element of MAXWELL’s formulation of electromagnetism in partial differential equations is denigrated, as they have no physical meaning, rather giving them only the significance of a mathematical intermediary construction, which, moreover, is even insufficient. Belief in MAXWELL’s formulation’s unconditional rectitude is not justified, not to mention that it can be shown[12], that the experimental basis is in a certain sense completely absent.

[11] See, e.g.: RIEMANN, B. and WEBER, *Partiielle Differentialgleichung,* t. **II** (Braunscheig, 1901) .

[12] RITZ, W., loc. cit. p. 427 and 462.