HAMILTONIAN MECHANICS CONTRA ASYMMETRIC AGING[*]

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Criticism of the logic behind asymmetric aging (or the "twin paradox") is presented. It is observed that there exists general proofs of the compatability of proper time as a variable conjugate to the Hamiltonian of systems of interacting particles. Further, many arguments supporting the interpretation of time dilation and length contraction as space-time perspective effects are reviewed.

I. THE DILEMMA

Perhaps the oldest astonishing feature of Special Relativity (SR) is asymmetric aging, also known as the "twin paradox." That objects appear different from what they are in fact, is not troubling; perspective is understood by absolutely all. It is an automatic aspect of vision. That objects at a distance appear smaller than they do up close, is common place; what would be truly astonishing would be, that an object when brought back close, remained smaller, or even got smaller still! This is what reputedly happens according to the nowadays conventional understanding of the consequences of SR.

That readings by means of light signals from a clock that is in motion relative to the 'reader,' are altered somehow in comparison to similar readings of a clock stationary with respect to the reader, is an understanding of time dilation that might be called 'MINKOWSKI perspective.' It is not hard to imagine, that because of the finite speed of the intermediary agent (light) in reading a moving clock, and the effect this has on dispatch and arrival time of signals, that intervals calculated on the basis of arrival times get distorted. However, if true that such alterations accumulate absolutely for a round trip of the clock, this indisputably would distinguish this effect from perspective.

What does the evidence show? Surprisingly, there is none![1] Virtually all experiments, when understood, turn out to be verifications of what could be just a perspective effect; they are essentially just half of a round trip, for example, muon life time experiments in the atmosphere or in accelerators. The famous "clocks-around-the-world" experiments, on the other hand, have all been found defective by cause of instability of the clocks; the late A. KELLY[2], among others, has argued convincingly that the stability of the atomic clocks used in the experiment was at least two orders of magnitude too low to permit detecting the timing asymmetry possible in the experiment.

This effect, addressing as it does, the single most critical phenomenon for mortals: time, is of limitless interest to all who learn about SR. Beyond this for the professional scientist there is another aspect that has very deep significance. It is the issue of proper time for a system of interacting particles in relativistic mechanics.

In conventional SR it is easy to show that a particle's proper time coincides with time shown on a clock traveling with the particle. This time is distinct from the fourth component of a MINKOWSKI event vector, which would be the time attributed to the particle by an exterior observer, i.e., signal arrival time. Further, it is a standard exercise in textbooks on SR to show that for a single particle bathed in an exterior field, the proper time, analogous to the situation in non relativistic mechanics, is the variable canonically conjugate to the particle's Hamiltonian. So far, this structure all fits together logically; that is, in the limit of the speed of light becoming infinite, this single-particle SR mechanics goes over to nonrelativistic Newtonian mechanics.

But, nowadays, many think that this story breaks down for multiple interacting particles, because the proper times for two or more particles are not mutually reconcilable.[3] In other words, the proper time intervals of two particles along orbits that cross twice are unequal between crossings, that is, there is asymmetric aging. If so, then proper time cannot serve as a variable conjugate to the Hamiltonian of the system.

On the face of it, this seems very unreasonable! Almost as much so as the unreasonableness of the existence of mutually exclusive rationalizations for asymmetric aging offered in standard works on SR. Mechanics, relativistic or not, need not be thought of as the science of motion of material objects; it can be abstracted to the study of differential equations on manifolds of a certain character. In that rendition, the central question then becomes: do there exist solutions to the basic differential equations, and if so, what properties do they have? Put technically: are the equations integrable?

Now, it turns out, this question in full abstraction has enjoyed long standing interest to many mathematicians. They have addressed the essential features involved at a level of generality that completely covers the requirements for a relativistic mechanics of material particles. They have found structure so general that it covers the relatively narrow needs of SR mechanics; in other words, they have what constitutes an existence proof for the type of equations needed for SR mechanics formulated at a level of generality that is independent of dimension and metric.[4; 5] This proof pertains then, even when the dimension is large enough for two or more particles and the metric is pseudo-RIEMANNian, i.e., exactly where conventional wisdom would have it that there is no such Hamiltonian structure.

Thus, this writer is brought to the conclusion that there is need for critical reevaluation of the standard explication of the

twin paradox. This is perhaps best done by reviewing the previous attempts to resolve the paradox; there are several such attempts, recent ones of which have not received wide spread publicity. Herein I shall review, therefore, attempts at dispatching this issue best known to me: namely those of PALACIOS, SACHS and myself.

II. IRRELEVANT ANNEXATIONS

A. Acceleration

One ubiquitous source of confusion in the literature on this matter arrises because historically there are two formulations of an asymmetric aging paradox. The first, and most often discussed (probably because it is easy to resolve), is the obvious antinomy from the asymmetry itself, i.e., by authority of the relativity principle, each twin can regard the other as the traveler who returns younger. Both can't be, hence a paradox. This paradox is easily resolved, however; as only one twin is subject to local accelerations, so the trip is not symmetric.[6] So far as it goes, fine. But, the very existence of an aging differential is also paradoxical, but of a somewhat more sophisticated and complex character. An asymmetry arises because the formulae yielding length contraction or time dilation with respect to moving objects observed by stationary observers, apply as well to stationary objects observed by moving observers, indeed, this is a restatement of the Relativity Principle. This is very perplexing because for every stationary object there are many moving observers; so, which one determines the scale of these effects? This matter goes directly to the self consistency of the very concept of aging, time progression and change in the material world, all deep philosophical issues, and, in the end, to the existence of a relativistic Hamiltonian form of mechanics, the initial source of concern for this writer.

The difficulty understanding the logic and mechanism of asymmetric aging has lead many to pore over the details of the anthropomorphized version involving the hypothetical trip into space after which a traveling twin is, according to standard texts, distinctly younger than a stay-at-home twin. Even EINSTEIN[7] has fallen for this tact, some 13 years after first suggesting the issue, albeit not in its anthropomorphic version, he then reverted to suggesting that the crucial physical aspect for the purported phenomenon is to be found in the practicalities of making a trip, namely in the accelerations involved in launching the trip, then turning around and finally decelerating to a stop at the end.[8]

Attentive analysis of the algebra involved for asymmetric aging, or time dilation, shows, however, that acceleration is not the issue. [1] This can be seen from the following formulation: Let us suppose that the traveler's journey is composed of two parts, one outbound, the other inbound. For both portions

let us consider putting all accelerations outside the bounds of that portion of the trip relevant to comparing proper time intervals. That is, the first or outbound trip shall involve a traveler preaccelerated on the opposite side of the stationary partner such that no further acceleration is involved until this traveler is past the turnaround point. As this traveler passes the stationary twin on his way to the turnaround point, let us suppose the stationary clock is started by instantaneous contact of antennas, say. Thereafter, when the outbound travel has reached the prearranged turnaround point, he makes instantaneous contact with a similarly preaccelerated inbound traveler whose identical clock is started. Finally when the inbound traveler passes the stationary twin, instantaneous contact signals both clocks to stop. Then the readings of these two clocks can be compared after-the-fact; by symmetry the inbound time interval is exactly half the elapse of total proper time made by the two travelers between contacts. [2]

The conventional interpretation of SR of this case, too, leads to the claim that the total elapsed proper time of the traveler(s) is less than that of the stationary twin. The calculation for this rendition does not involve any consideration of acceleration, and, therefore, of mechanics. It shows that time dilation, if it exists, is entirely a kinematic effect or an artifact of making observations using light; it does not depend in any way whatsoever on dynamics or on mechanics. All efforts, including EINSTEIN's, to resolve the issue on the basis of acceleration, i.e., General Relativity effects, are obviously misdirected.

B. Simultaneity

It is said categorically that SR precludes a covariant definition of 'simultaneity.' Is this actually so? The following little complexity may draw this assertion into doubt. Events in Minkowski space time can be of two characters: ontological and epistemological. The first are those associated with a particle's existence and should be thought of as signal source events. The latter are those as perceived by an observer of the first type of event and should be considered signal sink events. This distinction is so good as never explicitly made in connection with a discussion of simultaneity.

Clearly, simultaneously observed events all fall on the observer's past light cone; because each observer has a distinct light cone, relating one particle's set to another's is a complex matter. On the other hand, simultaneous events, as sources, simply by definition all lie on a space-like plane of whatever MINKOWSKI chart is being used. Even though they cannot be viewed using light instantaneously, they can be identified by

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[1] See UNNIKRISHNAN (Ref. 8) for detailed analysis of EINSTEIN's arguments; note, however, this writer does not share UNNIKRISHNAN's assessment of the utility of a privileged frame in understanding SR.

[2] Note that it makes no difference when and where the two travelers actually make contact; all that matters is that their speeds are identical and remain constant. This sort of formulation was first suggested by EINSTEIN himself and then VON LAUE too. They, and seemingly every authority thereafter, then have alternately used it and overlooked it; but, virtually no one has fully consistently applied conclusions drawn from it to length contraction and time dilation generally.

unambiguous observational procedures. Such a plane is common to all particles and its LORENTZ transformation between MINKOWSKI charts is well behaved. It can be self consistently and unambiguously defined for all observers. While it does not exist in the way a stone floor does, it is a well defined abstract concept and can be made to play a useful role in analysis of SR mechanics. Indeed, this concept appears naturally in cosmology, where it is well ensconced.

III. REVAMPINGS OF SPECIAL RELATIVITY

A. PALACIOS

JULIO PALACIOS MARTINEZ (1891-1970) was a Spanish physicist who came to a critical evaluation of SR firstly on the basis of dimensional analysis. In addition to many articles with partial results, by 1960 he published a book[9] explicating his version of relativity and then, shortly before his death published a lengthly article[10] with his critical analysis of conventional SR and recommended modifications.

PALACIOS' basic logic differers from EINSTEIN's in that he does not accept the postulate that all inertial frames are equivalent. Thus, assuming linearity and a universal constant speed of light, as did POINCARE, LORENTZ and EINSTEIN, he gets the transformations and inverses again with an arbitrary constant multiplier. Then where EINSTEIN used the Relativity Principle to argue that the arbitrary constant equals 1, Palacios' logic leads him to the following transformations.

$$x = x' + vt', \quad y = \alpha y', \quad z = \alpha z', \quad t = t' + \frac{v}{c^2}x',$$

where, again $\alpha = \sqrt{1 - (v/c)^2}$, and inverse transformations

$$x' = \frac{1}{\alpha^2}(x - vt), \quad y' = y/\alpha, \quad z' = z/\alpha, \quad t' = \frac{1}{\alpha^2}\left(t = \frac{v}{c^2}x\right).$$

The immediate consequence of Palacios' line of argumentation is that there is no time dilation or length contraction in the direction of motion, but a length augmentation in the orthogonal direction. In addition, his arguments leads to the conclusion that there is a privileged frame which he considers even physically identifiable.

PALACIOS' line of argumentation seems to this writer to be distinctly different from other critical analysis of time dilation, most all of which are foreshadowed one way or another by DINGLE's[11] contention, that the proper time intervals of both twins are identical by virtue of invariance of the differential of arc length. Because of this difference, I bring attention to it, but resist making an appraisal as it could well turn out to be equivalent to the standard formulation if well interpreted and correctly understood.

B. SACHS

In 1971 MENDEL SACHS managed to place an article in Physics Today in which he argued against the twin paradox as

a physical effect.[12] Based on the observation that the variables in any theory are not by themselves ontological items, but just epistemological aids, he argues that the results of a LORENTZ transformation yield information only about how the moving object appears to another inertial observer. This argument is in principle not new, but for reasons that are very opaque and seemingly based in part on arbitrary interpretations of experiments (most of them of the 'Gedanken' variety), have been rejected by most all authorities.

SACHS, on the other hand observes that the proper-length interval is a LORENTZ invariant in both Special and General Relativity, so its integral between two events in MINKOWSKI space is path independent. Since all clocks are read in their own proper frames, then they must show identical lapses of time, which means that the twins have not suffered asymmetric aging.

Sachs offers an unnecessarily complicated argument in this regard because he too, in principle, takes the accelerations into account and so finds it necessary to consider the equations of General Relativity. To make his exact point he calls on his own quaternion version of General Relativity to show that the differential of arc-length along a world line, or what is the same, the differential of proper time is single valued and analytic, which suffices to conclude that its integral between two points is path independent.

It could well be that this gratuitous complication robbed his paper of persuasiveness. This is very regrettable as had he formulated his argument as above, with the accelerations isolated outside the relevant portion of the trip, the differential of proper time would have been a constant which is obviously single valued[3] and analytic.

C. Spacetime perspective

This writer too has happened upon the basic argument used by SACHS, once inadvertently even and without recognizing the full consequences[13], and then again later in a graphical form.[14] The key point is that diagrams of MINKOWSKI planes are non Euclidean; under LORENTZ transformation invariant quantities change size and position in non intuitive ways when depicted on a Euclidean plane. Thus, a MINKOWSKI diagram of the traveling twin's trip differs in a very essential way from that of the stationary twin: the world line of the turn-around point (or pylon) is displaced on the traveler's chart, but left parallel to that on the other chart; see the figure. When this fact is taken into account, it is graphically seen that the proper-time intervals of both twins are equal, regardless of what transmissions of information back and forth between the twins would have stated. In other words, the ontology and the epistemology diverge.

This is, of course, absolutely nothing new. Perspective in Euclidean space presents exactly the same phenomenon. No

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[3] Sign ambiguity from taking a square root is only a matter of convention and once a choice is made, it does not change.
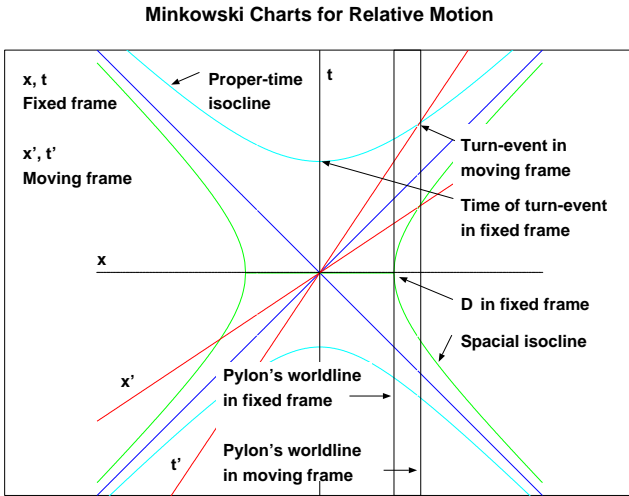
Minkowski Charts for Relative Motion

Figure 1 This figure is comprised of two MINKOWSKI charts super-imposed on each other. The world line of the turnaround point, or pylon, in the fixed frame passes through the point *D* on the x-axis. The corresponding point on the *x'*-axis is found by sliding up the eigenlength isocline to the intersection with the *x'*-axis. The world line of the pylon passes through this point on the prime chart. The intersection of the pylon's world line with the *t'*-axis is the point on the traveler's chart representing the 'turn-around' event. The eigen-time of the turn-around event in the fixed frame is found by sliding down that eigentime isocline which passes through the turn-around event to its intersection with the *t*-axis. It is clear that this value is identical with the time assigned by the fixed twin to the turn-around event as it may be projected horizontally over to the intersection of the pylon's world line in the fixed frame with the time axis of the traveler. The twin paradox arises by using, incorrectly, that eigen-time isocline which passes through the intersection of the traveler's and the pylon's fixed frame world-lines.

body or thing at a distance is perceived to have the same dimensions as they do up close. This is so well understood that we mortals make the appropriate compensations thoughtlessly. Time dilations and length contraction are fully analogous but essentially different because they arise as a consequence of the finite speed of light as used viewing moving objects. Relative motion introduces spacetime perspective on top of whatever Euclidean perspective effect might also pertain, which is a conception with a history, see:[15; 16] or[17], for example.

IV. THE EMPIRICAL RECORD

Nowadays time dilation is generally considered as having been verified empirically. In point of fact, however, the experiments are indecisive; in spite of their clear and sharp descriptions in pedagogical presentations, the laboratory realities of the actual experiments themselves destroy their persuasiveness.

Seemingly the most incisive experiment showing time dilation was the so-called "clocks-around-the-world" experi-

ment by HAFELE and KEATING.[18] They transported atomic clocks in commercial airplanes in both directions around the world for several days, and then compared the ostensible time differentials arising between the two sets of transported clocks and a third stationary set. Conceptually this experiment too is much less than ideal because the routes are circular and therefore subject continuously to accelerations, and the planes fly high enough to require a correction factor from General Relativity for variations of the gravitational potential. Such technicalities, besides introducing intricate numerical uncertainties, confuse the issue conceptually and detract from its decisiveness.

The strongest criticism, however, has to do with the required stability of the clocks needed in order to detect the effect at the scale of the experiment. This point was made generically by many early on, and finally on the basis of data from the experiment itself by the late A. G. KELLY. It seems that the raw data was classified by the US Government simply incidental to the fact that the funding for the experiment was provided by the Navy. In the mid 1990's, however, KELLY managed to pry it lose using a freedom-of-information request. His subsequent analysis showed that the experimental data analysis was corrupted by arbitrary "calibrations" and that all evidence showed that the stability of the clocks was at best two orders of magnitude too low to enable detecting the delays involved. For a subsequent experiment an improvement in stability by an order of magnitude was claimed, but the details appear never to have been published in the professional literature. In short, one cannot be assured that these results were anything but self delusion, more suitable for securing approval of the "curia" than serving as scientific evidence.

The pedagogical chestnut in this matter is the observation of mesons at sea level. It is argued that the stationary half-life of mesons is such that even at the speed of light they should all but disappear after circa 200 meters in the atmosphere if generated at the altitude of tens of kilometers by cosmic rays impacting atoms in the stratosphere. On the surface a logical argument; however, first it is to be noted that the equation for population density of mesons as a function of depth into the atmosphere is an exponential decay curve. This function is finite to infinity, so the only issue is: how many mesons are seen at sea level? This question is also a matter not only of the half-life, or what is the same, the half-path, but also of the initial density: $N(x=0)$,

$$N(x) = N(x=0)e^{-x/\lambda}.$$

The usual analysis of time dilation in reference to this equation asserts that in the transformed frame of the atmosphere, one must use in the equation

$$\lambda' = \lambda/\sqrt{1-(v/c)^2},$$

which, because $x-vt$, follows from $\tau' = \tau/\sqrt{1-(v/c)^2}$, i.e., time dilation 'predicts' observable meson densities at much lower altitudes, by cause of extended half-path, than naively expected. In fact, however, it is arguable that the numerator

too must be transformed, nullifying this effect. Except, and always overlooked, space-time perspective comes into play for the factor out front, which has the units [Quantity/volumn], or $[Q/L^3]$, and under LORENTZ transformation one length dimension must be multiplied by the factor $\sqrt{1-(v/c)^2}$, thereby altering the curve in favor of a higher meson density lower in the atmosphere, not by extended half-life, but by compacted density, a perspective effect.

In any case, it is clear that meson density observations do not test asymmetric aging; to do so, it would be necessary to divide an ensemble of mesons into two parts, and subject one of them to a round trip while leaving the other stationary, and comparing them thereafter in the same frame. Such is achieved neither in the atmosphere nor at particle accelerators, where the latter is also defective by fault of not being acceleration free motion.

In sum, the best evidence for time dilation is circumstantial at the very best and arguably incorrectly interpreted as anything but an observational artifact in any case.

V. CONCLUSIONS

Herein I have reattacked the age old issue of whether time dilation is 'real' or just apparent. This issue can be delineated also in terms of whether such dilation is an ontological differentiation of the observed object, or a epistemological effect of the observation of the object. In spite of the fact that current orthodoxy maintains that the effect is 'real,' all logic points to just an apparent effect. Of course, a good part of this dispute is lexicographical: what does "real" mean? Distortions caused by perspective are themselves 'real' too, as distortion in perception. The basic dictionary definition of 'real' most relevant here is: 'not imaginary.' On the other hand, in Physics literature, real physical effects inevitably involve energy transfer somehow. The latter notion is what this writer chooses to take as the meaning of 'real,' as he understands what other writers mean. Thus, non real (as differentiated from unreal, i.e, imaginary) effects, i.e., artifacts of observation using light, are those that do not involve energy transformations at or involving the observed object.

Each of us is comprised of matter that can for the purposes of this analysis be considered just a collection of charges. Observers, all of them, real or potential, also can be considered as charge collections. In this spirit, every charged particle is a fundamental observer of all other charges; so cosmic rays of extremely high and variated velocities going in every which way, are all 'observing' each and every one of us mortals! But we experience no 'real' time confusion or a multitude of length contractions. Unless addicted to mystical-illogical infatuations, this must be taken as simple direct evidence that time dilations and length contractions are artifacts of observation; in other words, they are space-time perspective effects.

Beyond the pedagogical advantage of having an openly logical presentation of Relativity Theory, this conclusion undermines the contention that proper time is not suitable as a system variable conjugate to the total system Hamiltonian. In other words, one must conclude that there are no preceeding or preempting objections to the general applicability of Hamiltonian structure to a covariant dynamics of interacting particles. For a candidate theory, see: [19].

Note

The writer's English translations of references [5; 9; 10]; and preprints of [13; 14; 19] can downloaded from http://www.nonloco-physics.000freehosting.com.

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