Antirelativistic dynamics

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La nueva dinámica antirrelativista, in translation from:
Rivista de la Real Academia de Ciencias (Madrid) , **LIX,** 69-132 (1968):

It is generally accepted that the equation

$$m_W = W/c^2,$$

which relates the energy W with its mass m_W, is a consequence of EINSTEIN's theory. The author shows that this equation may be obtained from MAXWELL's equations without invoking the principle of relativity. The mass of a moving particle, whose mass when at rest is m_r, becomes

$$m = m_r + T/c^2.$$

From this equation, combined with the principle of conservation of energy, PLANCK's expression for the kinetic energy

$$T = \frac{m_r c^2}{\sqrt{1-u^2/c^2}} - m_r c^2,$$

is obtained.
In a conservative field of force, the relativistic equation of motion

$$\vec{f} = m_r \frac{d}{dt} \frac{\vec{u}}{\sqrt{1-u^2/c^2}},$$

cannot be applied. Instead, the principle of conservation of energy

$$d(T+E_p) = 0,$$

leads to the following equations of motion for electric, magnetic and gravitational fields respectively:

$$-Q(1-u^2/c^2)^{3/2}\nabla\Phi = m_r \frac{d\vec{u}}{dt};$$

$$Q(\vec{u}\times\vec{B}) = \frac{m_r}{\sqrt{1-u^2/c^2}} \frac{d\vec{u}}{dt};$$

$$-(1-u^2/c^2)^{3/2}\nabla V = \frac{d\vec{u}}{dt}.$$

It follows from the last of these equations, that gravitational mass m_g and inertial mass m$_r$ of a moving particle are related by

$$m_g = G^{1/2}(1-u^2/c^2)^{2/3} m_r.$$

Consequently, the equivalence postulate of EINSTEIN's theory of gravitation must be rejected.

Contents

I. THE CONCEPTS OF MATTER: ENERGY AND MASS

One of the great achievements attributable to MAXWELL's theory of electrodynamics consists in having made it possible to foresee that energy posses inertial mass; that is, it is necessary only to convey energy to a material body in order to augment its mass. Thus, one can not say that the mass of a body is an invariant quantity or characteristic of it; and therefore, it follows that the concepts behind NEWTONian laws of mechanics must be amended.

By means of contemplation of all that which surrounds us, practitioners of Physics have elaborated two concepts: matter and energy, corresponding to the entities seemingly responsible for sensual perceptions. For a projectile, for example, one distinguishes the material from which it is made, lead or steel, say, on the one hand, and the energy it posses by virtue of its motion, its location in a gravitational field and its temperature acquired by friction with air.

Classical Physics is based on a clear distinction between matter and energy. It is believed, that this distinction is rooted in the fact that matter is inert and ponderous, while energy is supposed to be exempted from inertia and weight. Because this is in fact not so, however, it becomes necessary to identify other distinguishing features. The purpose of Physics is not to determine the essence of the entities with which it operates. Rather, for its purposes, the following definitions suffice:

The energy of a closed system is its capacity to do work on other objects.

The matter of a closed system is that which remains when it is exhausted of all its energy.

Another common concept needing a precise definition, is 'mass', in particular as in the vernacular mass is conflated with matter, and in works on Relativity it is held that mass can be converted into energy, and *visa versa*, an assertion which, as we shall show, is inadmissible.

NEWTON introduced inertial mass into Physics, but he did not specify exhaustively the significance of this important quantity. ERNST MACH in his work from 1927, gave a definition that, with more or less arbitrary variations, can be found in many physics books. MACH is one of the founders of modern philosophical positivism of the Vienna school; he supposes that his definition is purely operational. But, as MARGENAU has noted, it actually constitutes a curious mix of epistemological and tautological elements. MACH's definition, reduced to its operational elements, would be: Take body A as the unit mass. Consider that by whatever means it effects an attraction on another body B, in such a way that starting from rest, both bodies undertake movement; such as can be realized by a spring or electric charges, say. If, now, a_A and a_B are their respective accelerations as measured simultaneously, then the mass of B equals a_A/a_B.

For MACH's definition to be acceptable, it is necessary that the cause of the accelerations between these two bodies remain constant throughout the motion (for example, the elastic force from a spring). This hypothesis, which indisputably underpins NEWTON's mechanics, is actually unacceptable, and this suffices to reject MACH's definition.

Many authors, including EINSTEIN, define mass as the quotient of force by acceleration, without note of the fact, that with this definition mass loses its rank as a primary quantity, because this formulation ignores the possibility of determining its value by measuring other quantitates. For this reason, *inter alia*, EINSTEIN came to the misguided conclusion, that there are two types of mass: longitudinal and transversal, and thereby obtained an unacceptable formulation.

The objection we have made to MACH's definition is applicable to all operational definitions when they refer to primary quantities. In each case they introduce some implicit or tacit tautological aspect, that is, the definition depends on an implicit unenunciated law. In the end, this matter is so convoluted, that LENZEN (1931), after having studied it throughly, ordained the method of "successive definitions". It begins with abstract concepts, then proceeds to the discovery of laws, with the help of which these concepts are redefined with greater precision, with which, finally, one again reinterprets the primary concepts.

In my view of these matters, and in accord with my book *Dimensional Analysis*, one should start by giving a qualitative definition of each quantity, and then define the means of measurement, and give criteria for equality and sums thereof. For inertial mass the qualitative definition is as follows:

Inertial mass is that quantity for which a force is required to change its motion.

The criterion of equality is derived from this definition; it is obvious that two bodies have equal inertial masses when they respond the same to equal forces. To establish the criterion of summation, it suffices to admit that masses sum by accumulation such that if a mass is n times larger than another, when it behaves as if it were the sum of n times the first.

If one considers that primary quantities are those for which it is possible to establish criteria of equality and summation, then mass is a primary quantity.

II. THE INERTIA OF ENERGY

As is frequently the case in the history of science, many various researchers contributed to the discovery that energy posses mass.

THOMPSON (1881) published a series of papers based on the study of MAXWELLian electrodynamics, which subsequently were discussed by FITZ-GERALD, HEAVISIDE, SEARLE, MORTON, etc.[1] As a result of all of this, THOMPSON reached the conclusion that a charged conducting sphere, moving along a straight line, would experience an increase in mass per:

$$\triangle m = \frac{4}{3} W/c^2, \tag{II.1}$$

where W is the electrostatic energy of the charge, which is, as is well known, equal to the energy invested in transferring the

[1] WHITTAKER (1951, p. 306) presents a complete survey of these papers.

charge Q to the sphere:

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{4\pi\varepsilon r}, \tag{II.2}$$

where C is capacitance and ε is the permittivity or dielectric constant of the vacuum.[2]

POINCARÉ (1904) opined that in vacuo the momentum of electromagnetic waves equals the flux of the POYNTING vector times the factor $1/c^2$. This suggests, that an electromagnetic field for each unit of volume would have a mass equal to the product of energy density times the same factor, with which, in place of Eq. (II.1), one gets:

$$\triangle m = W/c^2. \tag{II.3}$$

This expression predicts, that an HERTZian oscillator emitting radiation in a particular direction, must recoil for the same reason that firearms do so.

The problem that concerns us here, also interested HASENHÖRL, who used MAXWELL's theory to study the comportment of a box with reflecting walls containing radiation and moving with uniform velocity. He deduced, that it is necessary to attribute to radiation a mass given by:

$$m = kW/c^2, \tag{II.4}$$

where k is a factor that he first estimated to be $8/3$ (1904), but later, (1905), corrected to $4/3$, a value in accord with THOMPSON's result.

In view of all these works, it must be said for certain that MAXWELL's theory of electromagnetism leads to Eq. (II.4), with only the value of k remaining dubious. Experimental experience supports this formula, albeit, not directly. Such confirmation was provided by KAUFMANN (1901). By measurements of the deviation of β-rays (electrons) emitted by various radioactive substances, he observed that they exhibited an apparent increase in mass given by:

$$m = \frac{m_r}{\sqrt{1-u^2/c^2}}, \tag{II.5}$$

where m_r is their mass when $u = 0$, i.e., when at rest. From Eq. (II.5), one deduces that an electron experiences an increase in mass:

$$\triangle m = m - m_r = m_r\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right), \tag{II.6}$$

which, can be seen, is just equal to the kinetic energy T, divided by c^2. As a consequence:

$$\triangle m = T/c^2, \tag{II.7}$$

where $k = 1$. Subsequent measurements have confirmed indisputably the validity of Eq. (II.5). This, in turn permits the conclusion that it is necessary to attribute inertia to electromagnetic energy, a major contribution of pre-relativistic Physics.

This was the situation when EINSTEIN (1905, p. 589) published his celebrated article: *Ist die Trägheit eines Körpers von seiner Energieinhalt abhängig?* (Does the inertia of a body depend on its energy?), in which, he posited the incontrovertibility of the principle of relativity, Eq. (II.4) with $k = 1$, and the claim that this is true whatever the form of energy. Thereafter, all have believed, including myself, that the formula:

$$m_W = W/c^2, \tag{II.8}$$

where the mass m_W corresponds to energy W, is due to EINSTEIN, i.e., that it is a consequence of his theory, not MAXWELL's, which has justified labeling it *Einstein's formula.*

The difficulty, however, of reconciling Eq. (II.8) with the principle of relativity springs into view, as by virtue of this formula, the state of absolute rest can be distinguished from any other, in that, for it, a body has an absolute minimal mass. It seems then, that EINSTEIN has fallen into some kind of error, and, in effect, as IVES (1952) proffered, his reasoning is false, because it assumes that which it strives to demonstrate.

I shall explicate a very sensible method that allows the derivation of Eq. (II.8) as a consequence of MAXWELL's theory without the necessity to call on the principle of relativity. In the following, it will be demonstrated that Eq. (II.8) and the principle of relativity are in fact incompatible.

Consider a plane wave train. From MAXWELL's theory we know that the vectors $\vec{E}$ and $\vec{H}$ are orthogonal and satisfy:

$$\vec{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{E}. \tag{II.9}$$

Furthermore, both vectors are perpendicular to the propagation velocity c , such that taken in the order $c, \vec{E}, \vec{H}$ they form a righthanded triad, see Fig. (1).

Suppose that this wave train perpendicularly impacts a conducting wall. The electromagnetic field induces currents in the wall, which, because they are immersed in these very waves, are subject to forces putting the wall itself into motion. If the system comprising the wave train and wall contained no mass but that of the wall, then internal forces would have put the center of mass in motion, which is a contradiction with the principle of inertia.

The reasoning presented above shows that it is necessary to attribute a mass m_W to waves that must depend on their energy W, their frequency ν and the vacuum constants ε_0 and μ_0. With these quantities, one can form only one monomial of dimension zero in which the frequency does not appear:

$$\frac{m_W}{\varepsilon_0\mu_0 W}. \tag{II.10}$$

From this result, by virtue of the π-theorem of Dimensional Analysis, there exists a relationship among these quantities of

[2] FERMI (1922), noted that, in addition to the electrostatic energy, if account is taken of the stress in the sphere, then the mass differential is: $\triangle m = W/c^2$.

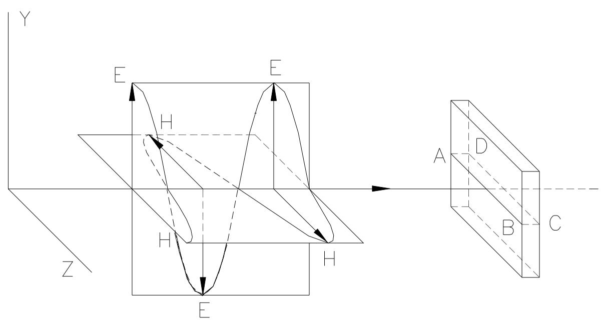

FIG. 1 A demonstration that waves posses mass

the form:

$$m_W = kW/c^2, \tag{II.11}$$

where k is a fixed number.

To find the value of k it is sufficient to consider a particular case, for example, the case in which the waves are totally absorbed by the wall, such that there is no reflection or tranmission. Arranging the axes as depicted in Fig. (1), gives $H_x = H_y = 0$, or $H_z = H$. Furthermore, if the wall is a conductor, the vectors $\vec{E}$ and $\vec{D}$ must be absolutely null, and MAXWELL's equation:

$$\nabla \times \vec{H} = \vec{i} + \frac{\partial H_x}{\partial x}, \tag{II.12}$$

becomes:

$$i_x = i_z = 0, \quad i_x = -\frac{\partial H_x}{\partial x}. \tag{II.13}$$

The result then, is an alternating current on the Y axis. To find its intensity, we apply STOKE's Theorem to a circuit that encloses the wall, such as $ABCD$. Since $H_x = 0$ and there is no reflection or transmission, one obtains $\vec{H}$ multiplied by the length, a, of side AB:

$$\vec{I} = a\vec{H}. \tag{II.14}$$

The force that the field exercises on the wall then equals:

$$\vec{f} = b\vec{I} \times \vec{B} = \mu_0 \vec{I} \times \vec{H}, \tag{II.15}$$

where b is the thickness of the wall. As $\vec{I}$ and $\vec{H}$ are perpendicular, this gives in view of Eq. (II.14), a force in the positive X direction:

$$f = \mu_0 abH^2 = \mu_0 AH^2, \tag{II.16}$$

where $A = ab$ is the area of a cross section of the wall. In view of Eq. (II.9), Eq. (II.15) can be written:

$$\vec{f} = \sqrt{\mu_0 \varepsilon_0} A\vec{H} \times \vec{E} = \frac{1}{c} A\vec{S}, \tag{II.17}$$

where $\vec{S}$ is POYNTING's vector. From this, we see that in the time interval dt the wall receives an impulse:

$$fdt = \frac{1}{c} ASdt = dW/c, \tag{II.18}$$

where dW is the energy deposited in the wall in the interval dt.

Suppose we consider the wall at rest before the arrival of the wave train, and let du be the velocity acquired by virtue of the impulse fdt. To calculate this velocity we call on NEWTON's Law written in the form

$$fdt = d(Mu), \tag{II.19}$$

where M is the mass of the wall, which is converted into $M + dm_W$, when it adsorbs energy W. Eliminating fdt between Eqs. (II.18) and (II.19) gives:

$$\frac{dW}{c} = Mdu + udM = Mdu, \tag{II.20}$$

as $u = 0$. Thus,

$$du = \frac{dW}{Mc}. \tag{II.21}$$

Applying now the principle of inertia, which requires that there be no forces exterior to the system comprising wave train and wall, implies conservation of motion of the center of mass. Because before the impact of the wave train, dm_W corresponded to the energy dW, which moves with the velocity c, and after the impact $M + dm_W$ has acquired the velocity du, it follows:

$$\frac{cdm_u}{M + dm_M} = du = \frac{dW}{Mc}, \tag{II.22}$$

or, since M is a finite mass:

$$dm_W = dW/c^2, \tag{II.23}$$

The above analysis demonstrates directly that electromagnetic waves have mass; but, the principle of inertia requires that Eq. (II.23) be applicable to all forms of energy. In effect, the system is a vessel that contains electromagnetic waves and moves with velocity u. Suppose that, without leaving the vessel, the energy of the waves is transformed into another species of energy. Following GALILEO's Principle, the velocity must remain constant. On the other hand, NEWTON's Law, written in the form:

$$fdt = d(mu), \tag{II.24}$$

requires $mu = \text{const.}$ because there are no exterior forces. As a consequence, the new species of energy must posses the same mass m_W, as possessed by the energy W of the waves.

The total mass of a body comprises the sum of material mass, or 'proper mass', m_m, and its 'energetic mass', m_W:

$$m = m_m + m_W = m_m + W/c^2. \tag{II.25}$$

In turn, the energetic mass is obtained by summing different forms of energy: internal U, kinetic T and potential E_v:

$$m = m_m + \frac{1}{c^2}(U + T + E_v). \tag{II.26}$$

In the absence of potential energy, and when the body is motionless, one has the 'rest mass':

$$m_r = m_m + U/c^2. \tag{II.27}$$

Tables give the rest masses of elementary particles and atomic nuclei. One must admit that stabile particles in a normal state, that is, when they are unexcited, lack internal energy. In contrast, one must attribute a certain internal energy to excited atoms and radioactive bodies, energy that can be liberated in the emission and transformation processes. Thus, atoms have a greater energy when excited than when in their normal state, and radioactive transmutation results in a lower mass than that of the primary particle.

To electrically charge a body, it is necessary to expend energy which is then stored in it. It is to be expected, therefore, that a charged body has more mass than an uncharged body. Confirmation of this expectation occurs for each of the mesons π^+ and π^-, for which the mass is 273 times greater than that of the electron while the neutral π^0 is only 264 times greater. That is, there is a difference of:

$$\triangle m = 9 m_e = 9 \times 9.1091 \times 10^{-25}\text{kg.}, \tag{II.28}$$

which corresponds to the stored energy of the charge.

The constant c has a double character. For space, it is the measured velocity of light in vacuum with respect to a system at absolute rest. From this point of view it may be considered a constant characteristic of the aether, for which the value depends on the circumstances that modify its index of refraction, depending on gravitation. However, in view of the formula:

$$\triangle W = c^2 \triangle m, \tag{II.29}$$

the referenced constant acquires the status of an ineluctable universal constant, embellished with abundant bombast, not only in optics, but throughout Physics. To measure c^2 it is not necessary to use meter sticks and clocks. Rather, a direct measurement of the augmentation $\triangle m$ of mass (or weight) a body exhibits whenever it communicates, in whatever form, an amount of energy $\triangle W$, for example as heat. Such a measurement which depends in no way on the structure of aether, figures into the factor $\sqrt{1 - v^2/c^2}$ and, as with all universal constants, depends on the system of units.

III. CONSERVATION PRINCIPLES

The Inertia of Energy Principle obliges us to reformulate the two conservation principles, that for mass and for energy. As new formulations, as presented in many works, suffer tortured interpretations, it behooves us to examine this matter with all thoroughness.

A. Conservation of Mass.

Consider a system of bodies which is subjected to a transformation as a consequence of the exit from the system of a certain amount of energy W. If m_a and m_d are the total masses before and after the transformation, and in so far as the energy W can be assigned the mass $m_W = W/c^2$, the Principle of Mass Conservation can be expressed as:

$$m_a = m_d + m_W, \tag{III.1}$$

such that one can not claim, as was done during LAVOISIER's time, that the mass of the system is held constant permanently. On the other hand, the total mass of the Universe does not vary, as losses suffered by any subsystem are gains in another.

Eq. (III.1) written in the form:

$$m_a - m_d = W/c^2, \tag{III.2}$$

is interpreted by many authors, for example by EINSTEIN, as if mass had been converted to energy, and thereby claim that mass and energy are equivalent quantities. This manner of speaking seems quite cavalier. Mass is neither matter nor energy, rather an attribute common to all species of matter and forms of energy. To claim that mass and energy are equivalent entities, is to confuse an object with its properties, with its volume, say. Finally, for an entity to transform into something else, it must cease to be that which it was and acquire a new sort of existence. What happens really is, that the energy W, found in a system in one or another form, is carried with it, to be expelled as mass W/c^2.

Heat and mechanical work can be said to be equivalent forms of energy as one can be converted into the other. Thus one attributes the same dimensional formulas to each and measures them in terms of the same units in all coherent systems. Mass and energy, contrariwise, are quantities in distinct classes, their dimensional formulas are different and each posses its own peculiar units in coherent systems, for example the kilogram and the Joule in the GIORGI system of units. It is, therefore, nonsense to say the energy transforms into mass and that they are equivalent entities.

In accord with the notions explicated in my book *Dimensional Analysis*, energy and mass are to be considered as "inseparable quantities", because between these two entities there is an ineluctable constant, which turns out to be $1/c^2$.

The new dynamics follows rigorously the Conservation of Mass Principle, expressed as

$$\sum m_m + \sum m_W = \text{const.}, \tag{III.3}$$

where $\sum m_m$ is the sum of all particle masses comprising the system, and

$$\sum m_W = \frac{\sum W}{c^2}, \tag{III.4}$$

is the mass corresponding to the total energy content in the system. Thus, for all transformations that take place in a closed system, that is one for which no material particles nor energy would cross an enclosing surface, comply with:

$$(\sum m_m + \sum W/c^2)_a = (\sum m_m + \sum W/c^2)_d. \tag{III.5}$$

So, for example, the weight of all mass losses in a nuclear reactor and the total mass—and its weight—of uranium, remains constant if the system is enveloped in a container that allows no radiation or particles to escape.

B. Conservation of matter and energy

According to Classical Physics it is a certainty that matter is indestructible, which was verified in the most violent of chemical reactions, even with large energy dissipation, by LAVOISIER who showed that the weight of bodies entering a reaction equaled the weight of those emerging. As matter can be measured by one of its attributes, and has the advantage of being independent of external circumstances, i.e., pressure and temperature, the principle of Conservation of matter can be expressed as:

$$(\sum m_m)_a = (\sum m_m)_d. \tag{III.6}$$

Likewise, Classical Physics admits the Principle of Conservation of Energy:

$$\sum(U + T + E_p) = \text{const.}, \tag{III.7}$$

where U , T and E_p represent internal, kinetic and potential energy respectively.

In the new dynamics, we abandon certitude in the separation of Eqs. (III.6) and (III.7) and, in its place, take Eq. (III.5), which is the sum of the two after dividing the second by c^2. Maybe this is the reason that relativists claim, thanks to EINSTEIN, that two conservation principles from Classical Physics, i.e., that for matter and energy, can be melded into a single principle:

$$\text{mass} = \text{energy} = \text{const.}. \tag{III.8}$$

This assertion is inadmissible, however, because one can not add mass with energy without violating the Principle of Homogeneity. Eq. (III.5), which is correct, makes manifest, that that which is to be added, is the mass equivalent of matter to the mass equivalent of energy. The factor c^{-2}, which is ineluctable, has as its mission to save the Principle of Homogeneity.

As for attempted fusions of the two principles, one can claim that while atomic nuclei remain unaffected, matter is conserved and Eqs. (III.6) and (III.7) are executed separately, as a consequence of Eq. (III.5), but not the reverse. Otherwise when there is change in an atomic nucleus, as happens by radioactive decay, each principle contributes independently to the transformation.

In radioactive transformations, large quantitates of energy are liberated as kinetic energy of decay products or as pure energy in the form of photons. One may plausibly suppose that the instability of a radioactive nucleus, or in general of fundamental particles, is due to stored internal energy U. For this reason it is not possible to confirm Eq. (III.6) directly, since that which is measured experimentally with a spectrometer, is *rest mass:*

$$m_r = m_m + U/c^2; \tag{III.9}$$

and, as is verified experimentally, liberated energy W is given by:

$$(m_r)_a - (\sum m_r)_d = W/c^2, \tag{III.10}$$

or, by virtue of Eq. (III.9):

$$(m_m + U/c^2)_a - (\sum m_m + U/c^2) = W/c^2. \tag{III.11}$$

The preceeding equations do not allow one to decide whether matter is conserved or not, but seem logically to imply that the total liberated energy is provided by internal energy:

$$W = U_a - U_d \tag{III.12}$$

with which, and with Eq. (III.11), one deduces:

$$(m_m)_a = (\sum m_m)_d. \tag{III.13}$$

It is in this sense, that the separation of the two conservation principles subsists, and thanks to this separation one can talk of energy levels in atoms and characterize them by their particular internal energy. As one sees, spontaneous radioactivity, uniquely recognized when EINSTEIN elucidated his theory, doesn't permit one to claim that there is a fusion of the principles of conservation of mass and energy. We shall see, that without exception, discoveries made subsequently oblige us to claim that matter (not mass) can be transformed into energy and *visa versa*, and that these transformations are regulated by a principle expressible as:

$$\text{matter} + \text{energy} = \text{const.} \tag{III.14}$$

As in these phenomena matter has to vanish, it is not possible to satisfy Eq. (III.13), and even less to satisfy Eq. (III.11) because the energy is released not from internal energy, but from the annihilation of matter. By contrast, Eq. (III.5) is satisfied, which can be expressed saying that the principles of conservation of matter and energy are fundamentally unified.

IV. RECIPROCAL TRANSFORMATION OF MATTER AND ENERGY

Phenomena mentioned in the preceeding sections consist of the production of twined particles (electron-positron, proton-antiproton, neutron-antineutron) by means of the consumption of the energy of a photon. And, the inverse phenomenon, that is, the mutual destruction of twined particles, results in the production of energetic radiation.

In this phenomenon, total mass is conserved, as what is lost in proper mass, is gained in radiation mass. Conservation of total mass subsists then, as expressed by $m_d = m_a$. Since:

$$m_a = m_{m1} + m_{m2} + \frac{1}{c^2}(T_1 + T_2); \quad m_d = W/c^2, \tag{IV.1}$$

giving:

$$W/c^2 = m_{m1} + m_{m2} + \frac{1}{c^2}(T_1 + T_2). \tag{IV.2}$$

Positing that matter has been transformed into energy, is to abandon the validity of conservation principles. One cannot, without exception, say that a single principle is united into conservation of mass and energy, when what has happened is the first is satisfied and not the second. On the other hand, Eq. (IV.2) suffices to resolve problems of examining energy W that is liberated by mutual annihilation of two particles, and still constrained by the conservation of momentum, as we shall see below.

The reciprocal transformation of matter into energy is a process comparable to the conversion of heat into work. In both cases it has to do with quantitates that transform one into another, such that there is a proportionality between the two quantitates. In thermodynamics one encounters a *principle of equivalence* by virtue of which heat, Q, is absorbed in a cyclic transformation and work, W, is done on bodies exterior to the system according to the equation:

$$W = JQ, \tag{IV.3}$$

wherein J is a universal constant.

Analogously, the transformation of matter into energy obeys a principle of proportionality between quantitates that, with one or another magnitude, are involved in the considered phenomenon:

$$\text{a quantity of matter} \sim \text{a quantity of energy}. \tag{IV.4}$$

Quantities of matter can be measured by various means, for example, for weight or for volume occupied under specified conditions. For our purposes, the most convenient measurement is that of proper mass, for which the the proportionality (IV.4) becomes:

$$m_m = kW, \tag{IV.5}$$

where the constant equals $k = 1/c^2$, if one uses a coherent system of units.

To clarify these notions, consider the nuclear reaction

$$\mathrm{Li}^6(\mathrm{n}, \gamma) \rightarrow \mathrm{Li}^7.$$

Since all the particles involved in this process are stable, we dispense with changes in internal energy, as well as others that are not involved, like gravitational energy, so that:

$$m = m_m + T/c^2. \tag{IV.6}$$

Proper masses have been determined using a mass spectrograph, and taking the unit of mass to be $u_m = 1,661 \times 10^{-21}$gr., yields the values:

$$\begin{aligned} m_m(\mathrm{Li}^6) &= 6.01697; \\ m_m(\mathrm{n}) &= 1.00893; \\ m_m(\mathrm{Li}^7) &= 7.01822. \end{aligned} \tag{IV.7}$$

Supposing that the kinetic energy of the impinging neutron is negligible with respect to the lithium nucleus, Eq. (III.1). which expresses the conservation of total mass, becomes:

$$m_m(\mathrm{Li}^6) + m_m(\mathrm{n}) = m_m(\mathrm{Li}^7) + \frac{1}{c^2}(T + W), \tag{IV.8}$$

where T is the kinetic energy of (Li^7), and W the photon energy.

Conservation of energy, expressed by Eq. (III.7) leads to:

$$T = -W, \tag{IV.9}$$

and in that this equation appears compatible with Eq. (IV.8) one has compatability with Eq. (III.6), that is, there must be conservation of proper mass. But, per the previous numerical results, there was a loss of proper mass given by:

$$\triangle m_m = -0.00768\, m_n = -1.276 \times 10^{-26}\mathrm{g}, \tag{IV.10}$$

so that, therefore, Eqs. (IV.8) and (IV.9) are incompatible.

Lose of proper mass indicates that a portion of matter was transformed into energy, so that Eqs. (III.7) and (IV.9) are not applicable. In its place one must use the principle of equivalence of matter and energy, that is:

$$-\triangle m_m = (T + W)/c^2, \tag{IV.11}$$

which gives:

$$\begin{aligned} T + W &= 1.276 \times 10^{-26} \cdot 9 \times 10^{20} = 1.148 \times 10^{-5}\mathrm{erg}. \\ &= 6.20\,\mathrm{MeV}. \end{aligned} \tag{IV.12}$$

This energy is distrubuted among the lithium atoms and photons. To determine the part that corresponds to each, one must apply the principle of conservation of momentum, the result shows that the portion in the lithium atoms is negligible.

V. THE NEW DYNAMICS' FUNDAMENTAL LAW

POINCARÉ (1904) has noticed that it is necessary to change NEWTON's Law:

$$\vec{f}\, dt = m\, d\vec{u}, \tag{V.1}$$

so that it can take account of the finite velocity of light. To convince oneself of the necessity to make such a modification, one need only consider the case when $u > c$, as then the factor $\sqrt{1 - u^2/c^2}$ becomes imaginary, which signifies that a moving body at this velocity loses real existence.

In the theory of relativity one substitutes for Eq. (V.1) the following:

$$\vec{f} = m_m \frac{d}{dt} \frac{\vec{u}}{\sqrt{1 - u^2/c^2}}, \tag{V.2}$$

and takes it that this is applicable whatever the nature of the force. We propose to examine the reasoning behind this equation to show that it pertains to contact forces such as elastic collisions and shock due to wave impact, but not to forces acting at a distance, e.g., from electric and magnetic fields.

LORENTZ (1916) studied theoretically the motion of an electron in an electromagnetic field and deduced that with increase of velocity there is an increase of mass up to infinity when $u = c$, and giving the remarkable circumstance, that it is also necessary to distinguish between *longitudinal* mass:

$$m' = \frac{m_r}{(1-u^2c^2)^{3/2}}, \tag{V.3}$$

valid when the force is parallel to the velocity, and *tranverse* mass:

$$m'' = \frac{m_r}{(1-u^2/c^2)^{1/2}}, \tag{V.4}$$

valid when the force is perpendicular to the instantaneous velocity of the body.

EINSTEIN (1915), claiming he did not know of LORENTZ's works, studied the same motion of a charge in an electromagnetic field, and applied the principle of relativity. He concluded that NEWTON's equation, (V.1), should be replaced by:

$$f = \frac{m_r}{(1-u^2/c^2)^{3/2}}\frac{du}{dt}, \quad f, \text{ and } u \text{ parallel}, \tag{V.5}$$

$$f = \frac{m_r}{(1-u^2/c^2)}\frac{du}{dt}, \quad (f, \text{ and } u \text{ perpendicular}). \tag{V.6}$$

Note that EINSTEIN obtained the same expression for longitudinal mass as did LORENTZ, but not for tranverse mass.

EINSTEIN's equations have the defect of being applicable only when time intervals are infinitesimally small, and then only in the particular case in which the force is constantly normal to, or parallel to the trajectory; in fact however, generally the angle between force and velocity is constantly changing.

PLANCK (1906) correctly gave the fundamental equation the form (V.2) for which it is not necessary to distinguish between longitudinal and transverse mass. This equation is accepted unanimously, and he is justifiably considered a great German savant as founder of relativistic mechanics. However, PLANCK deduced his equation based on the Principle of Relativity, so that all merit is attributed to EINSTEIN. We shall show that it is possible to derive Eq. (V.2) just by taking into account the inertia of energy, such that it is unnecessary to consider the Principle of Relativity. Further on we shall show even that Eq. (V.2) is actually incompatible with this principle.

VI. DERIVATION OF PLANCK'S LAW

We accept the validity of NEWTON's Law written in the form:

$$\vec{f}\,dt = d(m\vec{u}), \tag{VI.1}$$

and that the total mass of a body, instead of being constant as in classical dynamics, depends on the energy according to the formula:

$$m = m_m + (U + T + E_p)/c^2 \tag{VI.2}$$

which we obtained in §2 as a consequence of MAXWELL's theory.

To deduce the new law, we begin by supposing that there is no action-at-a-distance (fields) and that movement results from contact forces, likewise for those due to pressure exercised over the body in motion by external bodies. We also take it that internal energy is unaltered by motion. Under these conditions $E_p = 0$, and Eq. (VI.1) converts to:

$$m = m_r + T/c^2, \tag{VI.3}$$

where

$$m_r = m_m + U/c^2. \tag{VI.4}$$

The principle of energy conservation requires that all work done by these forces is transformed into kinetic energy:

$$\vec{f}\cdot d\vec{l} = dT. \tag{VI.5}$$

On the other hand, Eq. (VI.1):

$$\vec{f} = \frac{d(m\vec{u})}{dt}, \tag{VI.6}$$

substituted into Eq. (VI.5), gives:

$$\vec{u}\cdot d(m\vec{u}) = dT, \tag{VI.7}$$

or

$$\frac{1}{2}mdu^2 + u^2dm = dT. \tag{VI.8}$$

Replacing m by its value, Eq. (VI.3), yields:

$$\frac{1}{2}(m_r + T/c^2)du^2 + \frac{u^2}{c^2}dT = dT, \tag{VI.9}$$

so that separating variables leads to:

$$\frac{dT}{m_r + T/c^2} = \frac{du^2}{1-u^2/c^2}. \tag{VI.10}$$

Integrating with initial conditions $T = 0$ for $u = 0$, results in:

$$c^2 \ln\frac{m_r}{m_r + T/c^2} = -\frac{c^2}{2}\ln(1-u^2/c^2), \tag{VI.11}$$

or rearranged:

$$m = m_r + T/c^2 = m_r/\sqrt{1-u^2/c^2}, \tag{VI.12}$$

from which one need only substitute into Eq. (VI.1) to get PLANCK's equation:

$$\vec{f}dt = m_r\frac{d}{dt}\frac{\vec{u}}{\sqrt{1-u^2/c^2}}. \tag{VI.13}$$

Solving for T in Eq. (VI.12) one obtains for kinetic energy the value:

$$T = m_rc^2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right). \tag{VI.14}$$

Expanding this in a series of powers of u/c, gives one:

$$T = m_r c^2 \left(\frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4} + \cdots \right), \tag{VI.15}$$

so that for velocities much less than that of light, the result is the same as in classical dynamics.

The symbols m and m_r represent measurements obtained with the same units, the kilogram in S. In these conditions, the Metric Principle :

$$\text{quantity} = \text{measurement} \times \text{unit},$$

says that the measurements are proportional to their respective quantities, from which by Eq. (VI.12) one deduces:

$$< m > = < m_r > / \sqrt{1-u^2/c^2}; \quad \text{between quantitites,} \tag{VI.16}$$

and, as a consequence, *to transfer to a body the energy necessary for it to move with velocity u, its mass must be divided by the factor* $\sqrt{1-u^2/c^2}$, which is less that unity. In particular, to transfer a body from the system at rest S, to the moving reference system S', so that it is fixed in it, i.e., $u = v$, and Eq. (VI.16) becomes:

$$< m_r' > = < m_r' > /\alpha. \tag{VI.17}$$

Momentum (once know as the *quantity of motion*) is defined by the identity:

$$\vec{p} \equiv m\vec{u}, \tag{VI.18}$$

and, therefore, is a *secondary quantity* introduced with no more purpose than to abbreviate terminology. With its aid, the fundamental law can be stated as: *momentum equals the change produced by impulse:*

$$\vec{f}\,dt = d\,\vec{p}. \tag{VI.19}$$

In a system comprised of multiple bodies, with no forces but those between these bodies, which are always parallel, equal and opposed, the overall sum is null so that *in a system free of external forces, the total momentum is constant.*

In NEWTONian mechanics one must take it that mass is constant. In the new dynamics one must take it that mass varies with the body's stored energy. Thus, momentum is not simply proportional to velocity, as the proportionality factor is variable. In absence of action-at-a-distance, for example, Eq. (VI.14) is valid, and by comparison with Eq. (VI.16) gives momentum the form:

$$\vec{p} = \frac{m_r \vec{u}}{\sqrt{1-u^2/c^2}}. \tag{VI.20}$$

The quantity defined by:

$$m_{\text{mov.}} = \frac{m_r}{\sqrt{1-u^2/c^2}}, \tag{VI.21}$$

is denoted *mass in motion.*

The velocity a body acquires when augmented with energy W converted completely into kinetic energy, is obtained when $T = W$ in Eq. (VI.13) which when solved for u yields:

$$\frac{u^2}{c^2} = 1 - \frac{1}{1 - W/(m_r c^2)^2}, \tag{VI.22}$$

from which one deduces that $u = c$ for $W = \infty$. This explains why it is impossible to obtain *velocities* greater than light, as the total energy in the universe is insufficient to *achieve this velocity for even the smallest particle.*

As a function of kinetic energy, momentum equals:

$$p = m_r c \sqrt{(1 + T/(m_r c^2))^2 - 1}. \tag{VI.23}$$

VII. MOVEMENT OF A PARTICLE SUBJECTED TO A CONSTANT CONTACT FORCE

As an example, let us find the equation of motion for a body subjected to a constant contact force. Suppose that this body starts out at rest and moves along the X axis; integrating PLANCK's equation leads to:

$$gt = \frac{u}{\sqrt{1-u^2/c^2}}, \tag{VII.1}$$

where $g = f/m$, is the force per unit mass. The velocity increase per unit time equals

$$u = \frac{gt}{\sqrt{1+g^2t^2/c^2}}, \tag{VII.2}$$

which does not increase proportionally to time, but ever more slowly, tending to the limit c as time proceeds.

The distance traveled equals:

$$x = \int_0^t u\,dt = g\int_0^t \frac{dt}{\sqrt{1+g^2t^2/c^2}}, \tag{VII.3}$$

or:

$$x = \frac{c^2}{g}(\sqrt{1+g^2t^2/c^2} - 1). \tag{VII.4}$$

When gt is negligible in comparison to c, this expression reduces to:

$$u = gt, \quad x = \frac{1}{2}gt^2, \tag{VII.5}$$

which are the classical results for uniform acceleration.

VIII. MASS AS A TENSOR

With the form given by PLANCK to the fundamental equation, the conceptions of longitudinal and transverse mass, introduced primarily by LORENTZ, but later taken up again by EINSTEIN, have been rendered superfluous. In their place, rest mass m_r, for which the value is constant so long as internal energy is constant, plays a role. In many books, even in elementary treatises, one talks still as if longitudinal and transverse masses were cherished acquisitions from relativity theory. It behooves us, therefore, to focus on this matter.

The force $\vec{f}$ applied to a body and the acceleration $\vec{a}$ which it evokes are coexistent vectors, that, in general, have different directions. One may, therefore, write:

$$\vec{f} \equiv \mathbf{M}\vec{a}, \tag{VIII.1}$$

where $\mathbf{M}$ is a tensor. This expression is not a new law, rather the definition of a secondary quantity $\mathbf{M}$ as a function of the primary quantities $\vec{f}$ and $\vec{a}$. To determine the components of $\mathbf{M}$ we have to return to the law that relates $\vec{f}$ with $\vec{a}$. In rectangular coordinates the vector equation, (V.2), decomposes into three equations:

$$f_i = m_i \frac{d}{dy} \frac{u_i}{\sqrt{1-u^2/c^2}}, \quad i = x, y, z. \tag{VIII.2}$$

Carrying out the indicated derivations gives:

$$f_i = m_r \left(\frac{a_i}{\sqrt{1-u^2/c^2}} + \frac{uu_i/c^2}{(1-u^2/c^2)^{3/2}} \frac{du}{dt} \right). \tag{VIII.3}$$

Taking the instantaneous velocity parallel to the X axis, so that $u_x = u,\ u_y = u_z = 0$, results in:

$$\begin{aligned} f_x &= \left[\frac{1}{\sqrt{1-u^2/c^2}} + \frac{u^2/c^2}{\sqrt{(1-u^2/c^2)^3}} \right] a_x = \frac{a_x}{(1-u^2/c^2)^{3/2}}; \\ f_y &= \frac{a_y}{(1-u^2/c^2)^{1/2}}; \\ f_z &= \frac{a_z}{(1-u^2/c^2)^{1/2}}. \end{aligned} \tag{VIII.4}$$

For those not distinguishing laws and definitions, mass is a quantity defined by the identity:

$$\text{mass} \equiv \frac{\text{force}}{\text{acceleration}},$$

and from that, for them, it ceases to be mass as an intrinsic property of a body, and they distinguish between longitudinal and transverse mass, which depend on external circumstances, namely the angle between the force and velocity.

The components M_{ij} are defined by

$$f_i = \sum_j M_{ij} a_j, \tag{VIII.5}$$

i.e.,

$$\mathbf{M} = \begin{bmatrix} \frac{m_r}{(1-u^2/c^2)^{3/2}} & 0 & 0 \\ 0 & \frac{m_r}{(1-u^2/c^2)^{1/2}} & 0 \\ 0 & 0 & \frac{m_r}{(1-u^2/c^2)^{1/2}} \end{bmatrix}. \tag{VIII.6}$$

Obviously, this is a symmetric tensor, for which one principle axis is in the direction of the instantaneous velocity.

Secondary quantities are introduced into a theory for reasons of convenience. Longitudinal and transverse mass simplify nothing, in fact they constitute gratuitous complication.

IX. MOTION IN A FORCE FIELD

Relativists attribute validity to PLANCK's equation

$$\vec{f}\,dt = d\frac{m_r \vec{u}}{\sqrt{1-u^2/c^2}} \tag{IX.1}$$

and deduce from it that the mass of a body varies with the velocity according to:

$$m = \frac{m_r}{\sqrt{1-u^2/c^2}}. \tag{IX.2}$$

In the theory we have developed, Eq. (IX.1) is only applicable when the work realized by effect of the force $\vec{f}$ is invested in kinetic energy, without varying internal or potential energy. It is of interest, then, to investigate what would be the equation of motion in the new dynamics when the body finds itself in a force field.

To resolve this question, we base our considerations on the principle of conservation of energy, stated thus: *a body that neither adsorbs nor emits energy moving freely in a force field does so, such that the sum of kinetic and potential energy is a constant.* That is:

$$d(T + E_p) = 0. \tag{IX.3}$$

From this expression we deduce immediately, that if internal energy is constant, then the mass of a solid body is given by:

$$m = m_m + (U + T + E_p)/c^2 = m_r + (T + E_p)/c^2 = \text{const.}, \tag{IX.4}$$

which means, that in contrast to the relativistic formula, Eq. (IX.2), the mass of a body moving freely in a force field without adsorbing or emitting internal energy, is *independent of the velocity.*

To Apply Eq. (IX.3), it is necessary to investigate the value of dT as a function of velocity and of dE_p as a function of coordinates. In EINSTEIN's dynamics, as well as the new version, kinetic energy of a body with mass m, and velocity u, equals:

$$T = \frac{m_r c^2}{\sqrt{1-u^2/c^2}} - m_r c^2, \tag{IX.5}$$

resulting in:

$$dT = m_r \frac{\vec{u} \cdot d\vec{u}}{(1-u^2/c^2)^{3/2}}. \tag{IX.6}$$

To find an expression for dE_p requires knowledge of the nature of force fields, the most important being electrical, gravitational and magnetic.

A. Electric field.

In an electric field, if Φ is the potential, the potential energy of a charge equals:

$$E_p = Q\Phi, \tag{IX.7}$$

and as a consequence:

$$d\,E_p = Q\,d\Phi. \tag{IX.8}$$

Substituting (IX.6) and (IX.8) into Eq. (IX.5), and taking it that Φ depends solely on spacial coordinates, yields:

$$Q(\nabla\Phi\cdot d\vec{x})(1-u^2/c^2)^{3/2} = -m_r\vec{u}\cdot d\vec{u}. \tag{IX.9}$$

As $dx = u_x dt;\ dy = u_y dt$; and $dz = u_z dt$, the preceeding equation takes the vector form:

$$-Q(1-u^2/c^2)^{3/2}\nabla\Phi\cdot\vec{u} = m_r\frac{d\vec{u}}{dt}\cdot\vec{u}. \tag{IX.10}$$

From the equation of classical dynamics:

$$\vec{f} = m_r\frac{d\vec{u}}{dt}, \tag{IX.11}$$

on the other hand, one obtains:

$$-Q\nabla\Phi\cdot\vec{u} = m_r\frac{d\vec{u}}{dt}\cdot\vec{u}. \tag{IX.12}$$

Comparing this equation with (IX.10), one sees that in the new dynamics, force exercised by a field is not given by the product of the gradient of the potential and charge, rather by:

$$\vec{f} = Q(1-u^2/c^2)^{3/2}\nabla\Phi. \tag{IX.13}$$

As a consequence, the fundamental equation in the new dynamics, for a body moving freely in an electric field, is:

$$-Q(1-u^2/c^2)^{3/2}\nabla\Phi = m_r\frac{d\vec{u}}{dt}. \tag{IX.14}$$

If one lets:

$$\vec{f} \equiv \vec{f}_0(1-u^2/c^2)^{3/2}, \tag{IX.15}$$

Eq. (IX.10) takes the same form as in classical dynamics, namely:

$$\vec{f} = m_r\frac{d\vec{u}}{dt}. \tag{IX.16}$$

B. Gravitational field

If V is the gravitational potential, potential energy equals:

$$E_p = m_r V. \tag{IX.17}$$

Substituting, then, m for Q and V for Φ into Eq. (IX.14) gives:

$$-(1-u^2/c^2)^{3/2}\nabla V = \frac{d\vec{u}}{dt}. \tag{IX.18}$$

and one sees that in the new dynamics, force exercised by a gravity field equals[3]:

$$\vec{f} = -m_r(1-u^2/c^2)^{3/2}\nabla V. \tag{IX.19}$$

Eqs. (IX.13) and (IX.19) reveal a very interesting peculiarity of the new theory of dynamics. *Whatever force field affects a moving object diminishes as velocity increases until it vanishes at the speed of light.* As a consequence, no body can obtain the velocity $u = c$. Both velocities are related implicitly through the equation:

$$\vec{f} = \vec{f}_0(1-u^2/c^2)^{3/2}. \tag{IX.20}$$

The following considerations serve to make the difference between force at rest and force in motion manifest.

The weight of a body is the force that must be applied to keep it at rest in a gravity field. It has, therefore, the same value but of opposite sign to static force, Eq. (IX.19):

$$\vec{w} = -\vec{f}_0 = m_r\nabla V. \tag{IX.21}$$

On the other hand, the force which gravity exercises on the same body when passing the same location with velocity u, is lower, as it has the value:

$$\vec{f} = -\vec{w}(1-u^2/c^2)^{3/2}. \tag{IX.22}$$

In order that an electron shall acquire the potential of Φ volts, it must be vested with a quantity of energy equal to $e\Phi$, with which its mass must be converted into $m = m_r + e\Phi/c^2$. The velocity acquired by an electron on passage from the potential Φ to zero, is determined by the expression $T = e\Phi$, such that:

$$\frac{m_r c^2}{\sqrt{1-u^2/c^2}} - m_e c^2 = e\Phi, \tag{IX.23}$$

from which one deduces that this velocity satisfies:

$$\frac{u^2}{c^2} = \frac{2+e\Phi/c^2}{(1+e\Phi/m_e c^2)}\frac{e\Phi}{m_e c^2}. \tag{IX.24}$$

As a check of this result, consider, that when the energy $e\Phi$ is much less that $m_e c^2$, the last expression becomes:

$$u^2 = 2e\Phi/m_e, \tag{IX.25}$$

which coincides with the result from classical dynamics:

$$\frac{1}{2}m_r u^2 = e\Phi. \tag{IX.26}$$

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[3] Eqs. (IX.14) and (IX.18) were obtained starting from conservation of energy and the value of kinetic energy given by Eq. (IX.5). In EINSTEIN's theory the equations of motion would be:

$$-Q\nabla\Phi = \frac{d}{dt}\frac{m_r\vec{u}}{\sqrt{1-u^2/c^2}};\quad -\nabla V = \frac{d}{dt}\frac{m_r\vec{u}}{\sqrt{1-u^2/c^2}},$$

and, by being incompatible with those found in the new theory, implies that EINSTEIN's theory must be in contradiction with the Principle of Conservation of Energy.

Take note that in Eq. (IX.23) the rest mass m_e of an electron, rather than total mass $m = m_e + e\Phi/c^2$, plays a role. The latter mass represents the inertia of an electron, that is, the resistance it offers to any force tending to divert it from its trajectory.

C. Magnetic fields

In a magnetic field for which the magnetic induction equals $\vec{B}$, a charge Q is subject to a force:

$$\vec{f} = Q(\vec{u} \times \vec{B}), \tag{IX.27}$$

which, being normal to the trajectory, does no work and does not alter the tangential velocity. As a consequence, its potential energy is null and it leaves kinetic energy unchanged. Lacking potential energy, therefore, the equations obtained above do not apply, in their place the following is valid:

$$Q(\vec{u} \times \vec{B}) = m\frac{d\vec{u}}{dt}, \tag{IX.28}$$

where

$$m = m_r + T/c^2 = m_r/\sqrt{1 - u^2/c^2}, \tag{IX.29}$$

is the total mass, which remains constant.

If the field is homogeneous, acceleration, which is normal to the trajectory, would be constant, so that the motion is circular with radius satisfying:

$$f = mu^2/r, \tag{IX.30}$$

or, if $\vec{B}$ is perpendicular to $\vec{u}$:

$$r = mu/QB. \tag{IX.31}$$

This formula has been confirmed in modern accelerators, a fact which is considered by Relativists as confirmation of EINSTEIN's theory. But as Eq. (IX.31) coincides with that from his theory, it can not distinguish between the two.

X. THE MASS OF POTENTIAL ENERGY

If the reader consults works on Relativity in order to learn whether to attribute inertia to potential energy in EINSTEIN's theory, he will wind up perplexed, as they just pass over this question in silence. Moreover, since in applications this mass is not taken into account, they tacitly consider that it does not exist. In fact, this being so, they thereby actually reject the principles of conservation of mass and energy, as we shall argue below.

Suppose that a body is at rest where its potential energy is E_p. Under these circumstances, if potential energy reduces inertia, a body's mass is reduced to its rest mass m_r. Taking it that motion is unrestricted, potential energy E_p is transformed into a quantity equivalent to kinetic energy, for which the total mass of the body experiences the augmentation E_p/c^2, without having reduced mass or energy of other bodies. This would be, then, an increase of the mass of the universe, and a violation of the Principle of Mass Conservation.

Recently BRILLOUIN (1964) recognized the need to attribute inertia to potential energy, and as a consequence, to modify the foundation of relativistic mechanics. He remarked that the difficulty here, is to know where to locate such mass, since to have potential energy it is necessary to have at least two bodies, and that fact presents the issue of distribution of potential mass between them.

BRILLOUIN considered two charged bodies and concluded that potential energy must be attributed to their electrostatic interaction and distributed equally between them. This claim, however, is not acceptable, as according to an easy demonstration, whatever the interaction engendering equal and opposite forces, potential energy must be distributed in inverse proportion to their respective proper masses.

Suppose both bodies are in contact and at rest. To separate them one has to apply to each forces that, by hypothesis, are always equal and opposite. In so far as all work realized by both forces is converted into potential energy, it suffices after separation, that both bodies return to being at rest. Because it is not important how long the separation took to accomplish, we may suppose that it transpired with infinitesimal velocity, in the manner of reversible thermodynamic transformations. Under the condition $u \to 0$, and with an eye to §9, clearly a classical equation of motion pertains. In particular, the center of mass remains at rest, so that taking the origin of coordinates at that point, we can write:

$$m_m dx + m'_m dx' = 0. \tag{X.1}$$

The potential energy acquired by each body would be equal to the work realized through the force $\vec{f}_0$ exercised on both bodies if they were at rest. Thus, by effect of Eq. (X.1), one has:

$$dE_p = f_0 dx; \quad dE'_p = -f_0 dx' = f_0 \frac{m_m}{m'_m} dx. \tag{X.2}$$

As a consequence:

$$dE_p = \frac{m'_m}{m_m} dE'_p, \tag{X.3}$$

and, in so far as initially $E_p = E'_p = 0$,

$$\frac{E_p}{E'_p} = \frac{m'_m}{m_m}, \tag{X.4}$$

such that *potential energy due to the mutual interaction of two bodies is apportioned in inverse proportion to their proper masses.*

Given the miniscule masses of elementary particle, in comparison to bodies that engender electric and gravitational forces, it is practically so, that all potential energy is localized in the latter.

XI. INERTIAL AND GRAVITATIONAL MASS

In accord with ideas developed in the theory of dimensional analysis of physical quantities, (PALACIOS, 1956), it is necessary to distinguish between inertial mass and gravitational mass. The first is what is defined in §1, while the second is that responsible for the force acting when a body is in a gravitational field.

From experiments by NEWTON and EÖTVÖS, results show, that when a body is held at rest, there is a proportionality between inertial and gravitational mass:

$$m_g = G^{1/2} m_0. \tag{XI.1}$$

As a consequence, NEWTON's Law of Gravitation can be expressed as a function of gravitational masses:

$$f = \frac{M_g m_g}{r^2}, \tag{XI.2}$$

or as function of inertial masses:

$$f_0 = G\frac{M_0 m_0}{r^2}. \tag{XI.3}$$

The issue now is to investigate what happens with gravitational mass when the body is in motion. Following results from §9, the force exercised over the body by gravitation equals:

$$f = f_0 (1 - u^2/c^2)^{3/2}, \tag{XI.4}$$

where

$$f_0 = -m_0 \nabla V, \tag{XI.5}$$

is the force at rest, or that which is applied to a body to keep it at rest, i.e., the force measured by a dynamometer.

In view of Eq. (XI.3), Eq. (XI.4) can be written:

$$f = G\frac{M_0 m_0}{r^2}(1 - u^2/c^2)^{3/2}. \tag{XI.6}$$

If the field is constant, that is, if the body M is held at rest, then $M_g = G^{1/2} M_g$, and Eq. (XI.2) converts to:

$$f = G^{1/2}\frac{M_g m_g}{r^2}(1 - u^2/c^2)^{3/2}. \tag{XI.7}$$

Comparing this with Eq. (XI.2), one sees that:

$$m_g = G^{1/2}(1 - u^2/c^2)^{3/2} m_0; \tag{XI.8}$$

and, therefore, it has been shown that: *the relationship between inertial and gravitational mass is not constant but depends on the velocity.*

Measurement results of gravitational mass at increasing velocity diminishes such that at the speed of light they vanish altogether. In particular this means, that light itself has inertial mass but no gravitational mass.

The theory of relativity, however, is based explicitly on the *identity* of inertial and gravitational mass; thus, the implications of this are that this theory must be in principle false.

XII. CONSERVATION OF MOMENTUM PARTICLE COLLISIONS

In NEWTON's dynamics one takes it that the mass of a body is a constant characteristic of a body, which satisfies the equation:

$$\vec{f}\, dt = m\, d\vec{u}. \tag{XII.1}$$

Moreover, the principle of action-and-reaction holds, according to which if one body exercises a force f over another, the second exercises the force $-f$ on the first. This means that mutual forces are parallel, equal and opposite (although they can not be on the same line), such that their sum is null. As a consequence, a system of bodies subject to mutual interaction complies with the equation:

$$\sum_i m_i\, d\vec{u}_i = 0; \quad \text{or,} \sum_i m_i \vec{u}_i = \text{const.} \tag{XII.2}$$

If, to abbreviate terminology and notation, one introduces a vector called *momentum*, defined by:

$$\vec{p}_i \equiv m_i\, \vec{u}_i, \tag{XII.3}$$

then Eq. (XII.2) can be written as:

$$\sum_i \vec{p}_i = \text{const.}, \tag{XII.4}$$

which is the expression of the principle of *Conservation of Momentum.*

These equations are valid whenever the effect of mutual forces is simply to change velocity; but, they can not serve for study of particle collisions.

Study of the minutia of collisions is intractable because, in general, of extremely complex phenomena including: elastic and inelastic deformations, conversion of energy to thermal energy, excited chemical reactions and the like. But, what is of practical interest, is not really the vicissitudes of the collision itself, as much as its final outcome. To investigate how bodies move the instant after collisions, one introduces the principle of conservation of momentum of the *center of mass,* which is an extention of the principle of inertia, and which is valid in all systems exempt from exterior influences, whatever transpires during collisions.

As, by definition, the coordinates of the center of mass are:

$$r_d = \frac{\sum_i m_i r_i}{\sum_i m_i}, \quad d = x, y, z, \tag{XII.5}$$

the just mentioned principle is expressed as:

$$\sum_i m_i \vec{u}_i = \text{const.}, \tag{XII.6}$$

which shows that in collisions, Eqs. (XII.2) and (XII.4) remain valid.

In the new dynamics, in so far as mass is not constant, it is necessary to investigate the means of defining the center of mass. As the effects of collisions must be determined by velocities at the onset of collisions, plausibly any mass should be that of 'mass in motion', i.e., rest mass m_r augmented with kinetic energy T, in other words:

$$m = m_r + T/c^2 = m_r/\sqrt{1-u^2/c^2}. \tag{XII.7}$$

Thus, in the new theory, just as in EINSTEIN's, momentum must be defined by:

$$\vec{p} \equiv \frac{m_r \vec{u}}{\sqrt{1-u^2/c^2}}. \tag{XII.8}$$

Again, we consider three important cases.

A. Elastic collisions

Consider a body of mass m_r moving with velocity u_0 colliding with a second body of mass M, with velocity v_0. Further, suppose they are spherical and moving on the line joining their centers, where our purpose is to determine their velocities, u and v, after a collision.

Elastic collision means that both bodies conserve internal energy. Moreover, immediately preceeding and following the collision, the potential energy is null. Thus, only kinetic energy is involved, and the principles of conservation of energy and momentum are expressed as:

$$\begin{aligned}\frac{m_r}{\sqrt{1-\frac{u_0^2}{c^2}}} + \frac{M_r}{\sqrt{1-\frac{v_0^2}{c^2}}} &= \frac{m_r}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{M_r}{\sqrt{1-\frac{v^2}{c^2}}};\\ \frac{m_r u_0}{\sqrt{1-\frac{u_0^2}{c^2}}} + \frac{M_r v_0}{\sqrt{1-\frac{v_0^2}{c^2}}} &= \frac{m_r u}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{M_r v}{\sqrt{1-\frac{v^2}{c^2}}},\end{aligned} \tag{XII.9}$$

or, as well:

$$\frac{m_r}{M_r}\left[\frac{1}{\sqrt{1-\frac{u_0}{c^2}}} - \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}; \tag{XII.10}$$

$$\frac{m_r}{M_r}\left[\frac{u_0}{\sqrt{1-\frac{u_0}{c^2}}} - \frac{u_0}{\sqrt{1-\frac{u^2}{c^2}}}\right] = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{v_0}{\sqrt{1-\frac{v_0^2}{c^2}}}. \tag{XII.11}$$

Although these equations solve the problem, it is not easy to isolate u and v. However, for many practical applications in physics, the situation consists of a projectile, m_r, vastly lighter than a target, M_r. Under such assumptions, $v \simeq v_0$ and there is significant simplification. Dividing Eq. (XII.10) by (XII.11) gives:

$$\frac{\sqrt{1-\frac{u^2}{c^2}} - \sqrt{1-\frac{u_0^2}{c^2}}}{u_0\sqrt{1-\frac{u^2}{c^2}} - u\sqrt{1-\frac{u_0^2}{c^2}}} = \frac{\sqrt{1-\frac{v_0^2}{c^2}} - \sqrt{1-\frac{v^2}{c^2}}}{v\sqrt{1-\frac{v_0^2}{c^2}} - v_0\sqrt{1-\frac{v^2}{c^2}}}. \tag{XII.12}$$

In the limit, when $v \to v_0$, the second term becomes indeterminate in form, but can easily be seen to equal v_0/c^2, so that the value of u satisfies:

$$(1-u_0 v_0/c^2)\sqrt{1-u^2/c^2} = (1-uv_0/c^2)\sqrt{1-u_0^2/c^2}. \tag{XII.13}$$

Squaring and simplifying gives a second order equation in u, which, besides the trivial solution $u = u_0$, also yields:

$$u = \frac{2v_0 - u_0(1+v_0^2/c^2)}{1+\frac{v_0}{c^2}(v_0 - 2u_o)}. \tag{XII.14}$$

When the target is fixed, $v_0 = 0$, and then $u = -u_0$.

To justify the hypotheses underlying the preceeding calculation, which can appear exaggeratedly artificial, we shall obtain the same result applying our version of the *Covariance Principle, according to which, one can consider that all physics equations transform covariantly under* LORENTZ-EINSTEIN *transformations, even as expressed in new, well chosen, variables (or ficticious quantitates).*

We start supposing that the target is fixed in a system, S, at absolute rest and, that it and the projectile are perfectly elastic. Just as they make contact, both bodies begin to deform, more or less according to their modulus of elasticity. The deformation continues until all the projectile's kinetic energy is transformed into energy of elastic deformation. At this point, the process is reversed, so that when the projectile and target have recovered their original form, that is, when the cease to have contact, all energy of elastic deformation is reconverted back into kinetic energy, and as the target is fixed, the projectile must regain its original velocity, but with opposite sign. That is:

$$u = -u_0 \quad \text{if} \quad v_o = 0; \quad m_r/M \simeq 0. \tag{XII.15}$$

We now investigate what happens when the target, instead of being fixed, moves with velocity $v_0 \neq 0$. If one takes a reference frame moving with this same velocity v_0, and employs within it aberrated rulers and clocks artfully so as to satisfy LORENTZ-EINSTEIN transformations, then measurements will turn out incorrect, but Eqs. (XII.15) will be covariant with respect to said transformations, and they will yield:

$$u' = -u'_0; \quad \text{if} \quad \mathrm{v}' = 0. \tag{XII.16}$$

This solution, which relativists take as valid, is notably false, but is correct in S, which is what is obtained with no more than using formulas for a change of variables from S to S'. To do so, we must call on just the relativistic formula for the addition of velocities:

$$u = \frac{u' + v}{1 + u'v/c^2}, \tag{XII.17}$$

which leads directly to:

$$u_0 = \frac{u'_0 + v}{1 + v_0 u'_0/c^2}; \quad u = \frac{-u'_0 + v_0}{1 + v_0 u'_0/c^2}, \tag{XII.18}$$

and now by just eliminating u'_0 between them one obtains Eq. (XII.14).

When $u_0 = c$, which is the case for the photon, the result is $u = -c$, such that *reflected light in a moving frame propagates with the same velocity as if the mirror were at rest.* This deduction is in accord with experimental results obtained by MICHELSON (1913).

B. Inelastic collisions

Consider two bodies moving along the same line with velocities u_1 and u_2, which collide and fuse together. We now seek to investigate the proper mass M_m, the velocity u and the internal energy U, of the combined body resulting from the collision. We suppose that initially both bodies had no internal energy, and that there was no annihilation of matter, nor energy interchange with exterior bodies. This is the case, for example, when a neutron is captured by a nucleus.

In order to compact notation, let:

$$\alpha_{(\cdot)} = \sqrt{1 - u_{(\cdot)}^2/c^2}; \tag{XII.19}$$

conservation of total mass, then, is expressed as:

$$\frac{m_{m_1}}{\alpha_1} + \frac{m_{m_2}}{\alpha_2} = \frac{M_r}{\alpha}, \tag{XII.20}$$

and conservation of momentum:

$$\frac{m_{m_1} u_1}{\alpha_1} + \frac{m_{m_2} u_2}{\alpha_2} = \frac{M_r u}{\alpha}. \tag{XII.21}$$

Multiplying Eq. (XII.20) by u and subtracting from Eq. (XII.21) gives:

$$\frac{m_{m_1}}{\alpha_1}(u_1 - u) + \frac{m_{m_2}}{\alpha_2}(u_2 - u) = 0, \tag{XII.22}$$

which gives:

$$u = \frac{m_{m_1} u_1 \alpha_2 + m_{m_2} u_2 \alpha_1}{m_{m_1}\alpha_2 + m_{m_2}\alpha_1}. \tag{XII.23}$$

Knowing the value of u, permits evaluating Eq. (XII.20) for M_r:

$$M_r = \alpha(m_{m_1}/\alpha_1 + m_{m_2}/\alpha_2). \tag{XII.24}$$

If, as relativists claim, the principles of conservation of mass and energy are unified into a single principle, we would not be able to calculate the values of the proper mass M_m and internal energy U after a collision. But, calling on the results of §3, we can consider also the principle of conservation of proper mass:

$$M_m = m_{m_1} + m_{m_2}. \tag{XII.25}$$

As: $M_r = M_m + U/c^2$, Eq. (XII.24) gives:

$$\frac{U}{c^2} = m_{m_1}\left(\frac{\alpha}{\alpha_1} - 1\right) + m_{m_2}\left(\frac{\alpha}{\alpha_1} - 1\right), \tag{XII.26}$$

which is the solution to our problem.

A case of particular interest is one in which a body collides with a much larger target. If $m_{m_1} << m_{m_2}$ these equations give:

$$u \simeq u_2; \quad M_r \simeq m_{m_2}\left(\frac{\alpha_2}{\alpha_1}\frac{m_{m_1}}{m_{m_2}} + 1\right), \tag{XII.27}$$

$$\frac{U}{c^2} \simeq \left(\frac{\alpha_2}{\alpha_1} - 1\right). \tag{XII.28}$$

Since kinetic energy of the first body equals:

$$T_1 \simeq m_{m_1} c^2 \left(\frac{1}{\alpha_1} - 1\right), \tag{XII.29}$$

Eq. (XII.28) takes the form:

$$U \simeq (T_1 + m_{m_1} c^2)\alpha_2 - m_{m_1} c^2, \tag{XII.30}$$

which gives us the portion of kinetic energy stored in the form of internal energy. If the target remains at rest, then $u_2 = 0$ and $U \simeq T_1$. All the projectile's energy is converted into internal energy.

C. Collision of antiparticles

Another case of particular interest is that in which a particle collides with its antiparticle and all energy is converted to radiation. In this case the particles might have internal energy, of spin or charge say, which requires replacing their rest mass with

$$m_r = m_m + U/c^2. \tag{XII.31}$$

If W is the radiative energy liberated in the collision, conservation of total mass implies:

$$m_r\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right) = W/c^2, \tag{XII.32}$$

and if this should be a single photon, the energy equals:

$$h\nu = W = m_r c^2\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right). \tag{XII.33}$$

Conservation of momentum, however, requires:

$$m_r\left(\frac{u_1}{\alpha_1} + \frac{u_2}{\alpha_2}\right) = \frac{W}{c}, \tag{XII.34}$$

or

$$W = c m_r\left(\frac{u_1}{\alpha_1} + \frac{u_2}{\alpha_2}\right). \tag{XII.35}$$

Comparison of Eqs. (XII.33) and (XII.35) gives:

$$(c - u_1)/\alpha_1 + (c - u_2)/\alpha_2 = 0, \tag{XII.36}$$

which can not be satisfied because both terms are always positive. Thus, necessarily at least two photons result from annihilation so that the principle of mass conservation gives the equation:

$$m_r\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)=\frac{h}{c^2}(\nu_1+\nu_2). \tag{XII.37}$$

Conservation of momentum is expressed here by means of the vectorial equation:

$$m_r\left(\frac{\vec{u}_1}{\alpha_1}+\frac{\vec{u}_2}{\alpha_2}\right)=\frac{h}{c^2}(\nu_1\vec{c}_1+\nu_2\vec{c}_2). \tag{XII.38}$$

Taking coordinate axes in the plane determined by the velocities $\vec{u}_1$ and $\vec{u}_2$, converts Eq. (XII.38) to:

$$\begin{aligned} m_r\left(\frac{\vec{u}_{1x}}{\alpha_1}+\frac{\vec{u}_{2x}}{\alpha_2}\right) &= \frac{h}{c^2}(\nu_1\cos\theta_1+\nu_1\cos\theta_2); \\ m_r\left(\frac{\vec{u}_{1y}}{\alpha_1}+\frac{\vec{u}_{2y}}{\alpha_2}\right) &= \frac{h}{c^2}(\nu_1\sin\theta_1+\nu_2\sin\theta_2), \end{aligned} \tag{XII.39}$$

and one has three equations in the unknowns ν_1, ν_2, θ_1 and θ_2. Thus, one of the photon's directions remains undetermined. In any case, however, the total radiation energy is given by Eq. (XII.35).

XIII. PLANCK'S EQUATION IS NOT COVARIANT. TRANSFORMATION OF FORCE

NEWTON's dynamics is based on the equation:

$$\vec{f}=m\frac{d\vec{u}}{dt}, \tag{XIII.1}$$

and, being subject to composition of vector velocities, one uses the transformation:

$$\vec{u}=\vec{u}'+\vec{v}; \tag{XIII.2}$$

so that:

$$\frac{d\vec{u}}{dt}=\frac{d\vec{u}'}{dt}. \tag{XIII.3}$$

Moreover, mass is unaltered and one uses the same units in the frame S' as in S, that is:

$$m=m'; \quad \vec{f}=\vec{f}'; \quad t=t', \tag{XIII.4}$$

so that Eq. (XIII.1) converts to:

$$\vec{f}'=m'\frac{d\vec{u}'}{dt'}, \tag{XIII.5}$$

which means that this equation is covariant under Galilean transformations, therefore NEWTON's dynamics may be classified as *relativistic*.

We shall see, paradoxically, that EINSTEIN's dynamics is in this sense *antirelativistic*. His dynamics takes as the fundamental equation:

$$\vec{f}=m_r\frac{d}{dt}\frac{\vec{u}}{\sqrt{1-u^2/c^2}}, \tag{XIII.6}$$

and relativists claim that it is covariant under LORENTZ transformations, such that it in the frame S' it becomes:

$$\vec{f}'=m_r\frac{d}{dt'}\frac{\vec{u}'}{\sqrt{1-u'^2/c^2}}. \tag{XIII.7}$$

To examine covariance of the vector equation, (XIII.6), we start by considering its projection on the X axis:

$$f_x=m_r\frac{d}{dt}\frac{u_x}{\sqrt{1-u^2/c^2}}. \tag{XIII.8}$$

Covariance of this equation requires:

$$f'_x=m'_r\frac{d}{dt'}\frac{u'_x}{\sqrt{1-u'^2/c^2}}. \tag{XIII.9}$$

Let us see, however, what the right side of Eq. (XIII.8) is converted into in fact under LORENTZ transformations. Such transformations introduce local time t' which is a ficticious time defined by the equation:

$$t=(t'+vx'/c^2)/\alpha, \tag{XIII.10}$$

from which one deduces:

$$\frac{dt}{dt'}=\frac{1}{\alpha}(1+vx'/c^2). \tag{XIII.11}$$

Thus, given an arbitrary function, $\varphi(t)$, it follows that:

$$\frac{d\varphi}{dt}=\frac{d\varphi}{dt'}\frac{dt'}{dt}=\frac{\alpha}{1+vu'/c^2}\frac{d\varphi}{dt'}. \tag{XIII.12}$$

On the other hand, from LORENTZ-EINSTEIN transformation formulas, one deduces:

$$\frac{u_x}{\sqrt{1-u^2/c^2}}=\frac{u'_x+v}{\alpha\sqrt{1+vu'/c^2}}, \tag{XIII.13}$$

and by virtue of Eq. (XIII.12),

$$\frac{d}{dt}\frac{u_x}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{1}{1+\frac{vu'}{c^2}}\left(\frac{d}{dt'}\frac{u'_x}{\sqrt{1-\frac{u'^2}{c^2}}}+v\frac{d}{dt'}\frac{1}{\sqrt{1+\frac{u'^2}{c^2}}}\right). \tag{XIII.14}$$

To determine the transformation of rest mass, we return to Eq. (VI.17):

$$< m_r >= \alpha < m'_r > \quad \text{between quantitites}. \tag{XIII.15}$$

To go from an equation between quantities to one between measurements, one must specify the units used in system S'. In EINSTEIN's theory, it is tacitly taken that one employs a kilogram that has been calibrated against a standard in S and then transfered to S'. To transport this kilogram it is necessary to invest energy with its corresponding change of mass, such that, by virtue of Eq. (VI.17), one has:

$$1\,\mathrm{kg.\,in}\; S = \alpha\,\mathrm{kg.\,in}\, S', \qquad \text{(XIII.16)}$$

which means that a moving kilogram is less heavy than one at rest.

Applying the Metric Principle, one has:

$$m_r = \frac{< m_r >}{1\,\mathrm{kg.\,in}\; S}; \quad m'_r = \frac{< m'_r >}{1\,\mathrm{kg.\,in}\; S'}, \qquad \text{(XIII.17)}$$

and with Eqs. (VI.17) and (VI.21), results in:

$$m_r = m'_r \quad \text{between measurements.} \qquad \text{(XIII.18)}$$

As one sees, the quantities are different, but the measurements are equal because whatever alterations occur to the mass of a body, also occur to the standard kilogram.

By multiplying Eq. (XIII.14) by Eq. (XIII.18) we have on account of Eq. (XIII.9), the result:

$$m_r \frac{d}{dt} \frac{u_x}{\sqrt{1-u^2/c^2}} = \frac{1}{1+vu'_x/c^2} \left(f'_x + m'_r v \frac{d}{dt'} \frac{1}{\sqrt{1-u'^2/c^2}} \right). \qquad \text{(XIII.19)}$$

The second term of this expression can be simplified. To do so, expand the derivative:

$$\frac{d}{dt'} \frac{1}{\sqrt{1-u'^2/c^2}} = \frac{1}{2c^2(1-u'^2/c^2)^{3/2}} \frac{du'^2}{dt'}, \qquad \text{(XIII.20)}$$

and form the vectorial product

$$\vec{f}' \cdot \vec{u}' = m'_r \left(\vec{u}' \cdot \frac{d(\vec{u}'/\alpha')}{dt'} \right) = m'_r u'^2 \frac{d}{dt'} \frac{1}{\alpha'} + \frac{m'_r}{2\alpha'} \frac{du'^2}{dt'}, \qquad \text{(XIII.21)}$$

where $\alpha' = \sqrt{1-u'^2/c^2}$.

Eliminating du'^2/dt' in Eq. (XIII.21) using Eq. (XIII.20), one obtains

$$\frac{d}{dt'} \frac{1}{\alpha'} = \frac{1}{m'_r c^2} \vec{f}' \cdot \vec{u}', \qquad \text{(XIII.22)}$$

with which, Eq. (XIII.19) becomes

$$m_r \frac{d}{dt'} \frac{u_x}{\sqrt{1-u^2/c^2}} = \frac{1}{1+vu'^2/c^2} \left(f'_x + \frac{v}{c^2} \vec{f}' \cdot \vec{u}' \right). \qquad \text{(XIII.23)}$$

For the other components, proceeding in an analogous way, the results are

$$f_y = m_r \frac{d}{dt} \frac{u_y}{\sqrt{1-u^2/c^2}} = \frac{\alpha}{1+vu'_x/c^2} f'_y;$$
$$f_z = m_r \frac{d}{dt} \frac{u_z}{\sqrt{1-u^2/c^2}} = \frac{\alpha}{1+vu'_x/c^2} f'_z. \qquad \text{(XIII.24)}$$

These formulas reveal, that in order to conserve covariance of the fundamental equation of EINSTEIN's dynamics, it is necessary to introduce a new vector $\vec{f}'$ which, with an eye to its dimensional formula, can be called a force, and which is defined by the identities:

$$f_x = \frac{1}{1-vu'/c^2} \left(f'_x + \left(\frac{v}{c^2} \vec{f}' \cdot \vec{u} \right) \right);$$
$$f_y = \frac{\alpha}{1-vu'/c^2} f'_y; \quad f_z = \frac{\alpha}{1-vu'/c^2} f'_z. \qquad \text{(XIII.25)}$$

According to these equations, measurement of the *real* force $\vec{f}$ and measurement of the *fictitious* force $\vec{f}'$, are different. Since the ratios f_x/f, f_y/f, and f_z/f do not depend solely on v, the difference can not be attributed to use of distinct units. Consequently, the force $\vec{f}'$, defined by the identities Eqs. (XIII.25), has nothing to do with the force measured with a dynamometer, and shows that *covariance of the fundamental law of relativistic dynamics is obtained introducing a force which is nothing but a mathematical fiction.* Thus, we have demonstrated, one more time, that covariance is not a law of nature.

XIV. A CRITIQUE OF EINSTEIN'S REASONING

To develop a new dynamics we have taken a different route than that followed in relativity texts. Nonetheless, Relativists can not take objection to our reasoning, because we have based our arguments on a fact that they expressly accept: i.e., to total energy there corresponds a mass given by: W/c^2. In this way we have shown, that when there is no potential energy, the following law governs motion:

$$\vec{f} = \frac{d}{dt} \frac{m_r \vec{u}}{\sqrt{1-u^2c^2}}, \qquad \text{(XIV.1)}$$

from which it follows, that between a mass at rest and one in motion, there exists the relationship:

$$m = \frac{m_r}{\sqrt{1-u^2/c^2}}, \qquad \text{(XIV.2)}$$

which is valid only when the velocity u results from forces directly contacting the body and providing the necessary energy. Except for special cases, however, the preceeding formulas generally are inapplicable, and one must take recourse to the new dynamics. If, for example, the body moves unrestrained in a force field, it must exhibit variable velocity; but, if it has absorbed or emitted no energy, then its mass should have remained constant. In other words, it is not velocity, but energy that posses mass.

Our theory is an obvious contradiction to the theory of relativity, in so far as it implies acceptance of Eqs. (XIV.1) and (XIV.2), wherever there are forces changing the velocity. Let us proceed now, to examine the reasoning used by Relativists, in particular that of EINSTEIN.

To deduce the fundamental equation of relativistic dynamics, consider along with EINSTEIN 1905, §10) an electron that at time $t=0$ is at the origin of a system of coordinates S, moving with velocity u along the X axis. Clearly this electron remains at rest with respect to a reference system moving with the same velocity. Under these conditions, EINSTEIN accepts that NEWTON's Law is valid in reference system S', and writes:

$$\vec{f}' = m_r \frac{d^2\vec{x}'}{dt^2}. \tag{XIV.3}$$

Suppose further that a force arises from an electromagnetic field. For small values of t, the velocity will differ but little from u, and we may take it that this electron is at rest in S$'$, so that, seen from this reference system, it would be true that:

$$e\vec{E}' = m_r \frac{d^2\vec{x}'}{dt^2}, \tag{XIV.4}$$

where e is the electron's charge, taken to be inalterable.

For acceleration he used the relativistic transformation formulas. As at the instant $t=0$, one has $\vec{u}'=0$, this gives:

$$\begin{aligned} \frac{d^2x'}{dt'^2} &= \frac{1}{\alpha^3}\frac{d^2x}{dt^2}; \\ \frac{d^2y'}{dt'^2} &= \frac{1}{\alpha^2}\frac{d^2y}{dt^2}; \\ \frac{d^2z'}{dt'^2} &= \frac{1}{\alpha^2}\frac{d^2z}{dt^2}; \end{aligned} \tag{XIV.5}$$

where $\alpha = \sqrt{1-v^2/c^2}$ is the contraction factor due to the velocity $v \simeq u$, which can be taken constant during very short intervals.

For the intensity of the electric field, EINSTEIN made use of the formulas:

$$E'_x = E_x; \quad E'_y = \frac{1}{\alpha}(E_y - uB_z); \quad E'_z = \frac{1}{\alpha}(E_z + uB_z). \tag{XIV.6}$$

Substituting Eqs. (XIV.5) and (XIV.6) into (XIV.3), one obtains:

$$\begin{aligned} eE_x &= \frac{m_r}{\alpha^3}\frac{d^2x}{dt^2}; \\ e(E_y - uB_z) &= \frac{m_r}{\alpha}\frac{d^2y}{dt^2}; \\ e(E_z + uB_y) &= \frac{m_r}{\alpha}\frac{d^2z}{dt^2}. \end{aligned} \tag{XIV.7}$$

From these equations, which must be seen as valid in the system S if the reasoning behind them is correct, EINSTEIN drew no conclusions. However, ignoring Eqs. (XIV.6) and (XIV.7), and substituting (XIV.5) directly into (XIV.4), gives:

$$\begin{aligned} eE'_x &= \frac{m_r}{\alpha^3}\frac{d^2x}{dt^2}; \\ eE'_y &= \frac{m_r}{\alpha^2}\frac{d^2y}{dt^2}; \\ eE'_z &= \frac{m_r}{\alpha^2}\frac{d^2z}{dt^2}. \end{aligned} \tag{XIV.8}$$

Continuing, EINSTEIN stated: "The left sides constitute the components of ponderomotive force acting on the electron, as would be observed from a system moving with the same velocity $v=u$, at the considered instant (a force which, for example, may be measured with a spring dynamometer in this system)." With this argument he obtained the final equations:

$$f_x = \frac{m_r}{\alpha^3}\frac{d^2x}{dt^2}; \quad f_y = \frac{m_r}{\alpha^2}\frac{d^2y}{dt^2}; \quad f_z = \frac{m_r}{\alpha^2}\frac{d^2z}{dt^2}; \tag{XIV.9}$$

and drew the conclusion that the longitudinal and transversal masses equal, respectively:

$$m_l = \frac{m_r}{\alpha^3} = \frac{m_r}{(1-u^2/c^2)^{3/2}}; \quad m_t = \frac{m_r}{\alpha^2} = \frac{m_r}{(1-u^2/c^2)}. \tag{XIV.10}$$

As we have said, all relativists, in particular EINSTEIN himself, are in agreement with Eq. (XIV.9), which may be substituted for Planck's EQ. (XIII.1), and from which one may deduce, following the example of Eqs. (VIII.4):

$$m_l = \frac{m_r}{(1-u^2/c^2)^{3/2}}; \quad m_t = \frac{m_r}{(1-u^2/c^2)^{1/2}}, \tag{XIV.11}$$

which coincides with those given by LORENTZ, but not with those of EINSTEIN, as the values of m_t differ. This alone suffices to call EINSTEIN's reasoning into doubt. But, moreover, this also reveals objections that, to my judgment, totally invalidate his reasoning.

EINSTEIN's argumentation is unacceptable because it considers that acceleration transforms according to the usual relativistic formulas, which, as we have repeatedly seen, must be rejected because they presuppose the use of aberrant meter sticks and inappropriate clocks. On the other hand, given that MAXWELL's equations must be covariant passing from S to S', which compelled both introduction of the fictitious quantities E'_x, E'_y and E'_z, and acceptance, that in a moving system, the force acting on an electron is not f, but another, f', defined by Eq. (XIV.4).

The errors we have noted are caused by contradictions. It is seen directly from Eqs. (XIV.4) and (XIV.6), for example, that the ficticious force f' has to be different than the real force f, that is observed and measured by the dynamometer. Suppose, say, there is no magnetic field. Then, as $\vec{B}=0$, one has:

$$\vec{f} = e\vec{E}, \tag{XIV.12}$$

while from Eqs. (XIV.4) and (XIV.6) one deduces:

$$f_x = eE_x; \quad f_y = \frac{1}{\alpha}eE_y; \quad f_z = \frac{1}{\alpha}eE_z; \tag{XIV.13}$$

and so, as a consequence, $\vec{f}' \neq \vec{f}$. In spite of this, to obtain the final formula, namely Eq. (XIV.9), EINSTEIN takes it, with perfect logic but with notorious inconsistency, that in reality there is only one force, i.e., that one which a field actually exercises over an electron, say. On the other hand, there is no reason to complicate the argumentation by introducing electromagnetic fields, as it suffices simply to take it that $\vec{f} \equiv \vec{f}'$ and substitute Eq. (XIV.5) into (XIV.3) in order to obtain the final equation, Eq. (XIV.9).

PLANCK considered the case in which the electron's motion is along the X axis. Then, Eqs. (XIV.6) reduce to $E'_x = E$ and $\vec{f} = e\vec{E}$, $\vec{f}' = e\vec{E}'$, from which one deduces that $f = f'$, that is, in this particular case, no contradiction arrises. However, this does not establish the general validity of Eqs. (XIV.10). On the other hand, let us take it that there is only one force, such that its measurement as f and f' are to be equal, it is, then, necessary that in S' one is to use valid meter sticks and clocks adjustable in such a way that they show universal time. Under these conditions, the vectorial composition of velocities obtains, and acceleration respects the relativistic formulas, Eqs. (XIV.5).

Finally, EINSTEIN as well as PLANCK analyzed the motion of a charge in an electromagnetic field, a case for which, following our theory, Eq. (XIV.2) is not applicable; then, in order, that $T + E_p =$ const., and internal energy be fixed, the total mass must equal:

$$m = m_m + \frac{1}{c^2}(U + T + E_p) = \text{const.}, \qquad \text{(XIV.14)}$$

a result independent of the velocity acquired through free motion in the field.

In short, the poorly posed relativistic Eq. (XIV.1), applicable to contact forces, is a correct consequence of MAXWELL's equations, as we have seen in §5; but, it is impossible to derive it by means of relativity theory without incurring contradictions.

XV. THE METHOD OF LEWIS AND TOLMAN

The notorious shortcomings in rigor that EINSTEIN incurred while modifying the fundamental law of NEWTON's mechanics, has been, probably, the cause of the fact, that all authors, with mild variations, follow the methods of LEWIS (1909) and TOLMAN (1909), and (1934) to derive formulas giving the variation of mass with velocity.

These two authors, having based their derivation on the principle of relativity as applied to elastic shock interactions, obtain correct results, and therefore are able to infer the fundamental law of the new theory as a consequence of EINSTEIN's theory. But careful examination of TOLMAN's text from 1934, shows that their development is not irreproachable. I quote:

> In the first system of coordinate, for convenience the primed system S', let the two particles be moving before collision with the velocities $+u'$ and $-u'$ parallel to the $x-$axis in such a way that a head-on encounter can occur. Since by hypothesis the two particles are perfectly similar and elastic, it is evident that they will first be brought to rest on collision and then rebound under the action of the elastic force developed, moving back over their original paths with the reverse velocities $-u'$ and $+u'$ of the same magnitude as before but reversed in direction. In this system of coordinates the collision is obviously such as to satisfy the conservation laws of mass and momentum.
>
> Let us now change to a second system of coordinates S moving relative to the first in the $x-$direction with velocity V. Using this new system of coordinates, let us denote by u_1 and u_2 the velocities of the two particles before collision, and allowing for the possibility that mass may depend on velocity let us denote by m_1 and m_2 the masses of the two particles before collision. Furthermore, let us denote by M the sum of the masses of the two particles at the instant in the course of the collision when they have come to relative rest, and are hence both moving with velocity $+V$ with respect to out present system of coordinates, S.
>
> In accordance with the conservation laws. which must also hold in the new system of coordinates, the total mass and total momentum of the two particles must be the same before the collision and at the moment of relative rest, so that we can evidently write:
>
> $$m_1 + m_2 = M, \qquad \text{(XV.1)}$$
>
> and
>
> $$m_1 u_1 + m_2 u_2 = MV. \qquad \text{(XV.2)}$$
>
> In addition, however, using the [transformation equations for velocities,] we can write for the velocities u_1 and u_2, in terms of their values $+u'$ and $-u'$ with respect to the original coordinates S, the expressions:
>
> $$u_1 = \frac{u' + V}{1 + u'V/c^2} \quad \text{and} \quad u_2 = \frac{-u' + V}{1 - u'V/c^2}. \qquad \text{(XV.3)}$$
>
> And by combining these three equations and solving for the ration of the two masses, we easily obtain:
>
> $$\frac{m_1}{m_2} = \frac{1 + u'V/c^2}{1 - u'V/c^2}, \qquad \text{(XV.4)}$$
>
> which with the help of
>
> $$\left[\sqrt{1 - u^2/c^2} = \frac{\sqrt{1 - V^2c^2}}{1 - u'_x V/c^2}\sqrt{1 - u'^2/c^2},\right] \qquad \text{(XV.5)}$$
>
> gives us
>
> $$\frac{m_1}{m_2} = \frac{\sqrt{1 - u_1^2/c^2}}{\sqrt{1 - u_2^2/c^2}}. \qquad \text{(XV.6)}$$
>
> In accordance with this result the masses of the two particles, which by hypothesis have the same value, say m_0, when at rest, become inversely proportional to $\sqrt{1 - u^2/c^2}$ when moving with velocity u, so that we may now write
>
> $$m = \frac{m_0}{\sqrt{1 - u^2/c^2}}, \qquad \text{(XV.7)}$$

as the desired expression for the mass m of a moving particle in terms of its velocity u and mass at rest m_0.[4]

The route followed by TOLMAN differs fundamentally from ours. In our theory, the equation which gives mass as a function of velocity is only valid in the absence of potential energy and when internal energy is constant. In our case, to deduce this relation we have applied the formula $m_w = W/c^2$ to the particular case in which the energy W communicated to the body is converted completely to an increase of kinetic energy. In our reasoning it is not necessary to invoke the principle of relativity. On the contrary, since the mass of a body is minimum when $u = 0$, it provides a criterion to distinguish a frame at absolute rest, S, from any other frame at all.

TOLMAN claimed to have deduced Eq. (XV.6) starting from the principle of relativity and took for granted that it is valid whatever the disposition of forces producing a change in velocity.

He took it in his argumentation that the collision under consideration was perfectly symmetric with respect to a moving plane at $x' = 0$, which requires, evidently, that not only the velocities of the particles are equal, but also their masses, that is

$$u_1' = -u_2' = u'; \quad m_1' = m_2'. \tag{XV.8}$$

Without exception, and here is the contradiction, TOLMAN assumes that both particles poses the same mass, m_r, when they remain at rest in system S', which is in accord with Eq. (XV.5). From this, one deduces, that after the collision, the masses, measured in the units of S, equal

$$m_1 = \frac{m_r}{\sqrt{1-u_1^2/c^2}}; \quad m_2 = \frac{m_r}{\sqrt{1-u_2^2/c^2}}. \tag{XV.9}$$

To change from measurements made in S to those made in S', one need only keep in mind that

$$\frac{1 \text{ kg. in } S'}{1 \text{ kg. in } S} = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\alpha}, \tag{XV.10}$$

to obtain

$$\frac{m_1'}{m_1} = \frac{m_2'}{m_2} = \alpha, \tag{XV.11}$$

that is

$$m_1' = \frac{\alpha m_r}{\sqrt{1-u_1^2/c^2}}; \quad m_2' = \frac{\alpha m_r}{\sqrt{1-u_2^2/c^2}}, \tag{XV.12}$$

and since by virtue of Eq. (XV.3), $u_1 \neq u_2$, it follows

$$m_1 \neq m_2 \tag{XV.13}$$

which is the contradiction with Eq. (XV.8).

[4] TOLMAN, R. C. *Relativity, Thermodynamics and Cosmology*, (Dover, New York, 1987) p. 43.

XVI. TRANSFORMATION RULES FOR MASS AND MOMENTUM

In accord with our postulate of covariance, one can consider that all physics equations conserve their form passing from a system at rest to another inertial system S', applying to each quantity a particular transformation formula, which is equivalent to saying, that covariance in not a law of nature, but can be effected by introducing new variables, that can be called ficticious because they differ from the real observables and measurements. In the preceeding we have obtained transformation formulas for coordinates, time, velocities, accelerations and forces. Now we shall seek the correspondences of mass, energy and time.

For mass we start with the equation

$$m = \frac{m_r}{\sqrt{1-u^2/c^2}}, \tag{XVI.1}$$

and we seek a new variable, m', such that

$$m' = \frac{m_r'}{\sqrt{1-u'^2/c^2}}. \tag{XVI.2}$$

The measurement of rest mass, according to §13, is invariant

$$m_r = m_r'. \tag{XVI.3}$$

The formula giving the contraction factor under transformation is:

$$\sqrt{1-u^2/c^2} = \frac{\sqrt{1-v^2/c^2}}{1+vu_x'/c^2}\sqrt{1-u'^2/c^2}. \tag{XVI.4}$$

Substituting Eqs. (XVI.3) and (XVI.4) into (XVI.1), gives

$$m = \frac{m_r'}{\alpha\sqrt{1-u'^2/c^2}}(1+vu_x'/c^2)m', \tag{XVI.5}$$

thus, by virtue of Eq. (XVI.2):

$$m = \frac{1}{\alpha}(1+vu_x'/c^2)m', \tag{XVI.6}$$

which is the sought formula. One obtains the inverse formula, as always, by permuting the unprimed with primed symbols and changing the sign of v:

$$m' = \frac{1}{\alpha}(1-vu_x/c^2)m. \tag{XVI.7}$$

Momentum is a secondary quantity; in EINSTEIN's dynamics it is defined by the identity:

$$\vec{p} \equiv \frac{m_r\vec{u}}{\sqrt{1-u^2/c^2}}, \tag{XVI.8}$$

which, when projected on the X axis, becomes

$$p_x \equiv \frac{m_r u_x}{\sqrt{1-u^2/c^2}}. \tag{XVI.9}$$

The covariance of this expression is established by introducing a new variable defined by

$$p'_x \equiv \frac{m'_r u'_x}{\sqrt{1-u'^2/c^2}}. \qquad \text{(XVI.10)}$$

While the formulas

$$m_r = m'_r; \quad \frac{u_x}{\sqrt{1-u^2/c^2}} = \frac{u'_x+v}{\sqrt{1-u'^2/c^2}}, \qquad \text{(XVI.11)}$$

obtained in §13, give:

$$p_x = \frac{1+v/u'_x}{\alpha}\frac{m'_r u'_x}{\sqrt{1-u'^2/c^2}}, \qquad \text{(XVI.12)}$$

and requires only comparison with Eq. (XVI.10) to obtain

$$p_x = \frac{1}{\alpha}(1+v/u'_x)p'_x. \qquad \text{(XVI.13)}$$

Analogously, for the other components, one obtains:

$$p_y = p'_y; \quad p_z = p'_z. \qquad \text{(XVI.14)}$$

XVII. KINETIC AND TOTAL ENERGY

The kinetic energy of a body in motion is, by definition, the work which is obtained through application of the engendering force, or what is the same, the heat produced by deceleration. Above we have seen, that by virtue of the inertia of energy, the expression for kinetic energy in the new theory is

$$T = \frac{m_r c^2}{\sqrt{1-u^2/c^2}} - m_r c^2. \qquad \text{(XVII.1)}$$

This formula is valid in a system at absolute rest. Let us investigate how it would appear in an inertial system moving with velocity v.

Concerning T, it suffices to suppose that one uses a calorimeter moving with velocity v, for which the body, after being restrained, still conserves the kinetic energy corresponding to this velocity. That is,

$$T'_a = T - \left(\frac{m_r c^2}{\sqrt{1-v^2/c^2}} - m_r c^2\right), \qquad \text{(XVII.2)}$$

or, by virtue of Eq. (XVII.1):

$$T'_a = m_r c^2 \left(\frac{1}{\sqrt{1-u^2/c^2}} - \frac{1}{\sqrt{1-v^2/c^2}}\right), \qquad \text{(XVII.3)}$$

where the subscript a indicates that for measurement of T', one has used appropriate energy units.

The measurement u' of velocity with respect to system S' depends on the meter sticks and clocks that are utilized in this system; and, one must consider, the following two important cases:

a) The correct value of u' is obtained with corrected meter sticks and chronometers that indicate universal time. Under these conditions, velocities comport themselves as vectors, so that one has:

$$\vec{u} = \vec{v} + \vec{u}', \qquad \text{(XVII.4)}$$

or:

$$u^2 = u'^2 + v^2 + 2u'v\cos\theta', \qquad \text{(XVII.5)}$$

where θ' is the angle between $\vec{u}'$ and $\vec{v}$. In this way, Eq. (XVII.3) is converted into:

$$T'_a = m_r c^2 \left(\frac{1}{\sqrt{1-\frac{u'^2+v^2+2u'v\cos\theta'}{c^2}}} - \frac{1}{\sqrt{1-v^2/c^2}}\right). \qquad \text{(XVII.6)}$$

This expression, which is rigorously correct, indicates that in the moving reference system the formula:

$$T' = \left(\frac{m'_r c^2}{\sqrt{1-u'^2/c^2}} - m'_r c^2\right), \qquad \text{(XVII.7)}$$

is not valid, all of which shows, one more time, that the concept of inertia for energy is in contradiction with the principle of relativity.

b) According to relativists, to transform to a moving system one must utilize the LORENTZ transformations. From them, in turn, one deduces the following formula to transform the contraction formula:

$$\sqrt{1-u^2/c^2} = \frac{\alpha}{1+vu'/c^2}\sqrt{1-u'^2/c^2}, \qquad \text{(XVII.8)}$$

such that Eq. (XVII.3) becomes:

$$T'_a = \frac{m'_r c^2}{\sqrt{1-v^2/c^2}}\left(\frac{1+vu'/c^2}{\sqrt{1-u'^2/c^2}} - 1\right). \qquad \text{(XVII.9)}$$

This formula, which seems to be acceptable for relativists because it was obtained using LORENTZ transformations, contains nevertheless a crass contradiction with the principle of relativity, because

$$u'_x = u'\cos\theta', \qquad \text{(XVII.10)}$$

the energy T'_c measured with a calorimeter depends on the angle θ'. This means, that for measurements made in a moving frame, space is not isotropic, because the direction of the velocity is a privileged direction.

From Eq. (XVII.9) one deduces a new method to measure absolute velocity in a laboratory. It suffices to measure u' with clocks synchronized with light signals as if the velocity of light were the same in all directions and without correcting these measurements made with the standard meter. In this way, with energy measured with calorimeters, or its equivalent, Eq (XVII.9) permits calculating $\vec{v}$, as well as its direction.

To conceal the failure of the principle of relativity, one must introduce a new variable, T', that makes Eq. (XVII.1) covariant, that is, that transforms it into Eq. (XVII.8). To this end, we note that, when there is no force field, the *total energy* W contained in a body and comprised of three parts: kinetic energy T, internal energy U and that which comes from conversion of matter into energy:

$$W = T + U + m_m c^2 = T + m_r c^2, \qquad \text{(XVII.11)}$$

or, by virtue of Eq. (XVII.1):

$$W = \frac{m_r c^2}{\sqrt{1 - u'^2/c^2}}. \qquad \text{(XVII.12)}$$

In order that this expression should be covariant under LORENTZ transformation, one must introduce a variable W' defined by

$$W' \equiv \frac{m'_r c^2}{\sqrt{1 - u'^2/c^2}}. \qquad \text{(XVII.13)}$$

Thanks to Eq. (XVII.8), and recalling that $m_r = m'_r$, Eq. (XVII.12) is converted to

$$W = \frac{m'_r c^2}{\sqrt{1 - v^2/c^2}\sqrt{1 - u'^2/c^2}}(1 + u'_x v/c^2), \qquad \text{(XVII.14)}$$

and comparison with Eq. (XVII.13), gives:

$$W = \frac{1}{\sqrt{1 - v^2/c^2}}(1 + vu'/c^2)W', \qquad \text{(XVII.15)}$$

which is the relativistic formula for the transformation of total energy.

From the following two equations

$$W = T + m_r c^2; \quad W' = T' + m'_r c^2, \qquad \text{(XVII.16)}$$

one deduces, in view of Eq. (XVII.15), the following transformation formula for kinetic energy.

$$T = \frac{1}{\sqrt{1 - v^2/c^2}}(1 + vu'/c^2)(T' + m'_r c^2) - m'_r c^2. \qquad \text{(XVII.17)}$$

This transformation formula may always be applied, provided that one utilizes our principle of covariance.

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With great pleasure and gratitude I acknowledge the valuable help given to me by D. ANTONIO HERRANZ, both in discussions of my reasoning, and by reviewing the calculations. —J.P.M.