Queensland University of Technology
Brisbane Australia

This is the author's version of a work that was submitted/accepted for publication in the following source:

Atchison, David A. & Charman, W. Neil
(2011)
Thomas Young's contributions to geometrical optics.
Clinical and Experimental Optometry, *94*(4), pp. 333-340.

This file was downloaded from: https://eprints.qut.edu.au/49190/

© Copyright 2011 Optometrists Association Australia

Notice: *Changes introduced as a result of publishing processes such as copy-editing and formatting may not be reflected in this document. For a definitive version of this work, please refer to the published source:*

https://doi.org/10.1111/j.1444-0938.2010.00560.x

Thomas Young's contributions to geometrical optics

David A. Atchison[#] DSc and W. Neil Charman[$] DSc

School of Optometry and Institute of Health & Biomedical Innovation, Queensland University of Technology, Australia

$ Faculty of Health Sciences, University of Manchester, Manchester, United Kingdom

Abstract

In addition to his work on physical optics, Thomas Young (1773-1829) made several contributions to geometrical optics, most of which received little recognition in his time or since. We describe and assess some of these contributions: Young's construction (the basis for much of his geometric work), paraxial refraction equations, oblique astigmatism and field curvature, and gradient-index optics.

Keywords: oblique astigmatism, gradient index, geometrical optics, historical, image curvature, Thomas Young, Young's construction

INTRODUCTION

The English polymath Thomas Young (1773-1829) was a pioneer in many fields, including colour vision, hearing, mechanics, hydrodynamics, linguistics, construction, tides, actuarial science, and medical physiology. He also made several contributions to optics in the approximate period 1793-1807: this work can be divided into geometrical optics, physical optics, optical instrumentation and visual optics. Recently we presented a critique of his contributions to visual optics, nearly all of which can be found in his 1801 paper[1] "*On the mechanism of the eye*". In addition to his well-known investigations of ocular accommodation and astigmatism, in the critique[2] we highlighted those less familiar parts of the paper dealing with topics such as peripheral imagery, depth-of-focus, chromatic aberration, and change in spherical aberration with accommodation.

The present paper is concerned with his contributions to geometrical optics as applied to the eye. The work is described in the 1801 paper on pages 27-33[1] and in "*A course of lectures on natural Philosophy and the mechanical arts*" which was published in 1807: Vol I, pages 408-419[3] and Vol II, pages 70-83[4]. Note that Volume I was edited and republished in 1845 and the page numbers were changed.[5] The relatively brief optical material in the 1801 paper was included for use in Young's later discussion of ocular biometry and accommodation, primarily being applied to computation of the oblique astigmatism of the eye and to an explanation of how the index gradient contributed additional optical power to the crystalline lens. The meat of Young's geometrical optical contribution was in Volume II of The Lectures, which included definitions and theorems. Nearly all that was in the 1801 paper was superseded here, with Volume 1 containing general statements regarding optics without any proofs. This work is of great historical interest in relation to the design of spectacle lenses and the optics of the eye, and to our understanding of the role of index gradients, both within the crystalline lens and elsewhere.

Young's work on astigmatism and image curvature has been considered by Smith[6] and King[7]: in this paper we have summarised this material and brought it up to date. Young used the same symbol to represent different things at various points in his papers and books. Moreover, symbols which have well-recognised meanings in the current formulation of many relationships in geometrical optics, such as *r* for radius of curvature, were often used to represent quite different parameters in Young's work. To avoid confusion, we have tried to use symbols in a consistent way and to comply with modern notation on signs where appropriate. We use μ rather than *n* to represent refractive index since, when applying Snell's law, Young consistently used the ratio *m: n* to describe the ratio between the sine of the angle of incidence*, i*, and the sine of the angle of refraction, *i*′ (e.g. Proposition I, page 27 in the 1801 paper), that is

$$\sin i/\sin i' = m/n = \mu'/\mu. \qquad (1)$$

YOUNG'S CONSTRUCTION

Young is well known for his Theorem 425 that "*affords an easy method of constructing problems relative to spherical refraction*" at a surface. Young drew two circles concentric with the centre of curvature of the surface (Figure 1). In modern terminology, for a surface of radius of curvature *r* and the refractive indices of the incident and refracted media μ and μ′, respectively, a first circle associated with the incident ray had a radius of curvature $r\mu'/\mu$, and a second circle associated with the refracted ray had a radius of curvature of $r\mu/\mu'$ (see, for example[7-9]). An incident ray passing though A on the surface is extended to B on the other side of the first circle. When B is joined to the centre of curvature C, the intercept with the second circle at D indicates the path of the refracted ray AD.

The proof that the construction works is through its support of Snell's law. ΔABC is similar to ΔDAC, as CA/CD = CB/CA = μ′/μ. In triangle ΔDAC, ∠CAD is the angle of refraction i' and ∠ADC is the angle of incidence i (from its equivalence to ∠CAB in the larger triangle). In ΔDAC the law of sines gives

$\sin i/AC = \sin i'/CD$ or $\sin i/r = \sin i'/(r\ \mu/\mu')$

which gives Snell's law

$\mu \sin i = \mu' \sin i'$

Insert Figure 1 about here

Commentary

Young used this construction (or the results leading from its construction) to establish several other theorems. The construction can be extended to consider multiple lens systems including reflecting systems.

King[7] pointed out that Young missed the opportunity of exploring the nature of aplanatic imagery (imagery free of spherical aberration and coma) with this construction, but Young was aware that the tangential aberration at a surface would disappear if the object was on the far side of the first circle with the image forming on the second circle (see end of Theorem 434). This particular pair of aplanatic points is widely used in the design of high-aperture, oil-immersion microscope objectives, where the refractions through the first two components of the lens are usually arranged to be aplanatic[10]. Dowell[11] and Smith[12] gave a number of variants of Young's graphical approach to finding refracted rays under different circumstances and graphical techniques remained in use by lens designers[13] until the availability of computers and suitable ray-tracing programs made them largely obsolete.

PARAXIAL REFRACTION

Young developed several theorems (413-424) to deal with "perpendicular" refraction/reflection at plane and spherical surfaces and for single lenses. In the latter case, lenses were treated as being of "evanescent" (negligible) thickness. The restriction of these theorems to the paraxial regime was emphasised by his careful note (Theorem 414) that his equations only apply "*for rays falling on a spherical surface nearly in the direction of the axis"*.

When discussing thin lenses he first derived the conjugate object/image relationship for a single surface (Theorem 414) and then applied it in succession to the two surfaces of a thin lens (Theorem 421). This gave him the familiar relationship between the object and image distances of a thin lens in air from which, by setting the object at infinity, he derived the power of the lens in a relationship which is now often known as the Lensmaker's equation. e.g.[14]. This describes the power F of a thin lens of radii of curvature r_1 and r_2 and of refractive index μ' immersed in a medium of refractive index μ. In modern terms it is

$$F = (\mu' - \mu)(1/r_1 - 1/r_2) \qquad (2)$$

For a lens in air, this equation becomes

$$F = (\mu' - 1)/(1/r_1 - 1/r_2) \qquad (3)$$

Other theorems dealt with the principal foci of mirrors, combinations of thin lenses in contact and the optical centres of lens.

Commentary

Following earlier authors, Young used geometrical methods to establish most of his basic relationships. However, whereas previously others had been content to allow their readers to use graphical methods to explore imagery in different specific situations, Young was one of the first to formulate appropriate algebraic equations. As Smith[6] remarked "*However*

informative graphical methods may be, they are inadequate when great accuracy is needed. The requirement of high accuracy is fully met by Young's algebraic representation of the relations he established."

Nevertheless, one difficulty of dealing with the theorems was his sign convention. Signs in equations might need to be changed depending upon whether a surface presented a concave or convex perspective to its respective object or image point. Nowadays, refraction equations are usually fixed, but with the sign of distances from the surfaces being altered as necessary. Two conventions are 1) that distances are positive if measured from a surface into the relevant medium, and 2) that distances are positive if measured from a surface in the direction of light travel. The latter is used commonly in ophthalmic optics.

It is interesting that Young effectively recognised all the cardinal points of a thin lens in air, since he defined both the focal points (e.g. Definition 416) and noted (Theorem 424) "*All rays, which in their passage through the lens, tend to the centre, are transmitted in a direction parallel to their original direction.*", thus introducing the important concept of the optical centre of a lens (i.e. the coincident nodal points of a thin lens in air). However, to find the position of the paraxial image of an off-axis object point, he suggested using a mixture of calculation and geometry. This involved first finding the image plane by substituting the longitudinal object distance in the lens-maker's equation and then graphically locating the off-axis image by using the undeviated ray through the optical centre, rather than, for example, graphically determining the image point as the intersection point after refraction by the lens of two rays from the object point, one through the optical centre and one initially parallel to the axis which after refraction passes through the second focal point. Perhaps surprisingly, he never discussed the question of transverse magnification explicitly nor developed appropriate equations for it.

ASTIGMATISM AND IMAGE CURVATURE

Theorems and definitions 426-436 dealt with tangential and sagittal imagery at surfaces and lenses, that is, the focusing of narrow beams of light originating from an off-axis object. Tangential imagery refers to the section of the beam which is in the same plane as the chief (central) ray of the beam and the optical axis, with Young using the term "peripheric focus" to refer to the image and, in the 1801 paper (page 81), referring to the "nearer focus". Sagittal imagery refers to the section of the beam perpendicular to the tangential section, with Young referring to "collateral" rays and the "radial" focus and, in the 1801 paper, to the 'remoter" focus. Young referred to what we would call the least circle of confusion as being dioptrically half way between the sagittal and tangential foci. The term astigmatism had not as yet been coined.

Conversion of his terms and signs is needed to give the sagittal and tangential equations in their modern forms. For a surface of radius of curvature r we have

$$\mu'\cos^2 i'/t' - \mu \cos^2 i/t = (\mu'\cos i' - \mu\cos i)/r \quad (4)$$

$$\mu'/s' - \mu/s = (\mu'\cos i' - \mu\cos i)/r \quad (5)$$

while for a thin lens of radii of curvature r_1 and r_2 we have

$$\mu'\cos^2 i'/t' - \mu\cos^2 i/t = (\mu'\cos i' - \mu\cos i)(1/r_1 - 1/r_2) \quad (6)$$

$$\mu'\cos i'/s' - \mu\cos i/s = (\mu'\cos i' - \mu\cos i)(1/r_1 - 1/r_2) \quad (7)$$

Here t, s, t' and s' are tangential and sagittal object and image distances from a surface along the chief ray, i and i' are angles of incidence and refraction, and μ and μ' are again refractive indices.

Young determined a useful position in relation to raytracing which he referred to as the relative centre (Figure 2). This is "*the point of intersection of the right lines joining any two pairs of conjugate peripheral foci of pencils of oblique rays, falling on the same point of a curved surface in the same direction*" (theorem 429 and Proposition IV of the 1801 paper). To obtain this point, perpendiculars are dropped from the centre of curvature to the incident ray (at **L**) and to the refracted ray (at **L′**). The line through **L′L** is extended, and the perpendicular dropped from the centre of curvature intersects the line at the relative centre **K**. Now lines drawn from object points **A** and **B** through **K** intercept the refracted ray at the respective image positions **A′** and **B′**. Figure 2 shows the tangential image situation – for the sagittal situation the relative centre is the centre of curvature.

In theorem 436, Young found that the locus of tangential foci for a distant object for a thin equiconvex lens in air is a spherical surface of radius $\mu f'/(3\mu + 1)$, while the sagittal surface has a radius of curvature of $\mu f'/(\mu + 1)$, and the mean radius of curvature of the image surfaces is $\mu f'/(2\mu + 1)$: f is the focal length.Young wrote "*It has been usual to neglect the effect of the obliquity and to consider the effect the focal length as the radius of curvature of the image ; but it is obvious that this image is extremely erroneous*." When discussing the camera obscura, Young[3] considered having a curved image surface or having a correction to counteract this (page 425) (see Levene[15] for further discussion).

Insert Figure 2 about here

Commentary

Young's work followed that of other contributions to astigmatism of oblique incidence in the 17th and 18th centuries, most notably by Isaac Barrow (1630-1677)[16] and Isaac Newton (1643-1727) (see reviews by Levene[17] and Kingslake[18]). Barrow had determined the position of the tangential focus and Newton had added the sagittal focus[7]. The tangential and sagittal equations (4)-(7) above are generally attributed to Henry Coddington from "*A treatise on the reflexion and refraction of light*"[19], but Coddington acknowledged the prior work of Young and George Airy (pages i and 203). As well as the problematic sign convention issue mentioned above, Young's developments of the equations are difficult to follow because of the archaic expression and sometimes inadequate explanation. Sometimes the radius of curvature of a surface was given a value of 1, which necessitated some confusing changes when distances were converted to true values and cosines and sines had to be manipulated.

The position of the stop of a optical system received little attention in Young's work, but he was aware of its importance: in his raytracing through the eye he was careful to set it at the anterior lens vertex[1]. The equations for the tangential and sagittal image surfaces of a thin biconvex lens in Theorem 436 are correct in third-order approximations only if the stop of the system is at the lens. If the stop is at a different position, such as the centre of rotation of the eye, the radii of curvature of these surfaces are affected in a quadratic manner by the lens shape (Figure 3). For most spectacle lens powers, there are two shapes for which there is no astigmatism and the tangential and sagittal surfaces coincide with the Petzval curvature surface.

Insert Figure 3 about here

Although Young had provided much of the necessary theory, it does not appear that his equations had much direct influence on the later design of ophthalmic lenses[7]. Wollaston, who was a collaborator with Young in some of his work in visual optics[1], ought to have been familiar with the latter's work on oblique astigmatism and in particular his use of a tilted lens to correct his ocular astigmatism. It would therefore seem reasonable to suggest that this inspired Wollaston to avoid the problem of astigmatism in his largely empirical development of deeply-meniscus "periscopic" lenses.[20,21] He stated[21] "*The object of my invention is to remedy the following defect which has been observed in spectacles hithertofore in use, namely, that no objects appear distinct through them, but such as are seen through the centres of the glasses or nearly so….the indistinctness is greater in proportion as the rays of light pass more obliquely through the glass; having observed that by making the substance of the glass curved in the manner of a hollow globe, each portion of it might be situated nearly at right angles to the direction of sight, and would thereby render lateral objects distinct without impairing the distinctness at the centres*." He went on to say that with his "globular" lens "*a small oblique pencil of light makes equal angles with the two surfaces of a thin lens: the inclination of it to each is so small that its focal length will not sensibly differ from that of a central pencil*".

While this appears to support Wollaston's use of Young's ideas, a later paper on lens design for camera obscuras[22] makes it clear that his concern about imagery with oblique pencils was motivated by worries about field curvature rather than oblique astigmatism. Justifying his use of deeply meniscus lenses he wrote, in regard to blur in off-axis images, "*The causes of this indistinctness may be considered as twofold: for in the first part, all parts of the (image) plane, excepting the central point, are at a greater distance from the centre of the lens than its principal focus; and secondly, the point f, to which any pencil of parallel rays passing obliquely through the lens are made to converge, is less distant than the*

principal focus". There was no mention here of focal lines or circles of least confusion. Thus it would appear that, in spite of his close familiarity with Young and his work, Wollaston failed to realise the importance of astigmatism in relation to spectacle lens design[7].

In fact we have found no evidence for any immediate, direct impact of Young's work on other authors. Airy's later study[23] in which he gave a theoretical basis for Wollaston's lenses, published in 1830 although it was known to Coddington in 1829, seems to have been undertaken independently. Developments in "best form" lenses had to await the work of Muller, Ostwald, Percival and Tscherning nearly 100 years later.[17] Perhaps it was necessary that substantial progress should first be made in understanding and correcting ocular astigmatism (see, e.g. Levene's section 4 for a review[17] before the significance of oblique astigmatism and field curvature in spectacle design could be fully appreciated.

GRADIENT INDEX

Young was well aware of variations of refractive index in the atmosphere and in the ocular lens. He was interested in this in the context of his studies of the eye, not only in relation to the additional power conferred by the presence of a gradient of refractive index in the lens but also because Ramsden had suggested that the gradient, and in particular possible index-matching at the surface of the crystalline lens, might help to reduce reflections and aberrations.[24] He modelled gradient index in at least two forms, one of which appeared only in the paper[1] and the other which appeared in both the paper and the second volume of the lectures.[4] As we have recently discussed Young's gradient index modelling in considerable detail[25], a summary only of this is provided.

For the form appearing only in the paper (Proposition VI, page 32), Young presented an equation, without any explanation, for an axial gradient index on one side of a surface that would provide aberration-free imagery for an object at infinity. The equation is

$$m(v) = \sqrt{(m_0^2 \pm 2nv)} \quad (8)$$

where the refractive index immediately inside the surface along the axis has a relative value of m_0/n to the surrounding medium, n remains the same, v is the versed sine, and $m(v)$ is the value of m inside the surface corresponding to the versed sine. The versed sine is 1 minus the cosine of the angle between the optical axis and the normal to the surface point of interest, and is the same as the sagitta for a radius of curvature of 1. The equation applies also to an equiconvex lens for which the object and image are equidistant from the lens. We were unable to derive equation (8), obtaining instead the equation[25]

$$m(v) = \sqrt{[m_0^2 - 2nv(m_0 - n)]} \quad (9)$$

Figure 4 (top) shows this situation for an object at infinity and a gradient index medium followed by a surface of radius of curvature 1 mm, and Figure 4 (bottom) shows the situation for the equivalent equiconvex lens with finite object and image conjugates. Within the gradient index lens the rays do not change direction as they travel along the direction of the axial index gradient: this would not be true for an off-axis object point or a different distance.

Insert Figure 4 about here

Young also described a spherical gradient index in the paper (Proposition VII, corrections on pages 83-84) and in theorem 465 of the Lectures, in which a spherical nucleus of fixed refractive index was surrounded by an annular cortex in which the index gradually changed in a radial direction to finally match the medium on the other side of a spherical surface (theorem 465, pages 82-83). This followed work describing refraction in thin spherical shells, which he carried out to explain atmospheric refraction effects (theorem 461, pages 80-81). He

assumed that inside the nucleus the refractive index is μ′ and in the surrounding medium the refractive index is μ. In the cortex the refractive index is given by

$$\mu_R = \mu(R/a)^q \quad (10)$$

where R is the distance from the centre of the sphere in any direction and q is given by

$$q = \log(\mu'/\mu)/\log(b/a) \quad (11)$$

where b is the semi-diameter of the nucleus and a is the semi-diameter of the sphere. The focal length relative to the lens centre is

$$f = (q + 1)(\mu'/\mu)ab/\{2q[(\mu'/\mu)b - a]\} \quad (12)$$

We verified that this equation is correct[25] and provided examples.

Figure 5 shows the refractive index distribution of Young′s model as a function of axial position in an ocular lens in which b is 0.9 mm, a is 1.8 mm, μ is 1.37 and μ′ is 1.41. For comparison we show a more physiological distribution in which the refractive index distribution changes parabolically from the centre of the lens, rather than showing abrupt discontinuities as assumed by Young.

Insert Figure 5 about here

Commentary

Young was aware of the influence of gradient index distribution on lens power. He considered that the gradient index of the ocular lens could have an important part in peripheral imagery of the human eye, writing in the 1801 paper (page 47): "*It would appear that nothing more is wanting for their perfect coincidence* [that of the circle of least confusion and the retina]*, than a moderate diminution of density* [i.e. refractive index] *in the*

lateral parts of the lens". We have not pursued this possibility with his model because of its unphysiological nature (Figure 5).

Studies of atmospheric refraction date back two thousand years and the topic continues to be of interest e.g.[26]. Few seem to be aware of Young's early struggles to understand the optics of index gradients. As far as we can establish, his work has received little or no acknowledgement from later authors: for example, his contributions are not mentioned in Marchand's major textbook on gradient index optics.[27] According to Marchand, the possibility of using inhomogeneous media in optical systems dates to Maxwell, who in 1854 demonstrated that a medium with a suitable refractive distribution can have the properties of a lens[28]. This was the so-called Maxwell fish eye lens, with an index having spherical symmetry about a point. Young's work predated Maxwell by over fifty years, and it is possible that Young was the first to deal mathematically with gradient index media.

Various proposals have been made since the time of Young to use index gradients in spectacle, contact, and intraocular lenses, as an alternative or supplement to changes in surface curvature as a way of changing the local optical path through a lens e.g.[29-35]. For example bifocal or varifocal lenses could be produced in this way[36-39]. Advances in free-form surfacing have reduced the need to consider the gradient-index alternative.

DISCUSSION

One problem for any reader of Young's work is that his derivations were sometimes terse to the point of being incomprehensible. We have sometimes found it difficult to be sure that we have correctly interpreted his meaning. This facet of his work was well recognised by his contemporaries. What is difficult to establish is the extent to which he made use of the work of earlier authors. In his first paper to include geometrical optics[1], when discussing Snell's law and transmission through a series of parallel plates he referred to the earlier work of

Barrow, Newton (both the *Opticks* and the *Principia*), Wood and Smith, as well as that of Huygens and Euler. However there were no further references, nor were there any in his later lectures[3]. It is, however, of interest that he followed Barrow, Huygens and Euler in challenging Newton's opinion that the velocity of light is higher in media of high refractive index, correctly preferring the view that the velocity is lower in the denser medium.

It is tantalising that Young did not develop some of his ideas more fully: many of them made little impact on his contemporaries and it is only with hindsight that we can appreciate their significance. His work on the wave theory of light and accommodation is probably only well known because of the attention paid to it by Fresnel and Helmholtz. However as he himself remarked in his "Autobiographical Sketch"[40] "*His own idea was that the faculties are more exercised, and therefore probably more fortified, by going a little beyond the rudiments only, and overcoming the great elementary difficulties, of a variety of studies, than by spending the same number of hours in any one pursuit: and it was generally more his object to cultivate his own mind rather than to acquire knowledge for others in departments which were not his immediate concern…*"

Young's contributions to the theory of astigmatism and to physical optics have long been acknowledged. His work as a pioneer in the still undeveloped field of gradient-index optics deserves to be more widely recognised. We still do not fully understand the nature or role of index gradient in the crystalline lens and it may be that man-made index gradients will play an increasing role in the corrective lenses of the future. Perhaps the last word on Young and his work should go to Helmholtz [41]: "*He was one of the most acute men who ever lived, but had the misfortune to be too far in advance of his contemporaries. They looked on him with astonishment, but could not follow his bold speculations, and thus a mass of his most important thoughts remained buried and forgotten….until a later generation by slow degrees*

arrived at the rediscovery of his discoveries, and came to appreciate the force of his arguments and the accuracy of his conclusions."

Figure Captions

Figure 1. Young's construction. See text for details. Here $\mu' > \mu$.

Figure 2. The relative centre for tangential imagery. Lines drawn from object points **A** and **B** through the relative centre **K** intercept the refracted ray at the respective image positions **A′** and **B′**.

Figure 3. Ratios of the radii of curvature of the astigmatic image surfaces to the focal length as a function of lens shape X for a thin +5 D lens in air. The object is at infinity and the lens has a refractive index of 1.5. The stop is either at the lens or 27 mm behind it, corresponding to the position of the centre of rotation of the eye. Lens shape X is defined in terms of the front and back surface powers F_1 and F_2 as $(F_1 - F_2)/(F_1 + F_2)$. The Petzval curvature surface is unaffected by lens shape and stop position relative to the lens. The tangential and sagittal image surfaces r_t and r_s are unaffected by lens shape if the stop is at the lens, with $r_t/f' = \mu/(3\mu + 1)$ and $r_s/f' = \mu/(\mu + 1)$ as given by Young in his Theorem 436[4]. However, when the stop is not at the lens, the tangential and sagittal image surfaces are influenced by lens shape. These plots were obtained from raytracing and the equations $r_p = -H^2/(\mu S_4)$, $r_t = -H^2/[\mu(S_3 + S_4)]$ and $r_s = -H^2/[\mu(S_3 + S_4)]$[42], where H is the optical invariant, S_3 is Seidel astigmatism and S_4 is Seidel curvature.

Figure 4. (Top) Refraction at a surface for an infinite object conjugate. The refractive index medium has a thickness of 0.5 mm and an axial distribution described by $1.1339 + 0.4861z + 0.3117z^2 + 0.2175z^3 + 0.1737z^4 + 0.1719z^5 + 0.1040z^6$, where z is the axial distance from the front surface of the medium. The surface has a radius of curvature of 1 mm, the refractive index in image space is 1, and $m_0/n = 1.5$. The distribution is determined from a polynomial fit to the results of equation (9).

(Bottom) The related situation for an equi-convex lens of thickness 1.0 mm. Here the back half of the lens has the same refractive index as given above, but relative to the centre of the lens, and the front half has the refractive index $1.4999 - 1.1210z + 1.2108z^2 - 1.2547z^3 + 0.9936z^4 - 0.4840z^5 + 0.1040z^6$ relative to the front vertex of the lens.

Figure 5. Refractive index distribution along the axis of Young's spherical gradient index lens model ($b = 0.9$ mm, $a = 1.8$ mm, $\mu = 1.37$ and $\mu' = 1.41$). A more physiological refractive index distribution of the form $\mu_R = \mu' - 0.012346R^2$ is shown for comparison, where R is the radial distance from the lens centre.

References

1 Young T. On the mechanism of the eye. *Phil Trans Royal Soc Lond* 1801; 91: 23-88 + plates.

2 Atchison DA, Charman WN. Thomas Young's contribution to visual optics: The Bakerian lecture "On the mechanism of the eye". *J Vis* 2010; 10(12): 16,1-16.

3 Young T. A course of lectures on natural philosophy and the mechanical arts. Vol. I, 1st ed. London: Joseph Johnson, 1807.

4 Young T. A course of lectures on natural philosophy and the mechanical arts. Vol. II, 1st ed. London: Joseph Johnson, 1807.

5 Young T. A course of lectures on natural philosophy and the mechanical arts, 2nd ed. London: Taylor and Walton, 1845.

6 Smith T. The contributions of Thomas Young to geometrical optics, and their application to present-day questions. *Proc Phys Soc B* 1949; 62: 619-629.

7 King HC. Thomas Young's contribution to geometrical optics. *Brit J Physiol Opt* 1953; 10: 39-47.

8 Freeman MH, Hull CC. Optics, 11th ed. London: Butterworth-Heinemann, pp. 61-62, 2003.

9 Tunnacliffe AH, Hirst JG. Optics, 2nd ed. London: Association of Dispensing Opticians, pp. 22-23, 207, 1996.

10 Born M, Wolf E. Principles of Optics, 6th ed. Oxford: Pergamon, pp. 149-150, 253, 1980.

11 Dowell JH. Graphical methods applied to the design of optical systems. *Proceedings of the Optical Convention, Part II* 1926: 965. Cited by Smith, T. (1930-1931).

12 Smith T. Graphical reconstructions for a refracted ray. *Trans Opt Soc* 1930-1931; 32: 150-158.

13 Lewis RE. Graphical ray-trace and surface generation methods for aspheric surfaces. *J Opt Soc Am* 1951; 41: 456-459.

14 Jalie M. Ophthalmic Lenses and Dispensing, 3rd ed. London: Butterworth-Heinemann, p. 9, 2008.

15 Levene JR. Thomas Young's contribution to curved image surface theory. *Optician* 1966; 151: 111-112.

16 Barrow I. Isaac Barrow's Optical Lectures (Lectiones XVIII: in quibus opticorum phaenomenon genuinae rationes investidantur, ac exponuntur. London: Godbid, 1669.) Fay HC, trans; Bennett AG, Edgar DF, eds. London: Worshipful Company of Spectacle Makers, 1987.

17 Levene JR. Clinical refraction and visual science. London: Butterworth. pp. 96-102, 1977.

18 Kingslake R. Who? discovered Coddington's equations? *Opt Photon News* 1994; 5: 20-23.

19 Coddington H. A treatise on the reflexion and refraction of light. Part 1 of A system of optics. London: Simpkin and Marshall, 1829.

20 Wollaston WH. LXI. On an improvement in the form of spectacle lenses. *Phil Mag (Series 1)* 1804; 17: 327-329.

21 Wollaston WH. Specifications of the patent granted to William Hyde Wollaston, of Buckingham Street, in the County of Middlesex, for "An improvement in spectacles, by the application of concave-convex glasses to them" (dated February 9, 1804). Cited by Klughardt. On some new forms of spectacle lenses. *Trans Opt Soc* 1913; 14: 101-141.

22 Wollaston WH. On a periscopic camera obscura and microscope. *Phil Trans Roy Soc Lond* 1812; 102: 370-377.

23 Airy GB. On the spherical aberration of the eye-pieces of telescopes. *Trans Camb Phil Soc* 1830; 3: 1-63.

24 Home E. The Croonian lecture on muscular motion. *Phil Trans Roy Soc Lond* 1795; 85: 1-23.

25 Atchison DA, Charman WN. Thomas Young's investigations in gradient-index optics. submitted.

26 White R. New solutions of the refraction integral. *J Opt Soc Am* 1975; 65: 676-678.

27 Marchand EW. Gradient index optics, pp. 2, 10-12. New York: Academic Press, 1978.

28 Maxwell JC. Some solutions of problems. *Cam Dubl Math J* 1854; 8: 188-195.

29 Atchison DA, Smith G. First- and third-order optical theory of gradient index materials, with application to contact lenses. *Optom Vis Sci* 1998; 75: 840-846.

30 Charman WN. Gradient index optics. *Ophthal Optician* 1981; 21: 72-84.

31 Charman WN. Gradient-index optics. *Can J Optom* 1981; 44: 164-171.

32 Fischer DJ. Gradient-index ophthalmic lens design and polymer material studies. In. Institute of Optics: University of Rochester, 2002.

33 Fowler CW, Pateras ES. A gradient-index ophthalmic lens based on Wood's convex pseudo-lens. *Ophthal Physiol Opt* 1990; 10: 262-270.

34 Siedlecki D, Kasprzak H, Pierscionek BK. Radial gradient index intraocular lens: a theoretical model. *J Mod Opt* 2008; 55: 639-647.

35 Koike Y, Asakawa A, Wu SP, Nihei E. Gradient-index contact lens. *Appl Opt* 1995; 34: 4669-4673.

36 Charman WN. Theoretical aspects of concentric varifocal lenses. *Ophthal Physiol Opt* 1981; 2: 75-86.

37 Linares J, Montero C, Prieto X. Graded-index bifocal spectacle lenses produced by ion-exchange in glass: paraxial designing. *Pure Appl Opt* 1995; 4: 695-699.

38 Shitanoki Y, Tagaya A, Koike Y. Application of graded-index materials for aberration correction of progressive index lenses. *SPIE Proc* 2009; 7213: 7213-7220.

39 Volk DA. Multi-layered gradient index progressive lens. World patent WO/2008/051592, 2 May 2008. 2008.

40 Hilts VL. Thomas Young's "autobiographical sketch". *Proc Am Phil Soc* 1978; 122: 248-260.

41 Helmholtz HH. Popular Scientific Lectures. New York: Dover, p. 133, 1962.

42 Smith G, Atchison DA. The eye and visual optical instruments. New York: Cambridge University Press, pp. 638-639, 1997.

Figures

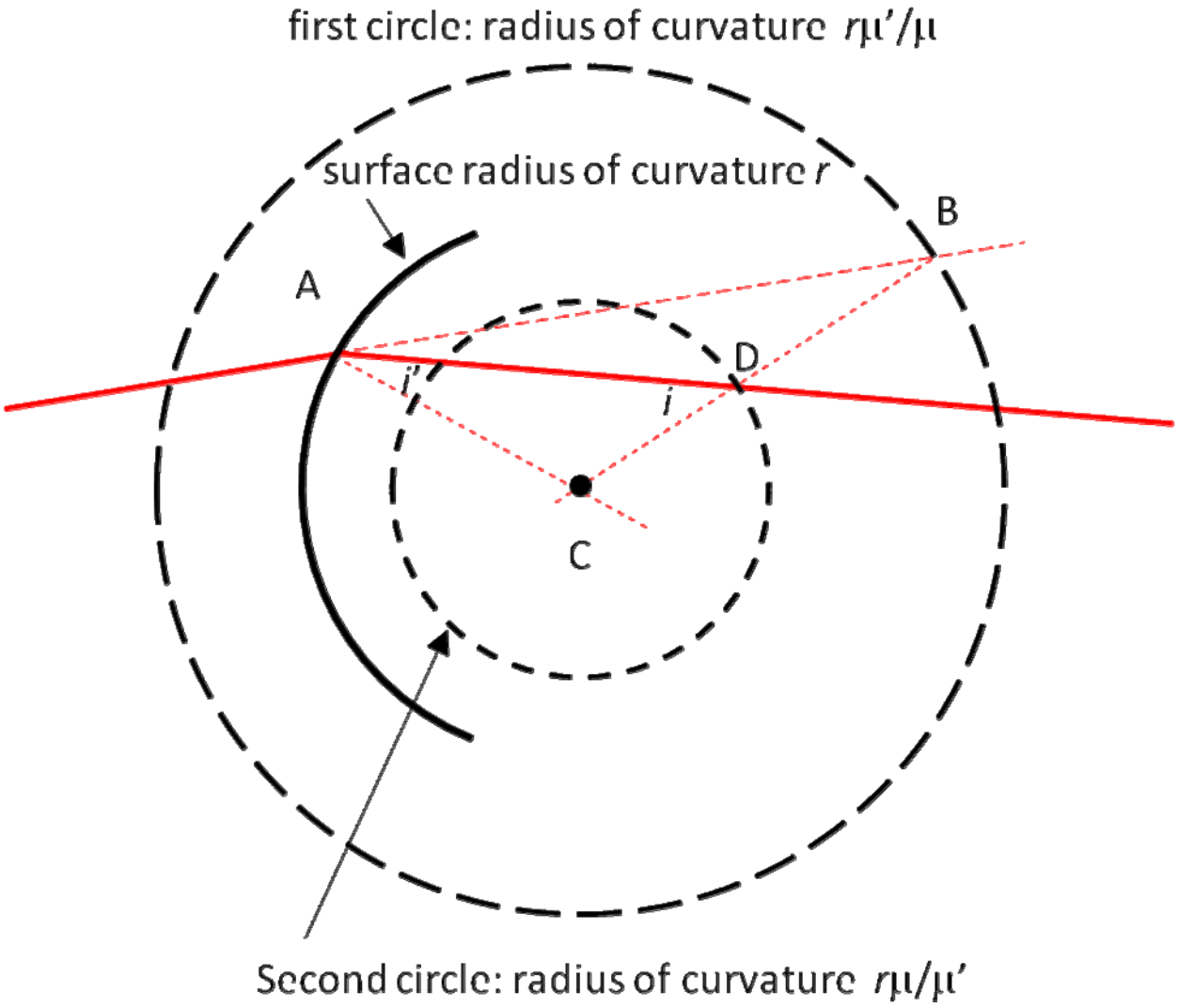

Figure 1

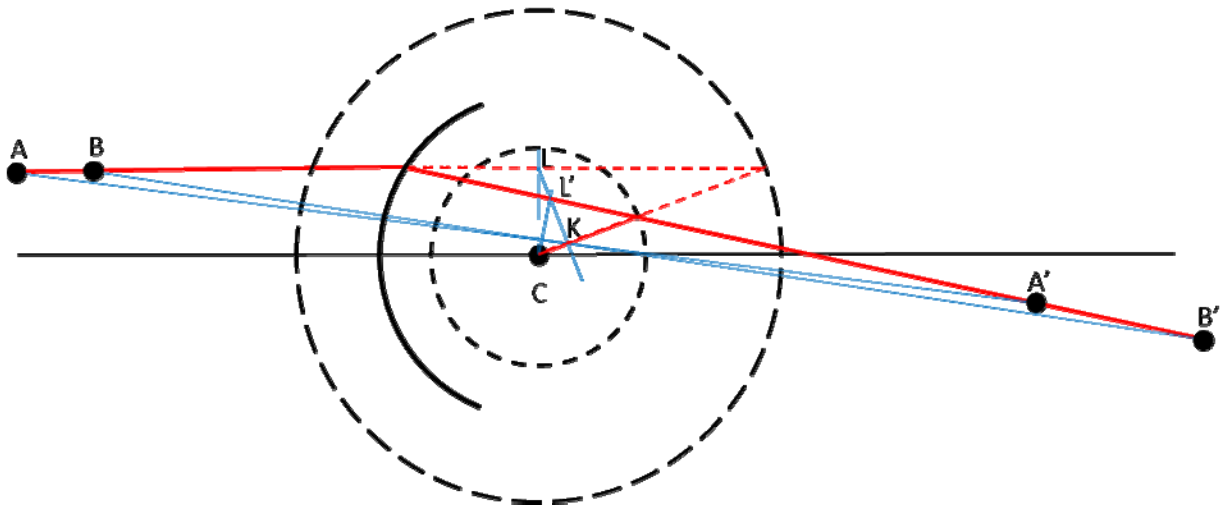

Figure 2

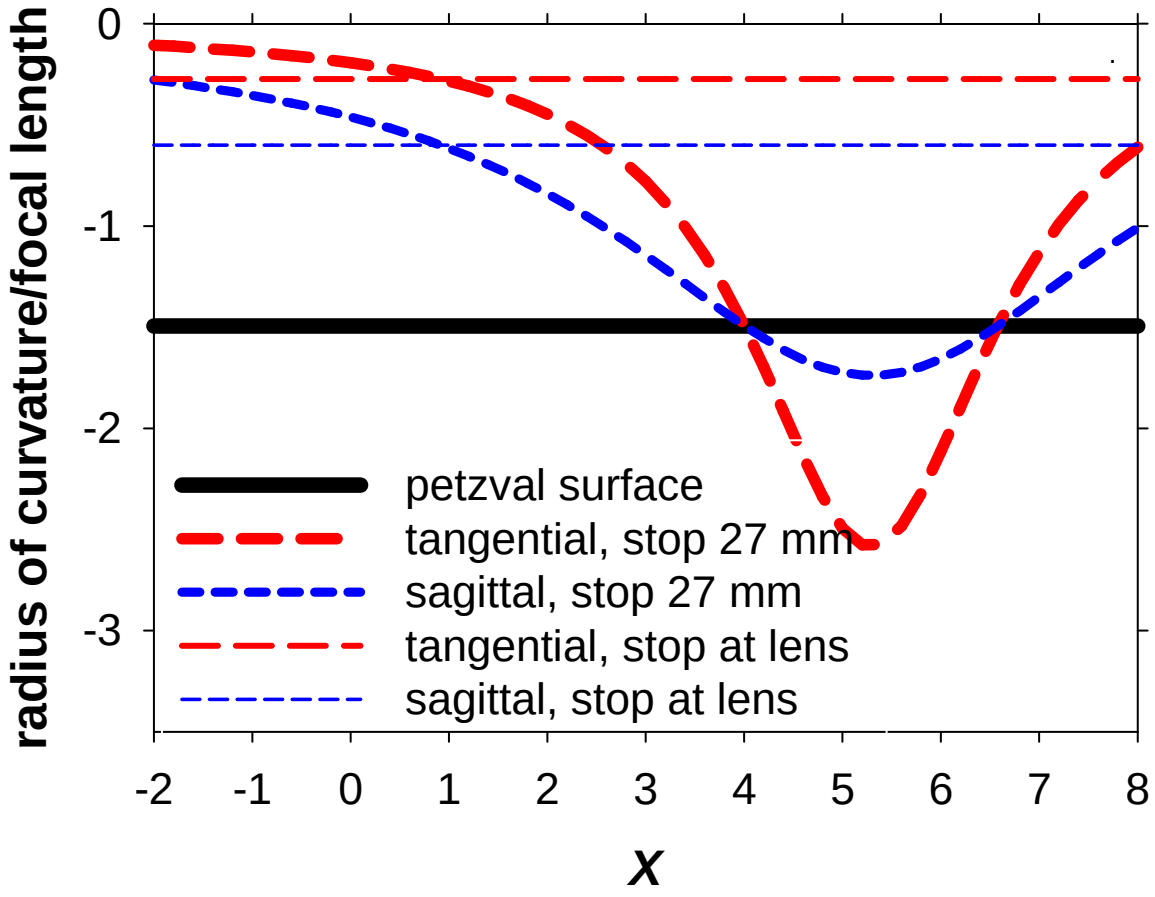
radius of curvature/focal length
0
-1
-2
-3
-2
-1
0
1
2
3
4
5
6
7
8
x
petzval surface
tangential, stop 27 mm
sagittal, stop 27 mm
tangential, stop at lens
sagittal, stop at lens

Figure 3

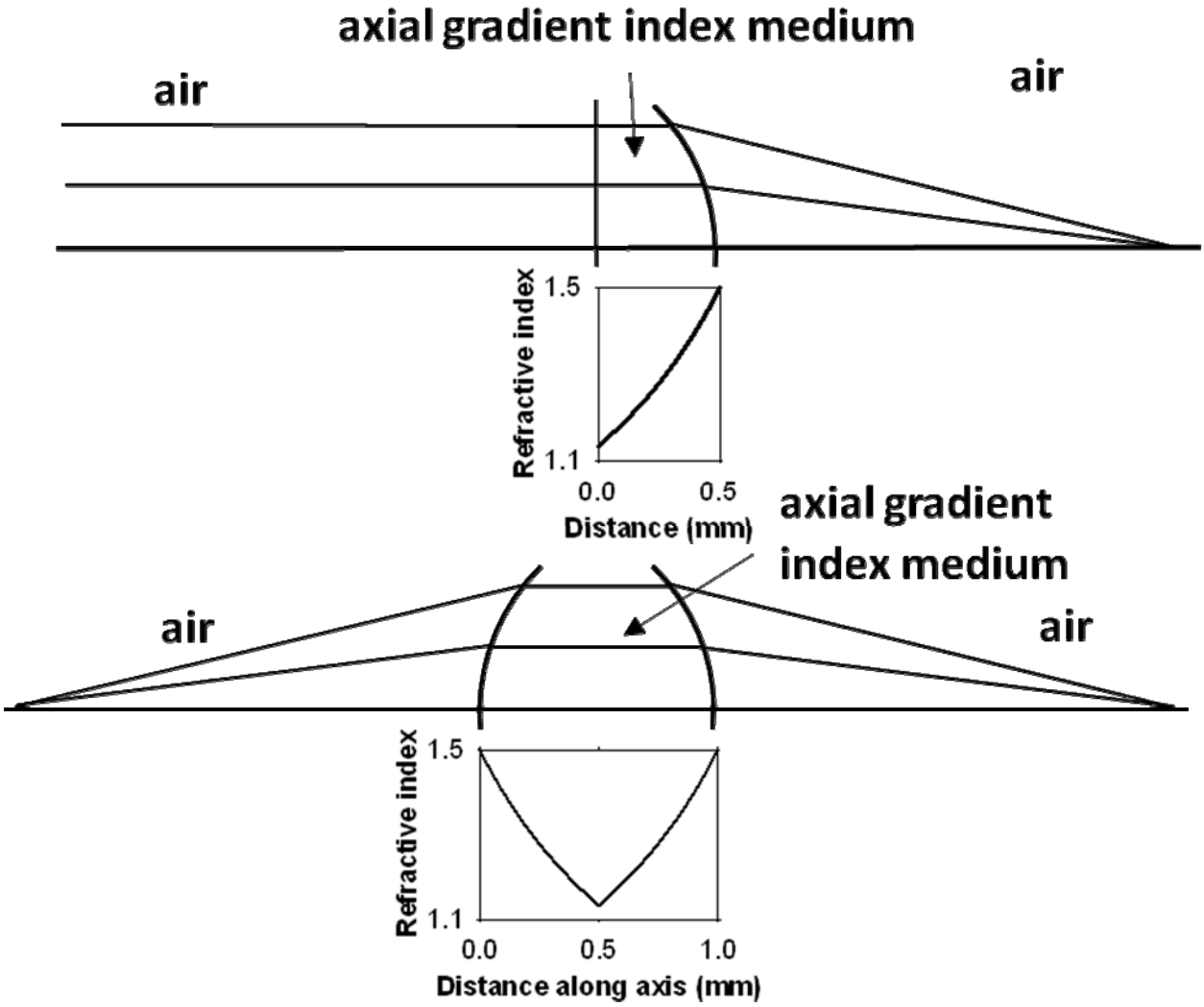
axial gradient index medium
air
air
Refractive index
1.5
1.1
0.0
0.5
Distance (mm)
axial gradient
index medium
air
air
Refractive index
1.5
1.1
0.0
0.5
1.0
Distance along axis (mm)

Figure 4

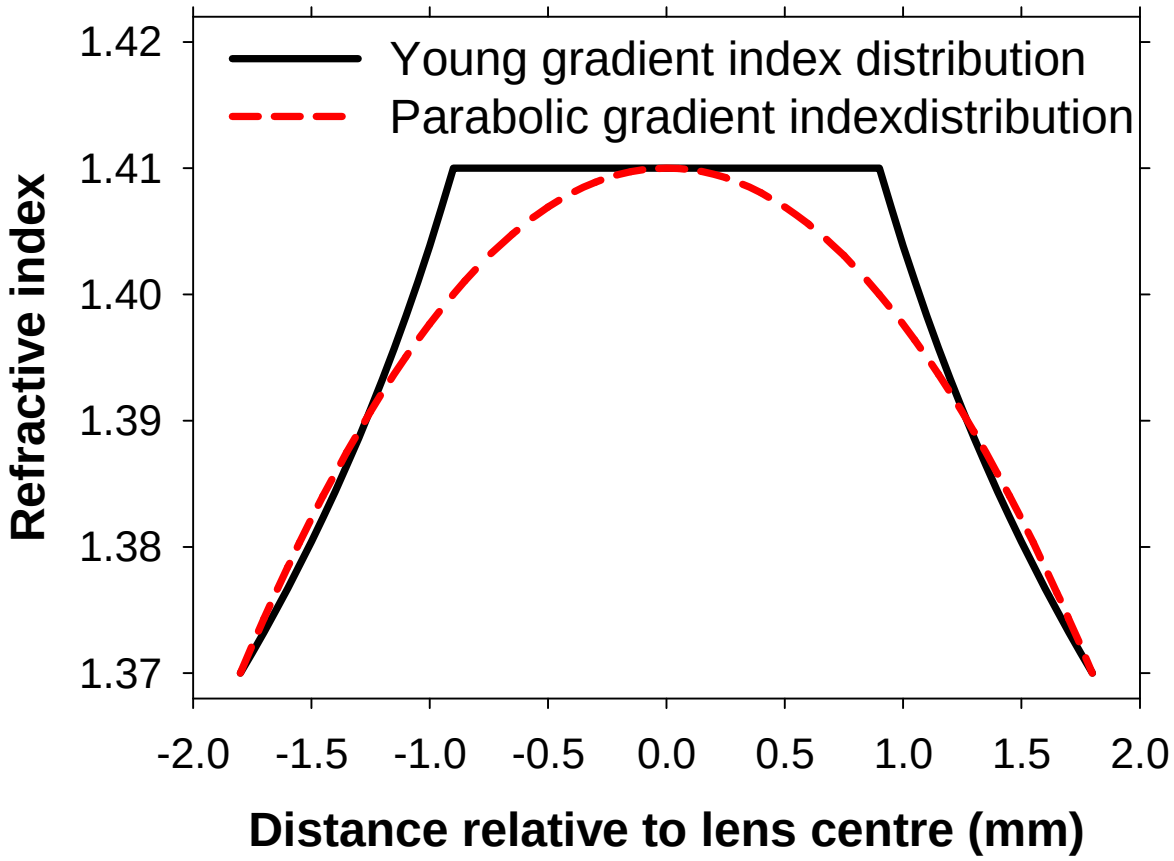
Young gradient index distribution
Parabolic gradient indexdistribution
Refractive index
1.42
1.41
1.40
1.39
1.38
1.37
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
Distance relative to lens centre (mm)

Figure 5