Lecture 2

Introduction to Data Flow Analysis

I. Introduction

II. Example: Reaching definition analysis

III. Example: Liveness analysis

IV. A General Framework
(Theory in next lecture)

Reading: Chapter 9.2

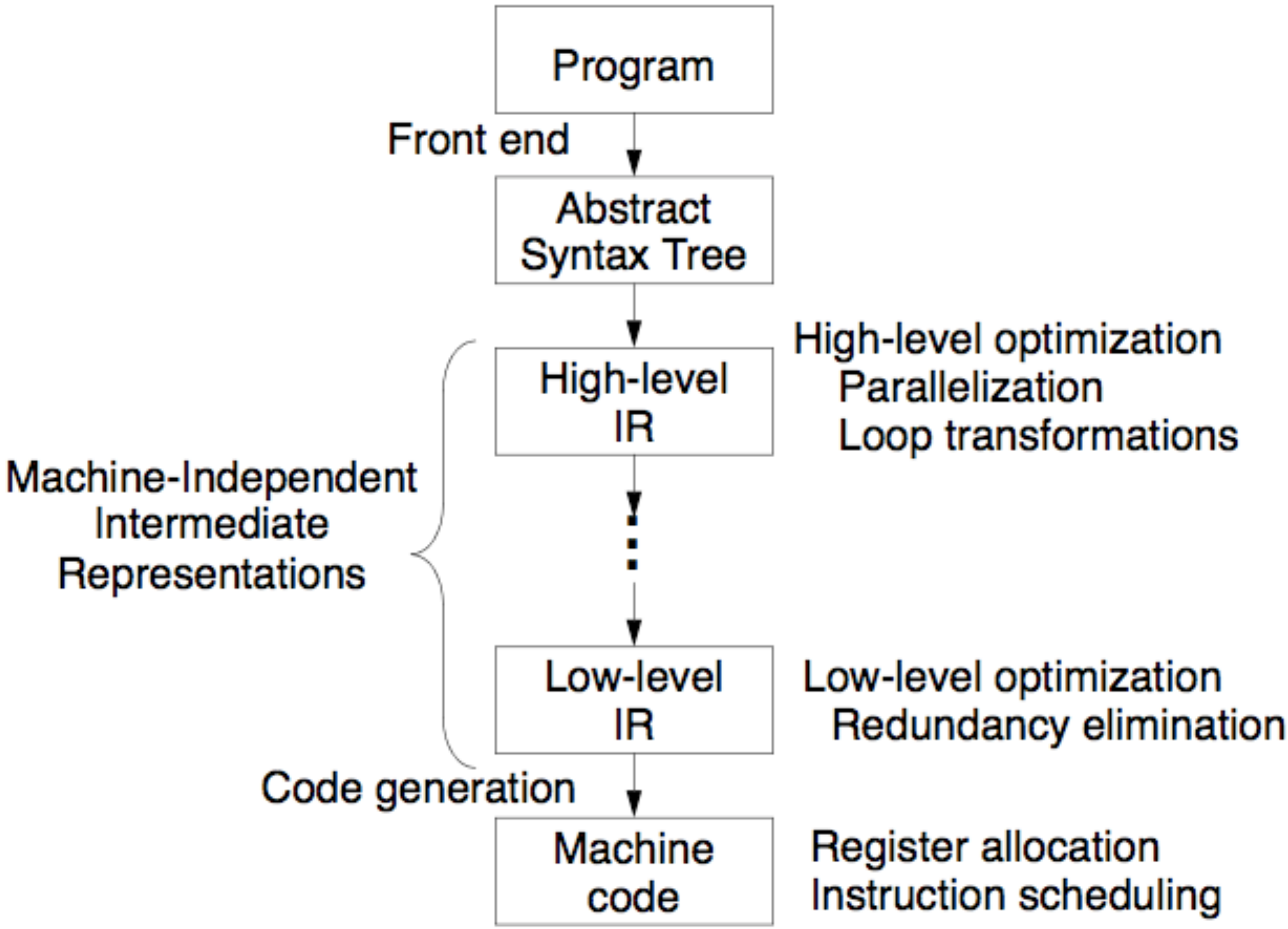
Overview of Data Flow Lectures 2-5

- High-level programming languages generate a lot of redundancy
- Many useful optimizations independently developed originally
  - Constant propagation
  - Common subexpressions
  - Loop invariant code motion
  - Dead code elimination
- A common framework: Dataflow (recurrent equations, fixed-points)
  - Theory: prove properties on the framework
  - Software engineering:
    implement / debug / optimize framework once
- Plan:
  - L2: Basic examples to build intuition about dataflow
  - L3: Theory
  - L4: Optimization examples
  - L5: Partial redundancy elimination (PRE)
    Subsumes multiple optimizations by setting up 4 DataFlow problems

Practice Today

- Many compilers use SSA (static single assignment) – an abstraction on top of dataflow
- Idea to be covered by the homework
- Useful for many optimizations, but cannot naturally support PRE

I. Compiler Organization

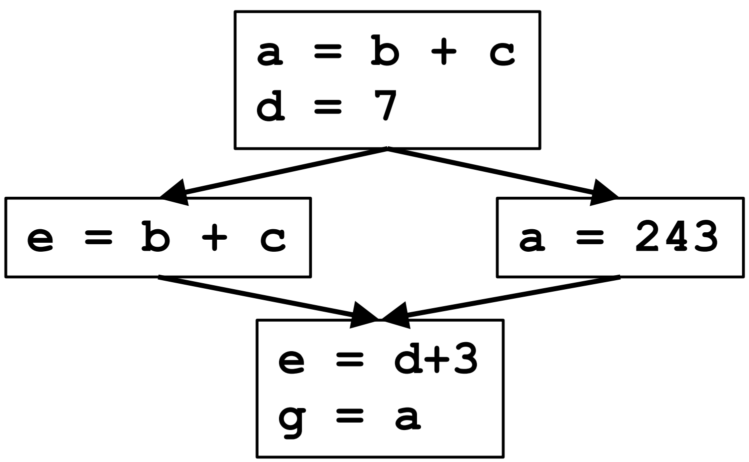


Flow Graph

- **Basic block = a maximal sequence of consecutive instructions s.t.**
  - flow of control only enters at the beginning
  - flow of control can only leave at the end
    (no halting or branching except perhaps at end of block)
- **Flow Graphs**
  - Nodes: basic blocks
  - Edges
    - *B_i --> B_j, iff B_j can follow B_i immediately in execution*

What is Data Flow Analysis?

- **Data flow analysis:**
  - Flow-sensitive: sensitive to the control flow in a function
  - intraprocedural analysis
- **Examples of optimizations:**
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination

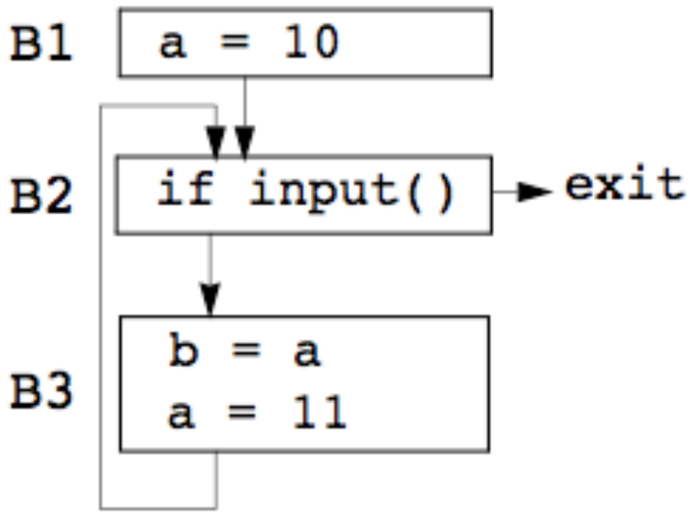


Value of *x*?

Which "definition" defines *x*?

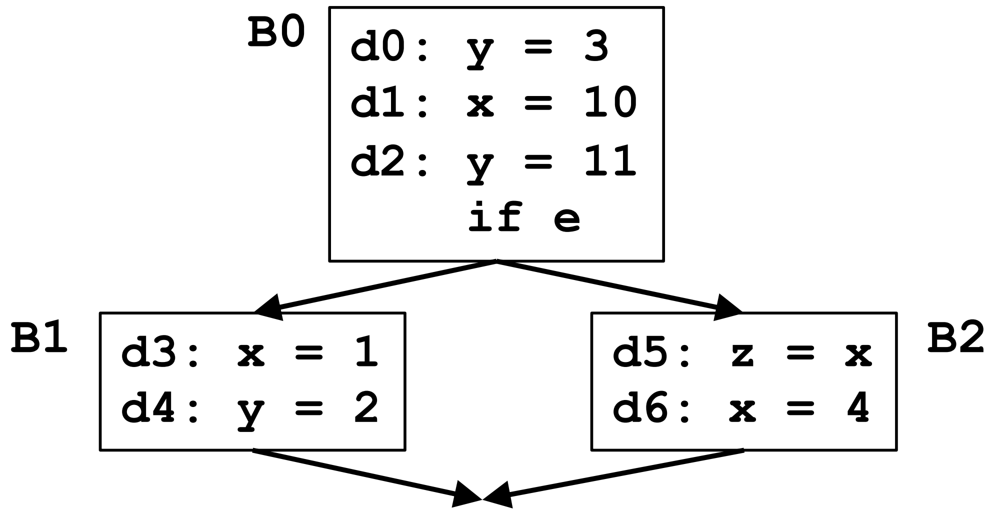
Is the definition still meaningful (live)?

Static Program vs. Dynamic Execution



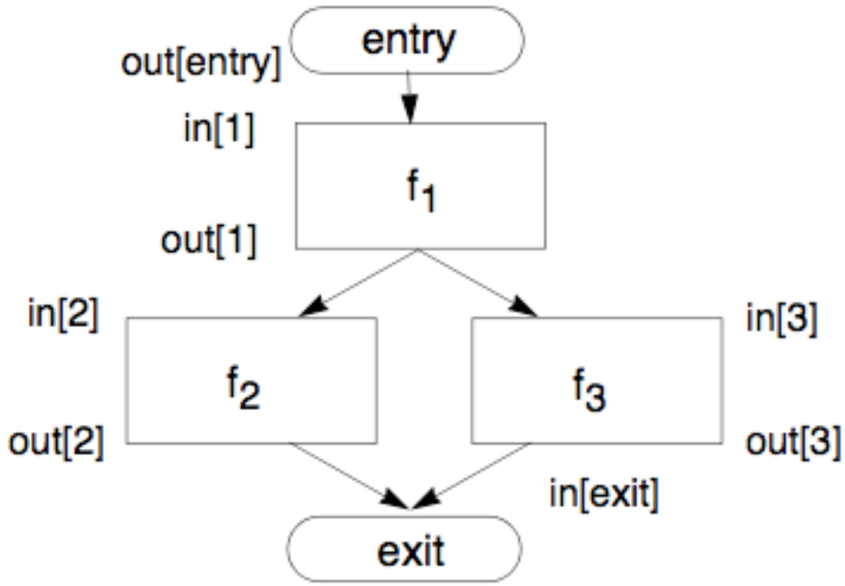
- **Statically**: Finite program
- **Dynamically**: Can have infinitely many possible execution paths
- **Data flow analysis abstraction:**
  - For each point in the program: combines information of all the instances of the same program point.
- **Example of a data flow question:**
  - Which definition defines the value used in statement "b = a"?

Reaching Definitions



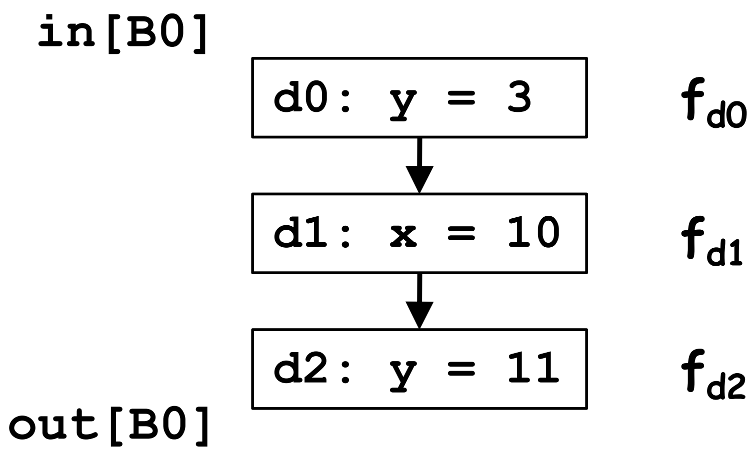
- Every assignment is a definition
- *A* **definition** *d* **reaches** a point *p*
  if **there exists** path from the point immediately following *d* to *p*
  such that *d* is not killed (overwritten) along that path.
- Problem statement
  - For each point in the program, determine
    if each definition in the program reaches the point
  - A bit vector per program point, vector-length = #defs

Data Flow Analysis Schema



- Build a flow graph (nodes = basic blocks, edges = control flow)
- Set up a set of equations between in[b] and out[b] for all basic blocks b
  - Effect of code in basic block:
    - Transfer function f_b relates in[b] and out[b], for same b
  - Effect of flow of control:
    - relates out[b_1], in[b_2] if b_1 and b_2 are adjacent
- Find a solution to the equations

Effects of a Statement



- f_s : A transfer function of a statement
  - abstracts the execution with respect to the problem of interest
- For a statement s (d: x = y + z)
  out[s] = f_s(in[s]) = Gen[s] U (in[s]-Kill[s])
  - **Gen[s]**: definitions generated: Gen[s] = {d}
  - **Propagated** definitions: in[s] - Kill[s],
    where **Kill[s]**=set of all other defs to x in the rest of program

Effects of a Basic Block

`in[B0]`

```
d0: y = 3
d1: x = 10
```

f_{d0}

f_{d1} $\quad f_B = f_{d1} \cdot f_{d0}$

`out[B0]`

- Transfer function of a statement s:
  - $out[s] = f_s(in[s]) = Gen[s] \cup (in[s] - Kill[s])$
- Transfer function of a basic block B:
  - Composition of transfer functions of statements in B
- $out[B] = f_B(in[B])$

  $= f_{d1} f_{d0}(in[B])$

  $= Gen[d_1] \cup (Gen[d_0] \cup (in[B] - Kill[d_0])) - Kill[d_1])$

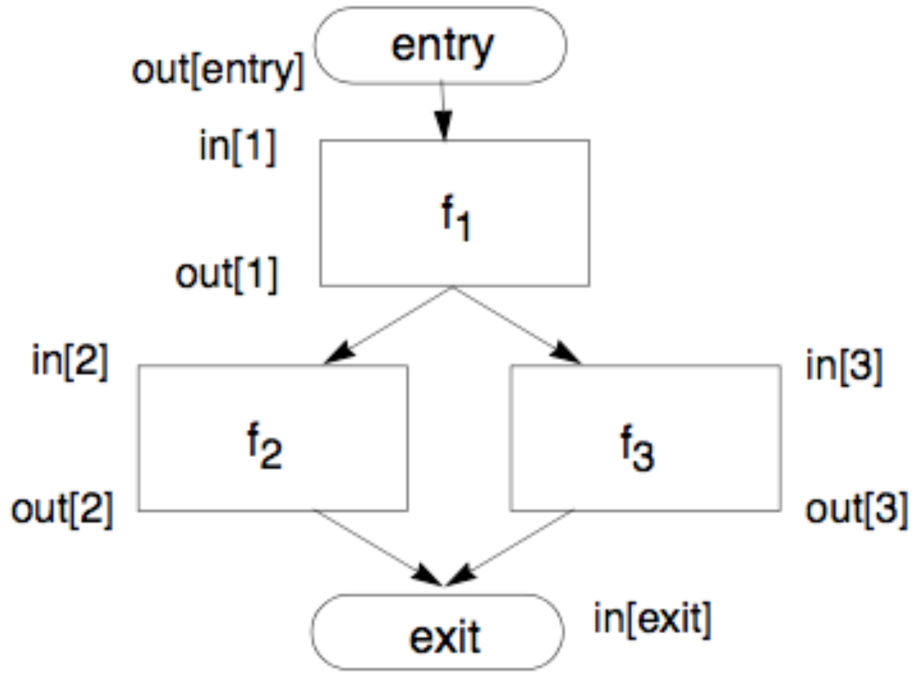
  $= (Gen[d_1] \cup (Gen[d_0] - Kill[d_1])) \cup in[B] - (Kill[d_0] \cup Kill[d_1])$

  $= Gen[B] \cup (in[B] - Kill[B])$

  Gen[B]: locally exposed definitions (available at end of bb)

  Kill[B]: set of definitions killed by B

Effects of the Edges (acyclic)

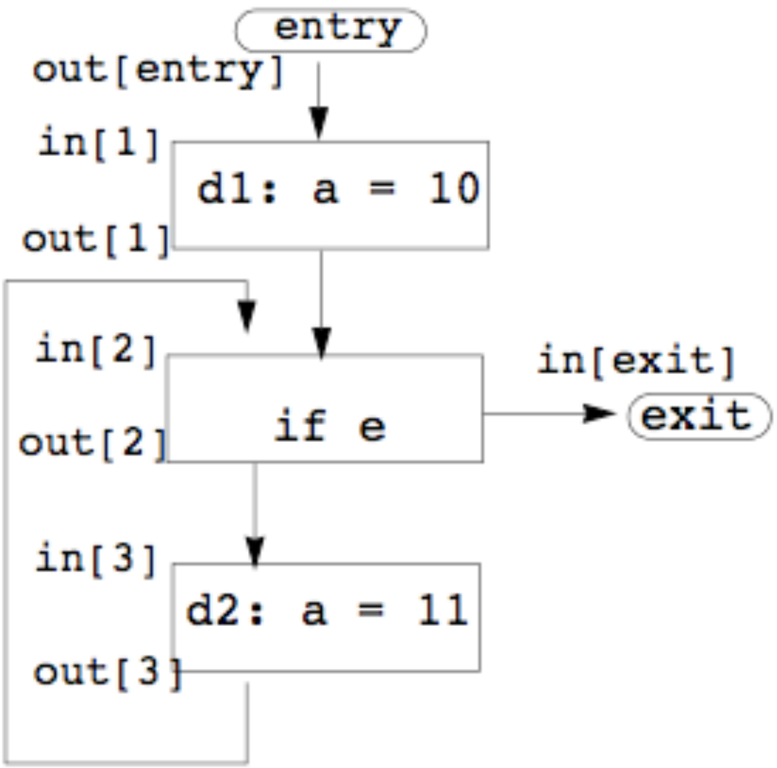


- Join node: a node with multiple predecessors
- **meet** operator ($\wedge$): ∪

  in[b] = out[p_1] ∪ out[p_2] ∪ ... ∪ out[p_n], where

  p_1, ..., p_n are all predecessors of b

Cyclic Graphs



- Equations still hold
  - out[b] = f_b(in[b])
  - in[b] = out[p_1] U out[p_2] U ... U out[p_n], p_1, ..., p_n pred.
- Find: fixed point solution

Reaching Definitions: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   out[Entry] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Entry
      out[B] = ∅

// iterate
   While (Changes to any out[] occur) {
      For each basic block B other than Entry {
         in[B] = ∪ (out[p]), for all predecessors p of B
         out[B] = f_B(in[B])    // out[B]=gen[B]∪(in[B]-kill[B])
      }
```

Summary of Reaching Definitions

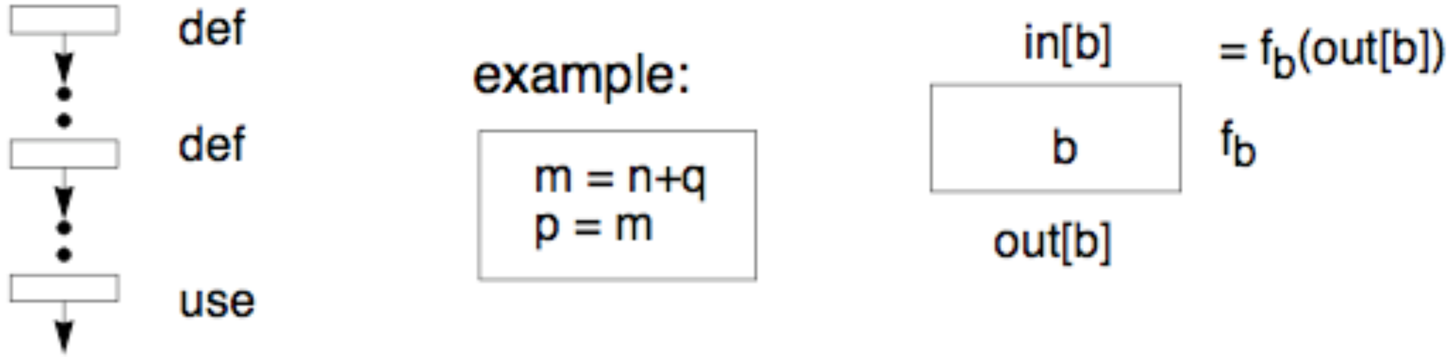
	Reaching Definitions
Domain	Sets of definitions
Transfer function $f_b(x)$	forward: $out[b] = f_b(in[b])$ $f_b(x) = Gen_b \cup (x - Kill_b)$ Gen_b: definitions in b $Kill_b$: killed defs
Meet Operation	$in[b] = \cup$ out[predecessors]
Boundary Condition	$out[entry] = \varnothing$
Initial interior points	$out[b] = \varnothing$

III. Live Variable Analysis

- **Definition**
  - A variable **v** is **live** at point *p* if
    - the value of **v** is used along some path in the flow graph starting at *p*.
  - Otherwise, the variable is **dead**.
- **Problem statement**
  - For each basic block
    - determine if each variable is live in each basic block
  - Size of bit vector: one bit for each variable

Effects of a Basic Block (Transfer Function)

- **Observation: Trace uses back to the definitions**

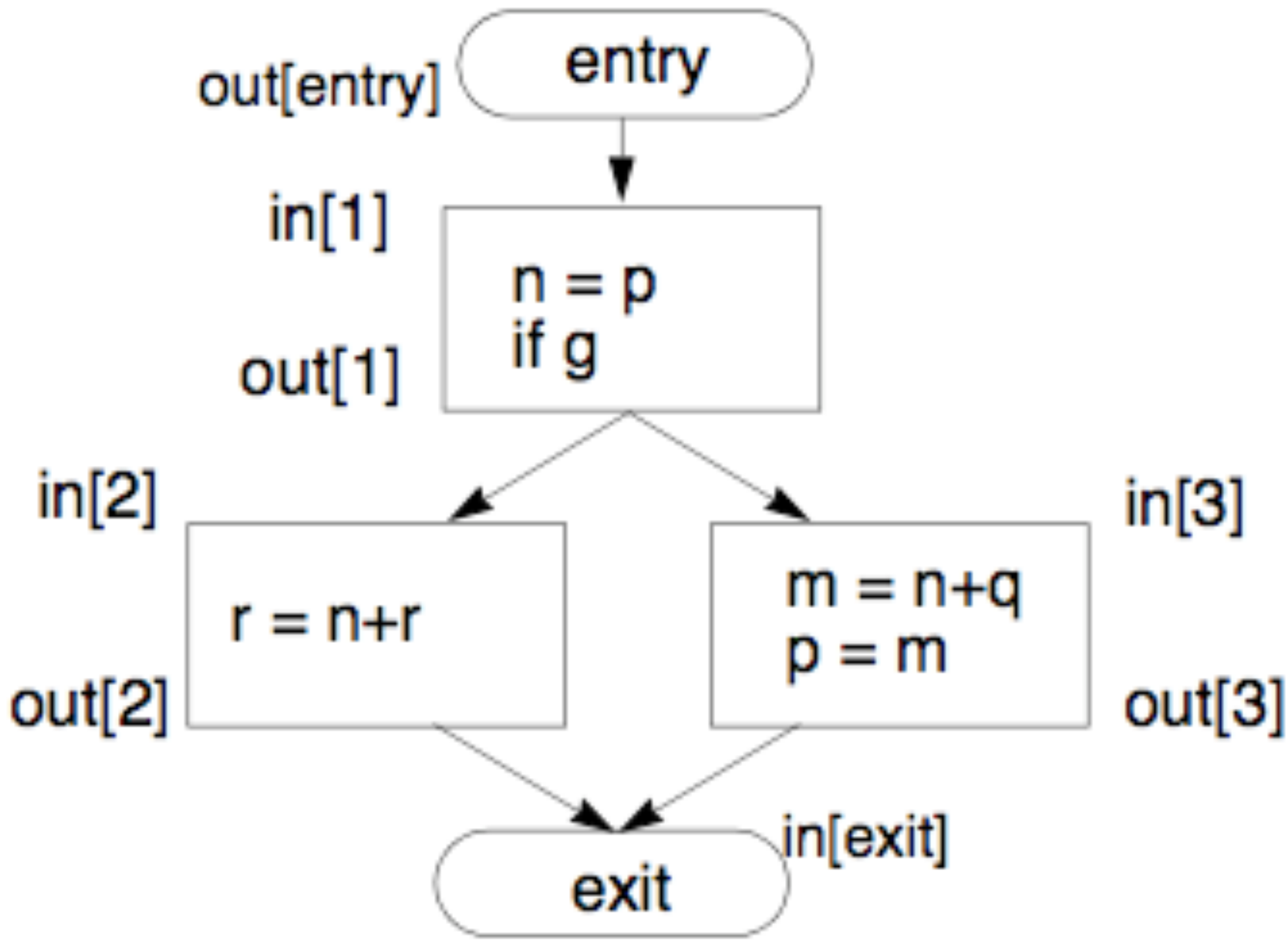


- **Direction: backward: in[b] = f_b(out[b])**
- **Transfer function** for statement s: x = y + z
  - generate live variables: Use[s] = {y, z}
  - propagate live variables: out[s] - Def[s], Def[s] = x
  - in[s] = Use[s] ∪ (out(s)-Def[s])
- **Transfer function** for basic block b:
  - in[b] = Use[b] ∪ (out(b)-Def[b])
  - Use[b], set of locally exposed uses in b, uses not covered by definitions in b
  - Def[b]= set of variables defined in b.

Across Basic Blocks

- **Meet operator ($\wedge$):**
  - $out[b] = in[s_1] \cup in[s_2] \cup ... \cup in[s_n]$, $s_1, ..., s_n$ are successors of b
- **Boundary condition:**

Example



entry
out[entry]
in[1]
n = p
if g
out[1]
in[2]
r = n+r
out[2]
in[3]
m = n+q
p = m
out[3]
in[exit]
exit

Liveness: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   in[Exit] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Exit
      in[B] = ∅

// iterate
   While (Changes to any in[] occur) {
      For each basic block B other than Exit {
         out[B] = ∪ (in[s]), for all successors s of B
         in[B] = f_B(out[B])    // in[B]=Use[B]∪(out[B]-Def[B])
      }
```

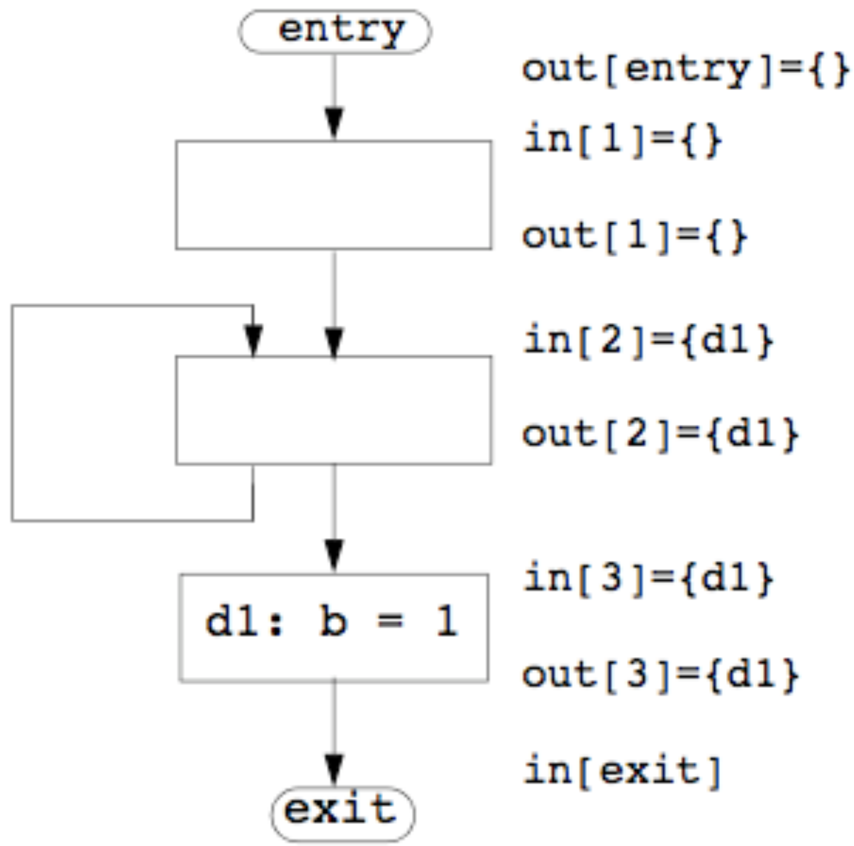
IV. Framework

	Reaching Definitions	**Live Variables**
Domain	Sets of definitions	Sets of variables
Direction	forward: $out[b] = f_b(in[b])$ $in[b] = \wedge\ out[pred(b)]$	backward: $in[b] = f_b(out[b])$ $out[b] = \wedge\ in[succ(b)]$
Transfer function	$f_b(x) = Gen_b \cup (x - Kill_b)$	$f_b(x) = Use_b \cup (x - Def_b)$
Meet Operation ($\wedge$)	$\cup$	$\cup$
Boundary Condition	$out[entry] = \emptyset$	$in[exit] = \emptyset$
Initial interior points	$out[b] = \emptyset$	$in[b] = \emptyset$

Thought Problem 1. "Must-Reach" Definitions

- **A definition D (a = b+c) <u>must</u> reach point P iff**
  - D appears at least once along on all paths leading to P
  - a is not redefined along any path after last appearance of D and before P
- **How do we formulate the data flow algorithm for this problem?**

Problem 2: A legal solution to (May) Reaching Def?



- Will the worklist algorithm generate this answer?

Problem 3. What are the algorithm properties?

- **Correctness**
- **Precision: how good is the answer?**
- **Convergence: will the analysis terminate?**
- **Speed: how long does it take?**