Dirichlet, Gustav Peter Lejeune | Encyclopedia.com

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(*b*. Düren, Germany, 13 February 1805; *d*. Göttingen, Germany, 5 May 1859)

mathematics.

Dirichlet, the son of the town postmaster, first attended public school, then a private school that emphasized Latin. He was precociously interested in mathematics; it is said that before the age of twelve he used his pocket money to buy mathematical books. In 1817 he entered the Gymnasium in Bonn. He is reported to have been an unusually attentive and well-behaved pupil who was particularly interested in modern history as well as in mathematics.

After two years in Bonn, Dirichlet was sent to a Jesuit college in Cologne that his parents preferred. Among his teachers was the physicist Georg Simon Ohm, who gave him a thorough grounding in theoretical physics. Dirichlet completed his *Abitur* examination at the very early age of sixteen. His parents wanted him to study law, but mathematics was already his chosen field. At the time the level of pure mathematics in the German universities was at a low ebb: Except for the formidable Carl Gauss in Göttingen, there were no outstanding mathematicians, while in Paris the firmament was studded by such luminaries as P.-S. Laplace, Adrien Legendre, Joseph Fourier, Siméon Poisson, Sylvestre Lacroix, J.-B. Biot, Jean Hachette, and Francoeur.

Dirichlet arrived in Paris in May 1822. Shortly afterward he suffered an attack of smallpox, but it was not serious enough to interrupt for long his attendance at lectures at the Collège de France and the Faculté des Sciences. In the summer of 1823 he was fortunate in being appointed to a well-paid and pleasant position as tutor to the children of General Maximilien Fay, a national hero of the Napoleonic wars and then the liberal leader of the opposition in the Chamber of Deputies. Dirichlet was treated as a member of the family and met many of the most prominent figures in French intellectual life. Among the mathematicians, he was particularly attracted to Fourier, whose ideas had a strong influence upon his later works on trigonometric series and mathematical physics.

Dirichlet’s first interest in mathematics was number theory. This interest had been awakened through an early study of Gauss’s famous *Disquisitiones arithmeticae* (1801), until then not completely understood by mathematicians. In June 1825 he presented to the French Academy of Sciences his first mathematical paper, “Mémoire sur l’impossibilité de quelques équations indéterminées du cinquième degré.” It dealt with Diophantine equations of the form

$$x^5+y^5=A.z^5$$

using algebraic number theory, Dirichlet’s favorite field throughout his life. By means of the methods developed in this paper Legendre succeeded, only a few weeks later, in giving a complete proof that Fermat’s equation

$$x^n+y^n=z^n$$

has no integral solutions ($x.y.z\neq0$) for $n = 5$. Until then only the cases $n = 4$ (Fermat) and $n = 3$ (Euler) had been solved.

General Fay died in November 1825, and the next year Dirichlet decided to return to Germany, a plan strongly supported by Alexander von Humboldt, who worked for the strengthening of the natural sciences in Germany. Dirichlet was permitted to qualify for habilitation as *Privatdozent* at the University of Breslau; since he did not have the required doctorate, this was awarded *honoris causa* by the University of Cologne. His habilitation thesis dealt with polynomials whose prime divisors belong to special arithmetic series. A second paper from this period was inspired by Gauss’s announcements on the biquadratic law of reciprocity.

Dirichlet was appointed extraordinary professor in Breslau, but the conditions for scientific work were not inspiring. In 1828 he moved to Berlin, again with the assistance of Humboldt, to become a teacher of mathematics at the military academy. Shortly afterward, at the age of twenty-three, he was appointed extraordinary (later ordinary) professor at the University of Berlin. In 1831 he became a member of the Berlin Academy of Sciences, and in the same year he married Rebecca Mendelssohn-Bartholdy, granddaughter of the philosopher Moses Mendelssohn.

Dirichlet spent twenty-seven years as a professor in Berlin and exerted a strong influence on the development of German mathematics through his lectures, through his many pupils, and through a series of scientific papers of the highest quality that he published during this period. He was an excellent teacher, always expressing himself with great clarity. His manner was modest; in his later years he was shy and at times reserved. He seldom spoke at meetings and was reluctant to make public appearances. In many ways he was a direct contrast to his lifelong friend, the mathematician Karl Gustav Jacobi.

The two exerted some influence upon each other's work, particularly in number theory. When, in 1843, Jacobi was compelled to seek a milder climate for reasons of health, Dirichlet applied for a leave of absence and moved with his family to Rome. A circle of leading German mathematicians gathered around the two. Dirichlet remained in Italy for a year and a half, visited Sicily, and spent the second winter in Florence.

Dirichlet's first paper dealing with Fermat's equation was inspired by Legendre; he returned only once to this problem, showing the impossibility of the case $n = 14$. The subsequent number theory papers dating from the early years in Berlin were evidently influenced by Gauss and the *Disquisitiones*. Some of them were improvements on Gauss's proofs and presentation, but gradually Dirichlet cut much deeper into the theory. There are papers on quadratic forms, the quadratic and biquadratic laws of reciprocity, and the number theory of fields of quadratic irrationalities, with the extensive discussion of the Gaussian integers $a + ib$, where and a and b are integers.

At a meeting of the Academy of Sciences on 27 July 1837, Dirichlet presented his first paper on analytic number theory. In this memoir he gives a proof of the fundamental theorem that bears his name: Any arithmetical series of integers

$an + b, n = 0, 1, 2, ...,$

where a and b are relatively prime, must include an infinite number of primes. This result had long been conjectured and Legendre had expended considerable effort upon finding a proof, but it had been established only for a few special cases, such as

$\{4n + 1\} = 1, 5, 9, 13, 17, 21, ...$

$\{4n + 3\} = 3, 7, 11, 15, 19, 23, ...,$

The paper on the primes in arithmetic progressions was followed in 1838 and 1839 by a two-part paper on analytic number theory, "Recherches sur diverses applications de l'analyse infinitésimale à la théorie des nombres." Dirichlet begins with a few general observations on the convergence of the series now called Dirichlet series. The main number theory achievement is the determination of the formula for the class number for quadratic forms with various applications. Also from this period are his studies on Gaussian sums.

These studies on quadratic forms with rational coefficients were continued in 1842 in an analogous paper on forms with coefficients that have Gaussian coefficients. It contains an attempt at a systematic theory of algebraic numbers when the prime factorization is unique, although it is restricted to Gaussian integers. It is of interest to note that here one finds the first application of Dirichlet's *Schubfachprinzip* ("box principle"). This deceptively simple argument, which plays an important role in many arguments in modern number theory, may be stated as follows: If one distributes more than n objects in n boxes, then at least one box must contain more than one object.

It is evident from Dirichlet's papers that he searched very intently for a general algebraic number theory valid for fields of arbitrary degree. He was aware of the fact that in such fields there may not be a unique prime factorization, but he did not succeed in creating a substitute for it: the ideal theory later created by Ernst Kummer and Richard Dedekind or the form theory of Leopold Kronecker.

Dirichlet approached the problem through a generalization of the quadratic forms, using the properties of decomposable forms representable as the product of linear forms, a method closely related to the method later used by Kronecker. One part of algebraic number theory, the theory of units, had its beginning in Dirichlet's work. He had earlier written a number of papers on John Pell's equation

$x^2 – Dy = N,$

with particular consideration of the cases in which $N = \pm 1$ corresponds to the units in the quadratic field But in the paper "Zur Theorie der complexen Einheiten," presented to the Berlin Academy on 30 March 1846, he succeeded in establishing the complete result for the Abelian group of units in an algebraic number field: When the field is defined by an irreducible equation with r real roots and s pairs of complex roots, the number of infinite basis elements is $r + s – 1$; the finite basis element is a root of unity.

After these fundamental papers, the importance of Dirichlet's number theory work declined. He published minor papers on the classes of ternary forms, on the representation of integers as the sum of three squares, and on number theory sums, together with simplifications and new proofs for previous results and theories.

In 1863, Dirichlet's *Vorlesungen über Zahlentheorie* was published by his pupil and friend Richard Dedekind. To the later editions of this work Dedekind most appropriately added several supplements containing his own investigations on algebraic number theory. These addenda are considered one of the most important sources for the creation of the theory of ideals, which has now become the core of algebraic number theory.

Parallel with Dirichlet's investigations on number theory was a series of studies on analysis and applied mathematics. His first papers on these topics appeared during his first years in Berlin and were inspired by the works of the French mathematicians whom he had met during his early years in Paris. His first paper on analysis is rather formal, generalizing certain definite integrals introduced by Laplace and Poisson. This paper was followed in the same year (1829) by a celebrated one published in *Crelle's Journal*, as were most of his mathematical papers: "Sur la convergence des séries trigonométriques qui servent à représenter une fonction arbitraire entre deux limites données." The paper was written under the influence of Fourier's theory of heat conduction as presented in his *Théorie analytique de la chaleur*.

Dirichlet and several other mathematicians had been impressed by the properties of the Fourier series on trigonometric series

$1/2a_0 + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + ...,$

particularly by their ability to represent both continuous and discontinuous functions. Such series, although now commonly named for Fourier, had already been used by Daniel Bernoulli and Leonhard Euler to examine the laws of vibrating strings. The convergence of the series had been investigated shortly before Dirichlet in a paper by Cauchy (1823). In the introduction to his own paper Dirichlet is sharply critical of Cauchy on two accounts: first, he considers Cauchy's reasoning invalid on some points second, the results do not cover series for which the convergence had previously been established.

Dirichlet proceeds to express the sum of the first n terms in the series corresponding formally to the given function $f(x)$ and examines the case in which the difference between $f(x)$ and the integral tends to zero. In this manner he establishes the convergence to $f(x)$ of the corresponding series, provided $f(x)$ is continuous or has a finite number of discontinuities. Dirichlet's method later became classic; it has served as the basis for many later investigations on the convergence or summation of a trigonometric series to its associated function under much more general conditions.

Dirichlet returned to the same topic a few years later in the article "Über die Darstellung ganz willkürlicher Functionen durch Sinus- und Cosinusreihen," published in the *Repertorium der Physik* (1837), a collection of review articles on mathematical physics on which his friend Jacobi collaborated. An outstanding feature of this article is Dirichlet's abandonment of the until then universally accepted idea of a function as an expression formulated in terms of special mathematical symbols or operations. Instead, he introduces generally the modern concept of a function $y = f(x)$ as a correspondence that associates with each real x in an interval some unique value denoted by $f(x)$. His concept of continuity is, however, still intuitive. For his continuous functions he defines integrals by means of sums of equidistant function values and points out that the ordinary integral properties all remain valid. On this basis the theory of Fourier series is then developed. In a related paper, "Solution d'une question relative à la théorie mathématique de la chaleur" (1830), Dirichlet uses his methods to simplify the treatment of a problem by Fourier: the temperature distribution in a thin bar with given temperatures at the endpoints.

Closely related to these investigations is the paper "Sur les séries dont le terme général dépend de deux angles et qui servent à exprimer des fonctions arbitraires entre des limites données" (1837). The Fourier series can be considered as expansions of functions defined on a circle. In this paper Dirichlet examines analogously the convergence of the expansion in spherical harmonics (*Kugelfonctionen*) of functions defined on a sphere. He later applied these results in several papers on problems in theoretical physics.

Dirichlet's contributions to general mechanics began with three papers published in 1839. All three have nearly the some content; the most elaborate has the title "Über eine neue Methode zur Bestimmung vielfacher Integrale." All deal with methods based upon a so-called discontinuity factor for evaluating multiple integrals, and they are applied particularly to the problem of determining the attraction of an ellipsoid upon an arbitrary mass point outside or inside the ellipsoid.

In the brief article "Über die Stabilität des Gleichgewichts" (1846), Dirichlet considers a general problem inspired by Laplace's analysis of the stability of the solar system. He takes the general point of view that the particles attract or repel each other by forces depending only on the distance and acting along their central line; in addition, the relations connecting the coordinates shall not depend on time. Stability is defined as the property that the deviations of the coordinates and velocities from their initial values remain within fixed, small bounds. Dirichlet criticizes as unsatisfactory the previous analyses of the problem, particularly those by Lagrange and Poisson that depended upon infinite series expansions in which terms above the second order were disregarded without sufficient justification. Dirichlet avoids this pitfall by reasoning directly on the properties of the expression for the energy of the system.

One of Dirichlet's most important papers bears the long title "Über einen neuen Ausdruck zur Bestimmung der Dichtigkeit einer unendlich dünnen Kugelschale, wenn der Werth des Potentials derselben in jedem Punkte ihrer Oberfläche gegeben ist" (1850). Here Dirichlet deals with the boundary value problem, now known as Dirichlet's problem, in which one wishes to determine a potential function $V(x,y,z)$ satisfying Laplace's equation and having prescribed values on a given surface, in Dirichlet's case a sphere. This type of problem plays an important role in numerous physical and mathematical theories, such as those of potentials, heat, magnetism, and electricity. Mathematically it can be extended to an arbitrary number of dimensions.

Among the later papers on theoretical mechanics one must mention "Über die Bewegung eines festen Körpers in einem incompressibeln flüssigen Medium" (1852), which deals with the motion of a sphere in an incompressible fluid; it is noteworthy for containing the first exact integration for the hydrodynamic equations. This subject occupied Dirichlet during his last years; in his final paper, "Untersuchungen über ein Problem der Hydrodynamik" (1857), he examines a related topic, but this includes only a minor part of his hydrodynamic theories. After his death, his notes on these subjects were edited and published by Dedekind in an extensive memoir.

In 1855, when Gauss died, the University of Göttingen—which had long enjoyed the reflection of his scientific fame—was anxious to seek a successor of great distinction, and the choice fell upon Dirichlet. His position in Berlin had been relatively modest and onerous, and the teaching schedule at the military academy was very heavy and without scientific appeal. Dirichlet wrote to his pupil Kronecker in 1853 that he had little time for correspondence, for he had thirteen lectures a week and many other duties to attend to. Dirichlet responded to the offer from Göttingen that he would accept unless he was relieved of the military instruction in Berlin. The authorities in Berlin seem not to have taken the threat very seriously, and only after it was too late did the Ministry of Education offer to improve his teaching load and salary.

Dirichlet moved to Göttingen in the fall of 1855, bought a house with a garden, and seemed to enjoy the more quiet life of a prominent university in a small city. He had a number of excellent pupils and relished the increased leisure for research. His work in this period was centered on general problems of mechanics. This new life, however, was not to last long. In the summer of 1858 Dirichlet traveled to a meeting in Montreux, Switzerland, to deliver a memorial speech in honor of Gauss. While there, he suffered a heart attack and was barely able to return to his family in Göttingen. During his illness his wife died of a stroke, and Dirichlet himself died the following spring.

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Several of Dirichlet's papers have been reissued in the series Ostwalds Klassiker der exacten Wissenschaften: no.19, *Über die Anziehung homogener Ellipsoide* (*Über eine neue Methode zur Bestimmung vielfacher Integrale*), which includes papers by other writers (1890); no. 91, *Untersuchungen über verschiedene Anwendungen der Infinitesimalanalysis auf die Zahlentheorie* (1897); and no. 116, *Die Darstellung ganz willkürlicher Functionen durch Sinus- und Cosinusreihen* (1900).

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