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Review Article

Hypergame Theory: A Model for Conflict, Misperception, and Deception

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When dealing with conflicts, game theory and decision theory can be used to model the interactions of the decision-makers. To date, game theory and decision theory have received considerable modeling focus, while hypergame theory has not. A metagame, known as a hypergame, occurs when one player does not know or fully understand all the strategies of a game. Hypergame theory extends the advantages of game theory by allowing a player to outmaneuver an opponent and obtaining a more preferred outcome with a higher utility. The ability to outmaneuver an opponent occurs in the hypergame because the different views (perception or deception) of opponents are captured in the model, through the incorporation of information unknown to other players (misperception or intentional deception). The hypergame model more accurately provides solutions for complex theoretic modeling of conflicts than those modeled by game theory and excels where perception or information differences exist between players. This paper explores the current research in hypergame theory and presents a broad overview of the historical literature on hypergame theory.

1. Introduction

"A conflict is a situation in which there is a 'condition of opposition' [1], and parties with opposing goals affect one another [2]." The study of how decision-makers interact during a conflict is known as game theory [3], while the study of how decision-makers make rational decisions is known as decision theory [4].

Game theory has been used to model diverse areas such as economics, natural selection, battles in past wars, and many other types of conflict [5]. The main influence behind the creation of game theory is the resolution of such competitions. Game theory models have many properties associated with them that influence the outcome and how game analysis proceeds but often fail to model the situation when one player has an advantage over the other in a conflict. When one or more players lack a full understanding or have a misunderstanding or incorrect view of the nature of the conflict, hypergame theory can be used to model the conflict.

Decision theory on the other hand is concerned with goal-directed behavior when options exist with different possible outcomes. The main influence behind the creation of decision theory is the rational behavior of the decision-maker [6]. Problems decision theory tries to answer include the following: "shall I bring an umbrella today?" or "I am looking for a house to buy shall I buy this one?" Decision theoretic models often fail to model the notion of fear, where another player may be able to outmaneuver during game play. Instead, models rely heavily on probability distributions to determine the preferred outcome.

Hypergame theory is an extension of game theory that addresses the kind of conflict games where misperception exists. The term hypergame was coined by Bennett in 1977. It seeks to explain how players in a game can have differing views of the conflict [7]. This advance in game theory shows how one player can believe that decisions of the other player are irrational, but the opponent is actually making a rational decision based upon the perceived game model.

Hypergame analysis extends game theory by providing the larger game that is really being played whether or not both players are aware of it. A different game model can represent each player's view of the conflict, but often the player's views will overlap where common knowledge exists. Figuring out what strategy a player will use is dependent upon not only his or her observation of the game, but also how that player believes their opponent is viewing the game. This creates many different game models that are examined for the solution to be obtained. The goal of hypergame analysis is to provide insight into real-world situations that are often more complex than a game where the choices of strategy present themselves as obvious.

After its introduction, hypergame theory was used to model past military conflicts, which are prone to having misperceptions and missing information in the process of their unfolding, to show how the outcomes were achieved. Analysis of past conflicts also lends itself to ease of understanding since the fog of war has cleared and the outcome has been determined. Options selected by each side in the conflict are shown to be the rational choice by way of defining the game that each side perceived. In this manner, the hypergame analysis shows why unexpected results were obtained when one or both sides misconstrued the conflict. Hypergame analysis offers advantageous reasoning of strategy selection through situational awareness.

Throughout this paper, the players are often referred to as human entities, engaged in a conflict. The perception or misperception of the players in all cases is the result of sensors, whether mechanical or human senses are combined with brain power. Human players, mechanical players, or artificial intelligence (AI) players are interchangeable in the hypergame models.

Examples in this paper are presented using two players with a limited number of actions/strategies. This allows the complexity of the example to be reduced and the visualization included in the figures to be clear. As more players and actions/strategies increase, the complexity will also increase.

2. Foundations of Hypergame Theory

Game theory, decision theory, and hypergame theory can be used to model conflicts as games. When very little is known about the opponents, game theory is used for adversarial reasoning. Decision theory is a better choice if the opponents are well known, which is often the case in complete information games. If one or more of the opponents are playing different games because they are not fully aware of the nature of the game, hypergames can be used to reason about subgames that are shared between opponents. The following provides a brief overview of decision theory and game theory.

Hypergames extend game theory by allowing for an unbalanced game model that contains different view of the game representing the differences in each player's information or beliefs. The unbalanced game model allows for a different game model for each player's view, while having overlap where there is common knowledge. Decision theory has been used in hypergames to model the fear of being outguessed. The fear of being outguessed is common in a game model where the different player's perceived games are unbalanced.

2.1. Game Theory. Game Theory is a set of analytical tools designed to help understand the phenomena that are observed when decision-makers interact [3]. The assumption is made in game theory that human beings are rational and always seek the best alternative when presented with a set of possible choices. Game theory is highly mathematical and assumes that all human interactions can be understood and navigated by presumptions. Game theoretic models seek to answer two questions about the interaction of the decision-makers [12]:

(i) How do individuals behave in strategic situations?

(ii) How should these individuals behave?

Answers to the two questions do not always coincide [13]. Often the answers to the two questions may be in conflict.

Game theory models have many properties associated with those that influence the outcome and how game analysis proceeds. Cooperative games can exist where there is communication between players, but more often games are seen as noncooperative where the players do not attempt to give up any information to their rivals. Simultaneous games are when players make decisions at essentially the same time and do not know of the opponent's move in advance (i.e., rock, paper, and scissors). Conversely sequential games are when the opponent's move is known before a decision is made (i.e., betting in poker or chess). Perfect information versus imperfect information is when all previous moves are known to all players instead of some being hidden. These previous two concepts are often confused with complete information and incomplete information, which are actually intended to be the knowledge of all player's strategies and payoffs. The payoffs, also known as utilities, have a common property of zero sum or nonzero sum.

The majority of game theory models are identified as either strategic games (normal form), used to represent a simultaneous game, or extensive games, more often used to represent a sequential game even though it can represent simultaneous one as well. A strategic (normal form) game is shown in Figure 1. Strategic games have utilities that are determined by which strategy is selected by each player. When a player's strategy is selected, all strategies are available to choose from and therefore the game is represented in a grid or matrix format. Extensive games' outcomes are instead represented as a tree structure where the initial player is at the top with branches leading off for each strategy available, as shown in Figure 2. After the first player chooses an action, the next player has strategies available creating branches for each of its strategies available. All strategies that a player can use may or may not be available at a particular node of the tree dependent upon the previous player's selected strategy.

The normal form is used to represent games where player moves, or turns, are simultaneous. This is often the case in games where information is imperfect and allows for better identifying strictly dominated strategies and Nash equilibria. The extensive form is better suited for games where player

		Nation B: Arm	Nation B: Disarm
Nation A	Arm	(2, 2)	(4, 1)
	Disarm	(1, 4)	(3, 3)

Figure 1: Cyber Arms Race, represented as a classical prisoner's dilemma shown as a normal form game.

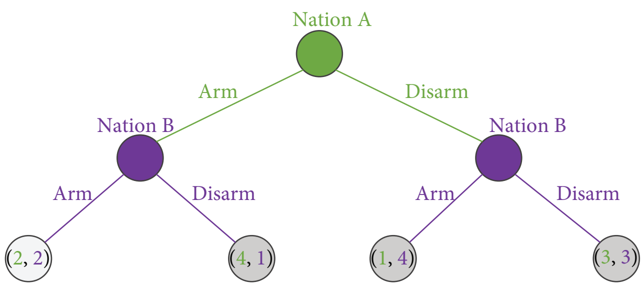

Figure 2: Prisoner's dilemma shown in extensive form.

moves are sequential (i.e., Player 1 moves first, observed by Player 2, and then Player 2 moves).

In the early 1950's, Nash contributed to noncooperative and cooperative game theory [40–42]. Nash [43] built on von Neumann and Morgenstern's work by assuming the absence of coalitions where each player acts independently. His work proves for each finite noncooperative game that there is at least one equilibrium point assuming the players are rational. A Nash equilibrium is a strategy where none of the players can improve their payoff by unilaterally changing their strategy. In a game of mixed strategies, every game will have at least one Nash equilibrium.

The Nash equilibrium means that none of the players can improve its own outcome (payoff/utility) by unilaterally changing strategies. The following definition of a Nash equilibrium is stated for two players but can be applied to any number of players [3]. The goal is to determine a unique outcome $[\sigma, \tau]$ for the game, given a strategy pair σ and τ for two players, denoted as P1 and P2. The unique outcome of the game is obtained by each player playing their respective strategy (P1 plays σ and P2 plays τ) against each other.

Definition (Nash equilibrium [44]). A strategy pair σ, τ is a Nash equilibrium if, for no $\sigma' \neq \sigma$, $[\sigma', \tau] >_{\mathrm{P1}} [\sigma, \tau]$ and for $\tau' \neq \tau$, $[\sigma, \tau'] >_{\mathrm{P2}} [\sigma, \tau]$.

Considering the Cyber Arms Race, shown in Figures 1 and 2, and applying the definition of a Nash equilibrium, the resulting solution to the Cyber Arms Race is $(2, 2)$ or both players choose "Arm" leading to an arms race. Neither nation can improve its own outcome or utility, by unilaterally changing strategies.

Deception in game theory has been mostly studied in turn-based or dynamic games, where a player chooses an action and then reports the action or outcome to the other player. This type of game is called signaling games, after the "signal" is sent between players. The signal is subject to deception, since the player can be truthful or deceptive or choose not to send a signal.

Carroll and Grosu [45] study network defense using deceptive signaling games. In their research, the defender can disguise a normal computer as a honeypot or a honeypot as a normal computer or use no disguising techniques. The attacker has the ability to test the system type and the defender sends the appropriate signal, deceptive or truthful. The authors showed that deception is an equilibrium strategy for the defender, either by disguising all honeypots as normal computers or all normal computers as honeypots, providing an increase in utility for the defender over using only truthful signals.

Multiturn attacker-defender games are used by Zhuang et al. to study deception [46]. In the game, a defender type is randomly selected from a set of possible defender types and at each turn of the game the defender selects a strategy and "signals" the attacker of the selected strategy. The defender may be either truthful or deceptive. The attacker then uses the signal to update his belief of the defender's true type and selects an attack strategy. After each turn, the payoffs are used to update the belief state until the game ends. The authors state that, given their game, deception can be a beneficial strategy for the defender.

Hespanha et al. [47] modeled an attacker-defender game where the defender has three units available to defend two locations. In the game, the defender signals the locations of the units by either sending a truthful or deceptive signal or not camouflaging the units revealed to the attacker. The authors also discuss the possibility of a malfunction of either the attacker's sensors or the defender camouflage, which may mean the signal seen may not be correct. The authors conclude that the use of deception can render the information collected from sensors and other methods to be useless to the attacker.

Deception has also been studied in repeated games. In this type of game, the players both choose an action and make their moves simultaneously. Depending on the game, the players may receive information about how the environment state changed between selecting moves. Pursuer-evader games are commonly modeled with this type of repeated game. Yavin [48] studies pursuer-evader deception, where both players choose a strategy based on the bearing of the other player and the distance between them, by corrupting the evader's bearing signal to the pursuer. The author's goal is to determine the optimal (or near-optimal) pursuit strategies for a pursuer when faced with deceptive or incomplete information.

2.2. Decision Theory. In any given situation, there are actions which a player can choose between making a choice in a nonrandom way. The choice between actions are goal-directed activities [49]. Given a set of actions, decision theory is concerned with goal-directed behavior to reach a desired outcome.

Decision theory is a formal mathematical theory about how decision-makers make rational decisions. It is also known as normative decision theory [4, 50], Bayesian decision theory [49], rational choice theory [51], and statistical decision theory [52]. Decision theory predates the development of game theory and can be divided into three parts: normative, descriptive, and prescriptive [53, 54].

(i) Normative decision theory [4, 50] studies the ideal agent and the decisions that this perfectly rational agent would make, often referred to as the study of how decisions should be made.

(ii) Descriptive decision theory [55] studies the nonideal agent, such as humans, and how they make decisions, often referred to the study of how decisions are made in reality.

(iii) Prescriptive decision theory [56] studies how nonideal agents, given their imperfections, can improve the decisions.

A person uses his own preferences to determine his action, according to rational choice theory [57]. A rational person selects his action according to the one that maximizes his preferences [58]. The Nash equilibrium builds on this concept, adding consideration for the other player(s) and what can be done to maximize the outcome unilaterally.

The problem is that preferences do not just represent the decision maker, but a rational person can consider moral, ethical, social (peer-pressure, social expectance, etc.) and/or other norms when establishing their preferences. Intentional or unintentional deceptions can also affect a player's preferences. This problem highlights the complexity of preferences, which may lead to odd choices in real-world situations.

Luce and Raiffa presented a classic example of this problem [59]. In the example, they compare two alternative scenarios of visiting a restaurant. In the first visit, only salmon and steak are offered on the menu. In this case, the customer decides to order salmon even though the customer normally prefers steak. The customer refrains from the steak because the small menu indicates the cook may not know how to prefer a steak. In the second visit, the menu includes lobster and clams in addition to salmon and steak. Here the customer chooses the steak. If the customer does not like lobster or clams, the fact they are offered on the menu indicates the restaurant is good and should know how to prepare a steak.

In this scenario, the customer may seem irrational since he did not choose the steak which he prefers. The reason why he is not irrational is because the menu does not only list the choices but also conveys information of value to the customer (which does not have to be true). The addition of lobster and clams on the second menu indicates the cook has the ability to prepare these delicate foods, while the first menu just has the basics. In this case, not only do the items offered on the menu get considered in the preferences, but also the type of restaurant (or the perceived type of restaurant).

Based on today's Internet connected pollution, the previous scenario can include deception. For example, instead of relying on the menu options, he may instead consult a community ranking website. Given that anyone can post their opinion, the restaurant may have paid for favorable rankings, therefore adding deception in the customer's preferences.

2.3. Bounded Rationality. Bounded rationality is where a player's rationality is limited in the decision-making process by the information the player has, cognitive limitations of their minds, and time available to make the decision [60]. Simon originally proposed the concept of bounded rationality as an improvement to the model of human decision-making [61]. Bounded rationality helps to explain why the most rational decision is not always the decision chosen by the player in game theory or decision theory.

Bounded rationality does not mean irrationality, since players want to make rational decisions, but cannot always do so [62]. Players are often very complex, but in order to be fully rational they need unlimited cognitive capabilities [63]. The cognitive capabilities of players are limited and therefore cannot conform to full rationality. Players will use the cognitive resources they have, with the information available, and often within time constraints to reach a decision that is as rational as possible. Bounded rationality allows the player to make a decision based on their perceived state of the game or environment, leading to multiple players having different perceptions of the game or interaction.

3. Hypergame Theory

Hypergame theory extends game theory by allowing for an unbalanced game model that contains a different view, representing the differences in each player's information, beliefs, and understating of the game. The unbalanced game model allows for a different game model for each player's view, while having overlap where there is common knowledge. The outcome or solution to the hypergame model is dependent on the player's perception of the game model, including how the player views the game and how the player believes the opponent is viewing the game. Because of multiple game models, each model has to be analyzed in order to determine the outcome to the hypergame. This allows hypergames to more accurately provide solutions for complex real-world conflicts than those modeled by game theory and excel where perception or information differences exist between players.

Two papers explain the transition from game theory to hypergames, early in the history of hypergames. The first paper [64] discusses the development apart from classical game theory towards hypergame theory. It explains the changes and focuses on descriptive modeling and does not cover the issue surrounding attempts to influence decision-makers. The second paper [65] provides illustrative case studies and presents a methodological framework for applying hypergames to complex decision problems.

3.1. Theory Foundations. Hypergames, first discussed by Bennett [7], are used to model the games where one or more players are playing different games [66]. Hypergame theory decomposes a single situation into multiple games. By reasoning about multiple games, the outcome to the single problem can be improved. Each player in a game has their

		Column	
		C_1	C_2
Row	R_1	0, 4	1, 3
	R_2	4, 0	3, 1

Column				
C_1	C_2	C_3		
0, 4	1, 3	1, 3	R_1	Row
4, 0	3, 1	3, 1	R_2	
3, 1	5, 0	1, 3	R_3	

Figure 3: An example of hypergame where each player has a different view of the game. Row's view is on the left, while column's view is on the right.

own perspective of how the other players view the game with regard to the possible actions and player preferences. Bryant [67, 68] discussed the difference in the set of players, pointing out that the set may vary in real life as players perceive differently. In a hypergame, each player may [2]

(i) have a false or misled understanding of the preferences of the other players,

(ii) have incorrect or incomplete comprehension of the actions available to the other players,

(iii) not have awareness of all the players in a game,

(iv) have any combination of the above: faulty, incorrect, incomplete, or misled interpretations.

A player's choice of actions (decisions) reflects the player's understanding of the game outcomes; the player chooses actions based on the way they perceive reality, which may not be the true state of reality. Figure 3 shows a basic two-player hypergame between "row" and "column," where C_i and R_i are different actions each player could take.

Hypergame analysis is conducted by first examining Row's belief about Column's reasoning, and then by examining Row's available actions [69, 70]. In Figure 3, the game on the left shows how Row believes Column will reason about the game. Based on this, Column will play C_2 while Row plays R_2, the Nash equilibrium concept from game theory. This allows the experience and intuition of the decision maker to be incorporated into hypergames. For example, this could apply to planning variables, such as a novel course of action for Row or Column's lack of time to plan or to situational variables, such as the hidden location of Row's resource [2].

Hypergames allow for domain knowledge incorporation; therefore, it does not require the game theory equilibrium condition [2]. Furthermore, the standard rationality arguments from game theory are replaced by knowledge of how the opponent will reason [2]. It is also valid to assume unequal availability of information in hypergames, when many players in games have imperfect information. In Figure 4, Kopp gives a graphical comparison of the general differences between a standard game model and a hypergame model. This representation depicts a general overview of how a hypergame incorporates different aspects of the conflict being modeled.

3.2. Hypergame Levels. Wang et al. [71] proposed different levels for developing mathematical hypergame models based on perceptions of the players. The lowest level (level 0) is a basic game with no misperceptions among the players. In a first level hypergame, players have different views of the game but are not aware of the other players' games. In a second level hypergame, at least one player is aware there are different games being played and that misperceptions exist. A third level hypergame is possible and occurs when at least one player is aware that at least one other player is aware that different games are being played. An nth level hypergame can be described, but the authors state this does not add to the hypergame model; instead it adds complication and excess information for the hypergame analysis. This allows the perceptions of the players to be incorporated into the hypergame model but with varying degrees of perceptions in order to reach a more complete game model.

3.2.1. First Level Hypergame. The levels of hypergames were originally presented by Fraser and Hipel [2]. A game G is defined by a set of preference vectors, V_n, for all game players, where n is the number of players and V_i is the preferences vector for player i:

$$G = \{V_1, V_2, \dots, V_n\}. \quad (1)$$

In game of complete information, all players know the other player's preference vectors; therefore, all players are playing the exact same game. In hypergames, one or more players may have incomplete information, which leads players to form slightly different versions of the same game or completely different games altogether. A game formed by player q includes any and all lack of information about the conflict, which is denoted by

$$G_q = \{V_{1q}, V_{2q}, \dots, V_{nq}\}, \quad (2)$$

where V_{iq} represents the preference vector of player i as understood (perceived) by player q.

A first level hypergame H is a set of games as understood from each player:

$$H = \{G_1, G_2, \dots, G_n\}. \quad (3)$$

An example of a hypergame in this form is shown in Table 1 in matrix form. Since players may have different misperceptions, each player may make a different decision which will result in a different outcome to the conflict. A mapping function can be used to relate the outcomes between the player's individual games. Bennett [72] gives an algebraic description of this problem, while an application is presented in Bennett et al. [20].

Game analysis is performed by treating each player's game separately. This means player q's game is analyzed from q's understanding about the conflict. The decisions made and the strategies chosen by q depend on q's interpretation of the conflict; therefore, a given player may not perceive all outcomes of a game. The player cannot unilaterally change from a perceived outcome, so, for the purpose of stability analysis, the outcome is stable for that player [2]. Therefore, an unknown outcome to a player can be stable in the

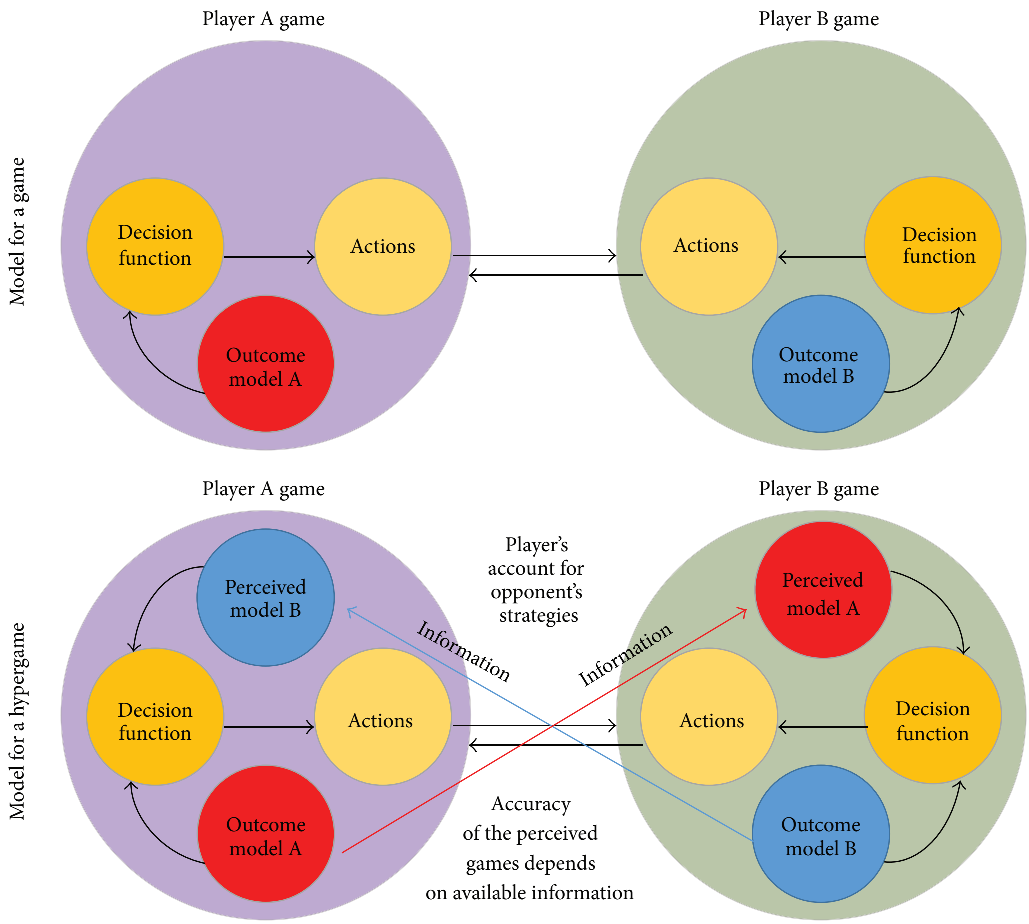

Figure 4: Comparison of a game theory and a hypergame model [8].

Table 1: An example of a hypergame in matrix form.

Player perceived	Game perceived by player			
1	V_{11}	V_{12}	$\cdots$	V_{1n}
2	V_{21}	V_{22}	$\cdots$	V_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	V_{n1}	V_{n2}	$\cdots$	V_{nn}
	G_1	G_2	$\cdots$	G_n

hypergame analysis. A strategic surprise occurs when a game contains an unknown outcome.

For player q's game, an outcome is stable if the outcome is stable in each of q's preference vectors. This means the equilibriums of q's game are only the outcomes q believes would resolve the conflict even if other equilibriums exist in the full game. Hypergame equilibriums depend on each player's perception of the stability of the outcomes. When determining equilibriums of hypergames, the equilibriums of each player's game are not needed, but these individual equilibriums can be useful to demonstrate what each player believes will happen.

3.2.2. Second Level Hypergame. A second level hypergame is a hypergame where at least one player is aware that a hypergame is being played. This situation can happen if at least one player perceives another player's misperception [2]. Player q's hypergames is defined as the (hyper)game perceived by player q. This hypergame is denoted as

$$H_q = \left\{G_{1q}, G_{2q}, \ldots, G_{nq}\right\}, \tag{4}$$

where G_{iq} is the game of the ith player as it is perceived by player q. It is not necessary for player q to be one of the players who are aware that a hypergame is being played. If set H_q is missing a player's game, it is because player q does not perceive the game.

A second level hypergame is a set of hypergames perceived by each player, denoted as

$$H_2 = \{H_1, H_2, \ldots, H_n\}. \tag{5}$$

Table 2 shows a second level hypergame in matrix form, where the hypergame for player p is the pth column. Each element of the matrix is a game made up of a preference vector for each player.

Similar to a first level hypergame analysis, game analysis of second level hypergames is performed by treating each

CMSs

Belief-contexts				C_Σ	S_1	S_2	S_3	$\cdots$	S_n
P_{K-1}	$\cdots$			C_{K-1}	C_{k1}	C_{k2}	C_{k3}	$\cdots$	C_{kn}
	$\cdots$			$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	$\cdots$	P_1		C_1	C_{11}	C_{12}	C_{13}	$\cdots$	C_{1n}
	$\cdots$		$P_0 = 1 - \sum_{i=1}^{K-1} P_k$	C_0	C_{01}	C_{02}	C_{03}	$\cdots$	C_{0n}
R_{K-1}	$\cdots$	R_1	R_0 = full game		Column 1	Column 2	Column 3	$\cdots$	Column n
r_{k1}	$\cdots$	r_{11}	r_{01}	Row 1	u_{11}	u_{12}	u_{13}	$\cdots$	u_{1n}
r_{k2}	$\cdots$	r_{12}	r_{02}	Row 2	u_{21}	u_{22}	u_{23}	$\cdots$	u_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
r_{km}	$\cdots$	r_{1m}	r_{0m}	Row m	u_{m1}	u_{m2}	u_{m3}	$\cdots$	u_{mn}

RMSs Full game

Figure 5: Hypergame Normal Form as proposed by Dr. Vane III.

Table 2: An example of a second level hypergame in matrix form.

Player perceived	Game perceived by player			
1	G_{11}	G_{12}	$\cdots$	G_{1n}
2	G_{21}	G_{22}	$\cdots$	G_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	G_{n1}	G_{n2}	$\cdots$	G_{nn}
	H_1	H_2	$\cdots$	H_n

player's game separately. This allows stability information to be determined for every preference vector in a conflict. This information can further be used to determine each game's equilibrium. The preference vectors of each player's game provides the stability information that determines the equilibriums of the second level hypergame. "Just as the equilibriums of a game within a hypergame are not needed to determine the equilibriums of that hypergame, the equilibriums of a hypergame within a higher level hypergame are not needed to determine the equilibriums of that higher level hypergame" [2].

3.3. Hypergame Normal Form (HNF). In [34], Dr. Vane III offers a different approach to hypergame modeling by providing the incorporation of a player's beliefs on an opponent's possible actions. He also provides a graphic representation of the hypergame that is reminiscent of the normal strategic form used in standard game theory analysis.

The new model is referred to as Hypergame Normal Form (HNF); see Figure 5. The full game is the familiar grid form with Row and Column strategies labeled and the utility values ($u_{11} - u_{nm}$) in the cells cross-referenced from the strategies. The additional sections are the hypergame situational information. The row mixed strategies (RMSs) are hyperstrategies gleaned from what the row player believes about the game being played by the Column player.

They are called hyperstrategies because they do not encompass the view of the full game, except for R_0 which is the full game Nash equilibrium (NE). The column mixed strategies (CMSs) are row's belief about the mixed strategy percentages that Column will play when selecting a strategy. C_0 is column's NE view of the full game. When a CMS cell contains a 0, this is an indication that there is a subgame that column is believed to be playing where the corresponding strategies are either unknown to Column or discounted as not worthwhile. The final section is the belief-contexts, which correspond to the percentage of which Row believes that the adjacent CMS will be played by column. Since they are percentages, the belief-contexts sum up to one, with the sum of values P_1 through P_{K-1} being at most 1 and the leftover constitutes the NE belief-context. Filling in the HNF with the values associated with the game provides the avenue for the hypergame analysis with HNF.

Determining the utility values allows for a NE for the full game to be calculated which provides the input into R_0 and C_0. CMSs are then entered in the section above the full game. A CMS can be determined manually; that is, knowing a player's preference for selecting rock in a game of rock, paper, and scissors or a NE for the Column player can be used from the analysis of the subgame that the Column player is believed to be using. Each CMS is assigned a belief-context value which serves to weight the Row player's belief that Column will choose that CMS. These values are used to calculate C_P, the aggregate amount which directly affects the expected utility that Row hopes to achieve. Row's hyperstrategies are then input into the RMS section. Expected utility values for

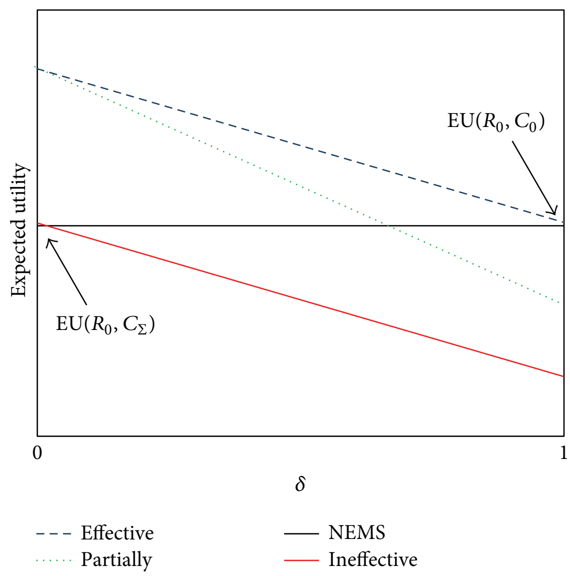

Figure 6: Hyperstrategies effectiveness in Hypergame Normal Form.

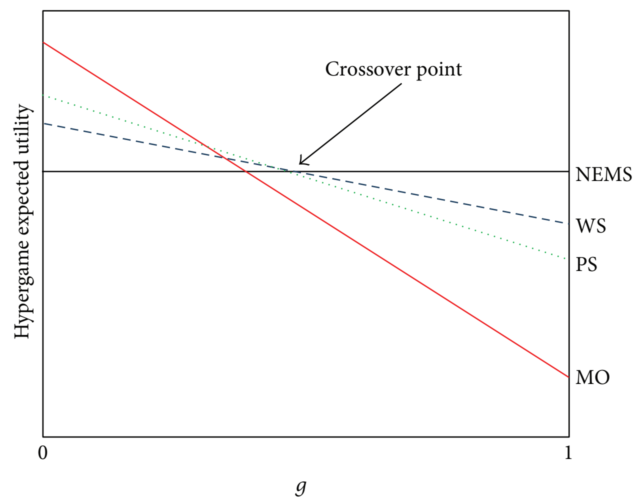

Figure 7: The value of HEU is dependent on the value of g.

each strategy set listed in the RMS section are calculated for the full game NE CMS, C_0, and the aggregate belief CMS, C_P. These values determine the effectiveness for which an RMS hyperstrategy is a practical selection for Row to apply.

RMS effectiveness is categorized into three levels of usefulness: fully effective, partially effective, and ineffective (Figure 6). A fully effective strategy set will provide at worst case the same expected utility that Row's R_0 strategy set achieves for C_0 but has a greater expected utility at C_P. Thus, given that row is viewing the game correctly, a fully effective RMS is always a good choice. Partially effective strategy sets also provide a greater expected utility at C_P than R_0 but have a lower utility expectation at C_0. Given Row's information, a partially effective RMS could provide a good outcome, but it is not always assured. The ineffective strategy set provides no increase in utility and at best can only get to that expected by the NE, so there is no reason to select it. It is reasonable to assume that fully effective strategies sets should always be used, but that does not mean there is not some inherent risk involved because the utility values are only expected and are not foolproof. Worst case scenarios can also be included in this determination to help mitigate risk.

Risk assessment is built into the hypergame analysis through a method termed quantified outguessing. This method introduces the fear of the player that he or she will be outmaneuvered and the worst case utility will be the end result. Three types of hyperstrategy sets are described for this analysis: modeling opponent (MO), pick subgame (PS), and weighted subgame (WS). MO is simply selecting the strategy for row that will provide the highest utility given all of row's strategy selections and when considering the belief of how column views the game. In contrast, the PS strategy set consists of the NE for the same game view that was considered for MO. WS uses the PS strategy values multiplied by the belief-context percentage for that CMS and adds the R_0 multiplied by the belief-context for C_0, which results in a hybrid strategy set between PS and the NEMS for the full game. Each hyperstrategy is then assessed against the full game to quantify the worst case utility (G) or the utility value expected when Column selects the correct counter strategy. The expected utility (EU) and G, once determined, allow the hypergame expected utility (HEU) to be calculated by also considering g, the percentage chance Row believes they will be outguessed (Equation (6)). As can be seen, the distance between EU and G has a quantifiable effect on the value of HEU for the hyperstrategy (hs):

$$\mathrm{HEU}(\mathrm{hs}) = \mathrm{EU}(\mathrm{hs}) - (\mathrm{EU}(\mathrm{hs}) - G(\mathrm{hs})) * g. \tag{6}$$

As the fear of being outguessed increases, the ability of any hyperstrategy to provide better utility (when compared to the NEMS solution of the full game) decreases (Figure 7). When the fear of being outguessed is low the MO hyperstrategy is the best selection, but as that fear increases eventually PS dominates for a short period until the crossover point where NEMS for the full game is dominant (note that WS is always dominated and does not provide a suitable choice). Therefore, with good information on the intent of the adversary, hyperstrategy selection that provides better utility than standard game analysis is achievable.

Further research in the use of the HNF has been conducted after its creation. The ideas about hypergame analysis are expanded upon in [73] by Russell Vane. The presence of luck and robustness of strategy plan are examined, but for the most part the research provides further evidence of the usefulness of hypergame analysis for the strategy selection process. A real-world example of how to use the HNF is provided in [74], which examines a terrorist attack. The analysis entails applying belief-context values to expected types of attackers so that a strategic decision can be made to best protect first responders. This reiterates the idea that uncertainty exists and needs to be assessed when planning.

Perhaps the most interesting application of the HNF is in [75] where it is used to model the fall of France in

1940. The model is compared to the dual standard game model presented in [16] and to a preference vector model like that in [76]. Specifically it is outlined that using the HNF approach allows all information to continue to be presented and not removed from the model. Even when a strategy is completely discounted by applying a percentage chance of use of zero, it remains in the total game NEMS analysis and is not completely removed from the model. These research efforts show insight into the usefulness of the HNF.

4. Hypergame Modeling

Huxham and Bennett [77] introduce the idea of preliminary problem structuring. In this phase the problem is explored and the relevant participants are identified, along with the possible interactions. The authors try to build up a structured picture in hypergame terms of the situation, instead of a hypergame model. The idea is to explore how the various pieces fit together. The structured picture will often be too complex to form into a formal hypergame model. It is therefore necessary to abstract farther, making simplifications by asking specific questions [77]:

(i) How do two different problem aspects relate?

(ii) Where are the complexities of the system?

(iii) Can simplifications be made while retaining the essential structure?

(iv) Which participants are most important or influential?

Hipel and Dagnino present an algorithm for modeling bargaining situations with two or more decision-makers where one or more of players have misperceptions [9]. The algorithm is called the hypergame cooperative conflict analysis system (HCCAS). HCCAS unifies work in hypergame theory [71, 72, 78–80], conflict analysis [2], and cooperative conflict analysis in bargaining [81, 82]. The HCCAS algorithm is shown in Figure 8.

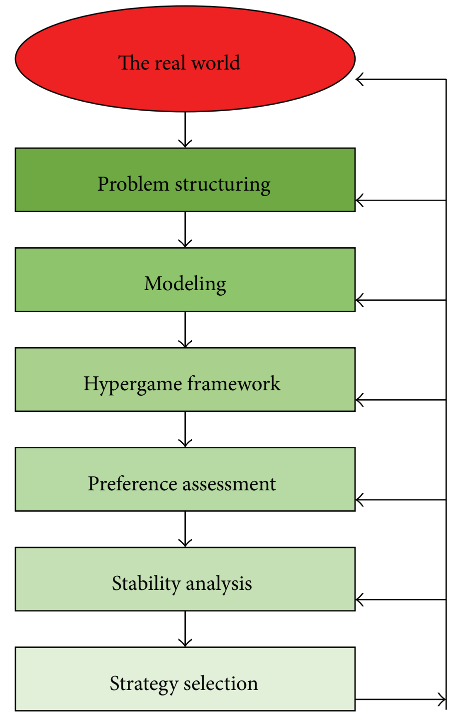

Figure 8: The HCCAS algorithm [9].

The real-world situation is represented at the top of the algorithm and provides critical information for the algorithm. The first step is to use the real-world information to define the structure of the bargaining situation. This stage involves selecting a point in time at which the analysis will be conducted, as well as identifying the participants and potential interactions. The second step in HCCAS is modeling, where the actions and outcomes of the players are identified. The third step of HCCAS is the hypergame framework where the bargaining situation structure and the levels of misperception for each player are identified. Following this step, the preference vectors for each player are formed using information from the previous steps; this is referring to the preference assessment in Figure 8. Stability analysis of the hypergame is performed in the fifth step. After this, a strategy is selected and can be used to explain the real-world events. The authors then apply the HCCAS algorithm to the Seymour landfill case, between Eau Claire city and the town of Seymour in Wisconsin.

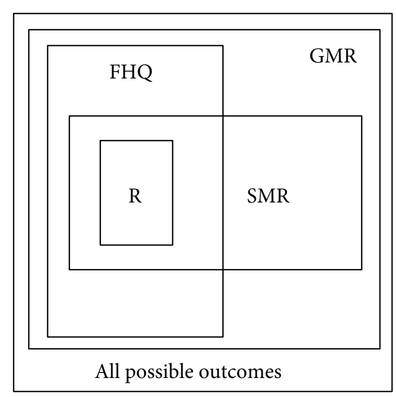

Figure 9: Venn diagram of stability analysis outcomes for n-players [10].

5. Other Related Works

In this section we summarize additional research related to hypergame theory. This work adds to the theory of hypergame and there are many contributions from previous researchers.

5.1. Stability Analysis. Wang et al. explores stability analysis for n-players in [10]. The authors present a relationship of possible outcomes, as shown in the Venn Diagram in Figure 9. Nash stability is when players make a rational decision based on the best outcome for the player; this type of outcome is considered rational (R). Nash stability is harder to achieve when misperceptions exist between players. A general metarational (GMR) outcome is where other players

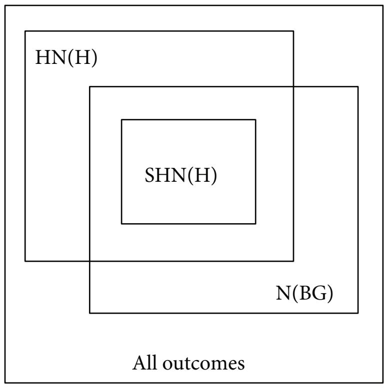

Figure 10: Venn diagram of stability relationships among outcomes [11].

have joint action for player i, and player i cannot achieve a better outcome than the original. A symmetric metarational (SMR) outcome is when there is one jointly sequential strategy selection that results in player i achieving the same outcome. If a response to a player's strategy results in that player not achieving a better outcome and the responding player not being able to possibly achieve a worse outcome, it is known as a sequential stable (FHQ). The contribution of this research is an FHQ outcome exists in all hypergame levels, which implies a GMR outcome also existing in all hypergame levels.

Another view of hypergame stability is given in [11]. When there exist hyper Nash equilibria in a hypergame, if all of them are not Nash equilibrium in the base game, there does not exist stable hyper-Nash equilibrium. An intuitive interpretation of the paper's theorem is that when we anticipate all outcomes which seem to happen actually (hyper-Nash equilibrium), each would not happen if all the misperceptions are eliminated, and those outcomes are necessarily unstable. Hence, the stability relationships among the solution concepts in a hypergame can be depicted by Figure 10. The relationships are defined as the hypergame (H), hyper-Nash equilibrium of H (HN(H)), base game (BG), and Nash equilibrium of H (SHN(H)). A hyper-Nash equilibrium is defined as a profile of such strategies that each agent plays according to their Nash strategy in their own subjective game. This allows for generalization of Nash's theorem about noncooperative games [40] to hypergames: in every finite hypergame with mixed strategies, there is at least one hyper Nash equilibrium [83]. A hyper-Nash equilibrium provides an equilibrium solution for a simple hypergame. This also allows for hypergames with cardinal utilities, while previous research only dealt with ordinal utilities.

5.2. Player Beliefs. Vane and Lehner [84] deal with beliefs over games. The hypergame framework allows a player to hedge its risk about what the other opponents are doing. This is done by selecting a set of possible games that represent the action the opponents may take, and then a probability distribution is built over this set of games and evaluated using the maximum expected utility. This allows the player to hedge its risk by using the probably that an opponent will select an action, increasing payoffs by lowering the effect of misperceptions on the hypergame model.

5.3. Perceptions/Deception. Hypergames have been used to model interactive decisions through matrices, trees, and tableaux [85, 86]. The authors expand this repertoire by showing preliminary problem structuring, where there are games within games, and build the perception in hypergames. They also expand the repertoire by combining hypergames with different methods to solve complex decisions.

Mateski et al. explore perception, misperception, and deception in conflict using hypergames [87]. They introduce a diagrammatic representation for hypergames called the hypergame perception model (HPM). The HPM was used to model misperception and deception during the Cuban Missile Crisis where perception played a critical role in the conflict. The HPM diagram is shown in Figure 11.

Gharesifard and Cortes [88] show that, for a game with rational players, where the past outcomes are perfectly observable, repeated play converges to equilibria. This results in the hypergame having an acyclic structure. They also present the notion of inconsistent equilibrium in the repeated play of first-level hypergames with two players [89]. Inconsistent equilibrium refers to the equilibria of the hypergame where at least one player expects the other to move away from the equilibria. Just the existence of inconsistent equilibrium means there is some misperception about the game among one of the players. A class of actions, called exploratory, are also identified by the authors to allow players to move away from inconsistent equilibria and decrease the misperception. If only one player in the game uses exploratory actions, then the hypergame will arrive at an outcome rational for the player. If both players use exploratory actions, then the repeated play may finish in a cycle.

They [90] also study the situations where the perceptions of players in the game are inconsistent and evolving. The authors present a new method, called swap learning, which allows the incorporation of information gained by observing their opponents actions into the player's beliefs. This method allows a player to decrease misperceptions, but at a cost of incorporating inconsistencies into their beliefs. Since the swap of preferences does not take into account the other outcomes, inconsistencies can form in the beliefs of player A. To eliminate the inconsistencies, the modified swap learning method is presented. This method assumes that the opponent has perfect information and plays their best strategy but yields consistent beliefs and decreases player misperception. The swap learning method place the origin of the misperception on the player performing the belief update.

Again, Gharesifard and Cortes [91, 92] focus on conflicts with incomplete information, where players may have different perceptions about the conflict. Specifically they focus on a 2-player hypergame where one player, the deceiver, has full information about his opponent's game and wants to introduce a certain belief in it. They use their previously developed H-digraph [93], a special class of digraph used to encode

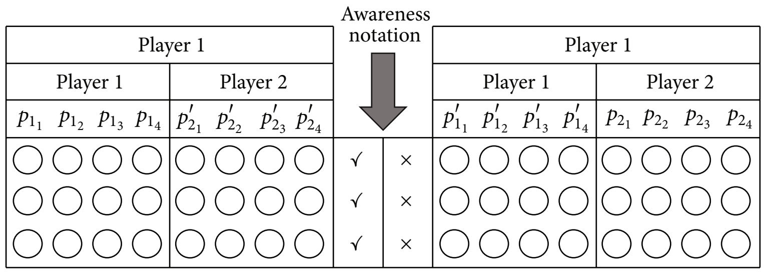

FIGURE 11: HPM diagrammatic representation.

Defender \ Attacker	Not attack	Attack	Zero-day exploit
Not defend	$(0, 0)$	$(-W_i, W_i - C_a W_i)$	$(-W_i, W_i - C_z W_i)$
Defend	$(-bC_f W_i - C_m W_i, 0)$	$(-(1-2a)W_i - C_m W_i, (1-2a)W_i - C_a W_i)$	$(-W_i, W_i - C_z W_i)$
Provide ruse	$(-W_i - C_r W_i, 0)$	$(V_a - C_r W_i, W_i - C_a W_i)$	$(-W_i, W_i - C_z W_i)$
Shutdown	$(-W_i - C_t W_i, 0)$	$(V_a - W_i - C_t W_i, W_i)$	$(V_a - C_t W_i, -W_i)$

FIGURE 12: Gibson's normal form game model.

the belief structure of the hypergame players. Using the H-digraph, they are able to characterize deception when stealthy actions are possible in the game. Their papers [90, 93, 94] also present two algorithms for updating perception in the hypergame. These methods can decrease the misperception between the player's perceived game and true payoffs.

5.4. Dynamic Payoff Functions. Gibson presents a model based on the intrusion model presented by Chen and Leneutre [95] and the Hypergame Normal Form model presented by Vane [33, 34]. Table 3 shows the symbols used in the payoff functions, while Figure 12 shows the game in normal form. The author achieves a model that has a changeable nonzero-sum utility values with a process for delineation of strategy selection [39]. In order to achieve this model, the Chen and Leneutre intrusion model is extended by adding strategies for both the attacker and defender, while the HNF model is used to hide or discount strategies from the other player.

TABLE 3: Gibson's experimental model, variables.

Variable symbol	Meaning
a	Detection rate
b	False alarm rate
C_a	Cost of attack
C_f	Cost of false alarm
C_m	Cost of monitoring
C_r	Cost of providing ruse
C_t	Cost of time down
C_z	Cost of zero-day Exploit
V_a	Value of attacker
W_i	Value of target

The attacker and defender are given additional strategies over the original model presented by Chen and Lenectre.

The most important contribution of Gibson's model is that, by combining the Chen and Leneutre model with HNF, dynamic variables are added to the payoff functions in HNF, as shown in Figure 12. This allows for dynamic play and updating of variables as the game is played.

5.5. Mutual Interaction. Inohara et al. discuss the ability of players to engage in multiple games simultaneously [96]. Each game a player engages in may have interactions with other games which can affect outcomes. They integrate different games in order to capture the interactions, which is realistic of real-life situations. An example is given using the hypergame methodology, in order to model hypergames that are mutually interactive and increase perception ability of players.

5.6. Fuzzy Logic. Song et al. [97, 98] present a novel method that uses fuzzy logic to obtain the outcome preference in first-level hypergame models. A fuzzy aggregate algorithm is applied to get the group fuzzy perception of the opponents' outcome preference. The preference sets are then obtained by solving linear programming models. The authors obtain the crisp perception for the opponents' outcome preference by using a defuzzification function and the Newton-Cotes numerical integration formula. The authors then use the concept of consensus winner to determine the preference vectors in the hypergame models. In [99], artificial neural networks (ANNs) are trained to learn the criteria for comparing fuzzy outcome preference numbers.

Qu et al. [100] use fuzzy pattern recognition to establish a nonlinear programming model. This model is used to integrate different outcome preferences for opponents perceived by different experts. Each expert perceives the outcome of the game and this information is processed using fuzzy pattern recognition to obtain a standard outcome.

Zeng et al. [101] develop an integration model for hypergames with fuzzy preference perceptions. In conflicts, players cannot perceive information about the opponent's game clearly, so an integration model of multiple perceived fuzzy games using hypergames is developed. Each player has fuzzy preference perceptions. The authors use linguistic values for the outcome preferences over the outcome space, which are represented as triangular fuzzy numbers. Hypergames with fuzzy preference perceptions are demonstrated with a military example about two country's navies.

5.7. Comparison to Bayesian Games. Sasaki and Kijima [102] propose a Bayesian representation of hypergames by using Harsanyi's theory that any game of incomplete information can be transformed into a game of complete information. The authors make the claim that "any hypergame can naturally be reformulated in terms of Bayesian games in a unified way." This claim is much stronger than the method they actually propose. There are limitations that result in hypergames that cannot be reformulated in terms of a Bayesian game. The authors discuss the limitations of their method, which limits the ability to reformulate a hypergame in terms of a Bayesian game. Sasaki and Kijima only apply Harsanyi's claims to the original hypergame model developed by Bennett [7]; they do not discuss or mention the extension to hypergame theory by Russell Vane in his doctoral dissertation published in 2000.

5.8. Multiagent Environments. Chaib-Dara [103] uses hypergames to analyze differences in perceptions in multiagent environments. The author shows how multiagents can interact using a third party, while having different views and perceptions of the game. The third party is used to observe the exact perceptions of the players from an external context. The players can then choose to trust the external observation and update their perceptions of the game.

5.9. Combining Approaches. Huxham and Bennett [104] explore combining hypergames with cognitive mapping, since they both deal with the subjective world of decision-makers. They start with the idea that maps could be built up, and then the players, preferences, and outcomes could be extracted. The authors determined this process was not straightforward. They then structure the problem in hypergame form and then used piecemeal maps to explore certain outcomes. The relationship between hypergames and cognitive mapping is explored theoretically by Bryant [67].

Bennett and Cropper [105] examine combining hypergames with Strategic Choice to provide an effective method for modeling decision problems. Strategic Choice deals with uncertainty [106], where a participant moves between the activities of problem shaping, generating alternatives, comparing solutions, and finally choosing how to act. While hypergames and Strategic Choice often deal with uncertainty, they both offer different perspectives. In Strategic Choice, the emphasis is on the need to coordinate between parties, where in hypergames the emphasis is on communication as a means to makes threats, bluffs, or deception [105].

Putro et al. [33, 107–109] combine hypergames with genetic algorithms to produce adaptive learning procedures. The genetic algorithm is used to choose nature's strategies in order to improve perceptions. They present three learning methods where each method varies a part of the genetic algorithm (such as fitness evaluation, modified crossover, and action choice). The authors present two experiments that analyze the effect of uncertainty and crossover rates on the outcome of the learning procedures.

Kanazawa et al. [110–112] study hypergames and evolutionary game theory. They use hypergames to add perceptions to evolutionary game theory, which result in evolutionary hypergames. Interpretation functions, which specify the relationship between the player's strategies and those of their opponent(s), from hypergames are introduced into evolutionary games. These interpretation functions are then used to create the replicator dynamics for the evolutionary game, which describe the selection process for the distribution of the strategies in a given population. This process is demonstrated using the original application by Bennett to soccer hooliganism [111].

5.10. LG Hypergames. While not directly related to hypergame theory as envisioned by Bennett, LG hypergames have

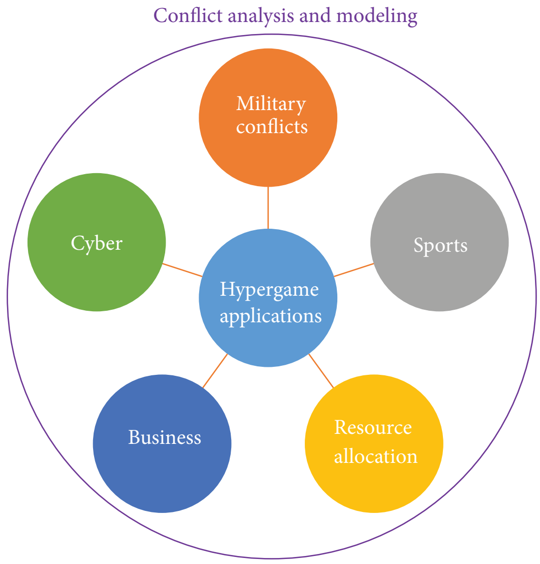

FIGURE 13: Hypergame application characterization.

a similar goal, to "account for drastic mutual influence of multiple subgames," and are applied to abstract board games (ASBs) [113]. Linguistic geometry (LG) hypergame was first demonstrated in [113], where it was used to infer the direct and indirect effects. Each ASB is dynamically linked together by interlinking maps, a concept similar to hyperlinks in an HTML document [114]. A detailed application of LG hypergames is given in [115].

6. Examples and Applications

Hypergame theory has been used to examine past military conflicts, which by their nature are conducted with missing information and misperceptions. Past conflicts lend to analysis because the excitement and fog of war have cleared as well as the outcome already being determined. Hypergame theory has also been applied to sports, resource allocation, and business, where competitive nature and proprietary information often lead to missing information and a desire to introduce misperceptions. Recently, hypergame theory has been applied to cyber in the form of attack/defender models.

We have separated applications of hypergames into these five topic categories, military conflict, sports, resource allocation, business, and cyber, holding the majority of the hypergame application work, as shown in Figure 13. An overview of the numerous applications in hypergame theory is summarized in Table 4. Each is listed chronologically and denoted with the corresponding year and topic category.

TABLE 4: Listing of hypergame applications, chronological.

1st Author and citation	Year	Category
Giesen [14]	1978	Business
Bennett [15, 16]	1979	Military conflicts
Giesen [17]	1979	Business
Wright [18], Shupe [19]	1980	Military conflicts
Bennett [20]	1980	Sports
Fraser [21, 22]	1980	Business
Bennett [23]	1980	Business
Bennett [24]	1981	Business
Fraser [25]	1981	Business
Said [26]	1982	Military conflicts
Bennett [27]	1982	Military conflicts
Fraser [28]	1983	Military conflicts
Stokes [29]	1983	Business
Okada [30]	1985	Resource allocation
Hipel [31]	1988	Military conflicts
Graham [32]	1992	Business
Vane [33]	1999	Cyber
Vane [34]	2000	Cyber
Maxime [35]	2002	Business
Kopp [8]	2002	Cyber
Hamandawana [36]	2007	Resource allocation
Novani [37]	2010	Business
House [38]	2010	Cyber
Gibson [39]	2013	Cyber

6.1. Military Conflicts. Bennett and Dando [15, 16] first applied hypergames to the first real-world application during their analysis of the Fall of France during WWII. They used hypergame theory to show how misperceptions between the two countries can lead to unexpected outcomes.

Wright et al. [18, 19] presented a more complex hypergame example in their analysis of the nationalization of the Suez Canal in the 1950s. This hypergame shows how one player waiting to participate in the conflict can lead to strategies changing over time. While this is a temporal concept, the analysis is only made for one point in time during the conflict.

Said and Hartley use hypergame theory to analyze the 1973 Middle East War [26]. Their analysis shows that each player behaves in a rational manner within their own perceptual beliefs. They also propose a methodology for applying hypergame theory to the crisis.

Rott [57] examines the Falkland/Malvinas conflict between Britain and Argentina in 1982. The author approaches the conflict from a different angle in the analysis of the conflict between Britain and Argentina. The hypergame analysis of the conflict is used to show how misperceptions dictated an outcome that was unexpected by all sides. This analysis uses three specific points in the conflict to conduct three different hypergame analysis. While multiple time points are used, each is picked and really does not contain any temporal aspects.

Bennett and Dando also model an arms race as a hypergame in [27]. Their analysis forces the modeler to consider the perceptions, beliefs, and actions of all parties involved, which they claim to lead to a more competent analysis.

Fraser et al. [28] apply five conflict analysis models to a possible nuclear confrontation between USA and USSR. The five conflict analysis models are normal form analysis from game theory, the extensive form of the game, metagame analysis [116], hypergame analysis [2, 117], and the state transition model [25, 118]. Their analysis determines that the hypergame analysis of conflicts is the best for modeling real-world conflicts.

6.2. Sports. Bennett et al. model soccer hooliganism [20] which appears in U.K. soccer around the late 1970s. They use the hooligan fans and the authorities as the players. Empirical studies were used to build up possible games that may be played between the players. The hypergame analysis showed that there were three critical variables: (1) the fans interpretation of how the authorities prepared for possible conflict; (2) how the authorities interpret the "play hooligan" strategy by the fans; (3) the effect previous incidents have on perception for future conflicts. The result of the analysis is that tolerance should be used by the authorities. This reduces the overpreparation and expectation that everyone is a hooligan and in time reduces the effect of previous incidents.

When the hypergame goes through a number of iterations, additional forces put pressure on players in the game. For example, previous incidents will place pressure on the authorities to be seen taking firm measures and may cause the authorities to expect trouble. If this is the case, then authorities will start using tougher measures. If the authorities expect malevolent fans, then there is the possibility that the fans will become malevolent and start playing the role after being categorized. Over several rounds, if each player is unhappy about the previous interaction, then they will start to see the other player as increasingly malevolent.

6.3. Resource Allocation. Okada et al. first applied hypergame analysis to water resource allocation in Japan's Lake Biwa conflict in the early 1970's [30]. The conflict is a water resource management problem, where the downstream users desire more water from the upstream water source, but the controllers of the water source are unresponsive. While each player in the Lake Biwa conflict had misperceptions about the other player's preferences, the hypergame analysis was able to correctly identify the compromise that resolved the conflict historically. This hypergame has three players: the Shiga Prefecture, downstream prefectures, and the national government. The authors use the notation from Howard [116] and the metagame analysis in [28] to solve the hypergame. While this game is unique in that it models three players, the details of the analysis are similar to [28].

Hamandawana et al. again applied a game theoretic analysis to a water management conflict [36]. They use a method similar to hypergame analysis to model the interstate conflict between Angola, Botswana, and Namibia over the shared water resource of the Okavango River. The authors use a hypothetical game to build a framework for developing sharing arrangements that minimize conflict, where players make compensatory sacrifices to offset the losses of other players.

Their model introduces the idea of perceived comprised strategic relationships. There are three types: fate control, reflexive control, and behavior control. In fate control, the player's outcome may be influenced by the actions of other players. With reflexive control, the player has some degree of control over the outcome regardless of the actions of other players. Behavior control is the case where the player's outcome is only feasible through interdependent actions of copartners. This idea follows that of Bennett with perceived games and Fraser with enforceable/credible equilibriums.

6.4. Business

6.4.1. Applications to Shipping. Hypergame theory was applied to a conflict in the oil shipping business in [14, 17]. The incident in 1954 almost led to the bankruptcy of Aristotle Onassis, an oil tanker fleet owner. The hypergame analysis showed that decisions made by a player, which appear to be irrational under a conventional game theory model, are actually rational when the perceptual limitations and differences in information are considered in hypergame theory.

Hypergame analysis was applied to an ongoing ship building conflict in [24]. The authors were invited by staff of a U.K. shipping company. Ship building had taken off in the 1970s in U.K., but due to developing countries building completing fleets and the oil crisis in 1973. The hypergame analysis helped to show how different countries supported the crisis in different ways. For example, Japan's profitable industries support the less profitable ones, which allow Japan to keep producing ships when the ship market went into a depression. Other developing countries had labor rates that were below those in U.K. and support the ship building industry which was lacking in U.K.

6.4.2. Negotiation and Contracting. Fraser and Hipel explore contract bargaining using hypergame theory [21]. They build a model using the information available to the bargainer and look at the effects of providing opponents with misinformation. They use the model to predict the expected course of events during a negotiation session. The authors provide the first implementation of hypergame analysis on a microprocessor called Conflict Analysis Program (CAP), discussed later.

Fraser and Hipel [25] explore labor-management negotiations, where they apply hypergame analysis to a hypothetical labor-management conflict. The hypothetical conflict is developed in detail in [22]. The authors again use the Conflict Analysis Program (CAP) to show that the best model does not always conform to the way things should be but sometimes will conform to how things actually are. For example, they build their model without considering union demands, fairness of salaries, benefits, or working conditions. Instead they model the power of the individual players.

Bennett used a hypergame analysis to explore a conflict where multiple bidders negotiate with a dispenser, who is able to accept the most generous offer [23]. This is a case of two nations bidding to get a multinational corporation to relocate to their jurisdiction. The model focuses on the ability of the dispenser to play bidders against each other.

Graham et al. [32] apply hypergame theory to study supply relationships and modify control systems. They use hypergames to identify misperceptions in the process that are causing inefficiency. These misperceptions are then identified and targeted for correction to improve efficiency in the supply relationships of the players. While the authors are studying twelve pairs of companies, they discuss the types of games created to study the relationship between a vendor of forgings and an engineering company.

6.4.3. Trade and E-Commerce. Stokes and Hipel use hypergame theory to study an international trade dispute over government subsidized export credits [29]. They model the awarding of large contracts to supply subway cars in New York City, which involves the U.S. and Canada, as well as the New York transit authority. Their analysis of the hypergame highlights the role of strategic deception in awarding contracts and presents logically reasonable resolutions.

Hypergame theory is applied to e-commerce by Leclerc and Chaib-draa in [35]. They use hypergame theory as an analysis tool for a multiagent environment. They show how multiple agents interact through communication and a mediator when each has differing views of the conflict. A discussion is also provided on how agents can take advantage of misperceptions.

Novani and Kijima [37] use a symbiotic hypergame model to examine the mutual understanding process between customer expectation and provider capability. They try to formalize the players' internal model dealing with the way each player identifies the situation subjectively and the interpretation function concerning how each player interprets the set of strategies. This model is then applied to different types of customers and providers that the authors develop.

6.5. Cyber. Unfortunately very little in the way of hypergame theory application has been done in the arena of cyber warfare. Although much recent work has used game theoretical applications in an attempt to model network security, the use of hypergame theory is not considered on the same level. Hints to its effectiveness have been suggested. Certainly, a computer-based tool can provide easy access to information [74]. It is easily conceivable that a cyber-defense system can be infused with a hypergame model in order to influence decisions on network defense. Even Vane, although not in a sense directed at defense agents, suggests that hypergame theory should be used for decision theory and game theory agents [119]. Providing a computer-based tool, the ability to analyze a situation with a hypergame model creates a quick and efficient technique for strategy selection.

6.5.1. Information Warfare. Kopp uses hypergame theory to model Information Warfare [8]. He uses a hypergame to describe how the manipulation of an information channel is reflected in the behavior of the adversaries. Figure 2 provides an overview of how hypergames improve upon the game theoretic model.

The focus is on the Information Warfare techniques of denial of information or degradation, deception and corruption, disruption and destruction, and subversion.

The hypergame provides a tool for understanding the nature of Information Warfare and allows for quantifying the effects of the action during warfare. The author determines that the hypergame theory can be used to model Information Warfare, because the strategies map directly into hypergame models.

6.5.2. Model with Obfuscation. There has been at least a small amount of work performed in the use of hypergame theory applied to cyber warfare. House and Cybenko in their paper [38] use hypergame theory to model a generic cyberattack where the defender can choose a specific subgame or the full game dependent upon the experience level of the current defending administrator. Their model is designed using Dr. Vane's HNF and static utility values and with the Row player as the attacker. Using learning models, it is shown that Row can eventually determine the percentages, which in the HNF are the belief-contexts, that each subgame is being used. In simulations, the results indicate that within 3000 to 5000 iterations these percentages are known within ±5% of the actual usage percentages. These results show that there is at least some measure of confidence that using a learning strategy one player can begin to understand the strategy selection of the other. This represents the ability to understand one's adversary in order to select one's own strategy to provide the best possibility at maximum utility.

The research does not conclude with these findings. Instead, the possibility of the defender (or Column player in this case) obfuscating the learning ability of the attacker is examined. It is shown that there does exist at least a limited ability for the Column player, using an obfuscation NEMS, to disrupt the ability of Row to learn Column's true usage percentages. This obfuscation NEMS is created by remodeling the utility values that were present in the initial Row learning experiments. The result is that Row will have a more difficult process to determine the strategy that has been played against it in each iteration. This approach is somewhat manufactured in order to provide Column with the ability to select strategies that may be misinterpreted. However, the fact that payoffs and subgame definitions are the heart of any hypergame scenario is presented as the reason for this approach.

The results of this research effort are of most interest due to the usage of hypergame theory in the attempt to model a cyber-warfare situation more than the actual content of the findings. House and Cybenko do not use the full HNF model, choosing to concentrate on learning the nature of the game through playing instead of attempting to outguess the opponent as discussed in [33, 34]. Despite this fact, the usage of the HNF to model the experiments shows the viability of the model to be incorporated into network defense initiatives. More avenues of the use of hypergame theory and specifically of models using the HNF are required to prove the effectiveness of the approach. The exploration of how hypergame theory can affect cyber defense and offense is only in its infancy.

6.5.3. Attacker-Defender Model. As discussed previously, Gibson [39] presents a hypergame model (based on HNF) of

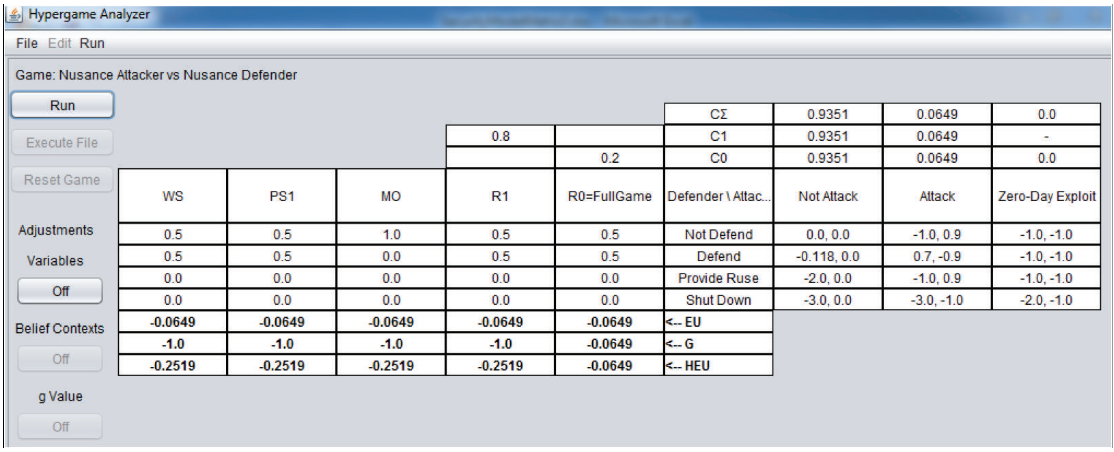

Figure 14: HNF analysis tool (HAT) software.

the work of Chen and Lenectre [95]. At the heart, this model is an attacker-defender game. It keeps the functional and non-zero-sum utilities from the Chen and Lenectre model.

With Gibson's model, the attacker is given a new strategy, zero-day exploit, which is an attack where there is no defense since the vulnerability is undiscovered. The defender is given two new strategies: providing ruse or shutdown. A defender may provide a ruse by fooling the attacker into attacking a honeypot, while collecting information about the type and style of the attack. The shutdown option allows the defender to remove the system from the network and stop the attack in its tracks but also removes the system from operation even for mission critical activities.

7. Hypergame Analysis Software

Hypergame analysis is possible by hand but it is not recommended; it is tedious belief-contexts between each game iteration. The ability of software to execute a hypergame analysis is discussed as different tools are explored for hypergame analysis. Microsoft Excel, Minitab, and other statistical software packages are able to model hypergames and can easily calculate utility functions from mathematical equations as shown at the IEE/WINFORMS Joint Program for Capital Science [120]. The main disadvantage is these programs have the inability to run multiple game iterations and update variables between iterations. Mathematical software, such as MATLAB or Mathematica, can calculate the utility functions from mathematical equations and run multiple game iterations and update variables as well as player beliefs between iterations, but this software is not specialized for hypergame analysis. This means that for each game model the entire model has to be built from scratch; there is no standardization of hypergames between researchers.

The first implementation of hypergame analysis on a microprocessor was done by Fraser and Hipel and called Conflict Analysis Program (CAP). Detailed information on CAP can be found in [21, 121, 122]. The authors use CAP to solve complex conflicts in [21, 25] during hypergame analysis. While this software was the first for hypergames, it may be outdated and there is no indication that it has been maintained.

Gambit is software designed for analyzing finite, noncooperative games using the strategic form [123]. Players and strategies can be added using the Gambit interface to quickly create a game for analysis. It has the ability to exchange game model to external tools, creating a standard for game theory model data. The main disadvantage of this software is lack of support for the complex hypergame model; there is no way in the Gambit interface to enter different games based on each player's perceptions or to use mathematical equations to calculate utility values during game analysis.

A software tool specifically designed for hypergame analysis, called HYPANT, was written by Brumley [124]. It uses a standard notation, referred to as a language, to represent hypergame models called Hypergame Markup Language (HML). The HML allows the hypergame model data to be saved, restored, and transported, as well as supported subgames based on the player's perceptions. The disadvantages of HYPANT are the lack of support for functional utility values; it only supports the stability and unilateral improvement values used by Frasier and Hipel in their analysis of the Cuban Missile Crisis [2].

Another hypergame analysis program based on Vane's HNF theory is called security policy assistant (SPA). SPA was created to assist in deciding if classified documents are released or withheld from foreign disclosure [125]. While the software manual was available, the software is not given its sensitive nature in decision-making with classified information. This software supports the application of hypergame theory beyond the previous applications of military, sports, and business conflicts, given this software's ability to assist in decision-making about classified documents.

The lack of suitable software meeting all the requirements for hypergame analysis caused Gibson to create the HNF analysis tool (HAT) software [39], as shown in Figure 14. HAT is written in Java and supports using the Extensible Markup Language (XML) to input and save game design. XML is an improvement over the HML used by HYPANT because XML is widely supported and has many tools to create, read, and verify, and it is not proprietary like HML as well. Once a game in XML is loaded, HAT allows multiple game iterations to be run, supporting static or random strategy selection. It also allows variables and belief-contexts to be updated between hypergame iterations. Given the availability of the HAT software and its ability to handle hypergames using HNF concepts developed by Vane [34], this software is used and updated throughout this research effort.

8. Hypergame Theory Fitness and Benefits

Evidence tends to point out that, given credible information, hypergame theory can do no worse than game theory models but has the potential to outdo them. With this in mind, very little research has been completed using hypergame theory to model cyber operations (attack, defense, etc.). Although there has been a limited research, it has been recommended that more attempts using hypergame theory applied to cyber warfare should be made. House and Cybenko have provided some limited insight about how hypergame theory can give credence to the approach. They were able to show that the HNF is a feasible modeling technique for experimentation in this arena. Further research into the use of hypergame theory for cyber defense and offense needs to be accomplished.

Little has also been done with hypergames in the temporal domain. Most conflicts develop over time and can have many rounds, by combining hypergames with the temporal domain; a more representative conflict model may be able to be created, leading to a better understanding about outcomes. Work with temporal game theoretic models indicates that a temporal model for hypergame theory is not only possible but would further be the foundation of the theory.

Hypergame analysis can become complex so software tools are necessary to help analysis complex models in a timely manner. Software tools are still in the maturing process with the first tool released by Alan Gibson specifically for hypergames based on Vane's HNF model. Continued software development and refinement will be necessary to support hypergame theory in the future, which will lead to improvements in conflict models under hypergame theory.

Disclaimer

The views expressed in this paper are those of the authors and do not reflect the official policy of the United States Air Force, Department of Defense, or the U.S. Government.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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