The Drag and Shape of Control Lines in Flight

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0   Introduction

This note documents (briefly) the development and implementation of the methods used computing for a control–line model aircraft the drag and shape of the lines in level flight. Wind and pilot technique (lead/lag) are considered. The wind is assumed to have constant velocity and direction and the pilot flies so as to make the handle rotate in a circle about a fixed point on the ground. The methods developed here have been implemented in a computer code in which the effect of Reynolds number on drag is accounted for as is a non standard atmosphere.

The problem that will be discussed has been thought about before and there are several references at the end of the paper. The first article and first paper I am aware of are: R. B. Johnston in *Model Airplane News* of March of 1948 (Reference[1]) and a paper by Henry Jex *et. alalia* at MIT in the summer of 1950 (Reference [2]). They not only had a good analysis but also the MIT paper backed it up with wind tunnel tests of both the control lines and of a control line speed model. Today it takes a deep-pockets hobby like radio control to draw this kind of attention. To read the Jex paper and see the wind tunnel results look at my website: `http://members.cox.net/microair3`

Don Monson (Reference [4]) and I (Reference[3]) have used this sort of analysis to produce results of various kinds. My analysis of (1) F2A World Champs results and the effect of line drag appeared in the *Aeromodeller Annual* and; (2) Flying technique in control line racing. This later material appeared in the *Control Line Gazette* under the pen name 'Marlon Gofast', in three issues of *Model Aircraft*, in the *Aeromodeller Annual* and in the Swedish *Model Flygnet* (Reference[5]). The editor of the *Aeromodeller*, Ron Moulton borrowed the article from *Model Aircraft*. He had trouble finding out who the Author was, thinking that Marlon Gofast was "surely a joke name". He discovered that I wrote the article too late to change the name of the author to mine, but wrote to thank me anyway.

Because of this interest, the fact that line drag is the largest item in speed and racing, and pilot technique being a very large contributor in racing, I take it there must be some continuing interest in the subject.

This note develops the results in steps. First, a simple analysis, essentially duplicating the results of Jex and Johnston but adding the line offset required. Then an empirical mathematical model of line drag coefficient as a function of Reynolds number ($\mathcal{R}_e$) based on extensive test data is developed next and used to improve the first model. The effect of $\mathcal{R}_e$ on C_D is most important for small models, however, with computers it is now easy to take many effects into account correctly for all cases. The Simple line drag analysis is then redone using the model of C_D variation with $\mathcal{R}_e$.

After these two sets of simplified analyses, the 'real' problem is taken on. The lines and drag are still there and in addition the pilot's handle position (leading or lagging the aircraft), radius of the handle from the center of rotation, coverage of a portion of the lines by the wing, and wind are all accounted for.

1 Analysis of a Simplified Case

Consider a simple case (the case R. B. Johnston and later Professor Larrabee and his students in the MIT "Tech Model Aircrafters" group started with): The aircraft is rotating about a point as it would if it were flown from a pylon. There is no wind and the lines have a constant drag coefficient. (The effect of Reynolds number will change this somewhat). Assume that the lines terminate at the aircraft c.g., that is, at the center of mass. This makes the line length equal to the flight circle radius, an assumption to be eliminated later with a better analysis. The drag of the lines is supported by the aircraft and the handle. What is the drag of the lines and how much of this drag is carried by the aircraft as compared with the load on the handle?

Standard aeronautical engineering symbols are used for density, drag coefficient, and so–on. In the list of symbols below, a consistent set of units must be used. If one uses the SI set, for example, density is in kilograms per cubic meter. If the USCS (U.S. Customary System) then density is in slugs per cubic foot. Examples of results will include these unit systems and, when mentioned, the "modeler" system used by model airplane magazines and others in the U.S. only in which a mixture of inches, pound mass, ounces (avoirdupois) and so–on are used.

Symbols

$\rho =$ atmospheric density

$\nu =$ kinematic viscosity

$d =$ line diameter

$v =$ velocity at an arbitrary point on the lines relative to earth

$v_p =$ velocity of aircraft relative to earth

$\mathcal{R}_e =$ Reynolds number. For lines $\mathcal{R}_e = vd/\nu$

$r =$ distance from the center of rotation to a point on the lines

$R =$ line length, here it is the flight circle radius

$\xi = \dfrac{r}{R}$ non dimensional distance from the flight circle center

$C_D =$ drag coefficient of a single line

$n =$ number of lines

$T =$ line tension

$D =$ drag force, subscript for handle and airplane $D = D_h + D_p$

$W =$ aircraft weight

$g =$ standard acceleration of gravity; mass to force conversion

$m =$ aircraft mass $W = gm$

$R_f =$ flight circle radius $R_f = R$ in the simple analysis

$\omega =$ angular velocity of aircraft about the center $\quad v_p = \omega R_f$

In the simple case the velocity of the lines at a radius x from the handle is $v = v_p x/R$. The total drag of the lines is :

$$D = n \int_{x=0}^{x=R} C_D d \frac{\rho}{2} \left(\frac{v_p x}{R} \right)^2 dx$$

$$D = C_D \frac{ndR}{3} \frac{\rho}{2} v_p^2$$

In other words, the "effective" frontal area of the lines is $ntdR/3$ when the drag calculation is based on airplane speed. The average drag coefficient for circular lines at the Reynolds number for model aircraft tether lines quite high, around 1.0.

To get the portion of drag that the aircraft has to carry, compute the moment of the drag forces about the center of rotation and then divide them by the aircraft flight circle radius. The result is the "drag at the aircraft".

$$D_p = \frac{n}{R} \int_{x=0}^{x=R} x C_D t d \frac{\rho}{2} \left(\frac{v_p x}{R} \right)^2 \, dx$$
$$D_p = C_D \frac{n d R}{4} \frac{\rho}{2} v_p^2$$

From the above it can be seen that the aircraft supports 3/4 of the line drag and the handle (pilot) supports 1/4 in this simple, constant C_D model. Although the math model used assumed the lines were straight, the curvature will be computed later and will be seen to be small.

The line tension, T, in this case (negligible line weight and curvature) is simply the centrifugal force required to keep the aircraft in a circular path.

$$T = \frac{W}{g} \frac{v_p^2}{R}$$

If the drag at the aircraft is divided by the line tension the result is an approximate tangent of what has been called the "line rake" angle, that is, the angle the lines sweep back from the nominal flight direction at the aircraft.

$$\tan(\beta) = \frac{D}{T} = \frac{C_D n R^2 d g \rho}{8W}$$

From this it is apparent that to first order the line rake angle β is independent of flight speed. Of course when wind and other factors are introduced there will be variation in this angle, but this is the "first order result".

With respect to line drag, this is as far as the Jex effort went with analysis. They did conduct wind tunnel tests of lines, however, and these indicated that solid lines had less resistance than cable. It is not known to what degree the tunnel turbulence affected the results. Later this note will use the results obtained by Weisselberger (Reference [5]) as a more accurate model of line drag coefficient than a constant, but to tell the truth the drag of cables and the drag of solid lines when vibrating is not backed up by reliable measurements.

2   Effect of $\mathcal{R}_e$ and a Line Drag Coefficient Model

The drag coefficient of circular cylinders changes markedly with $\mathcal{R}_e$. This section will account for that change, but in order to make line drag component calculations appear similar to those derived in the previous section and to enable calculations similar to those made for the rest of the airplane an effective drag coefficient and effective drag area is created for the lines. This coefficient will be the same as the actual C_D for the lines if C_D is constant and otherwise will essentially provide an equivalent constant called $\bar{C}_D$ to use when C_D varies along the line because of $\mathcal{R}_e$. The effective area will make up for the fact that the airspeed of the lines varies along the radius. For model aircraft the maximum $\mathcal{R}_e$ is in the low thousands. Most of the drag is created at $\mathcal{R}_e$ greater than 50. The variation of drag coefficient with $\mathcal{R}_e$ is shown below:
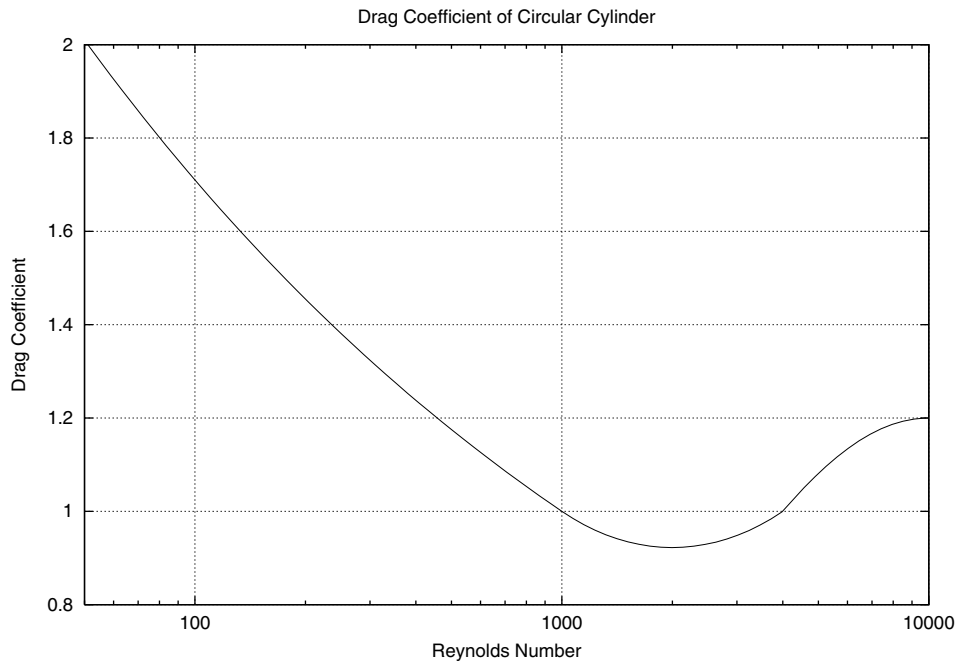
A word about assumptions: These calculations assume the lines are straight when calculating the drag. The lines are assumed to be straight lines connecting the handle and aircraft c.g. and that are shielded by a given amount of wing (this latter effect is considered in the next section). The curvature of the lines is computed and taken into account after the drag has been calculated so that in the event of large curvatures of the lines, some error due to approximation is introduced.

The drag coefficient of each of the lines is assumed to be identical. No reduction for shielding is accounted for, nor is there any estimate of the increased drag due to line vibration, which is known to occur within a range of Strouhal numbers.

The formula for the line drag the *aircraft* must support will be cast in the form below :

$$D = \bar{C}_{D_p} \frac{ndR}{4} \frac{\rho}{2} v_p^2$$

$$\bar{C}_{D_p} = \frac{8D_p}{\rho ndR v_p^2}$$

To make this even more like 'conventional' formulæ in aerodynamics use q for the dynamic pressure. For tethered airplanes use the dynamic pressure at the aircraft $q = \rho v_p^2 / 2$. For the drag reference area use $S_\pi = nRd/4$. Using these symbols, the standard notation for drag is $D = C_d q S_\pi$. By analogy for the line drag the aircraft must carry is:

$$D_p = \bar{C}_{D_p} q S_\pi$$

.

With this simple model $\mathcal{R}_e$ along the lines varies directly with speed. The $\mathcal{R}_e$ of the lines at the aircraft c.g. is :

$$\mathcal{R}_e|_p = \frac{v_p d}{\nu}$$

Let ξ be a non–dimensional radius variable:

$$\xi = \frac{r}{R}$$

In this case where $0 \leq r \leq R$ the $\mathcal{R}_e$ anywhere on the lines is:

$$\mathcal{R}_e = \xi \mathcal{R}_e|_p \qquad \text{where } 0 \leq \xi \leq 1$$

The drag coefficient will then have a functional form of:

$$C_D(\mathcal{R}_e) = C_D(\xi \mathcal{R}_e|_p)$$

Using the integral of the first section we find:

$$\bar{C}_{D_p} = 4 \int_{r=0}^{r=R} C_D(\mathcal{R}_e) \left(\frac{r}{R}\right)^3 \frac{dr}{R}$$

$$\bar{C}_{D_p} = 4 \int_{\xi=0}^{\xi=1} C_D(\xi \mathcal{R}_e|_p) \xi^3 \, d\xi$$

This has the characteristics required and, when an algorithm for generating $C_D(\mathcal{R}_e)$ is available, is readily integrated numerically.

The algorithm for generating a numerical approximation to $C_D(\mathcal{R}_e)$ will be presented and the resulting relation integrated with a robust adaptive quadrature routine, QUANC8, described in Reference [6] will be used.

The general results follow from the fact that for $\mathcal{R}_e < 1000$ the drag coefficient decreases with increasing $\mathcal{R}_e$ so that in this range $\bar{C}_D < C_D(\mathcal{R}_e|_p)$. For large speed models the $\mathcal{R}_e$ may be as high as 5000 and the effect can even be the opposite.

The results of the math model for C_D variation, previously shown in the graph above, that models the experimental curve on a log-linear scale that are shown on the web page http://members.cox.net/vcb13/CylDrag.htm . The model has second order contact (continuous first and second derivatives) between all but one of the fit segments. Also on the web page a plot of the corresponding $\bar{C}_D$ is shown.

As an example of S_π for FAI classes F2A, F2C, and an F2C proposal the table below presents computations based on the model given above:

**F2 Line Drag Area - for 2 lines**

Class/speed	thickness - *mm*	length - *m*	area - *cm*^{2}
F2A - 80*m/s*	0.40	17.68	27.0
F2C - 55*m/s*	0.30	15.92	24.2
F2C(proposal)	0.40	15.92	30.3

Note that the higher speed of the F2A class results in lower drag coefficients for the wire and, partly because of this, the drag coefficient for F2A on longer lines than the F2C proposal with the same line diameter is less. The main reason, however, is that the F2C estimates include the effect of the pilot lagging the aircraft and flying with the handle away from the true center of rotation. This latter effect and the effect of wind is considered in the following section.

3   Model Considering Wind and Handle Motion

The first elaboration on the model used above is to model the motion of the handle. In the previous section the handle was at the center of rotation. The next best approximation is to have the handle and the aircraft move in a circle around a common center. The handle will be at a radius R_h from the center of rotation. The lines will extend from the handle to the mass center of the aircraft, although the exposed portion of the lines will terminate at the wing tip. Two lines from the rotation center to the handle and aircraft, respectively, will make and angle θ with each other, $\theta > 0$ for pilot lagging the aircraft. This sign convention makes positive θ correspond to a line pull in the drag direction. The angle between the radial line from the center to the aircraft and the line from the handle to the aircraft is ϕ. If R_l is is length of the lines and R_f is the flight circle radius:

Flight Circle, Handle, and Line Radius
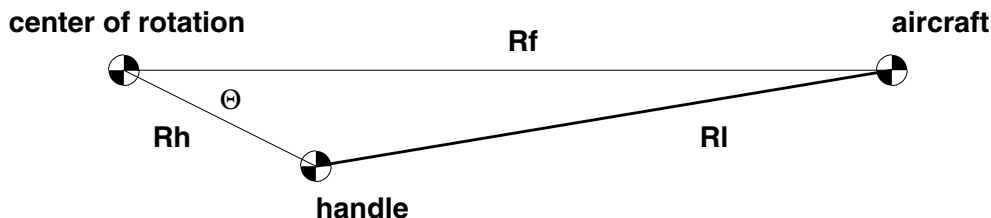
$$R_f = R_h \cos\theta + \sqrt{R_l^2 - (R_h \sin\theta)^2}$$

$$\sin\phi = \frac{R_h}{R_l}\sin\theta$$

The changes in the calculations above will only require the correct computation of velocity along the lines.

To the degree of approximation used here, the drag of any small segment of the lines is function only of the air velocity normal to the lines. To compute this velocity, define a new coordinate, x, measured from the intersection of a line perpendicular to the lines drawn from the center of rotation. Then the distance from this point to the handle is $x_1 \overset{\text{def}}{=} R_h \cos(\theta + \phi)$. The velocity (without wind) normal to the lines is :

$$v = \omega x \quad x_1 \leq x \leq R_l + x_1$$

The definition of x_1 is shown below

Define X1

center of rotation

Rf

Θ

handle

x1
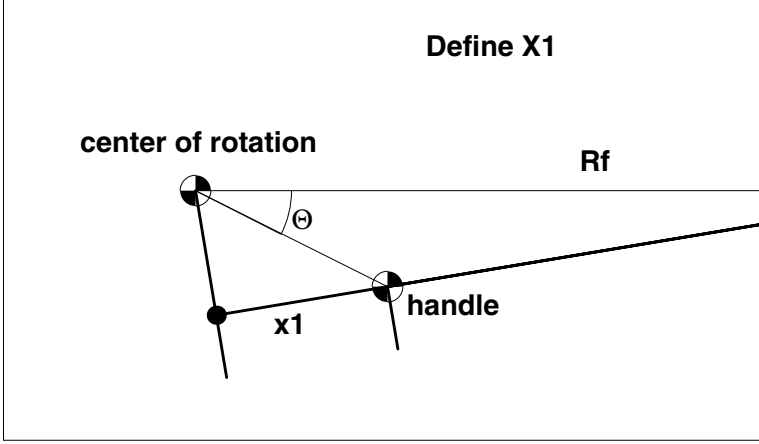
When ϕ is small and the approximation $\cos\phi \approx 1$ is made. In addition, some of the outer part of the line is covered by the wing, say from $R_l - R_w$ to R_l . This part of the lines creates no drag.

Again, using the same idea as the integral of the first section, but evaluated along the new nominal line direction. The moment arm of the drag at the handle is $(x - x_1)$. becomes:

$$\bar{C}_{D_p} = 4 \int_{r=x_1}^{r=R_l+x_1-R_w} C_D(\mathcal{R}_e) \left(\frac{r}{R_l}\right)^2 \left(\frac{x-x_1}{R_l}\right) \frac{dr}{R_l}$$

$$\bar{C}_{D_p} = 4 \int_{\xi=\xi_1}^{\xi=1+\xi_1-\xi_w} C_D(\xi\mathcal{R}_e|_p)\xi^2 \left(\xi - \xi_1\right) d\xi$$

$$\xi_1 \stackrel{\text{def}}{=} \frac{R_h}{R_l} \cos(\theta + \phi)$$

To show the effect of the handle in simple terms assume the drag coefficient is a constant equal to 1. and $\xi_w = 0$ so that the answer will compare with the simple analysis earlier, then

$$\bar{C}_{D_p} = 4 \int_{\xi=\xi_1}^{\xi=1+\xi_1} \xi^2 \left(\xi - \xi_1\right) d\xi$$

This can be integrated analyiticaly and becomes

$$\bar{C}_{D_p} = 1 + \frac{8}{3}\xi_1 + 2\xi_1^2$$

so that the line drag coefficient is increased unless the handle is nearly 'behind' the center of rotation, that is $\theta > \pi/2$ approximately.

Using this same technique at the handle end for a constant $C_D = 1$ for the lines:

$$\bar{C}_{D_h} = 1 + 4\xi_1 + 6\xi_1^2$$

Note that in this case the line drag is not all that must be computed when evaluating aircraft performance. The apparent drag or thrust of the lines due to the pilot lagging or leading the aircraft must be accounted for. In this case the thrust is from the line tension force at an angle ϕ with respect to the aircraft velocity vector. This component is proportional to the square of velocity as is the drag and it can be a more important factor than drag changes in the evaluation of lead/lag. The drag changes are due mainly to the change in flight circle radius.

$$T \tan \phi = \tan \phi \frac{W}{g} \frac{v_p^2}{R}$$

Note that if an aircraft is flown as the F2C rules imply, the angle θ is positive and the line tension component is in the drag direction. Increased weight requires increased power. Only when the aircraft is flown from a pylon is there no "pilot contribution" to line drag.

If this line thrust due to handle position is converted to a "drag coefficient" style model an equivalent "drag coefficient" increment for line thrust would be :

$$\Delta C_d = \frac{4T}{qR(nR_ld)}$$

$$\Delta C_d = \tan \phi \frac{8W}{\rho g R(nR_ld)}$$

In the relations above the velocity is the instantaneous groundspeed so the effect of wind will make the increment in effective drag coefficient not directly proportional to airspeed squared as is drag and lift.

When wind is present the component parallel to the lines is ignored. Tests have shown that drag of cylinders is a function of normal velocity only over a large range of yaw angles. The component normal to the lines is v_w and is constant along the lines. Adjusting the terms in the integral to account requires a new relation for velocity.

$$\zeta \overset{\text{def}}{=} \frac{v_w}{v_p}$$

$$V = \xi v_p + v_w = v_p(\xi + \zeta)$$

$$\bar{C}_{D_h} = 4 \int_{\xi=\xi_1}^{\xi=1+\xi_1-\xi_w} C_D((\xi+\zeta)\mathcal{R}_e|_p)(\xi+\zeta)^2(\xi-\xi_1)\,d\xi$$

In flying around the circle v_w will vary sinusoidally so that $\bar{C}_D$ will be a function of azimuth angle ψ. In general for a wind velocity of V_w the value used in the relation above is $v_w = V_w \sin(\psi)$

4 Angle of Lines at Aircraft and Handle

The "line rake", that is the angle the lines make at the attachment point on the aircraft measured from a straight line from the center of rotation to the aircraft, is of interest. It can be used as a first estimate to the line guide location. The angle at the handle can be used to estimate the total drag of the lines.

The part of the angle at the aircraft or handle that line drag is responsible for can be computed by dividing the drag load supported by the aircraft or handle by the line tension. To get the drag at the handle, moments are taken about the handle.

$$\bar{C}_{D_h} = 12 \int_{\xi=\xi_1}^{\xi=1+\xi_1-\xi_w} C_D((\xi+\zeta)\mathcal{R}_e|_p)\left((\xi+\zeta)-\xi_l\right)(\xi+\zeta)^2 \, d\xi$$

One expression will appear several times so the notation

$$\frac{ntg\rho}{2W} = \frac{1}{R_*^2}$$

will be used

At the aircraft the relation is the same as before.:

$$\tan(\beta) = \frac{D}{T} = \bar{C}_{D_p}\frac{nR_f^2 dg\rho}{8W} = \frac{\bar{C}_{D_p}}{4}\left(\frac{R_f}{R_*}\right)^2$$

At the handle :

$$\tan(\beta)_h = \frac{D}{T} = \bar{C}_{D_h}\frac{nR_f^2 dg\rho}{24W} = \frac{\bar{C}_{D_h}}{12}\left(\frac{R_f}{R_*}\right)^2$$

The effective drag coefficient at the handle with no–wind conditions is larger than the drag coefficient at the aircraft because the drag coefficient of the lines decreases with increasing Reynolds number for most control line model aircraft conditions. In general for all but the largest line Reynolds numbers $\bar{C}_{D_h} \approx k\bar{C}_D \qquad 1.1 < k < 1.2$

The large value of k is for low Reynolds numbers: $k = 1.2$ coresponds to $\mathcal{R}_e \simeq 200$.
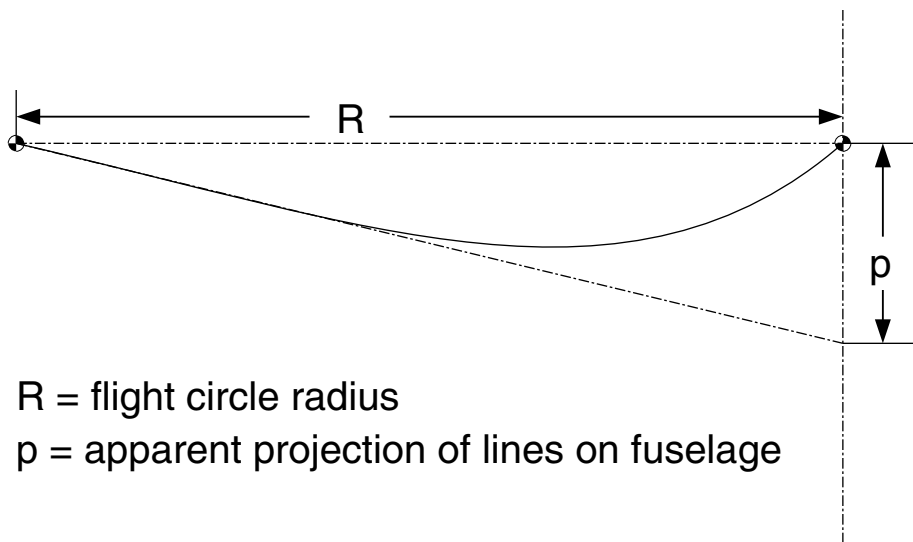
Line Average Drag Coefficient
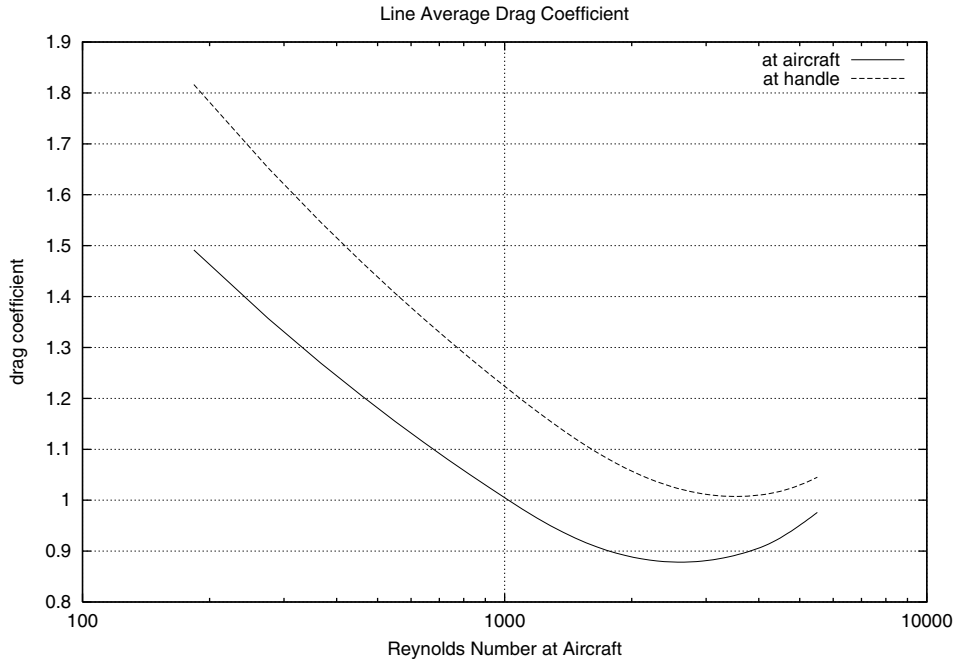
Note that the *projection* of the slope of the lines at the handle out to the aircraft p is approximately a function of R^3.

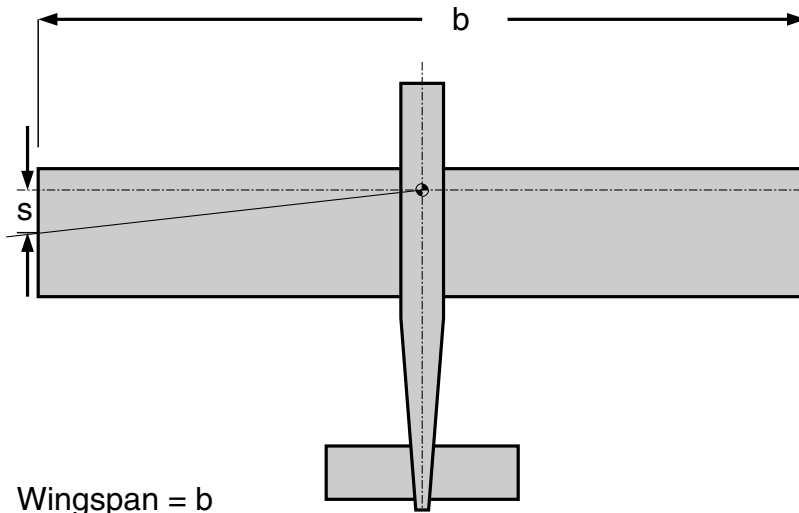$$p = \tan(\beta)_h R_l = \bar{C}_{D_h} R_f^2 R_l \frac{ntg\rho}{24W}$$

$$p = \frac{\bar{C}_{D_h}}{12} R_l \left(\frac{R_f}{R_*}\right)^2$$

The figure below shows this projection.

R = flight circle radius

p = apparent projection of lines on fuselage

The figure below shows the offset of the line guide outlet distance, s, aft of the the center of gravity.

Line Offset

Wingspan = b

Line Offset = s

From these figures and the work above one has:

$$\frac{s}{p} = \frac{b}{2R_l} \frac{\tan(\beta)}{3 \tan(\beta)_h}$$

$$\frac{s}{p} = \frac{b}{2R_l} 3 \frac{\bar{C}_{D_p}}{\bar{C}_{D_h}}$$

Also the measured value of p can be used to estimate $\tan(\beta)_h$ and from there, using the appropriate value of k, an experimental value of $\bar{C}_{D_p}$.

5   Miscelaneous Items

The timed speed in steady state flight with a wind may be estimated by assuming the airspeed to remain constant. This is correct within the assumprtions used here if there is no wind. With wind it is fairly accurate if the rate of change of windspeed relative to the aircraft is slow.

This models the aircraft speed relative to the earth (v_p) given the constant airspeed (v_a) and windspeed (v_w) as:

$$v_p = v_a + v_w \cos \beta$$

where β is the angle between the direction of flight and the direction of the wind.

The time required for one lap is

$$T = \int_{\beta=0}^{\beta=2\pi} \frac{dR_f}{v_p} d\beta$$
$$T = \frac{2\pi R_f}{\sqrt{v_a^2 - v_w^2}}$$

As can be seen for modest winds the effect of wind is small and always makes the average speed lower. 'Timed speed', however is based on the nominal line length. The actual flght radius can be significantly different so the timed speed is:

$$V = \sqrt{v_a^2 - v_w^2} \frac{R_l}{R_f}$$

6   Bibliography

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