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Influence of viscous dissipation and thermo-diffusion on double diffusive convection over a vertical cone in a non-Darcy porous medium saturated by a non-Newtonian fluid with variable heat and mass fluxes

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Abstract: This paper emphasizes the thermo-diffusion and viscous dissipation effects on double diffusive natural convection heat and mass transfer characteristics of non-Newtonian power-law fluid over a vertical cone embedded in a non-Darcy porous medium with variable heat and mass flux conditions. The Ostwald–de Waele power-law model is employed to describe the behavior of non-Newtonian fluid. Local non-similarity procedure is applied to transform the set of non-dimensional partial differential equations into set of ordinary differential equations and then the resulting system of equations are solved numerically by Runge-Kutta fourth order method together with a shooting technique. The influence of pertinent parameters on temperature and concentration, heat and mass transfer rates are analyzed in opposing and aiding buoyancy cases through graphical representation and explored in detail.

Keywords: Power-law fluid, Viscous dissipation, Double diffusive convection, Vertical cone, Thermo-diffusion, Non-Darcy porous medium

1 Introduction

From the past few years, the analysis of double diffusive convection through a porous medium has been the subject of a very intense research activity owing to its large number of applications such as chemical contaminant dispersion all the way through oil saturated by water, grain storage installations, geothermal reservoirs etc. Double diffusive convection is a convection induced by two different density gradients which retains distinct diffusion rates. In double diffusive convection the density of fluid mixture depends on concentration and temperature. An extensive research considered on double diffusive free convection flow over various geometries in porous medium with different physical conditions [1–4].

Due to the great extent of industrial importance of non-Newtonian fluid in porous medium, it is required to analyze the free convection flow of non-Newtonian power-law fluid through saturated porous medium. In this direction, Shenoy [5] discussed the heat transfer attributes of non-Newtonian power law fluids with/without yield stress embedded in porous media considering oil reservoir and geothermal engineering applications. Several researchers to mention a few [6–13] examined the non-Newtonian power-law fluid flow over various geometries embedded in Darcy or non-Darcy porous medium, in which the authors used the Ostwald–de Waele power-law model to describe the non-Newtonian fluid behavior.

Viscous dissipation plays a vital role in free convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes (Gebhart [14]). The effect of viscous dissipation is of pragmatic importance in natural convection embedded in porous medium in connection with their experimental correlation for the heat transfer in external flows (Fand and Brucker [15]). Most of the non-Newtonian fluids are extremely viscous which motivated the researchers to analyze the viscous dissipation effect in non-Newtonian fluid through porous medium. Several researchers discussed the effect of viscous dissipation in convective heat transfer flow over various geometries embedded in porous medium saturated with Newtonian/non-Newtonian fluid [16–19].

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Thermal diffusion, also called Soret effect in which molecules are carried in a multi-component mixture impelled by temperature gradients. Postelnicu [20] studied numerically the heat and mass transfer characteristics about natural convection over a vertical surface saturated porous media by taking chemical reaction and cross-diffusion effects. The Soret and melting effects on free convection flow of non-Newtonian power-law fluid saturated non-Darcy porous medium has been discussed by Kairi and Murthy [21]. Kairi and Murthy [22] analyzed the viscous dissipation and Soret effects on natural convection flow over a cone through a non-Darcy porous medium saturated with non-Newtonian fluid.

From the existing literature, it seems that the influence of viscous dissipation and thermo-diffusion on free convection flow over a vertical cone in non-Darcy porous medium saturated by non-Newtonian fluids with variable heat and mass fluxes has not been discussed so far. Motivated by the erstwhile studies, the main objective of the existing work is to get a local non-similarity solution for the aforesaid problem. Local non-similarity method is used to get the set of similarity equations and the resulting equations are worked out numerically to get the solution. The effects of various parameters on flow, temperature and concentration distributions, and heat and mass transfer rates are shown graphically.

2 Mathematical Formulation

Consider the two-dimensional, steady, laminar boundary layer flow due to natural convection over a vertical cone embedded in non-Darcy porous medium saturated with non-Newtonian power-law fluid. The surface of the cone is subject to variable heat and mass fluxes Q_T and Q_C, respectively. The ambient temperature and concentration are T_∞ and C_∞, respectively. The porous medium is assumed to be homogeneous and isotropic. The origin of the coordinate system is at vertex of cone and the cone is placed with its axis of symmetry along the vertical direction. The cone apex of angle is γ. Choose the coordinate system such that x-coordinate measures along surface of the cone and y-coordinate is perpendicular to the surface of cone, as exhibited in Fig. 1.

Making use of Boussinesq and boundary layer approximations, the governing equations for fluid flow problem (Kairi and Murthy [22]) may be written as:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

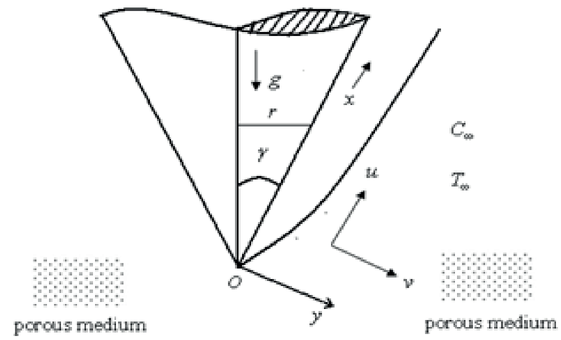


Fig. 1: Physical Model and coordinate system

$$\frac{\partial u^n}{\partial y} + \frac{\rho_\infty b K^*}{\mu^*}\frac{\partial u^2}{\partial y} = \frac{\rho_\infty g \cos\gamma K^*}{\mu^*}\left(\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu^*}{\rho_\infty K^* c_p} u \left(u^n + \frac{\rho_\infty b K^*}{\mu^*} u^2\right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1 \frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions are given by

$$y = 0 : v = 0, \frac{\partial T}{\partial y} = -\frac{Q_T}{k}, \frac{\partial C}{\partial y} = -\frac{Q_C}{D} \tag{5}$$

$$y \to \infty : u \to 0, T \to T_\infty, C \to C_\infty \tag{6}$$

In the above equations, u and v are velocity components along the x- and y- directions respectively, α indicates the effective thermal diffusivity, ρ_∞ is the reference density, T is the temperature, g represents the acceleration due to gravity, k is the effective thermal conductivity of the saturated porous medium, C represents the concentration, D is the effective solutal diffusivity, D_1 quantifies the contribution to the mass flux due to temperature gradient, β_T and β_C are the thermal and solutal expansion coefficients, respectively, c_p is the specific heat at constant pressure, b indicates the empirical constant related to the Forchheimer porous inertia term, μ^* represents the consistency index of power-law fluid, n represents the power-law index in which $n < 1$ refers to pseudo-plastics fluid, $n = 1$ represents Newtonian fluid and $n > 1$ indicates dilatants fluid. Christopher and Middleman [23] and Dharmadhikari and Kale [24] proposed the following relationships for the permeability of the flow as a function of the power-law index n as follows:

$$K^* = \frac{1}{2c_t}\left(\frac{n\varphi}{3n+1}\right)^n \left(\frac{50K}{3\varphi}\right)^{\frac{n+1}{2}}$$

where $K = \frac{\varphi^3 d^2}{150(1-\varphi)^2}$, is the porosity of the medium and

$$c_t = \begin{cases} \frac{25}{12} & \text{Christopher and Middleman [23]} \\ \frac{2}{3}\left(\frac{8n}{9n}+3\right)^n \left(\frac{(10n-3)}{6n+1}\right)\left(\frac{75}{16}\right)^{\frac{3(10n-3)}{10n+11}} & \text{Dharmadhikari and Kale [24]} \end{cases}$$

for $n = 1$, $c_t = \frac{25}{12}$.

In view of the continuity equation, we define the stream function $\psi(x, y)$ by $u = \frac{1}{r}\frac{\partial\psi}{\partial y}$ and $v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$.

Introducing the following transformations

$$\eta = \frac{y}{x} Ra_x^{\frac{1}{3}},\ \psi(\epsilon, \eta) = \alpha r Ra_x^{\frac{1}{3}} f(\epsilon, \eta),$$

$$T - T_\infty = \frac{Q_T}{k} x Ra_x^{-\frac{1}{3}} \theta(\epsilon, \eta),$$

$$C - C_\infty = \frac{Q_m}{D} x Ra_x^{-\frac{1}{3}} \phi(\epsilon, \eta),\ r = x\sin\gamma$$

and $\epsilon = \frac{g\beta_T \cos\gamma}{c_p}$ where

$$Ra_x = \left(\frac{x}{\alpha}\right)^{\frac{3n}{2n+1}} \left[\frac{\rho_\infty \beta_T \cos\gamma K^* Q_T x}{\mu^*}\right]^{\frac{3n}{2n+1}}$$

On substituting the above transformations in Eqs. (2)–(4), we get the following nonlinear system of partial differential equations.

$$(nf'^{n-1} + 2Gr^* f')f'' = (\theta' + N\phi') \tag{7}$$

$$\theta'' + \frac{3n+2}{2n+1} f\theta' - \frac{n}{2n+1} f'\theta + \epsilon f'\left[(f')^n + Gr^*(f')^2\right] = \epsilon\left(f'\frac{\partial\theta}{\partial\epsilon} - \theta'\frac{\partial f}{\partial\epsilon}\right) \tag{8}$$

$$\frac{1}{Le}\phi'' + \frac{3n+2}{2n+1} f\phi' - \frac{n}{2n+1} f'\phi + S_r\theta'' = \epsilon\left(f'\frac{\partial\phi}{\partial\epsilon} - \phi'\frac{\partial f}{\partial\epsilon}\right) \tag{9}$$

and the associated boundary conditions become

$$f + \frac{2n+1}{3n+2}\epsilon\frac{\partial f}{\partial\epsilon} = 0,\ \theta' = -1,\ \phi' = -1 \text{ at } \eta = 0 \tag{10}$$

$$f' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty. \tag{11}$$

In the above, $S_r = \frac{D_1 Q_T}{\alpha Q_C}$ is the Soret parameter, $Le = \alpha/D$ indicates the Lewis number, $\epsilon = Ge_x$ (= Gebhart number) represents the viscous dissipation parameter, $Gr^* = \left(\frac{\rho_\infty bK}{\mu}\right)\left(\frac{\alpha}{x}\right)^{2-n} Ra_x^{4-2n/3}$ is the modified Grashof number. $N = \beta_C Q_m k/\beta_T Q_T D$ is the buoyancy ratio with $N > 0$ represents the aiding buoyancy and $N < 0$ represents the opposing buoyancy.

Integrating Equation (7) once and using the boundary condition (11) gives:

$$f'^n + Gr^* f'^2 = \theta + N\phi \tag{12}$$

On introducing equation (12) the equation (8) transformed to:

$$\theta'' + \frac{3n+2}{2n+1} f\theta' - \frac{n}{2n+1} f'\theta + \epsilon f'(\theta + N\phi) = \epsilon\left(f'\frac{\partial\theta}{\partial\epsilon} - \theta'\frac{\partial f}{\partial\epsilon}\right) \tag{13}$$

The local Nusselt number $Nu_x = Q_T x/(k(T_w - T_\infty))$ and local Sherwood number $Sh_x = Q_m x/(D(C_w - C_\infty))$ are defined in non-dimensional form as:

$$\frac{Nu_x}{Ra_x^{\frac{1}{3}}} = \frac{1}{\theta(\epsilon, 0)} \tag{14}$$

$$\frac{Sh_x}{Ra_x^{\frac{1}{3}}} = \frac{1}{\phi(\epsilon, 0)} \tag{15}$$

3 Numerical Method

To solve the nonlinear coupled partial differential equations (7), (9) and (13) along with the boundary conditions (10) and (11), first apply a local non-similarity procedure (Minkowycz and Sparrow [25]) which has been studied by several researchers to solve various non-similar boundary value problems. The boundary value problem resulting from this procedure is solved numerically by Runge Kutta fourth order method with Shooting technique.

According to local non-similarity procedure, the system of partial differential equations considered here are first converted to a system of ordinary nonlinear differential equations by introducing new unknown functions of ϵ derivatives. In the first level of truncation the ϵ derivatives in equations (9) and (13) can be neglected because, the terms accompanied by $\epsilon = \frac{\partial}{\partial\epsilon}$ are assumed to be very small and this is true when $\epsilon \ll 1$. Thus we get the local similarity equations (first level of truncation) are

$$\left(nf'^{n-1} + 2Gr^* f'\right) f'' = (\theta' + N\phi') \tag{16}$$

$$\theta'' + \frac{3n+2}{2n+1} f\theta' - \frac{n}{2n+1} f'\theta + \epsilon f'(f'^n + Gr^* f'^2) = 0 \tag{17}$$

$$\frac{1}{Le}\phi'' + \frac{3n+2}{2n+1} f\phi' - \frac{n}{2n+1} f'\phi + S_r\theta'' = 0 \tag{18}$$

The corresponding boundary conditions are

$$\eta = 0 : f(\epsilon, \eta) = 0,\ \theta'(\epsilon, \eta) = -1,\ \phi'(\epsilon, \eta) = -1 \tag{19}$$

$$\eta \to \infty : f'(\epsilon, \eta) \to 0, \theta(\epsilon, \eta) \to 0, \phi(\epsilon, \eta) \to 0 \quad (20)$$

For the second level of truncation, we introduce new variables $G = \frac{\partial f}{\partial \epsilon}$, $H = \frac{\partial \theta}{\partial \epsilon}$ and $K = \frac{\partial \phi}{\partial \epsilon}$ restoring all the neglected terms at the first level of truncation. Thus, the governing equations at the second level are:

$$\left(nf'^{n-1} + 2Gr^* f'\right) f'' = (\theta' + N\phi') \quad (21)$$

$$\theta'' + \frac{3n+2}{2n+1} f\theta' - \frac{n}{2n+1} f'\theta + \epsilon f'\left(f'^{n} + Gr^* f'^{2}\right) = \epsilon\left(f'H - \theta' G\right) \quad (22)$$

$$\frac{1}{Le}\phi'' + \frac{3n+2}{2n+1} f\phi' - \frac{n}{2n+1} f'\phi + S_r\theta'' = \epsilon(f'H - \theta' G) \quad (23)$$

The corresponding boundary conditions are

$$\eta = 0 : f + \frac{2n+1}{3n+2}\epsilon G = 0, \theta'(\epsilon, \eta) = -1, \phi'(\epsilon, \eta) = -1 \quad (24)$$

$$\eta \to \infty : f'(\epsilon, \eta) \to 0, \theta(\epsilon, \eta) \to 0, \phi(\epsilon, \eta) \to 0 \quad (25)$$

At the third level of truncation, we differentiate Eqs. (21)–(23) with respect to ϵ and neglecting the terms $\frac{\partial G}{\partial \epsilon}$, $\frac{\partial H}{\partial \epsilon}$ and $\frac{\partial K}{\partial \epsilon}$ to get the following system of equations

$$\left(nf'^{n-1} + 2Gr^* f'\right) G'' + (n(n-1)f'^{n-2} + 2Gr^*)G'f'' = H' + NK' \quad (26)$$

$$H'' + f'(\theta + N\phi) - \frac{3n+1}{2n+1} f'H + \frac{3n+2}{2n+1} fH' + \frac{5n+3}{2n+1} G\theta' - \frac{n}{2n+1} G'\theta = \epsilon(G'H - H'G - G'(\theta + N\phi) - f'(H + NK)) \quad (27)$$

$$\frac{1}{Le}K'' - \frac{3n+1}{2n+1} f'K + \frac{3n+2}{2n+1} fK' + \frac{5n+3}{2n+1} G\phi' - \frac{n}{2n+1} G'\phi = \epsilon(G'K - K'G) \quad (28)$$

The corresponding boundary conditions are

$$\eta = 0 : G(\epsilon, \eta) = 0, H'(\epsilon, \eta) = 0, K'(\epsilon, \eta) = 0 \quad (29)$$

$$\eta \to \infty : G'(\epsilon, \eta) \to 0, H(\epsilon, \eta) \to 0, K(\epsilon, \eta) \to 0 \quad (30)$$

4 Results and discussion

The non-linear coupled differential equations (21–23) and (26–28) along with the boundary conditions (24–25) and (29–30) are worked out numerically using shooting technique. The integration length η_∞ varies with the parameter values and it has been suitably chosen every time such that the boundary conditions at the outer edge of the boundary layer are satisfied. In order to validate the accuracy of the solution, the existing results are compared with those obtained by Yih [11, 26] in some special cases and found that the results are in good agreement as shown in Table 1 and 2. Numerical calculations have been carried out for different values of the various dimensionless parameters. The variation in temperature, concentration, heat and mass transfer coefficient are shown graphically for some selected values of the parameters.

Table 1: Comparison of $\theta(0, 0)$ for vertical cone with $S_r = 0, \epsilon = 0, Gr^* = 0$ and $N = 0$.

n	Yih [11]	Present Work
0.5	1.1358	1.1357
0.8	1.0839	1.0836
1.0	1.0564	1.0563
1.5	0.9871	0.9873
2.0	0.9760	0.9764

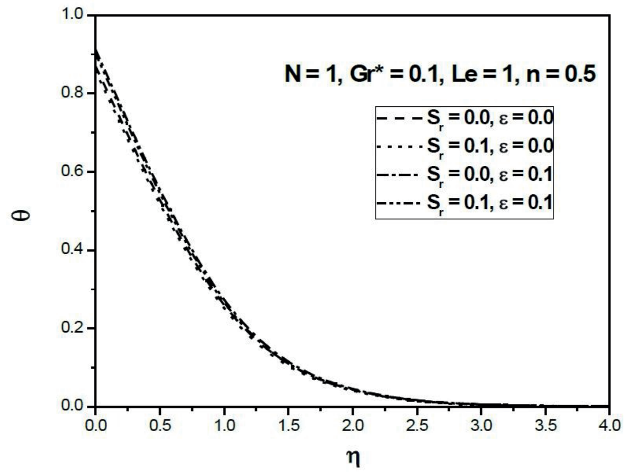
Table 2: Comparison of $\theta(0, 0)$ and $\phi(0, 0)$ with previously published article of Yih [26] for free convection flow of Newtonian fluid over a vertical cone in a saturated porous medium.

N	Le	$\theta(0, 0)$		$\phi(0, 0)$	
		Yih [26]	Present Work	Yih [26]	Present Work
1	1	0.8385	0.8385	0.8385	0.8385
1	10	1.0211	1.0209	0.2618	0.2617
4	1	0.6178	0.6177	0.6178	0.6176
4	10	0.9490	0.9492	0.2273	0.2278
0.5	1	1.3310	1.3313	1.3310	1.3313
0.5	10	1.0781	1.0785	0.2925	0.2929

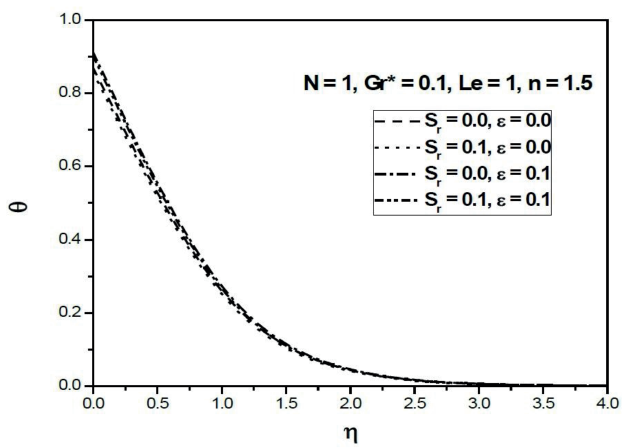
Figures 2(a) and 2(b) depict the variation of non-dimensional temperature distribution θ in aiding buoyancy $N > 0$ for $n = 0.5$ (pseudoplastics fluid) and $n = 1.5$ (dilatants fluid), respectively, for two different values of dissipation parameter ϵ and thermo-diffusion parameter S_r with fixed values of other parameters. It is observed that for both pseudoplastic and dilatant fluids, the temperature distribution inside the boundary layer increases with increased value of the dissipation parameter ϵ because the impact of viscous dissipation term in the energy equation performs as an internal distributed heat source generated due to the action of viscous stresses. Moreover, for both pseudoplastic and dilatant fluids the temperature decreases across the boundary layer with an increase of S_r. The variation of non-dimensional concentrationin ϕ aiding buoyancy ($N > 0$) for $n = 0.5$ (pseudoplastics fluid) and $n = 1.5$ (dilatants fluid) against the similarity variable η for different values of S_r and ϵ is shown in Figs. 3(a) and 3(b), respectively. The diffusive species with higher Soret values accelerates the concentration hence for both pseudoplastics and dilatants fluids, the concentration enhances with an increase in the values of S_r while it decreases with an increasing values of ϵ.

Figures 4 and 5 illustrate the variation of non-dimensional heat transfer coefficient (local Nusselt number) and non-dimensional mass transfer coefficient (local Sherwood number) against the power-law index parameter n in aiding buoyancy ($N > 0$) with varying ϵ and S_r. It is observed that the local Nusselt number increases with n and S_r while decreases with an increasing ϵ. This is because of the thermal boundary layer thickness increases with S_r and decreases with ϵ and η (Figs. 2a–2b). Moreover, it is shown in the equation (14), $\frac{Nu_x}{Ra_x^{\frac{1}{3}}}$ is directly proportional to $\frac{1}{\theta(\epsilon,0)}$. On the other hand, Sherwood number increases with n and ϵ while reduces with increasing S_r. The reason for such happening can be stated from the Figs. 3a–3b as concentration boundary layer thickness increases with S_r while decreases with ϵ. Moreover, it is shown in the equation (15), $\frac{Sh_x}{Ra_x^{\frac{1}{3}}}$ is directly proportional to $\frac{1}{\phi(0,\epsilon)}$. It is also noted that thermo-diffusion plays an important role to control heat and mass transfer rates due to the presence of viscous dissipation in the porous medium for all types of power-law fluids.

In Figs. 6a and 6b, the non-dimensional temperature profiles are plotted in the opposing buoyancy ($N < 0$) against the similarity variable η for $n = 0.5$ (pseudoplastic fluid) and $n = 1.5$ (dilatant fluid), respectively, for different values of S_r and ϵ. It is observed that the temperature profile grows with S_r while the temperature is diminished inside the boundary layer with the enhancing of the dissipation parameter ϵ. Figures 7a and 7b represent the variation of concentration profiles in opposing buoyancy ($N < 0$) against the similarity variable η for $n = 0.5$ and $n = 1.5$, respectively. From the definition, Soret number can be defined as the effect of temperature on concentration. This shows that diffusive species with higher Soret



(a)

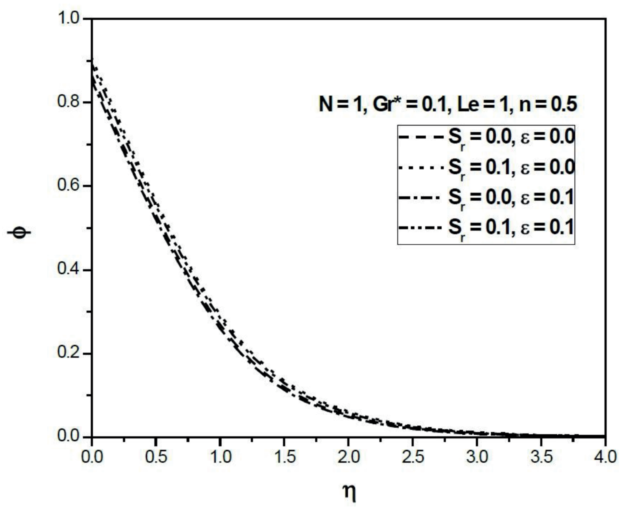


(b)

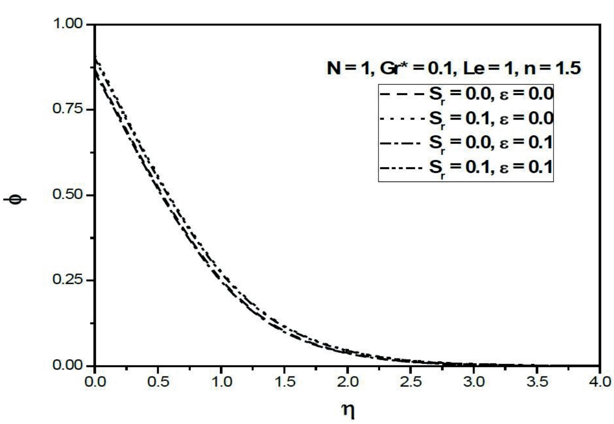
Fig. 2: (a) Variation of temperature profiles against η for $n = 0.5$ (Aiding Buoyancy); (b) Variation of temperature profiles against η for $n = 1.5$ (Aiding Buoyancy)

values accelerates the concentration profile. Hence, concentration profile raises with S_r while it reduces with inside the boundary layer. The behavior of temperature and concentration with effects of S_r and ϵ for $n = 0.5$ (pseudoplastic fluid) and $n = 1.5$ (dilatant fluid) resembles with existing results of Kairi and Murthy [22].

Figures 8 and 9 explain the variation of local Nusselt and Sherwood numbers against power-law index n in opposing buoyancy ($N < 0$) with varying Gr^* and ϵ. It is seen that local Nusselt number enhances with the increasing value of power-law index n but decreases with the increasing non-Darcy parameter. The role of non-Darcy parameter is very obvious because an increased value of non-Darcy parameter induced higher inertial effect (Forchheimer effect) into the flow that results retardation in the flow inside the boundary layer. On account of that there is a drop in



(a)



(b)

Fig. 3: (a) Variation of concentration profiles against η for $n = 0.5$ (Aiding Buoyancy); (b) Variation of concentration profiles against η for $n = 1.5$ (Aiding Buoyancy)

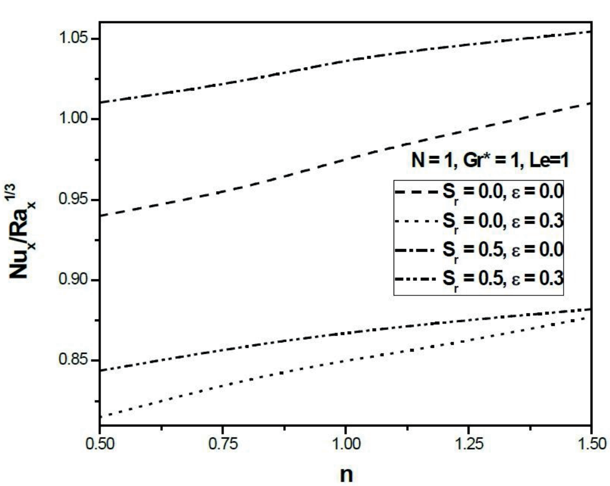


Fig. 4: Variation of heat transfer coefficient against n (Aiding Buoyancy.

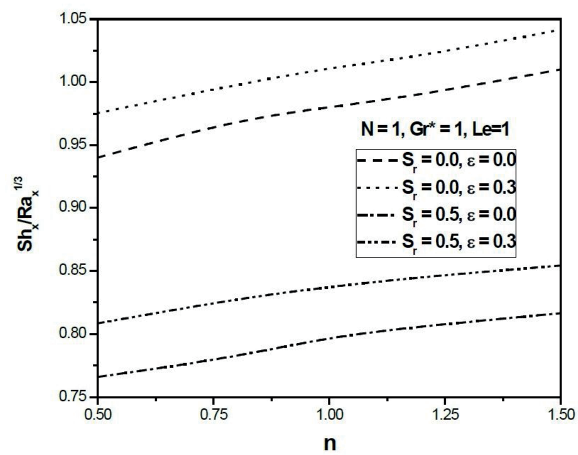
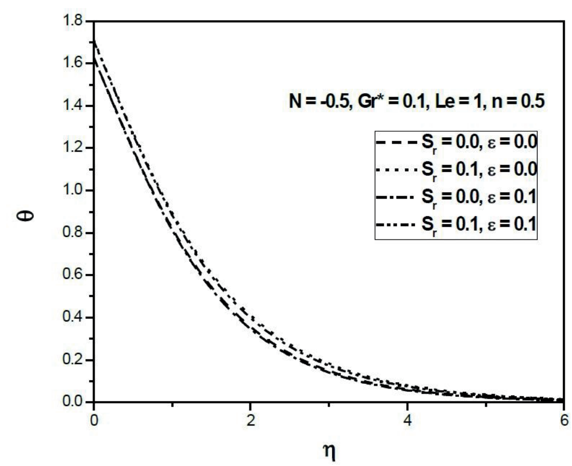
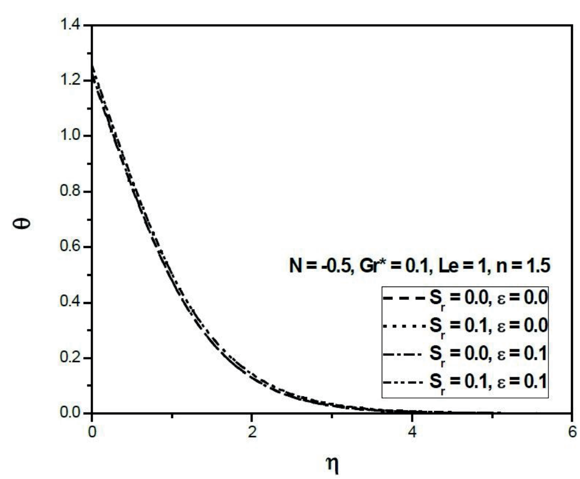


Fig. 5: Variation of mass transfer coefficient against n (Aiding Buoyancy.

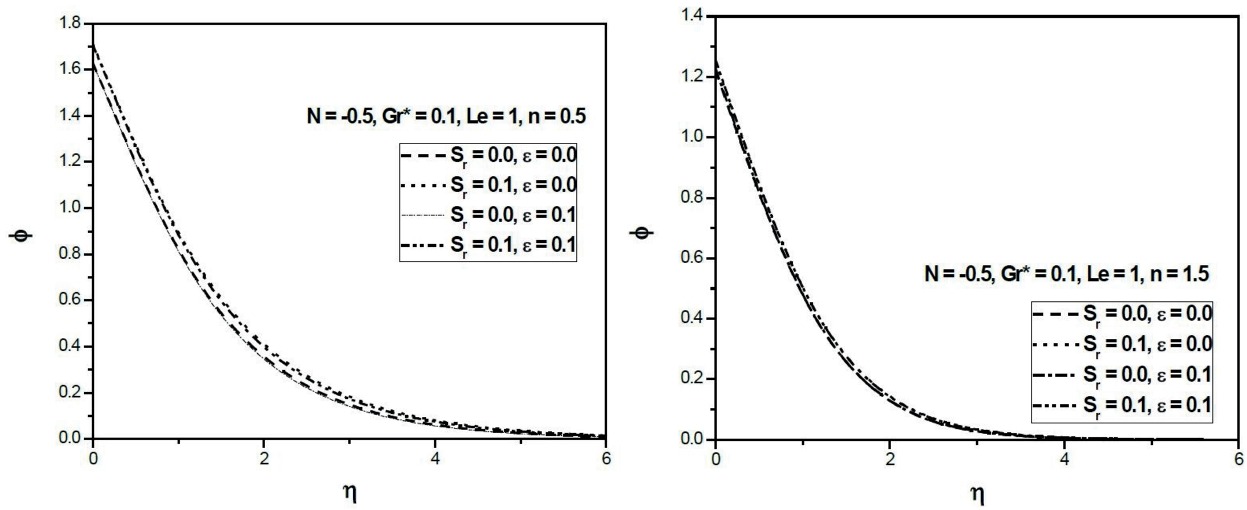


(a)



(b)

Fig. 6: (a) Variation of temperature profiles against η for $n = 0.5$ (Opposing Buoyancy); (b) Variation of temperature profiles against η for $n = 1.5$ (Opposing Buoyancy)



(a) (b)

Fig. 7: (a) Variation of concentration profiles against η for $n = 0.5$ (Opposing Buoyancy); (b) Variation of concentration profiles against η for $n = 1.5$ (Opposing Buoyancy)

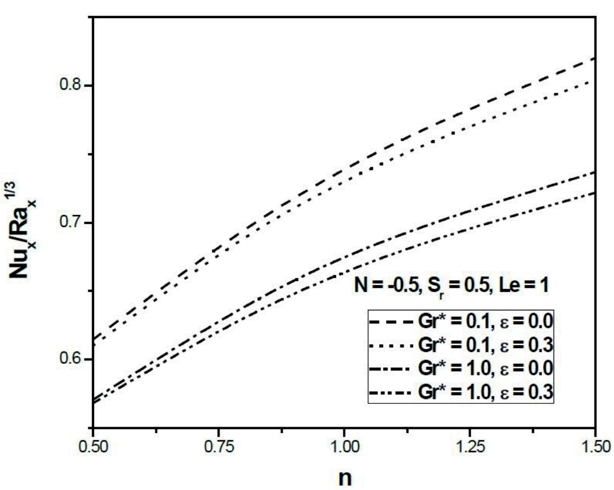


Fig. 8: Variation of heat transfer coefficient against n (Opposing Buoyancy.

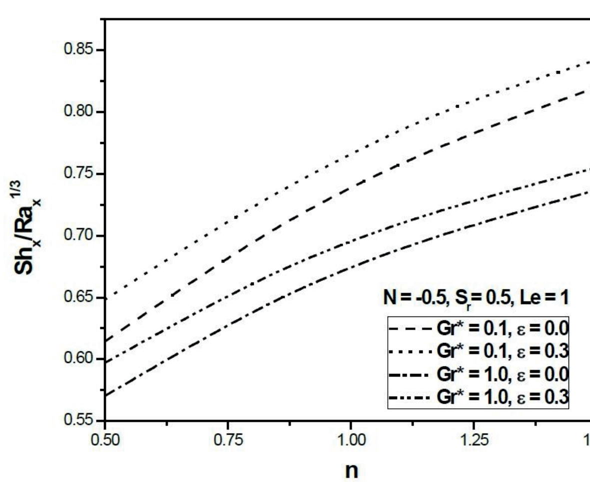


Fig. 9: Variation of mass transfer coefficient against n (Opposing Buoyancy.

heat and mass transfer rates. The effect of dissipation parameter on local Nusselt and Sherwood numbers is same as in aiding buoyancy (Figs. 4 and 5). The combined dissipation and Forchheimer effects are significant in dilatant as compared to pseudoplastic fluids.

5 Conclusions

This article considered the combined viscous dissipation and Soret effects on natural convection flow over a vertical cone in a non-Darcy porous medium with non-Newtonian power-law fluid using variable heat and mass fluxes. The various values of flow influencing parameters on concentration and temperature distributions, heat and mass transfer rates are exhibited graphically for both opposing and aiding buoyancy cases. For both the types of power-law fluids ($n < 1$ and $n > 1$) along with Newtonian fluid ($n = 1$), as the viscous dissipation parameter increases, the heat transfer rate reduces while the mass transfer rate enhances for both opposing and aiding buoyancy cases. The combined dissipation and Forchheimer effects are significant in dilatant as compared to pseudoplastic fluids. Moreover, thermo-diffusion plays an significant role in company with viscous dissipation through a porous medium for all types of power-law fluids (i,e., pseudoplastic, Newtonian and dilatant fluids).

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